

**LATE ABSTRACTS:  
V ANNUAL CONFERENCE  
OF THE GEORGIAN  
MATHEMATICAL UNION**

**Batumi, September 8-12, 2014**

# An Alternative Monotonicity Method in Quasi-variational Inequalities

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We consider the variational inequalities with unilateral and bilateral obstacles for second order linear elliptic operator. The domain is bounded and the obstacles may appear in domain and on the boundary as well. We prove some monotone dependence between the solutions and the data of the variational inequalities. It gives an opportunity to construct the monotonicity method for quasi-variational inequalities when the obstacle operator is not monotone in  $L_2$  sense. As an example we consider Implicit Signorini problem, the quasi-variational inequality with unilateral implicit obstacle on the boundary. We show the unique solvability of the problem and construct the iteration schemes for the solution. Then we consider the several statements of the mentioned problem; we consider this problem for the double boundary obstacles, also for the obstacles in domain, obtaining the similar results as for the classical statement of the problem.

Some of the results can be generalized for the evolutionary variational inequalities.

## Unilateral contact problems with friction arising along the normal

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we investigate the boundary-contact problem for micro-polar homogeneous hemitropic elastic solids with regard for friction. In this case, the friction forces arise not under tangential but under normal displacement. We consider two cases, the so-called coercive case (when elastic medium is fixed along some parts of the boundary), and the semi-coercive case (the boundary is fixed nowhere). Using the Steklov-Poincar'e type operator, we reduce equivalently these problems to boundary variational inequalities. Relying on the variational inequality approach, we prove the existence and uniqueness theorems for weak solutions. In the coercive case, the problem is unconditionally solvable and solutions continuously depend on the data of the initial problem. In the semi-coercive case, the necessary condition of solvability of the corresponding contact problem is written explicitly. This condition under certain restrictions is sufficient as well.

# Numerical radius inequalities of operators

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Let  $B(H)$  denote the  $C^*$ -algebra of all bounded linear operator on a complex Hilbert space  $H$  with inner product  $\langle \cdot; \cdot \rangle$ . For  $A \in B(H)$  let

$$w(A) = \sup \{ |\langle x; Ax \rangle| : \|x\| = 1 \}, \quad \|A\| = \sup \{ \|Ax\| : \|x\| = 1 \}$$

and  $|A| = (A^*A)^{1/2}$  denote the numerical radius, the usual operator norm of  $A$  and the absolute value of  $A$ , respectively. It is well know that  $w(\cdot)$  is a norm on  $B(H)$ , and that for all  $A \in B(H)$ ,

$$\frac{1}{2}\|A\| \leq w(A) \leq \|A\|.$$

It is shown that if  $A_j \in B(H)$ , then

$$\omega^r \left( \sum_{j=1}^n A_j \right) \leq (2n)^{r-1} \sum_{j=1}^n \left( \| |B_j|^{2r} + |C_j|^{2r} \| \right)^{\frac{1}{2}}$$

where  $A_j \in B(H)$  and  $A_j = B_j + iC_j$ ,  $1 \leq j \leq n$ .

In particular, If  $A \in B(H)$ , then

$$r(A) \leq 2^{-\frac{1}{4}} \sum_{j=1}^n \left( \| |A^2|^{2r} + |(A^*)^2|^{2r} \| \right)^{\frac{1}{4}} \leq \|A\|$$

## On Primitive Elements of free Lie $p$ -Algebras

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W. Magnus in 1930 proved that if  $F$  is a finitely generated free group and  $a \in F$ ,  $a$  is a primitive element (i.e.  $a$  belongs to some set of free generators of  $F$ ) if and only if  $F/\langle a \rangle$  is a free group ( $\langle a \rangle$  denotes a normal subgroup of  $F$  generated by  $a$ ). Later similar theorems were proved for free Lie algebras (G. P. Kukin, [1]), for free algebras, free commutative algebras and free anticommutative algebras (A. A.Mikhalev, U. U. Umirbaev, 2001). Mikhalev, Shpilrain and Umirbaev in 2004 conjectured that similar theorem for Lie  $p$ -algebras is also true.

Let  $k$  be a field of the characteristic  $p > 0, p \neq 2$ . Let  $F = k\langle X \rangle$  and  $L_p\langle X \rangle$  are a free associative algebra without identity and, consequently, a free Lie  $p$ -algebra, both with  $X$  as a set of free generators; let  $\langle a \rangle$  be an ideal generated by  $a$ . We assert, that

**Theorem.** If  $X$  is finite, then  $L_p\langle X \rangle / \langle a \rangle$  is free if and only if  $a$  is primitive.

For the proof of this theorem we apply method from [1] and Bokut's result from paper [2].

A major intermediate result is a following statement:

**Lemma.** If  $a, b \in k\langle X \rangle$  and  $\langle a \rangle = \langle b \rangle$ , then  $a$  and  $b$  are linearly dependent.

[1] G. P. Kukin: Primitive elements of free Lie algebras, *Algebra i Logica*, 9 (1970), 458–472.

[2] L. A. Bokut: Embeddings in free associative algebras, *Algebra i Logica*, 15 (1976), 117–142.

## On the Number of representations of positive integers by the diagonal quadratic forms with coefficients that are ones and twos

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The modular properties of generalized theta-functions with characteristics and spherical polinomials are used to build cusp forms corresponding to the quadratic forms of 8 level. It gives the opportunity of obtaining formulas for the number of representations of positive integers by all diagonal quadratic forms with coefficients that are ones and twos.

## Submanifolds of a Riemannian Product Manifold admitting a Quarter-Symmetric Metric Connection

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We study the relations between the geometry of the semi-invariant submanifolds of Riemannian product manifold with Levi-Civita connection and the geometry of the semi-invariant submanifolds of Riemannian product manifold with

quarter-symmetric metric connection. We obtain fundamental properties of the semi-invariant submanifolds of Riemannian product manifold with quarter-symmetric metric connection such as the integrability of the distribution  $D$  and  $D^\perp$  and mixed-geodesic property for a quarter symmetric metric connection.

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