საქართველოს მეცნიერებათა ეროვნული აკადემია
Georgian National Academy of Sciences

საქართველოს მათემატიკოსთა კავშირი
Georgian Mathematical Union

ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტი
Batumi Shota Rustaveli State University

საქართველოს მათემატიკოსთა კავშირის VI ყოველწლიური საერთაშორისო კონფერენცია

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რედაქტორები: გურამ გოგიშვილი, მაია კვინიკაძე

გარეკანის დიზაინი: დავით სულაქველიძე

Editors: Guram Gogishvili, Maia Kvinikadze

Cover Design: David Sulakvelidze

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პროფესორი დავით გორდეზიანი

დავით გიორგის ძე გორდეზიანი დაიბადა 1937 წლის 9 დეკემბერს ქ. თბილისში. 1945 წელს დაიწყო სწავლა თბილისის ვაჟთა I საშუალო სკოლაში, რომელიც ოქროს მედალზე დაამთავრა 1956 წელს. იმავე წელს გახდა თბილისის სახელმწიფო უნივერსიტეტის მექანიკა-მათემატიკის ფაკულტეტის სტუდენტი. 1961 წელს დაამთავრა უნივერსიტეტი წარჩინებით და გააგრძელა სწავლა ამავე უნივერსიტეტის ასპირანტურაში მიახლოებითი ანალიზისა და გამოთვლითი ტექნიკის კათედრაზე. ასპირანტურაში მის მუშაობას ხელმძღვანელობდა ცნობილი მეცნიერი აკად. შალვა მიქელაძე.

საკანდიდატო დისერტაცია გამოთვლით მათემატიკაში დაიცვა საქართველოს მეცნიერებათა აკადემიის ა. რაზმაძის სახელობის მათემატიკის ინსტიტუტში. სადისერტაციო საბჭოს თავმჯდომარეობდა მსოფლიოში აღიარებული მათემატიკოსი და მექანიკოსი აკად. ნ. მუსხელიშვილი. მათემატიკის ინსტიტუტში მან 1964 წლიდან დაიწყო მუშაობა. 1968 წელს კი გადავიდა თბილისის სახელმწიფო უნივერსიტეტის სამეცნიერო-კვლევით პრობლემურ ლაბორატორიაში, რომელიც იმავე წელს გადაკეთდა გამოყენებითი მათემატიკის ინსტიტუტად. 1970 წლიდან ხელმძღვანელობდა ამ ინსტიტუტის რიცხვითი მეთოდების განყოფილებას. 1979 წლიდან 1985 წლამდე იყო ამავე ინსტიტუტის დირექტორის მოადგილე სამეცნიერო დარგში. ამ დროს ინსტიტუტი უკვე ატარებდა თავისი დამაარსებლის, აკად. ი. ვეკუას სახელს. 1985 წლიდან 2006 წლის ბოლომდე იყო აკად. ი. ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტის დირექტორი. 1984 წლიდან 2006 წლამდე ასევე იყო თსუ-ს მექანიკა-მათემატიკის ფაკულტეტის გამოთვლითი მათემატიკისა

და ინფორმატიკის კათედრის გამგე. 2006-დან 2009 წლის სექტემბრამდე - თსუ-ს სრული პროფესორი. 2009 წლიდან სიცოცხლის ბოლომდე კი თსუ-ს ემერიტუს-პროფესორი. 1996-2006 წლებში იყო სოხუმის სახელმწიფო უნივერსიტეტის, ხოლო 2008-2015 წლებში საქართველოს საპატრიარქოს წმ. ანდრია პირველწოდებულის სახელობის ქართული უნივერსიტეტის მიწვეული პროფესორი.

1971-1972 წლებში გადიოდა სტაჟირებას საფრანგეთში პარიზ VI უნივერსიტეტის რიცხვითი ანალიზის ლაბორატორიასა და გრენობლის გამოყენებითი მათემატიკის ინსტიტუტში თანამედროვეობის ერთერთი უდიდესი მათემატიკოსის, მექანიკოსისა და ინფორმატიკოსის აკად. ჟ.-ლ. ლიონსის ხელმძღვანელობით.

1982 წელს მ. ლომონოსოვის სახელობის მოსკოვის სახელმწიფო უნივერსიტეტში დაიცვა სადოქტორო დისერტაცია გამოთვლითი მათემატიკის სპეციალობით. სადისერტაციო საბჭოს თავმჯდომარეობდა მსოფლიოში აღიარებული ერთერთი უდიდესი მათემატიკოსი აკად. ა.ნ. ტიხონოვი. 1985 წელს მიენიჭა პროფესორის წოდება.

პროფესორ დავით გორდეზიანს გამოქვეყნებული აქვს 200-მდე ნაშრომი, მათ შორის 4 გამოგონება, 2 პატენტი (აშშ, დანია), 3 მონოგრაფია. მიღებული აქვს მრავალი სამეცნიერო გრანტი. ყავს აღზრდილი მეცნიერებათა 7 დოქტორი, მეცნიერებათა 17 კანდიდატი. მისი ხელმძღვანელობით შესრულებულია ბევრი სამაგისტრო ნაშრომი. წლების განმავლობაში ხელმძღვანელობდა სხვადასხვა სამაგისტრო და სადოქტორო პროგრამას. იყო მრავალი საინტერესო სილაბუსის ავტორი ან თანაავტორი. მისი ხელმძღვანელობით შესრულებულ სამეცნიერო ნაშრომებში სტუდენტებს, ასპირანტებსა და დოქტორანტებს მოპოვებული აქვთ სხვადასხვა სახის დიპლომები, სიგელები და პრემიები, როგორც ადგილობრივ, ისე საერთაშორისო საგანმანათლებლო პროგრამებით ჩატარებულ კონფერენციებში. აღზრდილებს შორის არის პრეზიდენტის 7 და სოროსის 4 სტიპენდიანტი. მის უშუალო მოწაფეთაგან ზოგიერთი ნაყოფიერად მუშაობდა და მუშაობს საზღვარგარეთ (პროფ. ე. ევსეევი - ისრაელი, დოქტორი ვ. იუცისი - აშშ, დოქტორი თ. ჯიოევი - რუსეთი, დოქტორი ი. ჯანაშვილი - ისრაელი და სხვ.).

არის უამრავი საერთაშორისო და სხვა ფორუმების მონაწილე, მათ შორის მათემატიკოსთა ვარშავისა (1983 წ.) და ციურიხის (1994 წ.) მსოფლიო კონგრესების, IUTAM-ის სიმპოზიუმების, ათენის ინტერდისციპლინარული ოლიმპიადისა და სხვ. დ. გორდეზიანი არაერთხელ იყო მიწვეული ლექციების წასაკითხად და ერთობლივი სამეცნიერო მუშაობის ჩასატარებლად პარიზის, რომის, გრენობლის, ათენის, იენის, მოსკოვის, კიევის, მინსკისა და სხვა ქალაქების ცნობილ უნივერსიტეტებში.

პროფესორმა დ. გორდეზიანმა მონაწილეობა მიიღო მრავალი საერთაშორისო თუ ადგილობ-რივი კონგრესის, სიმპოზიუმის, კონფერენციის, სკოლის ორგანიზებასა და ჩატარებაში გამოთვლითი მათემატიკის, მექანიკის, გარსთა თეორიის, ჰიდროდინამიკის, მაგნიტური ჰიდროდინამიკის, ინფორმატიკის სფეროებში (საერთაშორისო კონგრესი მათემატიკაში, IUTAM სიმპოზიუმი და ა.შ.).

სხვადასხვა დროს საკანდიდატო და სადოქტორო დისერტაციების ოპონირებისათვის მიწვეული იყო საფრანგეთის, გერმანიის, იტალიის, რუსეთის, დანიის, პოლონეთის, საბერძნეთის, უკრაინის, ბელორუსიის, უზბეკეთის, აზერბაიჯანისა და მოლდავეთის სამეცნიერო და სასწავლო ცენტრებში. ხშირად იყო საკანდიდატო და სადოქტორო დისერტაცი-

ების ოპონენტი და ექსპერტი საქართველოშიც მათემატიკის, ინფორმატიკისა და მექანიკის დარგებში.

1993 წელს აირჩიეს კომპიუტერულ მეცნიერებათა და სისტემების საერთაშორისო აკადემიის წევრად, იყო საქართველოს საბუნებისმეტყველო აკადემიის წევრი და მისი საპატიო პრეზიდენტი. იყო საქართველოს საინჟინრო აკადემიის წევრი. ასევე სსრკ მეცნიერებათა აკადემიის მათემატიკური მოდელირების საკოორდინაციო საბჭოს წევრი, საქართველოს მეცნიერებათა აკადემიის მათემატიკური მოდელირების საბჭოს თავმჯდომარის მოადგილე, ივ. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის სამეცნიერო საბჭოს წევრი (1985-2006 წწ.) და სხვ.

პროფ. დ. გორდეზიანის სამეცნიერო თემატიკა ძირითადად ეხება:

- ეკონომიური სასრულ-სხვაობიანი ალგორითმების აგებისა და კვლევის საკითხებს;
- ი. ვეკუას გარსებისა და ფირფიტების თეორიის დაფუძნებასა და განვითარებას;
- მათემატიკური ფიზიკის ზოგიერთი არაწრფივი ამოცანის შესწავლას;
- კერძოწარმოებულებიანი დიფერენციალური განტოლებებისათვის დასმული არაკლასიკური საწყის-სასაზღვრო და სასაზღვრო ამოცანების შესწავლას.

მის შრომებში განვითარებულია ახალი და თანამედროვე მეთოდები ფიზიკის, ქიმიის, ეკოლოგიის, სამშენებლო მექანიკისა და ჰიდრო-გაზო დინამიკის კონკრეტული პრაქტიკული ამოცანების მათემატიკური და კომპიუტერული მოდელირებისათვის.

მისი შრომები ციტირებულია მსოფლიოში აღიარებულ უდიდეს მეცნიერთა შრომებში, მონოგრაფიებსა და სახელმძღვანელოებში, ენციკლოპედიურ და მიმოხილვითი ხასიათის ნაშრომებში.

დ. გორდეზიანის შრომების ციტირებების შესახებ - მონოგრაფიებში, სახელმძღვანელოებსა და ენციკლოპედიური ხასიათის ნაშრომებში ინფორმაცია განთავსებულია მისამართზე: `http://www.books.google.com/books?q=gordezianid`, სადაც 30-მდე წიგნია მითითებული და ისინი არ ფიგურირებს სხვა ტიპის ციტირების ჩამონათვალში, მაგალითად, როგორებიცაა Scholar Google და Harzing's Publish or Perish. აქ ცალკეა გამოყოფილი გამოთვლითი, გამოყენებითი მათემატიკისა და მექანიკის პრობლემების მსოფლიოში აღიარებულ მკვლევართა ის მონოგრაფიები, სახელმძღვანელოები, აგრეთვე მიმოხილვითი და ისტორიული მასალები, ენციკლოპედიები, სადაც აღნიშნული და ციტირებულია დ. გორდეზიანის შრომები:

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7. П.И. Вабищевич, А.А. Самарский, *Вычислительная теплопередача*. М., Изд. УРСС, 2003 (მონოგრაფია);

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დ. გორდეზიანს წილად ხვდა ბედნიერება ერთობლივი მუშაობისა შალვა მიქელაძესთან, ილია ვეკუასთან, ანდრია ბიწაძესთან, ალექსანდრე სამარსკისთან, ანდრეი ტიხონოვთან, ჟაკ ლუი ლიონსთან, ფილიპ სიარლესთან, რომლებმაც დიდი როლი ითამაშეს მისი, როგორც გამოყენებითი მათემატიკისა და ინფორმატიკის ცნობილი სპეციალისტის ჩამოყალიბებაში. პირველი სამეცნიერო ნაშრომი მის მიერ შესრულებული იქნა აკად. შ. მიქელაძის ხელმძღვანელობით და მთელი ცხოვრების განმავლობაში დიდ პატივს მიაგებდა მასწავლებლისა და გამოჩენილი მეცნიერის ნათელ ხსოვნას. საერთოდ, პროფ. დ. გორდეზიანი ყოველთვის დიდად აფასებდა და სიყვარულით მოიხსენიებდა ყველა პედაგოგს, რომელთაც მის ჩამოყალიბებაში გარკვეული წვლილი მიუძღოდათ.

დ. გორდეზიანის მიერ მიღებულია პრინციპული მნიშვნელობის მქონე შედეგები ეკონომიური სასრულ–სხვაობიანი ალგორითმების აგებისა და კვლევისას, ი. ვეკუას გარსებისა და ფირფიტების თეორიის დაფუძნებისა და განვითარებისას, მათემატიკური ფიზიკის არაწრფივი ამოცანების შესწავლისას, დიფერენციალური განტოლებებისათვის დასმული არაკლასიკური ამოცანების გამოკვლევისას, მის მიერ განვითარებული მათემატიკური და კომპიუტერული მოდელირების მეთოდების საინჟინრო და საბუნებისმეტყველო ამოცანათა შესასწავლისას და სხვ.

აქ წარმოდგენილია მსოფლიოში აღიარებული მეცნიერის, გამოყენებითი და გამოთვლითი მათემატიკის უდიდესი სპეციალისტის, აკადემიკოს ა. სამარსკის დახასიათება, რომელიც ეხება პროფესორ დავით გორდეზიანის მეცნიერულ მოღვაწეობას. მიუხედავად იმისა, რომ ამ დახასიათების დაწერიდან გარკვეული პერიოდია გასული, მათ არ დაუკარგავთ თავიანთი მნიშვნელობა და ძირითადად სრულად აღწერენ პროფესორ დავით გორდეზიანის მეცნიერულ მოღვაწეობას და მის მიერ მიღებული შედეგების მნიშვნელობას.

''დავით გიორგის ძე გორდეზიანი არის გამოთვლითი და გამოყენებითი მათემატიკის ცნობილი სპეციალისტი, რომელმაც თავისი შრომებით მნიშვნელოვანი წვლილი შეიტანა მეცნიერების განვითარებაში. მისი შრომები ეძღვნება თანამედროვე მათემატიკის ისეთ აქტუალურ საკითხებს, როგორიცაა მათემატიკური ფიზიკის წრფივი და არაწრფივი ამოცანების ამოხსნის რიცხვითი მეთოდების (მათ შორის ეკონომიური ალგორითმების) შექმნა და დაფუძნება, ილია ნესტორის ძე ვეკუას გარსებისა და ფირფიტების მათემატიკური მოდელების გამოკვლევა, არაკლასიკურ სასაზღვრო ამოცანათა ახალი კლასის (არალოკალური სასაზღვრო ამოცანების) კორექტულობისა და მათი დისკრეტიზაციის საკითხების შესწავლა; ფიზიკის, ქიმიის, სამშენებლო მექანიკის, გაზიფიკაციის, ღვარცოფების და სხვ. ამოცანების ეგმ-ზე მათემატიკური მოდელირების პრობლემათა გამოკვლევა და სხვ. დავით გორდეზიანის ადრეული შრომები ეძღვნება ზოგიერთი არასტაციონარული, წრფივი და არაწრფივი პარაბოლური ტიპის განტოლებათათვის სხვადასხვა ტიპის ბადეებზე (მართკუთხოვანი, რომბისებური) სასრულსხვაობიანი სქემების (მათ შორის მაღალი სიზუსტის სქემების) კონსტრუირებასა და გამოკვლევას.

ამ შრომებში ნაჩვენებია აგებული სხვაობიანი სქემების კრებადობა, გამოკვლეულია მათი მდგრადობა და სიზუსტე. ეს გამოკვლევები წარმოადგენს სითბოგამტარებლობის განტოლებისათვის აკად. შალვა მიქელაძის მიერ მიღებული შედეგების განვითარებას. ამავე პერიოდში დავით გორდეზიანმა დაიწყო თავისი გამოკვლევები ეკონომიურ სხვაობიან სქემებში (ლოკალურად-ერთგანზომილებიანი მეთოდი).

მის მიერ 1965 წელს გამოქვეყნებული ნაშრომი, რომელიც $2m$ რიგის პარაბოლური განტოლებებისათვის ლოკალურად-ერთგანზომილებიანი სქემების გამოკვლევას ეძღვნება, ფაქტობრივად პირველია ამ ტიპის ნაშრომებს შორის. მასში ნაჩვენებია, რომ საკმაოდ ზოგადი სახის მაღალი რიგის განტოლებებისათვის სპეციალური ერთგანზომილებიანი სისტემის (ადიტიური მოდელის) ამონახსნი ბადის კვანძებში მრავალგანზომილებიანი ამოცანის ამონახსნს ემთხვევა. აქვე მოცემულია $2m$ რიგის განტოლებებისათვის ლოკალურად-ერთგანზომილებიანი სქემების მდგრადობის სრული დაფუძნება. აღნიშნულმა შრომამ მიიპყრო ყურადღება მრავალი მკვლევარისა, რომლებმაც მას მისცეს ძალიან მაღალი შეფასება (ა.ა. სამარსკი, ნ.ნ. იანენკო) და ციტირებულია სპეციალისტთა მთელ რიგ შრომებში, მათ შორის ჩემს მონოგრაფიაშიც.

უნდა აღინიშნოს, რომ სახელდობრ დავით გორდეზიანის პირველი შრომები დაედო საფუძვლად საქართველოში გამოთვლითი მათემატიკის ისეთი უმნიშვნელოვანესი თანამედროვე მიმართულებების განვითარებას, როგორიცაა მათემატიკური ფიზიკის ამოცანების ამოხსნისათვის ეკონომიური ალგორითმების გამოკვლევა (ლოკალურად-ერთგანზომილებიანი მეთოდი, წილად ბიჯთა მეთოდი, დეკომპოზიციის მეთოდი, გახლეჩვის მეთოდი, ცვალებად მიმართულებათა მეთოდი და სხვ.). ამ მეთოდების თეორიული საფუძვლები მოცემული იყო 50-იანი წლების ბოლოს და 60-იანი წლების დასაწყისში ამერიკელი და საბჭოთა მეცნიერების შრომებში (ჯ. დუგლასი, გ. რეკფორდი, დ. პისმენი, ნ.ნ. იანენკო, ა.ა. სამარსკი, გ.ი. მარჩუკი და სხვ.).

ამ ციკლის მომდევნო შრომებში დავით გორდეზიანმა ააგო და გამოიკვლია სხვადასხვა ტიპის ლოკალურად-ერთგანზომილებიანი სქემები და გახლეჩილი სქემები. კერძოწარმოებულებიანი არასტაციონარული წრფივი განტოლებებისათვის ცვალებადი კოეფიციენტებით მრავალგანზომილებიან შემთხვევაში და აგრეთვე არაწრფივი პარაბოლური და ჰიპერბოლური განტოლებებისათვის. აუცილებლად უნდა აღინიშნოს ერთი მნიშვნელოვანი გარემოება, რომელიც ახასიათებს დავით გორდეზიანის შრომებს - მათში ალგორითმების გამოკვლევა წარმოებს აბსტრაქტულ დონეზე, ფუნქციონალური ანალიზის თანამედროვე მეთოდების გამოყენებით. ამასთან, მიღებული შედეგები ატარებენ კონკრეტულ და გამოყენებით ხასიათს. შემდგომში დავით გორდეზიანმა ზემოთ ნახსენები საკითხების კვლევაში ჩართო ახალგაზრდა სპეციალისტები, რომლებმაც მაღალ მეცნიერულ დონეზე დაამუშავეს რიცხვითი ანალიზისა და მათემატიკური მოდელირების ზოგიერთი თანამედროვე პრობლემა.

1968-72 წლებში დავით გორდეზიანის მიერ აგებული იქნა ახალი ტიპის ეკონომიური ალგო-რითმები მათემატიკური ფიზიკის არასტაციონარული ამოცანების ამოსახსნელად. ამ ალგორითმებს მან „გასაშუალებული“ მოდელები უწოდა. მათი მეშვეობით აგებული და გამოკვლეული იქნა პარალელური თვლის სქემები. აღნიშნული გამოკვლევების ზოგიერთი შედეგი ნაწილობრივ 1970 წელს ნიცაში ჩატარებულ მათემატიკოსთა კონგრესზე იქნა წარმოდგენილი (ა.ა. სამარსკის მოხსენება „სხვაობიან სქემათა თეორიისადმი მიძღვნილი ნაშრომების შესახებ“). შედეგების ნაწილი გამოქვეყნებული იყო საფრანგეთში 1971-72 წლებში დავით გორდეზიანის პარიზის უნივერსიტეტში მივლინებისას, რიცხვითი ანალიზის ლაბორატორიაში აკად. ჟ.-ლ. ლიონსის ხელმძღვანელობით მუშაობის პერიოდში. ეს შრომები ციტირებულია არაერთგზის (ჟ.ლ. ლიონსი, პ. ტემამი, ვ.ლ. მაკაროვი და სხვ.).

გასაშუალებულმა მოდელებმა (პარალელური თვლის ალგორითმები) განსაკუთრებული მნიშვნელობა შეიძინეს მრავალპროცესორიანი ელექტრონულ გამომთვლელი მანქანების გამოჩენასთან დაკავშირებით. ისინი წარმატებით გამოიყენება დრეკადობის თეორიის, ფირფიტებისა და გარსების თეორიის, მაგნიტური ჰიდროდინამიკისა და სხვ. კონკრეტული გამოყენებითი ამოცანების ამოსახსნელად. აღსანიშნავია, რომ დავით გორდეზიანის გასაშუალებული ადიტიური მოდელებისა და სქემების ამომხსნელი ფორმულები ახალია და წარმოადგენს დიდ თეორიულ ინტერესს ნახევარჯგუფთა თეორიის თვალთახედვით, ლი-ტროტერ-კატო-ჩერნოვის ცნობილი ფორმულების მსგავსად, რომელთა დაფუძნებას ზოგად ფუნქციონალურ სივრცეებში ეძღვნება ამერიკელი სწავლულის მ. ლაპიდუსის გამოკვლევები.

დავით გორდეზიანის ნაშრომთა დიდი ციკლი ეძღვნება აკად. ილია ვეკუას ფირფიტებისა და გარსთა თეორიის კვლევას. კერძოდ, ამ თეორიის საფუძველზე ფირფიტებისა და გარსების მათემატიკური მოდელის აგებას, კონკრეტული გამოყენებითი ამოცანების (თხელი გარსები, თაღოვანი კაშხლები და სხვა სამშენებლო კონსტრუქციები) სტრუქტურული და ხარისხობრივი თვისებების (სასაზღვრო ამოცანების ამოხსნადობა, ჩამოჭრილი მოდელების სიზუსტე და ა.შ.) საფუძველზე თანამედროვე დისკრეტული ალგორითმების დამუშავებას ელექტრონულ გამომთვლელ მანქანებზე რეალიზაციის მიზნით. ნაშრომთა მოხსენიებული ციკლი დაწყებული იქნა აკად. ილია ვეკუას ზეგავლენითა და ხელმძღვანელობით. ამ გამოკვლევათა შედეგები (1969-80 წლები) მოხსენებული იქნა სხვადასხვა საერთაშორისო და საკავშირო ფორუმებზე და გამოქვეყნდა ორი სტატიის სახით სსრკ მეცნიერებათა აკადემიის მოხსენებებში (1974 წ.), აგრეთვე სხვა ჟურნალებში. აღნიშნული თემატიკის ირგვლივ მოხსენებები გაკეთდა 1977 წელს პარიზის ავტომატიკისა და ინფორმატიკის ინსტიტუტში.

1983 წელს მათემატიკოსთა კონგრესზე ვარშავაში დავით გორდეზიანმა გააკეთა მოხსენება ახალი ტიპის არაწრფივი პარაბოლური განტოლების რიცხვითი ამოხსნის შესახებ. ამ გამოკვლევათა შედეგებმა პრაქტიკული გამოყენება ჰპოვა და დანერგილ იქნა ი.ვ. კურჩატოვის სახელობის ატომური ენერგიების ინსტიტუტში და სოხუმის ფიზიკა-ტექნიკურ ინსტიტუტში.

უნდა აღინიშნოს, რომ ტექნიკასა და სახალხო მეურნეობაში პრაქტიკული გამოყენება ჰპოვეს დავით გორდეზიანის მიერ თავის მოსწავლეებთან და თანამშრომლებთან ერთად ჩატარებულმა სხვა გამოკვლევებმა (ღვარცოფის გავრცელების გათვლა, საქალაქო ქსელებში გაზის დინების ანგარიში და ოპტიმიზაცია, სითბური დანადგარები).

სამეცნიერო მოღვაწეობასთან ერთად დავით გორდეზიანი მონაწილეობას ღებულობს სპეციალური სამშენებლო მექანიზმების შექმნასთან დაკავშირებულ მუშაობაში, აქვს გამოგონებები, რომლებიც დაპატენტებულია აშშ-სა და შვეციაში.

ნაყოფიერი იყო დ. გორდეზიანის, როგორც პროფესორის მუშაობა ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტში, სადაც ათეული წლების მანძილზე ხელმძღვანელობდა ინფორმატიკისა და გამოთვლითი მათემატიკის კათედრას და მიყავდა სალექციო კურსები, გადასცემდა რა სტუდენტებს თავის ცოდნასა და გამოცდილებას, მოწინავეებს კი თავიდანვე აზიარებდა სამეცნიერო კვლევით მუშაობას. მისი ლექციები ძალიან მრავალფეროვანი იყო: პროგრამირება ეგმ-ზე; გამოთვლითი მათემატიკა; მათემა-

ტიკური მოდელირება; ფუნქციონალური ანალიზი და გამოთვლითი მათემატიკა; გარსებისა და ფირფიტების ი. ვეკუას თეორია; კერძოწარმოებულებიანი დიფერენციალური განტოლე-ბების ამოხსნის სხვაობიანი მეთოდები; დეკომპოზიციის მეთოდები; რიცხვითი ანალიზი; სამეცნიერო გამოთვლები; მათემატიკური მოდელირება და გამოთვლითი მათემატიკა; წრფივი ალგებრის რიცხვითი მეთოდები; კომპიუტერული ალგებრა და სხვა. იგი იყო რამდენიმე სამაგისტრო და სადოქტორო პროგრამის ხელმძღვანელი.

განსაკუთრებით უნდა აღინიშნოს დ. გორდეზიანის მოღვაწეობა უნივერსიტეტის ი. ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტში, სადაც გაიარა გზა უფროსი მეცნიერი თანამ-შრომლის თანამდებობიდან ინსტიტუტის დირექტორის თანამდებობამდე, ბოლო ათი წლის მანძილზე კი ხელმძღვანელობდა ერთ-ერთ ძირითად სამეცნიერო მიმართულებას (მათემატიკური მოდელირება და გამოთვლითი მათემატიკა).

დ. გორდეზიანი სხვადასხვა წლებში იყო სამეცნიერო ხარისხებისა და წოდებების მიმნიჭებელი საბჭოების წევრი; საქართველოს მეცნიერებათა აკადემიის მათემატიკური მოდელირების საბჭოს თავმჯდომარის მოადგილე; რუსეთის მეცნიერებათა აკადემიის მათემატიკური მოდელირების საპრობლემო საბჭოს წევრი; გერმანიის ინფორმატიკოსთა საბჭოს წევრი; საქართველოს საბუნებისმეტყველო აკადემიის პრეზიდენტი; კომპიუტერულ მეცნიერებათა და სისტემების საერთაშორისო აკადემიის ნამდვილი წევრი; რამდენიმე ავტორიტეტული სამეცნიერო ჟურნალის სარედაქციო კოლეგიის წევრი. იგი იყო საქარ-თველო–ესპანეთის ერთობლივი სამეცნიერო ჟურნალის ,,გამოყენებითი მათემატიკა და ინფორმატიკა‘‘ ერთ–ერთი დამაარსებელი და მთავარი რედაქტორი.

კოლეგები

Professor David Gordeziani

David Gordeziani was born in Tbilisi, in the family of engineer and inventor George Gordeziani on December 9, 1937. His mother Natalia Kireulishvili was a pediatrician. Since 1945 he had been studying at Boys' Gymnasium No 1 (further known as Secondary School No 1 and currently Tbilisi Classical Gymnasium), which he graduated with golden medal for extraordinary achievements in 1956. In the same year he became a student of the faculty of Mechanics and Mathematics at Iv. Javakhishvili Tbilisi State University (TSU). In 1961 he graduated TSU with honor and continued his education as a post graduated student at the department of Approximate Analysis and Computational Technics of the same university. During the postgraduate period he had been working under the supervision of outstanding Georgian scientist, academician Shalva Mikeladze.

David Gordeziani had done his PhD in Computational Mathematics at A. Razmadze Institute of Mathematics of Georgian Academy of Sciences. Dissertation council was chaired by worldwide famous mathematician and a professional of mathematical mechanics Academician Niko Muskhelishvili. David Gordeziani began his scientific work at the Institute of Mathematics from 1964. In 1968 he had moved to TSU Scientific-Research Laboratory, which later on became Institute of Applied Mathematics in the same year. Since 1970 he had been heading the department of Numerical Analysis of this institute. From 1979 till 1985 he was Deputy Director responsible for a Scientific Area. By that time the instituted was already named after academician Ilya Vekua. From 1985 till the end of 2006 he was a Director of Ilya Vekua Institute of Applied Mathematics. From 1984 till 2006 David Gordeziani was also heading the Department of Computational Mathematics and Informatics at the faculty of Mechanics and Mathematics of TSU. From 2006 till September 2009 he was full professor, and from 2009 till the end of his life he was Professor Emeritus at TSU. He was invited professor at Sokhumi State University and St. Andrew the First Called Georgian University of Patriarchate of Georgia correspondingly in 1996-2006 and in 2008-2015.

In 1971-1972 David Gordeziani took internship course at the laboratory of Numerical Analysis in Paris VI University and in Grenoble Institute of Applied Mathematics in France. His internship was supervised by the greatest contemporary mathematician, engineer and informatician, academician Jaque-Luis Lions.

In 1982 David Gordeziani defended doctoral dissertation by specialization of Computational Mathematics at M. Lomonosov Moscow State University. The doctoral council was chaired by one of the greatest mathematicians in the world, academician A.N. Tikhonov. David Gordeziani was granted a title of a Professor in 1985.

Professor David Gordeziani has published about 200 scientific works, including 4 inventions, 2 patents (USA, Denmark) and 3 monographs. He has received large number of research grants. He has roused 7 scientific doctors and 17 candidates. He has supervised many scientific works for Master's Degree. During many years he was heading various masters and doctoral programs. He was the author and co-author of many interesting syllabuses. Under- and Post-graduate and doctoral works carried out under his supervision have gained variety of diplomas, certificates and prizes on international as well as local conferences held within the framework of educational programs. 7 of his pupils have won presidential scholarship and 4 - scholarship of Soros Foundation. Among his students been under his direct supervision are those, who successfully worked and have still been working abroad (Prof. E. Evseev – Israel, Doc. V. Iucys – USA, Doctor T. Jioev – Russia, Doctor I. Janashvili – Israel, etc.).

David Gordeziani participated in numerous international and other forums, among those the World Congress of Mathematicians in Warsaw (1983) and Zurich (1994), IUTAM Symposium, Athens Interdisciplinary Olympiad, etc. He was many times invited to famous Universities of Paris, Rome, Grenoble, Athens, Jena, Moscow, Kiev, Minsk and other cities to give lectures and carry out joint scientific researches.

Professor Gordeziani has taken part in organizing and carrying out many international and local congresses, symposiums, conferences, schools in the area of computational mathematics, mechanics, shell theory, hydrodynamics, magnetic hydrodynamics, informatics (e.g. International congress in Mathematics, IUTAM Symposium, etc.).

Professor Gordeziani was invited as an official opponent of a defense party in scientific-study centers of France, Germany, Italy, Russia, Denmark, Poland, Greece, Ukraine, Belorussia, Uzbekistan, Azerbaijan and Moldova. Often he was an opponent and expert in Georgia in the areas of Mathematics, Informatics and Mechanics.

In 1993 Professor Gordeziani was elected as a member of International Academy of

Computer Sciences and Systems; he was a member and honorary president of Georgian Academy of Natural Sciences, the member of Georgian Engineering Academy. He was also a member of Coordination Council of the USSR Academy of Sciences in Mathematical Modeling, deputy head of Coordination Council of Georgian Academy of Sciences in Mathematical Modelling, member of Iv. Javakhishvili Tbilisi State University Scientific Council (1985-2006), etc.

David Gordeziani was supervisor and main performer of many international grants, was owner of various scientific and governmental awards, prizes, medals and diplomas. For his scientific, educational and scientific-organizational works he has received the following prizes, awards and diplomas: Iv. Javakhishvili Medal for important contribution in development of scientific and educational processes; Medal of Honer of Georgian Republic; Medal of USSR Presidium of the Supreme Council for “The Dedicated Work” and Sigel of Honor of Minister of Council for “High Educational Performance and Special Success in Work”; Sigel of Honor of Ministry of Higher and Secondary Special Education; The First Degree Sigel and Prize at Georgian National Economy Performance; Diploma for winning in republican Scientific-Technical Conference-Exhibition SofTEC’99 (for created program package); Medal for Dedicated Work; The First Prize of Georgian Council of Ministers for wining in competition of industrialized high effective works done in high education institutions in 1984; The first degree sigel on exhibition of Achievements of Georgian National Economy, etc.

Scientific themes of Professor David Gordeziani mainly deal with:

- Development and investigation of economic finite-difference algorithms;
- Establishment and development of the theory of plates and shells of I. Vekua;
- Investigation of some problems of mathematical physics;
- Research of nonlocal initial-boundary and boundary value problems for partial-differential equations.

His works develop new contemporary methods for mathematical and computational modelling of certain practical problems of Physics, Chemistry, Ecology, Construction Mechanics and Hydro-Gas Dynamics. Works of Professor Gordeziani are cited in papers, monographs and handbooks of the worlds’ greatest scientists, encyclopedias and overviews.

The information on the citations of Professor Gordeziani’s works in monographs, handbooks and encyclopedias can be found on the following link: `http://www.books.google.com/books?q=gordezianid`. This link provides for a list of more than 30 books that are not mentioned anywhere else (e.g. on Google Scholar or Publish or Perish Databases on Harzing.com). Here one can find the list of those monographs, handbooks, reviews and historical works, encyclopedias of well known researchers (see further below) of Computational, Applied Mathematics and Mechanics, where papers of Professor David Gordeziani are cited:

1. M. Bernadou, *Finite Element Methods for Thin Shell Problems.* John Wiley, 1996 (textbook);
2. Ph. G. Ciarlet, *The Finite Element Method for Elliptic Problems.* Studies in Mathematics and its Applications, Vol. 4. North-Holland Publishing Co., Amsterdam-New York-Oxford, 1978 (monograph). Russian translation: Ф. Сьярле, *Метод конечных элементов для эллиптических задач.* М., Изд. Мир, 1980;
3. Ph. G. Ciarlet, Mathematical Elasticity. Vol. II. Theory of Plates. Studies in Mathematics and its Applications, 27. North-Holland Publishing Co., Amsterdam, 1997 (monograph);
4. Ph. G. Ciarlet, Mathematical Elasticity. Vol. III. Theory of Shells. Studies in Mathematics and its Applications, 29. North-Holland Publishing Co., Amsterdam, 2000 (monograph);
5. M. Vogelius, I. Babuška, On a dimensional reduction method. I. The optimal selection of basis functions. *Math. Comp.* **37** (1981), no. 155, 31–46; II. Some approximation-theoretic results. *Math. Comp.* **37** (1981), no. 155, 47–68; III. A posteriori error estimation and an adaptive approach. *Math. Comp.* **37** (1981), no. 156, 361–384 (review);
6. А. А. Самарский, *Теория разностных схем.* М., Наука, 1983 (textbook);
7. П.И. Вабищевич, А.А. Самарский, *Вычислительная теплопередача.* М., Изд. УРСС, 2003 (monograph);
8. Международная конференция математиков в Ницe, Доклады Советских Математиков, Москва 1972 (review of important results of the scientific area);
9. А.А. Самарский, П.Н. Вабищевич, П.П. Матус, *Разностные схемы с операторными множителями.* Минск, 1998 (monograph);
10. Т.Ю. Хома, *Обобщенная теория оболочек.* Киев, Наукова Думка, 1986 (monograph);
11. Monique Dauge, Erwan Faou, Zohaz Yosibash, *Plates and Shells Asymptotic Expansions and Hierarchical models.* Encyclopedia for Computational Mechanics, 2004, Edited by Ervin Steinm Rene de Borst Thomas J.R. Highes (encyclopedia);
12. А.В. Бицадзе, *Некоторые классы уравнений в частных производных.* М., Наука, 1981 (monograph);
13. E. Obolashvili, *Higher Order Partial Differential Equations in Clifford Analysis. Effective Solutions to Problems.* Progress in Mathematical Physics, 28. Birkhäuser Boston, Inc., Boston, MA, 2003 (monograph);
14. S. Jensen, Adaptive-dimensional reduction and divergence stability. *Mat. Model.* **8** (1996), no. 9, 44–52 (review);
15. *История отечественной математики*, т. 4, кн. 2, СССР, 1917-1967;

16. M. Dikmen, *Theory of Thin Elastic Shells.* Surveys and Reference Works in Mathematics, 8. Pitman (Advanced Publishing Program), Boston, Mass.-London, 1982 (monograph type textbook).

David Gordeziani had honor of working jointly with Shalva Milkeladze, Ilia Vekua, Andria Bitsadze, Alexsander Samarskii, Andrei Tikhonov, Jaque-Luis Lions, Phillip Ciarlet. They played important role in his establishment as a specialist of Applied Mathematics and Informatics. His very first scientific work was performed under the supervision of Academician Shalva Mikeladze. Further, during his whole life he respected his supervisor and kept great memory of this outstanding mathematician. In general, Professor David Gordeziani always much appreciated and mentioned with love all his teachers, who contributed to his forming as a researcher.

Results of a principle importance were obtained by David Gordeziani during constructing and investigating economic finite-difference algorithms, establishing and developing I. Vekua Theory of Plates and Shells, studying nonlinear problems of Mathematical Physics, investigating nonlocal problems stated for differential equations, researching methods of Mathematical and Computer Modelling for problems of engineering and natural sciences developed and stated by himself, etc.

Further below is brought David Gordeziani's characteristic given by Academician A. Samarskii, the world's one of the outstanding mathematician, greatest specialist of Applied and Computational Mathematics. Nevertheless this characteristic is dated quite a long ago, these words have not lost their meaning and they completely describe Prof. David Gordeziani's scientific activity and importance of his results.

"David Gordeziani is a famous specialist in Computational and Applied Mathematics, who has made a considerable contribution to the development of science. His works are dedicated to such problems of current importance in modern mathematics as development and establishment of numerical methods (economic algorithms as well) for resolving linear and nonlinear problems of Mathematical Physics, investigating mathematical models for I. Vekua plates and shells, studying correctness of a new class of non-classical boundary problems (nonlocal boundary value problems) and developing methods of their discretization research of problems for computer realization of mathematical models of physics, chemistry, engineering mechanics, gasification, landslide, etc. In his earlier works David Gordeziani has constructed and studied the finite-difference schemes (having high precision of approximations) in various types of grids (right-angled, diamond type) for some non-stationary linear and nonlinear parabolic equations.

In these works Professor Gordeziani has shown convergence of constructed finite-difference schemes, studied stability and precision issues. These researches present enhancement of results obtained by academician Shalva Mikeladze for heat-conductivity equations. During the same period David Gordeziani has began his studies in economic difference schemes (locally one-dimensional methods). His paper published in 1965, which was devoted to the investigation of locally one-dimensional schemes for 2m-order parabolic equations, was practically first among this type of works. In this paper the author has shown that for rather general high order equations the solution of a special one-dimensional system (additive model) in grid-points coincide with the solution of a multi-dimensional problem. The same paper studies stability issues of locally one-dimensional schemes for 2m-order equations. The mentioned paper has attracted attention of many researchers, who have qualified it as of high importance (A.A. Samarskii, N.N. Ianenko). The work has got large amount of citations in other scientific works and monographs.

It should be noted that the first works of David Gordeziani became the bases for the development of such an important modern direction of Computational Mathematics in Georgia as investigation of economic algorithms for the problems of Mathematical Physics (locally one-dimensional method, fractional step method, decomposition method, split methods, method of variable directions, etc.). Theoretical basis of these methods was given in the papers of scientists from USA and USSR (J. Douglas, G. Reckford, D. Peacman, N.N. Ianenko, A.A. Samarskii, G.I. Marchuk, etc.) at the end of 50-th and beginning of 60-th.

In the next works of that cycle, David Gordeziani has constructed and investigated various types of locally one-dimensional and split schemes for partial differential non-stationary linear equations with variable coefficients in multi-dimensional cases a well as for nonlinear parabolic and hyperbolic equations. One important fact, characterizing David Gordeziani's works, should be noted here: that – investigation of algorithms is carried out on an abstract level applying modern methods of functional analysis. In addition, obtained results are of a specific as well as applicable character. Later on, David Gordeziani has involved young scientists in studying the above mentioned problems.From their part, the young scientists have managed to develop some modern problems of numerical analysis and mathematical modeling.

In 1968-72 David Gordeziani constructed new economic algorithms for the resolution of non-stationary problems of mathematical physics. He called those algorithms "averaged" models. Applying these algorithms algorithms of parallel calculation were built and studied. Mentioned results were partially presented at the Congress of Mathematicians held in Nice in 1970 (report of A.A. Samarskii "On Works about Solution of Finite-Difference Schemes"). Part of the results was published during the period when Devid Gordeziani worked at Laboratory of Numerical Analysis under the supervision of J.-L. Lions in the University of Paris, France in 1971-72. These works have many citations (J.-L. Lions, P. Themam, V.L. Makarov, etc.). Averaged models (algorithms of parallel calculation) gained special attention after creation of computers with parallel processors. They are widely used for solution of certain applied problems of the theory of elasticity, plates and shell theory, magnetic hydrodynamics, etc.

It should be noted here that formulas for solution of David Gordeziani's averaged additive models and schemes are new and are of a great importance for semi-groups theory, like Lee-Troter-Kato-Chernov well known formulas, to the establishment of which in general functional spaces was devoted researches of American scientist M. Lyapunov's.

Large cycle of works of David Gordeziani deals with the studies plates and shell theory of Academician Ilia Vekua. In particular, those works are dedicated to the construction of mathematical models of plates and shells, using structural and qualitative properties (solvability of boundary value problems, precision of cut models, etc.) of certain applied problems (thin shells, arch dams, etc.), development of modern discrete algorithms in order to make their realization on electronic computational machines. Mentioned cycle of problems was begun under the influence and supervision of Academician I. Vekua. Results of those researches (1969-80) were reported on various international and Soviet Union forums and were also as two articles in Reports of the Academy of Sciences of USSR (1974) as well as in other journals. Reports around the mentioned theme were made in Paris Institute of Automatics and Informatics in 1977. In 1983 at the Congress of Mathematicians in Warsaw David Gordeziani made report on numerical resolution of a new type of nonlinear parabolic equation. Results of this research were applied in practice and were established in I.V. Kurchatov Institute of Atomic Energy and Sokhumi Physical-Technical Institute.

It worth noting that the researches carried out by David Gordeziani together with his students and colleagues (landslide spread calculation, calculation of flow in city gas-network and its optimization, heat facilities) have found practical applications in technique and agriculture. Along with scientific activity, David Gordeziani had been participating in works dealing with creation of special building mechanisms; he has had inventions patented in USA and Sweden.

D. Gordeziani's work as a professor at Iv. Javakhishvili Tbilisi State University was particularly prodictive.He had been working there for decades: he was a head of department of Informatics and Computational Mathematics and was delivering lectures, sharing his knowledge and experience with his students; he was carrying out joint scientific works with most successful students. Content of his lectures was very colorful:programming on Computer, Computational Mathematics, Mathematical Modelling, Functional Analysis and Computational Mathematics, I. Vekua Plates and Shell Theory, Finite-Difference Methods for Solution of Partial Differential Equations, Decomposition Methods, Numerical Analysis, Scientific Calculation, Mathematical Modeling and Computational Mathematics, Numerical Methods of Linear Algebra, Computer Algebra, etc. He was a supervisor of several master and doctoral programs.

Particular note should be made pertinent to David Gordeziani's activities at I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University. There he passed the way starting from the position of a senior researcher up to an Director of the Institute. During the last ten year he was leading one of the main scientific directions (Mathematical Modeling and Computational Mathematics) of the Institute.

During different time periods Professor David Gordeziani had been a board member of Scientific Quality and Title Council, Deputy Director of the Council of Georgian Academy of Sciences in Mathematical Modeling, Member of the Council on Problematic Issues at Russian Academy of Sciences in Mathematical Modeling, member of German Council of

Informaticians, president of Georgian Academy of Natural Sciences, Honorable Member of the Council of International Academy of Computer Sciences and Systems, member of Editorial Boards of several authoritative scientific journals. He had been one of the Founders and Chief Editor of Georgian-Spain joint scientific journal "Applied Mathematics and Informatics".

Colleagues

Abstracts of Plenary and Invited Speakers

პლენარული და მოწვეული მომხსენებლების თეზისები

Sharp Spectral Stability Estimates for Uniformly Elliptic Differential Operators

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We consider the eigenvalue problem for the operator

$$Hu=(-1)^m\sum_{|\alpha|=|\beta|=m}D^\alpha\left(A_{\alpha\beta}(x)D^\beta u\right),\quad x\in\Omega,$$

subject to homogeneous Dirichlet or Neumann boundary conditions, where $m\in\mathbb{N}$, Ω is a bounded open set in $\mathbb{R}^N$ and the coefficients $A_{\alpha\beta}$ are real-valued Lipschitz continuous functions satisfying $A_{\alpha\beta}=A_{\beta\alpha}$ and the uniform ellipticity condition

$$\sum_{|\alpha|=|\beta|=m}A_{\alpha\beta}(x)\xi_\alpha\xi_\beta\geq\theta|\xi|^2$$

for all $x\in\Omega$ and for all $\xi_\alpha\in\mathbb{R}, |\alpha|=m$, where $\theta>0$ is the ellipticity constant.

We consider open sets Ω for which the spectrum is discrete and can be represented by means of a non-decreasing sequence of non-negative eigenvalues of finite multiplicity $\lambda_1[\Omega]\leq\lambda_2[\Omega]\leq\cdots\leq\lambda_n[\Omega]\leq\dots$ Here each eigenvalue is repeated as many times as its multiplicity and $\lim\limits_{n\to\infty}\lambda_n[\Omega]=\infty$.

The aim is sharp estimates for the variation $|\lambda_n[\Omega_1]-\lambda_n[\Omega_2]|$ of the eigenvalues corresponding to two open sets Ω_1, Ω_2 with continuous boundaries, described by means of the same *fixed* atlas $\mathscr{A}$.

There is vast literature on spectral stability problems for elliptic operators. However, very little attention has been devoted to the problem of spectral stability for higher order operators and in particular to the problem of finding explicit qualified estimates for the variation of the eigenvalues.

Our analysis comprehends *operators of arbitrary even order*, *with homogeneous Dirichlet or Neumann boundary conditions*, and *open sets admitting arbitrarily strong degeneration.*

Three types of estimates will be under discussion: for each $n\in\mathbb{N}$ for some $c_n>0$ depending only on n, $\mathscr{A},m,\theta$ and the Lipschitz constant L of the coefficients $A_{\alpha\beta}$

$$|\lambda_n[\Omega_1]-\lambda_n[\Omega_2]|\leq c_n d_{\mathscr{A}}(\Omega_1,\Omega_2),$$

where $d_{\mathscr{A}}(\Omega_1,\Omega_2)$ is the so-called *atlas distance* of Ω_1 to Ω_2,

$$|\lambda_n[\Omega_1]-\lambda_n[\Omega_2]|\leq c_n\omega(d_{\mathscr{H}\mathscr{P}}(\partial\Omega_1,\partial\Omega_2)),$$

where $d_{\mathscr{H}\mathscr{P}}(\partial\Omega_1, \partial\Omega_2)$ is the so-called *lower Hausdoff-Pompeiu deviation* of the boundaries $\partial\Omega_1$ and $\partial\Omega_2$ and ω is the common modulus of continuity of $\partial\Omega_1$ and $\partial\Omega_2$, and, under certain regularity assumptions on $\partial\Omega_1$ and $\partial\Omega_2$,

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n \mathrm{meas}\,(\Omega_1 \Delta \Omega_2)\,,$$

where $\Omega_1\Delta\Omega_2$ is the symmetric difference of Ω_1 and Ω_2.

Joint work with P. D. Lamberti.

Supported by the RFBR grant (project 14-01-00684).

Dynamical Interface Crack Problems for Metallic and Electro-Magneto-Elastic Composite Structures

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We investigate the solvability and asymptotic properties of solutions to 3-dimensional dynamical interface crack problems for metallic and electro-magneto-elastic composite bodies. We give a mathematical formulation of the physical problems when the matallic and electro-magneto-elastic bodies are bonded along some proper parts of their boundaries where interface cracks occur.

Using the Laplace transform, potential theory and theory of pseudodifferential equations on a manifold with boundary, the existence and uniqueness theorems are proved. We analyse the regularity and asymptotic properties of the mechanical and electro-magnetic fields near the crack edges and the curves where the different boundary conditions collide. In particular, we characterize the stress singularity exponents and show that they can be explicitly calculated with the help of the principal homogeneous symbol matrices of the corresponding pseudodifferential operators. For some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and show that they essentially depend on the material parameters. The questions related to the so called oscillating singularities are treated in detail as well.

This is joint work with T. Buchukuri and D. Natroshvili.

Acknowledgements: This research was supported by Rustaveli Foundation grant No. FR/286/5-101/13: “Investigation of dynamical mathematical models of elastic multi-component structures with regard to fully coupled thermo-mechanical and electro-magnetic fields”.

Rotation of Coordinate Axes and Differentiation of Integrals with respect to Translation Invariant Bases

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The dependence of differentiation properties of an indefinite integral on a rotation of coordinate axes is studied, namely: the result of J. Marstrand on the existence of a function the integral of which is not strongly differentiable for any choice of axes is extended to Busemann-Feller and homothecy invariant bases which does not differentiate $L(\mathbb{R}^n)$; it is proved that for an arbitrary translation invariant basis B formed of multi-dimensional intervals and which does not differentiate $L(\mathbb{R}^n)$, the class of all functions the integrals of which differentiate B is not invariant with respect rotations, and for bases of such type it is studied the problem on characterization of singularities that may have an integral of a fixed function for various choices of coordinate axes.

Toeplitz plus Hankel Operators with Matching Generating Functions

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Equations with Toeplitz plus Hankel operators $T(a)+H(b)$, $a, b \in L^\infty$ are considered. If the generating functions a and b satisfy the matching condition

$$a(t)\,a(1/t) = b(t)\,b(1/t), \quad t \in \mathbb{T}, \tag{1}$$

an effective description of the structure of the kernel and cokernel of the corresponding operator is given. Moreover, an efficient method for solution of the non-homogeneous operator equations

$$(T(a)+H(b))\phi = f, \quad f \in L^p(\mathbb{T}), \quad 1 < p < \infty, \tag{2}$$

with generating functions $a, b \in L^\infty$ satisfying relation (1). It turns out that the solvability and the number of solutions of the equation (2) depends on the indices of the scalar

Toeplitz operators $T(c)$ and $T(d)$ generated by the functions $c(t) := a(t)\, b^{-1}(t)$ and $d := a^{-1}(1/t)\, b(t)$.

This talk is based on joint work with Bernd Silbermann [1], [2].

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Spectral Stability for the Dirichlet–Laplace Operator in Conformal Regular Domains

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We prove that the eigenvalues problem for the Dirichlet-Laplace operator in bounded simply connected plane domains $\Omega \subset \mathbb{C}$ can be reduced by conformal transformations to the weighted eigenvalues problem for Dirichlet-Laplace operator in the unit disc $\mathbb{D}$. It permits us to estimates variation of eigenvalues of Dirichlet-Laplace operators in terms of energy type integrals for a large class of domains (so-called confomal regular domains)that includes all quasidiscs, i.e. images of the unit disc under a quasiconformal homeomorphism of the plane onto itself. Boundaries of such domains can have any Hausdorff dimension between one and two.

We call a bounded simply connected plane domain $\Omega \subset \mathbb{C}$ as a conformal regular domain if there exists a conformal mapping $\varphi : \mathbb{D} \to \Omega$ of the class $L^{1,p}(\mathbb{D})$ for some $p > 2$. Note, that any conformal regular domain has finite geodesic diameter.

It is known that in a bounded plane domain $\Omega \subset \mathbb{C}$ the spectrum of the Dirichlet-Laplace operator is discrete and can be written in the form

$$0 < \lambda_1[\Omega] \leq \lambda_2[\Omega] \leq \cdots \leq \lambda_n[\Omega] \leq \cdots .$$

One of the main result is:

Theorem. *Let $\Omega_1, \Omega_2 \subset \mathbb{C}$ be conformal regular domians. Then for every $n \in \mathbb{N}$*

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n A_p \|\varphi_1 - \varphi_2 \mid L^{1,2}(\mathbb{D})\|.$$

where $c_n = \max\{\lambda_n^2[h_1], \lambda_n^2[h_2]\}$*. The constant* A_p *depends on the integrability exponent* p *of derivatives of conformal mappings* $\varphi_k : \mathbb{D} \to \Omega_k$ *only,* $k = 1, 2$*.*

Similar results are correct for the Neumann–Laplace operator.

The same machinery permit us to obtain lower estimates for first eigenvalues of the Neumann–Laplace operators in conformal regular domains.

The work is done jointly with *V. I.Burenkov and A. Ukhlov.*

Higher K-Theory of Toric Varieties

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In the talk we report on recent progresses in understanding higher K-theory of general toric varieties, accomplished in a series of works of several people. We will also discuss a conjectural description of higher K-groups of these varieties, representing a far reaching – in a sense the ultimate extension of the known results. In general terms, the theory develops around controlling the failure of homotopy invariance of Quillen's theory and the conjecture is a multi-graded refinement of the previously known results. The starting point here is our positive results for the Grothendieck group of vector bundles on toric varieties, known since the 1980s.

Phenomena of Projectivity and Freeness in Classical and Quantum Functional Analysis

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The concept of the projectivity, together with its dual version (injectivity) and and a weaker version (flatness) is extremely important in algebra. The variants of this concept, now established in operator theory, in particular, in representation theory of "classical" and "quantum" Banach algebras, are also of considerable importance. In analysis, however, there exist several different approaches to this concept, corresponding to different problems of lifting of operators.

We shall discuss several (comparatively rigid and comparatively tolerant) variants of projectivity in operator theory and show that all of them can be included in a certain general scheme. This scheme allows to study projectivity by means of the so-called freeness. The relevant free objects are defined by the same way as free groups, free modules, free Banach spaces etc. Projective objects are direct summands (in a proper sense) of free objects.

We shall describe free objects, corresponding to each of the discussed versions of the projectivity. In particular, we shall characterize free operator spaces in terms of spaces of nuclear operators. In the "classical" context we shall characterize metrically free normed spaces: they turn out to be subspaces in $l_1(\Lambda)$, consisting of functions with finite supports. As a corollary, all metrically projective normed spaces are free.

Investigation and Numerical Resolution of Two Types Nonlinear Partial Integro-Differential Models

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Two type of partial integro-differential models are considered. These models arise at mathematical simulation of process of electro-magnetic field penetration into a substance. In the quasi-stationary approximation this process is described by following system of Maxwell equations:

$$\frac{\partial H}{\partial t} = -\mathrm{rot}\left(\nu_m \mathrm{rot}\, H\right), \quad c_\nu \frac{\partial \theta}{\partial t} = \nu_m \left(\mathrm{rot}\, H\right)^2, \tag{1}$$

where $H = (H_1, H_2, H_3)$ is a vector of magnetic field, θ is temperature, c_ν and ν_m characterize correspondingly heat capacity and electroconductivity of the medium. If c_ν and ν_m depend on temperature θ, i.e. $c_\nu = c_\nu(\theta)$, $\nu_m = \nu_m(\theta)$, then the system (1) can be rewritten in the following form (D. G. Gordeziani, T. A. Dzhangveladze, T. K. Korshia, Existence and Uniqueness of a Solution of Certain Nonlinear Parabolic Problems. *Differential'nye Uravnenyia* **19** (1983), no. 7, 1197–1207):

$$\frac{\partial H}{\partial t} = -\mathrm{rot}\left[a\left(\int_0^t \left|\mathrm{rot}\, H\right|^2 d\tau\right)\mathrm{rot}\, H\right], \tag{2}$$

where coefficient $a = a\,(S)$ is defined for $S \in [0, \infty)$.

Modeling of the same process some generalization of system of type (1) is proposed (G. I. Laptev, *Quasilinear evolution partial differential equations with operator coefficients.* Doctoral Thesis, Moscow, 1990). Assuming the temperature to be constant through considered body following so-called averaged system of integro-differential equations is obtained:

$$\frac{\partial H}{\partial t} = a\left(\int_0^t \int_\Omega |\mathrm{rot}\, H|^2\, dx d\tau\right) \Delta H. \tag{3}$$

Many scientific works are devoted to the investigation and numerical resolution of (2) and (3) type models. There are still many open questions in this direction.

Here we study some properties of initial-boundary value problems for one-dimensional (2) and (3) type models as well as numerical solution of those problems. We compare theoretical results to numerical ones.

Acknowledgement. The author thanks Shota Rustaveli National Science Foundation and France National Center for Scientific Research (grant # CNRS/SRNSF 2013, 04/26) for the financial support.

Commutators of Convolution Type Operators with Piecewise Quasicontinuous Data and Their Applications

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Applying the theory of Calderón-Zygmund operators, we study the compactness of the commutators $[aI, W^0(b)]$ of multiplication operators aI and convolution operators $W^0(b)$ on weighted Lebesgue spaces $L^p(\mathbb{R}, w)$ with $p \in (1, \infty)$ and Muckenhoupt weights w for some classes of piecewise quasicontinuous functions $a \in PQC$ and $b \in PQC_{p,w}$ on the real line $\mathbb{R}$. Then we study two C^*-algebras Z_1 and Z_2 generated by the operators $aW^0(b)$, where a, b are piecewise quasicontinuous functions admitting slowly oscillating discontinuities at ∞ or, respectively, quasicontinuous functions on $\mathbb{R}$ admitting piecewise slowly oscillating discontinuities at ∞. We describe the maximal ideal spaces and the Gelfand transforms for the commutative quotient C^*-algebras $Z_i^\pi = Z_i/\mathscr{K}$ $(i = 1, 2)$ where $\mathscr{K}$ is the ideal of compact operators on the space $L^2(\mathbb{R})$, and establish the Fredholm criteria for the operators $A \in Z_i$.

The talk is based on joint work with Isaac De la Cruz-Rodríguez and Iván Loreto-Hernández.

Two Conditions Related with Unconditional Convergence of Series in Banach Spaces

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In this talk we plan to discuss a sufficient and a necessary conditions for unconditional convergence of a series in a Banach space with an unconditional basis.

The talk is based on [1], the first draft of which was prepared in collaboration with Professor Nicholas Vakhania (1930-2014). N. Vakhania was always deeply interested in questions of unconditional convergence and his challenging ideas were helping us to understand the topic better.

Supported in part by the Shota Rustaveli National Science Foundation grant no. FR/539/5-100/13.

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On Generalized Poisson–Nernst–Planck Equations

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A strongly nonlinear system of Poisson-Nernst-Planck partial differential equations is considered. The governing relations are provided by the Gauss and Fickian multiphase diffusion laws coupled with the Landau grand potential for entropy variables within the Gibbs simplex.

The model describes a plenty of electro-kinetic phenomena in physical, chemical, and biological sciences.

The generalized model is supplemented by the positivity and volume balance constraints, quasi-Fermi electrochemical potentials depending on the pressure, and inhomogeneous Robin boundary conditions representing reactions at the micro-scale level.

We aim at the proper variational modelling, well-posedness, dynamic stability, optimization, and asymptotic analysis as well as homogenization of the model at the macro-scale level.

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Field Theoretical Approach which Saves Probability Interpretation of the Wave Function

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The wave function is the most fundamental concept of quantum mechanics. According the standard interpretation of the wave function today the square of its absolute value represents the probability density for particles to be measured in certain locations.

However none of existing "quantum mechanical" approaches developed within quantum field theory (incorporating quantum mechanics) confirms such interpretation. Indeed all of them offer expressions for the charge density of a few body system which is altered by interaction between them in spite of that the probability interpretation would require the charge density of a few-body system to be only the sum of single particle charge densities.

Here the quantum field theoretical approach is presented for the description of strongly interacting particles where the expression for the charge density is consistent with the probability interpretation of the particles' wave function. A key bases of this achievement is the fundamental property of gauge invariance which is kept manifest up to the last step of our derivation.

Apart from the obvious conceptual importance of this result it is extremely useful for practical applications. For example it significantly simplifies high accuracy first principle calculations of electromagnetic properties of few nucleon systems which are extensively studied in the proposed by S. Weinberg chiral effective field theory [1].

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On the Similarity of Holomorphic Matrices

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Let $D \subseteq \mathbb{C}$ be a domain, $L(n, \mathbb{C})$ the algebra of complex $n \times n$ matrices, $GL(n, \mathbb{C})$ its group of invertible elements, and $A, B : D \to L(n, \mathbb{C})$ two holomorphic maps.

R. M. Guralnick proved in 1981 that A and B are globally holomorphically similar (meaning there exists a holomorphic map $S : D \to GL(n, \mathbb{C})$ such that $A = SBS^{-1}$ on D) if and only if they are locally holomorphically similar. He obtained this theorem as a special case of a more general algebraic result (which I do not understand). In this talk, a direct analytic proof will be outlined. Also we observe that local holomorphic similarity is equivalent to local continuous similarity, so that at the end the following is obtained: A and B are globally holomorphically similar if and only if they are locally continuously similar.

Possibly, we will also discuss generalizations to holomorphic functions of several complex variables as well as to general complex matrix Lie groups (in place of $GL(n, \mathbb{C})$), which are still "under construction".

On a Mazur Problem from "Scottish Book" Concerning Second Partial Derivatives

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In 1935 Mazur ("Scottish Book", Problem 66) posed following question:

The real function $z = f(x, y)$ of real variables x, y possesses the 1st partial derivatives f'_x, f'_y and the pure second partial derivatives f''_{xx}, f''_{yy}. Do there exist then almost everywhere the mixed 2nd partial derivatives f''_{xy}, f''_{yx}? According to a remark by p. Schauder, this theorem is true with the following additional assumptions: The derivatives f'_x, f'_y are absolutely continuous in the sense of Tonelli, and the derivatives f''_{xx}, f''_{yy} are square integrable. An analogous question for n variables.

We present two results concerning this problem.

1. If a function $f(x,y)$ of real variables defined on a rectangle has continuous derivative with respect to y and for almost all y the function $F_y(x) := f'_y(x,y)$ has finite variation, then almost everywhere on the rectangle there exists the partial derivative f''_{yx}.

2. There exists a separately twice differentiable function, whose partial derivative f'_x is discontinuous with respect to the second variable on a set of positive measure.

This solves in the negative the Mazur problem.

Modeling of Elastic Rods Torsion with Large Deformations

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In this paper, experimental investigation and computer modeling for processes of free and bending torsion of elastic cylindrical rods of polyurethane material are performed, including instability and definition of postbuckling deforming configurations. This modeling is based on usage of Hencky's isotropic hyperelastic material model with new Lagrangian formulation of constitutive relations. This relations are stated with usage of compact expressions for symmetric Lagrangian second Piola–Kirchhoff stress tensor $\mathbf{S}^{(2)}$ and new representation of the fourth-order elasticity tensor $\mathbb{C}$, that possesses both minor symmetries, and the major symmetry. This fourth-order elasticity tensor realize linear connection between material rates of stress tensor $\mathbf{S}^{(2)}$ and Green–Lagrange strain tensor $\mathbf{E}^{(2)}$ through eigenvalues and eigenprojections of right Cauchy–Green strain tensor $\mathbf{C}$. Obtained equations of tensors $\mathbf{S}^{(2)}$ and $\mathbb{C}$ for Hencky's isotropic hyperelastic material model are suitable for use in finite element analysis software packages.

It is well known, that application of complex material's models that are efficient in all the range of elastomers deforming needs accurate experiment definition and huge work of parameters searching for description of experimental curves. It is shown, that the Hencky's isotropic hyperelastic material model provide good approximation of elastomers deformations up to 50 % of scale and, for processes of elastic rods torsion, let to obtain certain critical values of torsion angles and postbuckling deforming configurations, which are in good agreement with experimental data.

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Functional Calculus for almost Commuting Self-Adjoint Operators and an Extension of the Helton–Howe Trace Formula

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Self-adjoint operators A and B are called *almost commuting* if $AB - BA$ belongs to trace class S_1. Helton and Howe established the following trace formula:

$$\operatorname{trace}(\varphi(A,B)\psi(A,B) - \psi(A,B)\varphi(A,B)) = \iint \iint \left(\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\varphi}{\partial y}\frac{\partial\psi}{\partial x}\right) g(x,y)dxdy,$$

for all polynomials φ and ψ of two variables, where g is an integrable function on the plane that is uniquely determined by the pair of almost commuting operators A and B. The function g is called the *Pincus principal function.*

We construct a functional calculus $f \mapsto f(A,B)$ for arbitrary functions f in the Besov space $B^1_{\infty,1,1}$ of functions on the plane. This functionsl calculus is almost multiplicative, i.e., $f(A,B)$ and $g(A,B)$ almost commute for arbitrary f an g in $B^1_{\infty,1,1}$. Moreover, we extend the Helton–Howe trace formula to functions in $B^1_{\infty,1,1}$.

The main tool is triple operator integrals.

The Short Overview of the Aims, Methods and Main Theoretical Results of the Logical Grammar of the Georgian Language

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In 2013, in the Center for the Georgian Language Technology at the Georgian Technical University, it was started a project "Foundations of Logical Grammar of Georgian Language and Its Application in Information Technology", which is one of the fundamental sub-project of the long-term project "Technological Alphabet of the Georgian Language" [1].[1]

The logical grammar of the Georgian language aimed to construct isomorphic mathematical theory of the Georgian language system. This, in turn, is aimed to construct the technological alphabet of the Georgian language, in other words, the Georgian thinker and speaker system. Our main methodological line lies on the exhaustively recognition of the thinking laws, which are exist in the Georgian language independently from us i.e. naturally, and, after, on their exhaustively description as mathematical theory [2].

The main philosophical result of the logical grammar of the Georgian language is that there is proved a universal subconscious existence of the operator of the declarative sentences in all languages that proves the existence of subconscious mathematical languages in all humans. The main mathematical i.e. lingvo-logical result of the logical grammar of the Georgian language is to prove that the Core Part of the Georgian language is Sh. Pkhakadze's type formally developable mathematical theory [3], which has its own formal alphabet, its own systems of syntactic, semantic, and logical axioms, its own system of the inference and extension rules, and, also, its own system of the translation rules by the help of which any well-formed expression of the Core Part of the Georgian language can be translate i.e. rewrite into natural i.e. subconscious Georgian mathematical language. Also, from anthropological scientific points of views it is very important the concept of the speech alphabet of the Georgian language [2].

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[1]In 2012, the project "Technological Alphabet of the Georgian Language" was elaborated by K.Pkhakadze on the base of the State Priority Program "Free and Complete Programming Inclusion of a Computer in the Georgian Natural Language System", which, in turn, was running in previous years with his leadership in Tbilisi State University.

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Polynomial Functors on Free Groups

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Polynomial functors have a long history. The first paper was published in 1901 by Issai Schur, far before the notion of a functor was formalized by Eilenberg and MacLane in the 40's. Then the theory was developed by Albrecht Dold, Dietrich Puppe, James Green, Ian Grant Macdonald. In the early 90's two important applications were found: The first due to Hans-Werner Henn, Jean Lannes and Lionel Scwartz relates polynomial functors to the representation theory of Steenrod algebra, and the second application relates polynomial functors to algebraic K-theory. In the first half of our talk we will give an overview of these and other classical results due to Eric Friedlander, Andrei Suslin and others. The second half of our talk will be devoted to the recent joint work of the author together with Christine Vespa and Aurélian Djament.

Wiener–Hopf Factorization through an Intermediate Space

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An operator factorization conception is investigated for a general Wiener-Hopf operator $W = P_2 A|_{P_1 X}$ where X, Y are Banach spaces, $P_1 \in \mathscr{L}(X), P_2 \in \mathscr{L}(Y)$ are projectors and $A \in \mathscr{L}(X, Y)$ is boundedly invertible. Namely we study a particular factorization of $A = A_- C A_+$ where $A_+ : X \to Z$ and $A_- : Z \to Y$ have certain invariance properties and $C : Z \to Z$ splits the "intermediate space" Z into complemented subspaces closely related to the kernel and cokernel of W, such that W is equivalent to a "simpler" operator, $W \sim PC|_{PX}$. The main result shows equivalence between the generalized invertibility of the Wiener-Hopf operator and this kind of factorization (provided $P_1 \sim P_2$) which implies a formula for a generalized inverse of W. It embraces I.B. Simonenko's generalized factorization of matrix measurable functions in L^p spaces, is significantly different from the cross factorization theorem and more useful in numerous applications. Various connected theoretical questions are answered such as: How to transform different kinds of factorization into each other? When is W itself the truncation of a cross factor?

Motivated by the classical Sommerfeld diffraction problem we consider interface problems in weak formulation for the n-dimensional Helmholtz equation in $\Omega = \mathbb{R}^n_+ \cup \mathbb{R}^n_-$ (due to $x_n > 0$ or $x_n < 0$, respectively), where the interface $\Gamma = \partial\Omega$ is identified with $\mathbb{R}^{n-1}$ and divided into two parts, Σ and Σ', with different transmission conditions of first and second kind. These two parts are half-spaces of $\mathbb{R}^{n-1}$ (half-planes for $n = 3$). It is possible to construct explicitly resolvent operators acting from the interface data into the energy space $H^1(\Omega)$ by using the present factorization conception. In a natural way, the intermediate space turns out to be a non-isotropic Sobolev space which reflects the wedge asymptotic of diffracted waves.

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Covariance Operators before and after N. Vakhania

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This talk is dedicated to the 85th birthday of Professor Nicholas Vakhania (August 28, 1930–July 23, 2014). In it we plan to make a survey of the theory founded by him: the Theory of Covariance Operators.

Acknowledgement. This talk is partially supported by Rustaveli National Science Foundation grant No. FR/539/5-100/13.

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The Francis Matrix Eigenvalue Algorithm

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This talk looks at the genesis of John Francis' QR eigenvalue algorithm and the connections with Vera Kublanovskaja's version thereof; followed by the story of how Gene

Golub and I found John in 2007/2008 and a few recent extensions of Francis' ideas and work.

Here is the outline of its four parts:

(I) **Francis' Algorithm, its inner workings**

a) Reduction to Hessenberg form H

b) Francis' algorithm with implicit steps

  b1) Francis shifts

  b2) The shift induced unitary similarities on H create a Hessenberg matrix with bulge

  b3) Chasing the bulge to regain Hessenberg form

c) Convergence

(II) **Why the name 'Francis Algorithm' now?**

a) Classical QR algorithm of John Francis and Vera Kublanovskaja (1961/62), or $m = 1$

b) Implicit QR algorithm of Francis, $m = 2$ and the Implicit Q Theorem

c) Wilkinson shift strategy, (actually due to Francis)

d) Multishift Francis alg. of Karen Braman, Ralph Byers, Roy Mathias (2002), $m > 2$

(III) **Who is Francis ?**

a) His vita

b) His surroundings and influences of the times and at the time

(IV) **An Extension to generalized Orthogonality and Polynomial Roots**

a) The DQR Algorithm for generalized tridiagonal matrices (Uhlig 1997)

b) A crude comparisons of Francis and DQR on Tridiagonals:

c) Polynomial roots via Francis and via Euclid plus DQR: (Uhlig 1999)

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Abstracts of Participants' Talks

მონაწილეთა მოხსენებების თეზისები

The Consistent Criteria for Checking of Hypotheses

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Let(E,S) be a measurable space with a given family of probability measures $\{\mu_i, i \in I\}$. Let us give some definitions (see [1, 3]).

Definition 1. We consider the notion of hypothesis as any assumption that defines the form of the distribution selection.

Let $\{H\}$ be set of hypotheses and $B(\{H\})$ be σ-algebra of subsets of $\{H\}$ which contains all finite subsets of $\{H\}$.

Definition 2. The family of probability measures $\{\mu_H, H \in \{H\}\}$ is said to admit a consistent criteria for checking of hypotheses if there exist at last one measurable map $\delta : (E,S) \to (\{H\}, B(\{H\})$ such that $\mu_H(\{x|\delta(x) = H\}) = 1$ for all $H \in \{H\}$.

Definition 3. The probability $\alpha_H(\delta) = \mu_H(\{x|\delta(x) \neq H\})$ is called the probability of error of H-th kind for a given criterion δ.

Let $\xi(t) = h(t) + \triangle(t)$, $t \in T \subset R^n$, where $h(t)$, $t \in T$, is determinate function, $\triangle(t)$ is a gaussian homogenous fields with zero means and $f(\lambda_1, \lambda_2, \dots, \lambda_n) = \lambda_1^{2k_1}, \lambda_2^{2k_2}, \dots, \lambda_n^{2k_n}$, $k_1 + k_2 + \dots + k_n \geq \frac{n+1}{2}$ be spectral densities of this fields, T be closed bounded domain in R^n. Let $\{\mu_H, H \in \{H\}\}$ be the corresponding probability measures.

We prove the following theorems:

Theorem 1. *The family of probability measures $\{\mu_H, H \in \{H\}\}$ admits a consistent criteria δ for checking of hypotheses if only if the error of all kinds is equal to zero for the criterion δ.*

Theorem 2. *The family of probability measures $\{\mu_{H_k}, k \in N\}$, $N = 1, 2, \dots, n, \dots$ admits a consistent criteria for checking of hypotheses if only if for the function $h(t)$ or for some derivation of $h(t)$ the relation*

$$\int_T \left[\frac{D^{m_1+m_2,+\dots+m_n} h_k(t_1, t_2, \dots, t_n)}{Dt_1^{m_1} Dt_2^{m_2} \cdot Dt_n^{m_n}} \right]^2 dt_1 dt_2 \cdot dt_n = \infty$$

is fulfilled, where $0 \leq m_1 \leq k_1, \dots, 0 \leq m_n \leq k_n$ and $k \in N$.

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The Global Solvability Cauchy Problem for the Fourth Order Semilinear Pseudohyperbolic Equation with Structural Damping

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We consider the Cauchy problem for the semilinear pseudohyperbolic equation with structural damping

$$u_{tt} - \Delta u_{tt} + \Delta^2 u + (-\Delta)^\alpha u_t = f(u), \tag{1}$$

$$u(0,x) = \varphi(x), \quad u_t(0,x) = \psi(x), \tag{2}$$

where $0 \le \alpha \le 1$, $(-\Delta)^\alpha \cdot = F^{-1}[|\xi|^{2\alpha} F[\,\cdot\,]]$, F is Fourier transform.

Theorem 1. *Assume that $2 \le n \le 6$, $f(\cdot) \in C^1(R)$ and $|f(u)| \le c\,|u|^p$ where*

$$p \in \left(1 + 2(1-\alpha)^{-1}, +\infty\right), \ 0 \le \alpha < \frac{1}{2}, \quad \text{if } n = 2;$$

$$p \in \left(1 + 4(3-2\alpha)^{-1}, 6\right), \ 0 \le \alpha < \frac{3}{4}, \quad \text{if } n = 3;$$

$$p \in \left(1 + \frac{4}{n-2\alpha}, \frac{n+4}{n-2}\right) \ 0 \le \alpha < 1, \quad \text{if } 4 \le n \le 6.$$

Then there exists a real number $d > 0$ such that for any

$$(\varphi,\psi) \in U_d^3 = \Big\{(\varphi,\psi) : \varphi \in W_2^3 \cap L_1, \psi \in W_2^2 \cap L_1 \, \|\varphi\|_{W_2^3} + \|\varphi\|_{L_1} + \|\psi\|_{W_2^2} + \|\psi\|_{L_1} < d\Big\}$$

the problem (1), (2) has a unique solution $u \in C([0,\infty); W_2^1) \cap C^1([0,\infty); L_2)$ which satisfies the following estimates:

$$\|u(t,\cdot)\|_{L_2(R^n)} \le c(d)(1+t)^{-\gamma_0}, \quad t \in [0,\infty);$$

$$\|\nabla u(t,\cdot)\|_{L_2(R^n)} \le c(d)(1+t)^{-\gamma_1}, \quad t \in [0,\infty);$$

$$\|u_t(t,\cdot)\|_{L_2(R^n)} \le c(d)(1+t)^{-\eta}, \quad t \in [0,\infty),$$

where $\gamma_i = \frac{n}{2(2-\alpha)} + \frac{i-2\alpha}{2-\alpha}$, $i = 0, 1$, $\eta = \min\left\{\gamma_0 + 1, \frac{(p-1)n-2p\alpha}{2-\alpha}\right\}$, $c(\cdot) \in C(R_+, R_+)$, $R_+ = [0, \infty)$.

Theorem 2. *Let* $0 \leq \alpha \leq 1$, $\varphi(x) = 0$, $\int\limits_{R^n} [\psi(x) + \Delta\psi(x)]\, dx \geq 0$, $f(u) \geq c\,|u|^p$, *where* $1 < p \leq 1 + \frac{4}{n-2\alpha}$. *Then the problem* (1), (2) *has no nontrivial weak solutions.*

Solution of Problem of Set Covering by means of Genetic Algorithm

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The problem of set covering is a NP-complex problem of combinatorial optimization. Algorithms of precise solution of this problem use techniques of branches and limits[1]. Time necessary for realization of such algorithms rapidly increases and when dimension of a problem also increases, it is impossible to get optimal values in real time [2]. This problem is a mathematical model for practical tasks, such as location of service centers, development of transport schedule, location of sources of power systems, etc. Therefore, it is very important to solve this problem in real time. When such problems are solved, heuristic algorithms are often used that find near optimal solutions in reasonable time interval. Approximate algorithms mainly imply partial selection of covering sets. In genetic algorithms, this process is similar to development of biological populations [3].

Heuristic algorithm of solution of problem of covering minimal summary weight of given set with subsets of non-uniform values is offered. The algorithm is based on genetic algorithm with operators of crossover and mutation. The results of computational experiments are given on the basis of knows test problems.

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On One Two-Dimensional Nonlinear Integro-Differential Equation Based on Maxwell System

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Process of penetration of the magnetic field into a substance is modeled by Maxwell system of partial differential equations. If the coefficient of thermal heat capacity and electroconductivity of the substance depend on temperature and vector of magnetic field has one component $U = U(x, y, t)$, then Maxwell's system can be rewritten in the following integro-differential form:

$$\frac{\partial U}{\partial t} = a\left(\int_0^t \int_\Omega \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2\right] dx\, dy\, d\tau\right) \Delta U, \tag{1}$$

where $\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}$, and $\Omega = [0,\, 1] \times [0,\, 1]$.

Asymptotic behavior of solution of initial-boundary value problems for two-dimensional model (1) as well as numerical solution of those problems are studied.

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On the Well-Possed of the Cauchy Problem for Linear Generalized Differential Systems

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We present the results concerning to the well-possed question for the Cauchy problem for linear generalized (in the J. Kurzveil sense) differential system

$$dx(t) = dA(t) \cdot x(t) + df(t) \tag{1}$$

under the Cauchy condition

$$x(t_0) = c_0, \tag{2}$$

where $A : [a, b] \to R^{n\times n}$ and $f : [a, b] \to R^n$ are, respectively, matrix and vector functions with bounded variation components on $[a, b]$; $t_0 \in [a, b]$ and $c_0 \in R$.

A vector function $x : R \to R^n$ is said to be a solution of the generalized system (1) if is has bounded variation on $[a, b]$ and

$$x(t) - x(s) = \int_s^t dA(\tau) \cdot x(\tau) + f(t) - f(s) \quad \text{for} \quad a \le s < t \le b,$$

where the integral is understand in the Kurzweil–Stieltjes sense.

To a considerable extent, the interest to the theory of generalized ordinary differential equations has also been stimulated by the fact that this theory enables one to investigate ordinary differential, impulsive and differential equations from a unified point of view.

The necessary and sufficient conditions are presented (for each of the following three cases) for the sequence of the Cauchy problem

$$dx(t) = dA_k(t) \cdot x(t) + df_k(t),$$
$$x(t_k) = c_k \quad (k = 1, 2, \dots)$$

to have a unique solution x_k for sufficient large k and

$$\lim_{k\to+\infty} x_k(t) = x_0(t), \quad \lim_{k\to+\infty} x_k(t-) = x_0(t-) \quad \text{and} \quad \lim_{k\to+\infty} x_k(t+) = x_0(t+)$$

uniformly on $[a, b]$, where $A_k : [a, b] \to R^{n\times n}$ $(k = 1, 2, \dots)$ and $f_k : [a, b] \to R^n$ $(k = 1, 2, \dots)$ are, respectively, matrix and vector functions with bounded variation components on $[a, b]$; $t_k \in [a, b]$ $(k = 1, 2, \dots)$, $c_k \in R$ $(k = 1, 2, \dots)$, and x_0 is the unique solution of the problem (1), (2). The analogous results are established in [1] for the case one.

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On the Optimal Stopping of Conditional Gaussian Process with Incomplete Data

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The problem of optimal stopping of conditional Gaussian process with incomplete data is reduced to the optimal stopping problem with complete data and the convergence of payoffs is proved when the small parameter of observable process tends to zero.

Let us consider the partially observable conditional Gaussian process $(\theta,\xi)=(\theta_t,\xi_t)$, $0\le t\le T<\infty$,

$$d\theta_t = a(t,\xi)\theta_t dt + b(t,\xi)dw_1(t),$$

$$d\xi_t = A(t,\xi)\theta_t dt + \epsilon dw_2(t),$$

where $\epsilon>0$, w_1 and w_2 are independent standard Wiener process [1]. Let the gain function $g(t,x)=f(t)+h(t)x$. Introduce the payoffs [2]:

$$S_T^0 = \sup_{\tau\in M_T^\theta} Eg(\tau,\theta_\tau), \quad S_T^\epsilon = \sup_{\tau\in M_T^\xi} Eg(\tau,\theta_\tau),$$

and the notations:$m_t = E(\theta_t|\mathscr{F}_t^\xi)$ $\gamma_t = E((\theta_t-m_t)^2|\mathscr{F}_t^\xi)$.

Theorem 1. *The payoff S_T^ϵ has the following form:*

$$S_T^\epsilon = \sup_{\tau\in M_T^\theta} Eg(\tau,\tilde{\theta}_\tau),$$

where

$$\tilde{\theta}_t = \int\limits_0^t a(s,\xi_s)\tilde{\theta}_s ds + \frac{1}{\epsilon}\int\limits_0^t A(s,\xi_s)\gamma_s dw_1(s).$$

Theorem 2. *The following convergence is valid*

$$\lim_{\epsilon\to 0} S_T^{\epsilon} = S_T^0.$$

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კვლევაზე დაფუძნებული სწავლება მათემატიკაში

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თანამედროვე სასკოლო განათლებაში განსაკუთრებული ყურადღება ეთმობა მაღალი დონის კოგნიტური უნარების განვითარებას. რაც იმას ნიშნავს, რომ მოსწავლეებს კი არ უნდა მივაწოდოთ მზა ფაქტები, არამედ მათ თავად უნდა შეძლონ პრობლემის გადაჭრა, ფაქტის აღმოჩენა, კანონზომიერების დადგენა. ამ პროცესში მათ უნდა შეძლონ არსებული ცოდნის გამოყენება და ახალი საჭირო ფაქტების მოძიება. მასწავლებლის მიზანს წარმოადგენს მოსწავლეებს განუვითაროს სწორედ ზემოთ ჩამოთვლილი უნარები. ასეთი ტიპის უნარების განვითარებას ხელს უწყობს სწორედ შესაბამისი დავალებების – აქტივობების შესრულება მოსწავლეების მიერ. ქვემოთ მოვიყვანთ რამდენიმე დავალებას – აქტივობას, რომლებიც სწორედ აღნიშნული უნარების განვითარებას უწყობს ხელს.

ასეთი ტიპის დავალების მიცემისას მასწავლებელმა უნდა მისცეს მოსწავლეებს მითითებები, თუ რა მოეთხოვებათ მოსწავლეებს და რა უნდა გააკეთონ მათ. შესაბამისად, დავალებები თავადაც უნდა შეიცავდეს დეტალურ ინსტრუქციებს, რასაც უნდა გაჰყვეს მოსწავლე და ბოლოს გააკეთოს სათანადო დასკვნა.

ჩვენი მიზანია სწორედ წარმოვადგინოთ ერთის მხრივ ისეთი ტიპის აქტივობები – დავალებები, რომლებიც ხელს უწყობენ მოსწავლეებში კვლევითი ტიპის უნარების განვითარებას და მათემატიკური პროექტები. ორივე ზემოთ ნახსენებ აქტივობებზე მუშაობისას მოსწავლეები იძენენ კვლევითი და სხვა მნიშვნელოვან არსებით უნარებს.

Characterizing the k-Space Property in Free Objects of Topological Algebra

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A topological space X is called a *k-space* (resp. a k_ω-space) if (there exists a countable cover $\mathscr{K}$ of X by compact subsets such that) each non-closed subset $A \subset X$ has non-compact intersection $A \cap K$ with some compact set $K \subset X$ (that belongs to $\mathscr{K}$). The k-space property is the weakest one in the chain of properties:

metrizable $\Rightarrow$ first-countable $\Rightarrow$ Frechét-Urysohn $\Rightarrow$ sequential $\Rightarrow$ **k-space** $\Leftarrow$ k_ω-space.

In the talk we shall detect the k-space property in some free objects of topological algebra over $\aleph_\omega$-spaces (which form a wide class of generalized metric spaces). A regular topological space X is called an *$\aleph_\omega$-space* if X has a compact-countable k-network, i.e., a family $\mathscr{N}$ of subsets such that for each open set $U \subset X$ and compact subset $K \subset U$ the set $\mathscr{N}_K = \{N \in \mathscr{N} : N \cap K \neq \emptyset\}$ is countable and $K \subset \bigcup \mathscr{F} \subset U$ for some finite subfamily $\mathscr{F} \subset \mathscr{N}_K$. The class of $\aleph_\omega$-spaces includes all $\aleph$-spaces (i.e., regular spaces with σ-locally finite k-network) and consequently, all $\aleph_0$-spaces (i.e., regular spaces with countable k-network) and all metrizable spaces.

Theorem. *Let X be a sequential $\aleph_\omega$-space X. Then*

- V: *the free topological vector space $V(X)$ over X is a k-space iff X is a k_ω-space;*
- L: *the free locally convex space $L(X)$ over X is a k-space iff X is countable and discrete;*
- F: *the free topological group $F(X)$ over X is a k-space iff X is discrete or a k_ω-space;*
- A: *the free topological Abelian group $A(X)$ over X is a k-space iff X is a topological sum of a discrete space and a k_ω-space;*
- FP: *the free paratopological group $FP(X)$ over X is a k-space iff X is either discrete or a countable k_ω-space;*
- AP: *the free paratopological Abelian group $AP(X)$ over X is a k-space iff X is a topological sum of a discrete space and a countable k_ω-space.*

SL: *the free topological semilattice $SL(X)$ over X is a k-space iff X is a topological sum of k_ω-spaces;*

LSL: *the free Lawson topological semilattice $LSL(X)$ is a k-space iff X is metrizable;*

I: *the free (Clifford or Abelian) topological inverse semigroup over X is a k-space iff X is a topological sum of k_ω-spaces.*

In metrizable spaces X the characterization (F) and (A) of this theorem were proved by Arhangelski, Okunev and Pestov in 1989 and the characterization (I) by Banakh, Guran and Gutik in 2001. The equivalence (L) holds for all (not necessarily $\aleph_\omega$) spaces and was proved by Gabriyelyan in 2014. The characterizations (FP) and (AP) answer Open Problems posed by Arhangelski and Tkachenko in their classical monograph "Topological groups and related structures" published in 2008.

Separation Axioms in Paratopological Groups

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We prove that each regular paratopological group is completely regular and each Hausdorff paratopological group is functionally Hausdorff. This resolves two long standing open problems in the theory of paratopological groups.

Also we prove that each (first-countable) Hausdorff paratopological group admits a continuous bijective map onto a (metrziable) Tychonoff quasi-topological group. This answers a question of Arhangelskii posed in 2002.

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Brück Conjecture and Its Generalization

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Value distribution theory of meromorphic functions, is one of the most important tool to deal with the properties of meromorphic functions. In this theory one studies in what frequency an entire or a meromorphic function assumes some values and based on which one can make some idea about the form of the functions. Now-a-days the uniqueness theory of meromorphic functions, have become an extensive subfield of value distribution theory. In this theory one studies the relationship between two non-constant entire or meromorphic functions when they satisfy some prescribed conditions.

Nevanlinna's uniqueness theorem shows that two meromorphic functions f and g share 5 values ignoring multiplicities are identical. Rubel and Yang [2] first showed for entire functions that in the special situation where g is the derivative of f, one usually needs sharing of only two values CM for their uniqueness. Natural question would be to investigate the relation between an entire function and its derivative counterpart for one shared value. In 1996, in this direction the following famous conjecture was proposed by Brück [1]:

Conjecture. *Let f be a non-constant entire function such that the hyper order $\rho_2(f)$ of f is not a positive integer or infinite. If f and f' share a finite value a counting multiplicities, then $\frac{f'-a}{f-a} = c$, where c is a non zero constant.*

Brück himself proved the conjecture for $a = 0$. Gradually the research in this direction gained pace and today it has become one of the most prominent branch of uniqueness theory. In this talk, we propose to highlight the development on the results of Brück starting from the initial stage to the latest one in connection to our humble contribution. We also want to point out future scope of research in this particular aspect.

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On Dual Paradoxical Objects – Luzin Sets and Sierpiński Sets

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It is well known that, under some additional set-theoretical axioms, many interesting and exotical objects on real line $\mathbf{R}$ can be constructed. In this thesis our discussion is devoted to certain paradoxical subsets of $\mathbf{R}$, in particular, Luzin sets and Sierpiński sets. These sets have many applications in real analysis, measure theory, general topology, and modern set theory. Luzin sets were constructed by Luzin in 1914, and Sierpiński sets were constructed by Sierpiński in 1924. Both Luzin and Sierpiński worked under the assumption of the Continuum Hypothesis (**CH**). These sets are dual objects from the point of view of Lebesgue measure and Baire category (see, for instance, [1] and [3]).

We consider the above-mentioned paradoxical subsets of $\mathbf{R}$ and analyze these sets from the point of view of measure and category.

(a) There exists a translation invariant measure μ on $\mathbf{R}$ which extends the Lebesgue measure λ and has the property that all Sierpiński subsets of $\mathbf{R}$ are measurable with respect to μ; moreover, all of them are of μ-measure zero.

(b) If X is a λ-thick Sierpiński subset of $\mathbf{R}$ and λ_X is the induced measure on X, then the completion of the product measure $\lambda_X \otimes \lambda_X$ is not isomorphic to λ_X.

(c) Any Luzin set Y is universal measure zero but no uncountable subset of Y has the Baire property.

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On Increasing the Convergence Rate of Difference Solution to the Third Boundary Value Problem of Elasticity Theory

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We consider the third boundary value problem of static elasticity theory (stiff contact problem) in a rectangle. On the first stage we solve the difference scheme $\mathscr{L}_h U = \varphi$, which has the second-order accuracy [1, 2]. On the second stage, using approximate solution U, it is constructed correcting addend $\mathscr{R}U$, and on the same grid we solve the problem $\mathscr{L}_h \tilde{U} = \varphi + \mathscr{R}U$.

Using the methodology of obtaining the consistent estimates (see, e.g. [3, 4]) it is shown that the solution $\tilde{U}$ of the corrected scheme converges at the rate $O(|h|^m)$ in the discrete L_2-norm, provided that the solution of the original problem belongs to the Sobolev space $W_2^m(\Omega)$ with exponent $m \in [2, 4]$.

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On the Solvability of the Three-Dimensional First Dynamic Boundary-Value Problem of Hemitropic Elasticity

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We consider the first main dynamic boundary-value problem for a three-dimensional piecewise homogeneous hemitropic micropolar medium. By using the Fourier method under sufficiently general assumptions, we prove the solvability of the problem in the classical sense.

On a Method of Constructing a Basis for a Banach Space

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In this work, we consider a direct expansion of a Banach space with respect to subspaces. We offer a method for constructing a basis for a space proceeding from bases for subspaces. We also consider the cases when the bases for subspaces are isomorphic and the corresponding isomorphisms may not hold. And we study the completeness, the minimality and the uniform minimality of corresponding systems.

Let us recall the definitions of completeness, minimality of a system in a banach space.

Let X be some Banach space. A system $\{u_n\}_{n\in\mathbb{N}} \subset X$ is called complete in X if $\overline{L\left[\{u_n\}_{n\in\mathbb{N}}\right]} = X$.

A system $\{x_n\}_{n\in\mathbb{N}} \subset X$ is called minimal in X if

$$x_k \notin \overline{L\left[\{x_n\}_{n\in\mathbb{N}_k}\right]}, \ \ \forall k \in \mathbb{N}, \ \ \text{where} \ \ \mathbb{N}_k = \mathbb{N}\backslash\{k\}.$$

Let the following direct sum hold

$$X = X_1 \oplus \cdots \oplus X_m,$$

where X_i, $i = \overline{1, m}$, are some B-spaces, and let some system $\{u_{in}\}_{n\in\mathbb{N}}$ be given in the space X_i for every $i \in \{1, \dots, m\}$. Consider the following system in the space X:

$$\omega_{in} = (a_{i1}u_{1n}; \dots ; a_{im}u_{mn}), \quad i = \overline{1, m}; \quad n \in \mathbb{N}, \tag{1}$$

where a_{ij} are some numbers. Let

$$A = (a_{ij})_{i,\,j=\overline{1,\,m}}; \quad \Delta = \det A.$$

The following theorem is true.

Theorem 1. *Let the system $\{u_{in}\}_{n\in\mathbb{N}}$ be complete (minimal) in the space X_i, $i = \overline{1, m}$. If $\Delta \neq 0$, then the system $\{\omega_{in}\}_{i=\overline{1,m};n\in\mathbb{N}}$ is also complete (minimal) in the space X.*

In case $\Delta = 0$ we have the following theorem.

Theorem 2. *Let the system $\{u_{in}\}_{n\in\mathbb{N}}$ be minimal in X_i for every $i \in 1 : m$. If $\Delta = 0$, then the system $\{\omega_{in}\}_{i=\overline{1,m};n\in\mathbb{N}}$ defined by (1) is not minimal in X.*

On Riemann Boundary Value Problem and Its Application in Morrey Spaces

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We consider the Riemann boundary value problem with piecewise continuous coefficients in Hardy–Morrey classes. Under certain conditions, on coefficient of the problem being studied Fredholm property of this problem and general solution as homogeneous as inhomogeneous problem in Hardy–Morrey classes. The results of applied to the study of bases properties of exponential systems with piecewise linear phase on Lebesgue–Morrey space.

On the Nonlinear Analogue of the Darboux Problem

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In this talk for the well-known nonlinear oscillation equation we consider a problem which is a nonlinear analogue of the Darboux problem and consists in the simultaneous definition of a solution and its regular propagation domain. The question of solvability of the formulated problem is solved by the method of characteristics.

The talk is based on the paper [1].

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w-Isolated Subgroups

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The main result of this paper is that $I(H)$ consists of all elements w-group G, such that $g^n \in H^w$ for some non-zero element η of w, thus generalising a known result for locally nilpotent groups. With this result as a basis, we prove the generalizations to w-groups of a number of known theorems on locally nilpotent groups.

Functional Differential Inclusions Generated by Delay Differential Equations with Discontinuities

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Given a functional differential equation with a discontinuity, a construction of its extension in the sense of functional differential inclusions is offered. This construction can be regarded as a generalization of the well-known Filippov framework [3] to study ordinary differential equations with discontinuities. Some basic properties of the solutions of the introduced functional differential inclusions are studied in the manner described in the paper [1].

The developed framework is applied to analysis of gene regulatory networks with general delays. An important feature of gene regulatory networks is the presence of thresholds causing switch-like interactions between genes. Such interactions can be described by smooth monotone functions rapidly increasing in a vicinity of their thresholds. The resulting smooth nonlinear system can however be too complicated to be studied theoretically and even numerically, as it can contain thousands of variables. To simplify the functional form of the equations, it is common to represent interactions by the step functions, which gives a system of differential equations with discontinuous right-hand sides. To prove that the dynamics of the simplified system is close to the dynamics of the original smooth system, one may use the Filippov framework, at least in the case of nondelay genetic networks [2].

On the other hand, it is well-known that delay effects are an important issue in genetic models. The challenge in this case is to combine delays with the discontinuities arising from the simplification of the models. In order to implement the central idea of Filippov's theory, we suggest a formal procedure of obtaining a functional differential inclusion from a general discontinuous functional differential equation. This gives a possibility to define an analog of a Filippov solution for discontinuous functional differential equations and, finally, to apply the developed theory to gene regulatory networks with general delays.

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About Correspondence between Proof Schemata and Unranked Logics

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The proof theory takes its roots from G. Gentzen, when he introduced a sequent calculus for first-order logic. Since then, proofs are heavily used in computer science. It is well known that first-order logic is undecidable, therefore all complete proof-search procedures are non-terminating.

The concept of *term schematization* was introduced in [2] to avoid non-termination in symbolic computation procedures and to give finite descriptions of infinite derivations. Later, *formula schemata* for propositional logic was developed [1] to deal with schematic problems (graph coloring, digital circuits, etc.) in more uniform way. In [3, 4] the language of formula schemata was extended to first-order logic and a sequent calculus was defined, introducing a notion of *proof schema*.

Another very expressive formalisms used in computer science are unranked languages, which have unranked alphabet, i.e. function and/or predicate symbols do not have a fixed arity. Since such languages can naturally model XML documents and operations over them, they are more and more often used for knowledge representation. Thus increasing demand for designing and improving deduction methods that would permit to automatize reasoning in unranked languages.

It is easy to see similarities between schematic logical operators and unranked logical operators, defined in [5]. Therefore the question rises: whether it is possible to use proof schemata for knowledge representation. To tackle this problem we try to find correspondence between these two formalisms. As a result we obtained that proof schemata contains unranked logics, i.e. every unranked formula can be represented as a formula schema, but not vice versa.

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Influence of the Background Inhomogeneous Wind on Large Scale Zonal Flow Generation by ULF Modes

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In the work the features of generation of the large scale flows in the ionosphere on the background of inhomogeneous non-stationary winds is considered. From the equation of magnetized (modified by the geomagnetic field) Rossby type waves using multi-scale expansion the nonlinear equation of interaction of amplitudes of five different scale modes is obtained. These modes are: ultra low frequency (ULF) primary magnetized Rossby wave, its two satellites, long wavelength zonal mode and large scale background mode (inhomogeneous wind). The effects of nonlinearities (scalar, vector) in formation of the large scale zonal flows by magnetized Rossby waves with finite amplitudes in the dissipative ionosphere is studied. In this case modified parametric approach is used. On the basis of theoretical and numerical analysis of the corresponding system (generalized problem on the eigen values) the new features of energy pumping from comparably small scale ULF magnetized Rossby wave and the background flow into the large scale zonal flows and nonlinear self-organization of collective activity of above mentioned five modes in the ionosphere medium is revealed. Generation of the zonal flow is caused by the Reinolds stress of the magnetized Rossby wave with finite amplitude and effect of the background shear flow. It is shown, that amplitude of the background flow affects the increment of modulation instability and the zonal flow generation. The satellite observation data is also analyzed by means of linear and nonlinear methods.

პროექტის ,,ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული მეტყველების სინთეზი და ამოცნობა“ მიზნების, მეთოდებისა და პირველი შედეგების მოკლე მიმოხილვა

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2015 წლიდან სტუ ქართული ენის ტექნოლოგიზების ცენტრში შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის საგრანტო მხარდაჭერით ამოქმედდა პროექტი ,,ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული მეტყველების სინთეზი და ამოცნობა“.[1]

მოხსენებისას მოკლედ მიმოვიხილავთ პროექტის მიზნებსა და მეთოდებს [1]. ასევე, პროექტის პირველი შედეგების სახით მიმოვიხილავთ, კ.ფხაკაძის ქართული ენის ლოგიკური გრამატიკის [2] ფარგლებში შემუშავებული ქართული სამეტყველო ენის ანბანზე [3] დაყრდნობით აგებულ ქართული მეტყველების მასინთზებელ სისტემებს. ასევე, ამავე მიდგომებზე დაყრდნობით, მიმოვიხილავთ ქართული მკითხველ-მსმენელი სისტემის ამგებ მეთოდებსა და ჩვენ მიერ უკვე შემუშავებულ ქართული მეტყველების ამომცნობ და ხმოვანი მართვის სისტემებს [4].

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის DO/305/4-105/14 პროექტზე გაღებული საგრანტო მხარდაჭერით.

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2015 წლიდან სტუ ქართული ენის ტექნოლოგიზების ცენტრში შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის საგრანტო მხარდაჭერით ამოქმედდა პროექტი „ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული გრამატიკული მართლმწერი (ანალიზატორი)“.[2]

[2] პროექტი სტუ ქართული ენის ტექნოლოგიზების ცენტრში კ.ფხაკაძის ხელმძღვანელობით მოქმედი ხანგრძლივვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [4] ერთ-ერთი შემადგენელი ქვეპროექტია.

მოხსენებისას მოკლედ მიმოვიხილავთ პროექტის მიზნებსა და მეთოდებს [1]. ასევე, პროექტის პირველ შედეგების სახით მოკლედ მიმოვიხილავთ კ.ფხაკაძის ქართული ენის ლოგიკური გრამატიკის [2] ფარგლებში ქართული ენისათვის შემუშავებული ახალ მიდგომებზე დაყრდნობით აგებულ ქართული სამწერლებო ენის სიტყვების, ფრაზების, წინადადებებისა და ტექსტების მაანალიზებელ, მაგენერირებელ მართლმწერის შემმოწმებელ სისტემებს [3].

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ DO/308/4-105/14 პროექტზე გაღებული საგრანტო მხარდაჭერით.

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Nonlinear Mathematical Model of the Two-Level Assimilations

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In work the new nonlinear mathematical model of two-level assimilation taking into account demographic factors is offered. In model three subjects are considered: the population and powerful government institutions with very widespread language, influencing by means of the state and administrative resources the population of two states or the autonomy for the purpose of their assimilation; the population and government institutions with widespread second language which underwent assimilation from the powerful state, but in the turn, influencing by means of the state and administrative resources the third population with some less widespread language for the purpose of their assimilation; the third population (autonomy) which underwent bilateral assimilation from two rather powerful states.

In model existence of negative demographic factor (natural decrease in the population) at the most powerful state and positive demographic factor (a natural increase of the population) at the autonomy which underwent bilateral assimilation is supposed.

In special cases, constancy of coefficients of model, for Cauchy's task of system of three nonlinear differential equations is found the first integrals. In the first case, the first integral in phase space of solutions represents a hyperbolic paraboloid, and in the second case, a cone. By means of the first integral the required task of Cauchy is reduced to Cauchy's task for nonlinear system of two differential equations for which the stationary point lying in the first quadrant of the phase plane of solutions is found. With use of a criteria of Bendikson, the theorem, about existence in the first quadrant of the phase plane of solutions of some area in which there is a solution in the form of the closed trajectory which is completely lying in this area is proved.

Thus it is proved that when performing some conditions, there is no full assimilation of the population of the autonomy to less widespread language.

Nonlinear Mathematical Model of Bilateral Assimilation with Zero Demographic Factor of the Assimilating Sides

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In work mathematical modeling of nonlinear process of the assimilation taking into account positive demographic factor which underwent bilateral assimilation of the side and zero demographic factor of the assimilating sides is considered. In model three objects are considered:

1. The population and government institutions with widespread first language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation;

2. The population and government institutions with widespread second language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation;

3. Population of the third state formation which is exposed to bilateral assimilation from two powerful states or the coalitions.

For nonlinear system of three differential equations of the first order are received the two first integral. Special cases of two powerful states assimilating the population of small state formation (autonomy), with different initial number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in all cases there is a full assimilation of the population to less widespread language. Thus, proportions in which assimilate the powerful states the population of small state formation are found.

Nonlinear Mathematical Model of Two-Party Elections in Case of Linear Functions of Coefficients

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In work the nonlinear mathematical model describing dynamics of voters of pro-governmental and opposition party (two selective subjects, the coalitions) is offered. In model three objects are considered: the government and administrative institutions influencing citizens by means of administrative resources (first of all, on voters of opposition party) for their attraction on the side of pro-government party; the citizens with a selective voice now supporting opposition party; the citizens with a selective voice now supporting pro-government party.

Cases when coefficients of attraction of votes of pro-government and oppositional parties are linearly increasing functions of time, and administrative impact on voters of opposition party from government institutions, is constant from elections to elections are considered. Cauchy's problem for nonlinear system of the differential equations with variable coefficients of attraction of votes is solved numerically by means of the program environment Matlab. Cases as maximum and certain voter turnout on elections, and also the set falsification of voices of opposition party, the election commission which is partially controlled by government institutions are considered.

The following qualitatively various results are received:

– despite superiority of coefficient of attraction of votes of opposition party over pro-governmental, due to constant administrative impact on voters of opposition party from government institutions, the pro-government party will win the next elections;

– despite superiority of the voters supporting opposition party by the election day due to the best mobilization on elections of the voters, the pro-government party will win the next elections;

– despite superiority of the voters supporting opposition party by the election day at a lonely voter turnout on elections, due to a certain falsification of elections, the pro-government party will win the next elections;

– the opposition party, despite the best appearance on elections of voters of pro-government party, all the same will win the next elections;

– the opposition party, despite the best appearance on elections of voters of pro-government party and a certain falsification of elections, nevertheless will win the next elections.

Nonlinear Mathematical Model of Elections with Variable Coefficients of Model

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Mathematical modeling and computing experiment in the last decades gained comprehensive recognition in science as the new methodology which is roughly developing and widely introduced not only in natural-science and technological spheres, but also in economy, sociology, political science and other public disciplines. Considerable interest represents creation of the mathematical model, allowing to define dynamics of voters of political subjects.

In work the nonlinear mathematical model describing dynamics of voters of pro-governmental and opposition party (two selective subjects, the coalitions) is offered. In model three objects are considered: the government and administrative institutions influencing citizens by means of administrative resources (first of all, on voters of opposition party) for their attraction on the side of pro-government party; the citizens with a selective voice now supporting opposition party; the citizens with a selective voice now supporting pro-government party.

Cases when coefficients of attraction of votes of pro-government and oppositional parties, and also administrative impact on voters of opposition party from government institutions, are exponential increasing functions from elections to elections are considered.

Cauchy's problem for nonlinear system of the differential equations with variable coefficients of model is solved numerically by means of the program environment Matlab. Cases as maximum and certain voter turnout on elections, and also the set falsification of voices of opposition party, the election commission which is partially controlled by government institutions are considered. The following qualitatively various results are received:

– despite superiority of coefficient of attraction of votes of opposition party over pro-governmental, due to administrative impact on voters of opposition party from government institutions, the pro-government party will win the next elections;

– despite superiority of the voters supporting opposition party by the election day due to the best mobilization on elections of the voters, the pro-government party will win the next elections;

– despite superiority of the voters supporting opposition party by the election day at a lonely voter turnout on elections, due to a certain falsification of elections, the pro-government party will win the next elections;

– the opposition party, despite the best appearance on elections of voters of pro-government party, all the same will win the next elections;

– the opposition party, despite the best appearance on elections of voters of pro-government party and a certain falsification of elections, nevertheless will win the next elections.

The Magnetic Boundary Layer of the Earth as an Energy-supplying Channel for the Processes inside the Magnetosphere

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Quasi-viscous interaction between the solar wind plasma and the geomagnetic field regularly takes place at the boundary of the magnetosphere. Like the effect of reconnection of force lines of the Earth magnetic field and the interplanetary magnetic field (IMF) transported by the solar wind the intensity of the quasi-viscous interaction depends on the magnetic viscosity of the plasma. Anomalous increase of the value of this parameter in the MHD boundary layer of the Earth, the magnetopause is analogized with which, is connected with the variation of the solar wind perturbation. In such circumstances for presenting the development process of the magnetopause dynamics the numerical and analytical methods of mathematical modeling have been used. Their effectiveness depends on the quality of the model describing the energy transmission process from the solar wind to the magnetopause. Usually, adequacy of a model for the development dynamics of the phenomena inside the magnetosphere is assessed in this way. In this work one of such theoretical models is considered. This model is based on the Zhigulev "magnetic" equation of the MHD boundary layer

$$\frac{\partial H_y}{\partial t} + u\frac{\partial H_y}{\partial x} + v\frac{\partial H_y}{\partial y} - H_y\frac{\partial v}{\partial y} = \lambda_m\frac{\partial^2 H_y}{\partial x^2},$$

which is simplified by means of the Parker velocities kinematic model

$$u = -\alpha x, \quad v = \alpha y,$$

where α is the reverse value of the time characteristic for the overflow of the magnetosphere day side.MHD equations involve magnetic viscosity λ_m as a coefficient that is defined by σ specific electric conductivity (c is light speed):

$$\lambda_m = \frac{c^2}{4\pi\sigma}.$$

In order to clearly show the physical mechanisms stipulating the energy transmission process from the magnetosphere boundary to its inner structures some new characteristics of the MHD boundary layers are presented: thicknesses of magnetic field induction and the energy driven into the magnetopause. Besides, in the magnetic field induction equation several models of impulsive time variation of the magnetic viscosity of the solar wind is used

$$1)\ \lambda_{m=}\lambda_{0m}\left[1+\beta\sin\left(\pi t/\tau_0\right)\right]; \quad 2)\ \lambda_{m=}\lambda_{0m}e^{-\frac{t}{\tau_0}}; \quad 3)\ \lambda_{m=}\lambda_{0m}\left(1-e^{-\frac{t}{\tau_0}}\right),$$

where λ_{0m} is the value characterizing the magnetic viscosity, τ_0 is the time characterizing the impulsive variation of the magnetic viscosity, β is the coefficient of the impulsive strengthening. By means of the sequent approximation method an analytical image of quasi-stationary variation of the magnetopause parameters correspondent to these models is presented.

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Rotation of Coordinate Axes and Integrability of Maximal Functions

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A mapping B defined on $\mathbb{R}^n$ is said to be a *differentiation basis* if for every $x \in \mathbb{R}^n$, $B(x)$ is a family of bounded measurable sets with positive measure and containing x, such that there exists a sequence $R_k \in B(x)$ $(k \in \mathbb{N})$ with $\lim_{k\to\infty} \operatorname{diam} R_k = 0$.

For $f \in L(\mathbb{R}^n)$ the *maximal function* $M_B(f)(x)$ *corresponding to a basis* B is defined as the supremum of integral means $\frac{1}{|R|}\int_R |f|$, where $R \in B(x)$.

In what follows the dimension of the space $\mathbb{R}^n$ is assumed to be greater than 1.

Denote by $\mathbf{I} = \mathbf{I}(\mathbb{R}^n)$ the basis of intervals, i.e., the basis for which $\mathbf{I}(x)$ $(x \in \mathbb{R}^n)$ consists of all n-dimensional intervals containing x.

A basis B is called: *translation invariant* if $B(x) = \{x + R : R \in B(0)\}$ for every $x \in \mathbb{R}^n$; *sub-basis of a basis* B' if $B(x) \subset B'(x)$ for every $x \in \mathbb{R}^n$.

Let us introduce the following notation: $\mathfrak{B}_{\mathrm{TI}}$ is the class of all translation invariant bases; $\mathfrak{B}_B$ is the class of all sub-bases of a basis B; $\mathfrak{B}_{\mathrm{NL}}$ is the class of all bases which does not differentiate $L(\mathbb{R}^n)$(i.e. there exists a function $f \in L(\mathbb{R}^n)$ the integral of which is not differentiable with respect to B).

For a basis B by Λ_B denote the class of all functions $f \in L(\mathbb{R}^n)$ for which the maximal function $M_B(f)$ is locally integrable.

A class of functions F is called *invariant with respect to a class of transformations of a variable* Γ if $(f \in F, \gamma \in \Gamma) \Rightarrow f \circ \gamma \in F$.

Denote by Γ_n the family of all rotations in the space $\mathbb{R}^n$.

From the results of G. G. Oniani (see [1] or [2]) it follows that the class $\Lambda_{\mathbf{I}}$ is not invariant with respect to rotations(i.e., with respect to the class Γ_n). The following theorem shows that the similar conclusion is valid for bases from a quite general class.

Theorem. *If $B \in \mathfrak{B}_{\mathbf{I}} \cap \mathfrak{B}_{\mathrm{TI}} \cap \mathfrak{B}_{\mathrm{NL}}$, then the class Λ_B is not invariant with respect to rotations.*

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On the Development of the Growth Properties of Composite Entire and Meromorphic Functions from Fifferent Angle of View

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The value distribution theory deals with various aspects of the behavior of entire and meromorphic functions one of which is the study of comparative growth properties. For any entire function f, $M(r,f)$, a function of r is defined as follows:

$$M(r,f) = \max_{|z|=r} |f(z)|.$$

Similarly for another entire function g, $M(r,g)$ is defined. The ratio $\frac{M(r,f)}{M(r,g)}$ as $r \to \infty$ is called the growth of f with respect to g in terms of their maximum moduli.

The maximum term $\mu(r,f)$ of f can be defined in the following way:

$$\mu(r,f) = \max_{n \geq 0} (|a_n| r^n).$$

In fact $\mu(r,f)$ is much weaker than $M(r,f)$ in some sense. So from another angle of view $\frac{\mu(r,f)}{\mu(r,g)}$ as $r \to \infty$ is also called the growth of f with respect to g where $\mu(r,g)$ denotes the maximum term of entire g.

But for meromorphic $f, M(r,f)$ and $\mu(r,f)$ are not defined. To overcome this situation, the theory due to Rolf Nevanlinna (1926) may be considered. The quantity $T(r,f) = m(r,f) + N(r,f)$ is called the Nevanlinna's Characteristic function of f which plays an important role in the theory of meromorphic functions where $m(r,f)$ is the proximity function of f and $N(r,f)$ is the integrated counting function of f. If $T(r,g)$ denotes the Nevanlinna's Characteristic function (abbreviated as N.C.F.) of meromorphic g, the ratio $\frac{T(r,f)}{T(r,g)}$ as $r \to \infty$ is called the growth of f with respect to g in terms of their N.C.Fs. In the talk an extensive study of growth properties of composite entire and meromorphic functions have been made from different angle of view. Several researchers have made close investigations on this topic. In this talk we have made a brief survey of the existing literature involved in the growth properties of composite entire and meromorphic functions and then have mentioned the improvement of some of the results in connection with the original one.

Hydraulic Calculation of Branched Gas Pipeline by Quasi-stationary Nonlinear Mathematical Model

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At present pipelines become one of the main sources of liquid and gas substances transportation and play a vital role in our daily lives. That is way study of gas and liquid substances flow behavior in horizontal and inclined branched pipelines became topical problem of today and had attracted attention of a number of scientists. Recently, many gas flow equations have been developed and a number are using by the gas-liquid industry but as accounting practices have shown none of them are universal. In spite of the fact that most of those have been based on the result of gas-liquid flow experiments as yet they needs to be carefully analyzed, retreated, reworked and checked by the flow pattern. It has been shown in many modern publications that the most complicated part describing practical methods of modeling especially are branched pipeline networks and mathematical models describing flow in the pipelines having outlets containing essential mistakes, which are owing significant simplification of the modeling environment and processes. For this reason development of the detailed numerical models adequate describing the real non-stationary not isothermal processes processing and progressing in the branched pipeline systems and study the problem by analytical methods are actual. In the present paper pressure and gas flow rate distribution in the branched pipeline based on the one quasi-stationary nonlinear mathematical model using analytical methods is investigated. For realization of that purposes the system of partial differential equations describing gas quasi-stationary flow in the branched pipeline was studied. We have found effective solutions of the quasi-stationary nonlinear mathematical model(pressure and gas flow rate distribution in the branched pipeline). For learning the affectivity of the method quite general test was created. Preliminary data of numerical calculations have shown efficiency of the suggested method.

Integro-Differential Equations with Piecewise-Continuous Coefficients

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The purpose of the present research is to investigate the integro-differential equations with piecewise-continuous coefficients and obtained Fredholm criteria in the Bessel potential spaces.

We reduce the integro-differential equations to an equivalent system of the equation of Mellin convolution type. Applying recent results on Mellin convolution equations with meromorphic kernels in Bessel potential and Sobolev-Slobodeckij (Besov) spaces obtained by V. Didenko & R. Duduchava [1] and R. Duduchava [2], criteria of the unique solvability (the Fredholm criteria) of the above mentioned integro-differential equations in classical and non-classical setting are obtained.

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ლაიბნიცი – მათემატიკური ლოგიკის ფუძემდებელი

ნინო დურგლიშვილი

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ლაიბნიცის ლოგიკურ მემკვიდრეობაში იკვეთება თანამედროვე ფორმალური ლოგიკის ჩანასახები. მკვლევართა აზრით, ლაიბნიცის ნაშრომის «ზოგადი გამოკვლევები ცნებებისა და ჭეშმარიტებების ანალიზის შესახებ» ბაზაზე რეკონსტრუირებული ცნებათა ლოგიკის განზოგადება ბულის სიმრავლეთა ალგებრის პირველსახეა.

ლაიბნიცის მიერ ნაგულისხმევ ინტერპრეტაციებში მარტივი კატეგორიული წინადადებების შესატყვისი გამოსახულებები წარმოადგენს კლასთა აღრიცხვის მეტათეორიის მტკიცებებს. ეს აღრიცხვა ლოგიკური თეორიაა და, შესაბამისად, მისი მტკიცებები ლოგიკური თეორიის ფაქტებს აღწერენ.

სავარაუდოა, რომ კლასთა აღრიცხვის ფორმულები, ანუ თვისებათა «შიშველი» სტრუქტურები, ლაიბნიცისათვის უფრო ბუნდოვანი მენტალური ობიექტები იყო, ვიდრე - ლინგვისტური. ეს ბუნებრივიცაა, რადგან კლასთა აღრიცხვის პირველსახეს ლაიბნიცი პარალელურად ქმნიდა ზოგადი აღრიცხვის სახით. მიუხედავად ამისა, შეიძლება ითქვას, რომ ლაიბნიცის იდეების ნამდვილი აზრი, რომელიც საფუძვლად დაედო სილოგისტიკის ენის მის მიერ შემუშავებულ არითმეტიკულ ინტერპრეტაციებს, არის სილოგისტიკის ინტერპრეტაცია კლასთა აღრიცხვის მეტათეორიაში და, სწორედ ამ აზრით შეიძლება მივიჩნიოთ ლაიბნცი მათემატიკური ლოგიკის ფუძემდებლად.

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Uniform Convergence of Integrated Double Trigonometric Fourier Series

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The following theorem is valid:

Theorem. *For the exponential series of a 2π-periodic in each variable and summable function f on* $[0,2\pi]^2$

$$f \sim c_{00} + \sum_{|m|\geq 1} c_{m0}e^{imx} + \sum_{|n|\geq 1} c_{0n}e^{iny} + \sum_{|m|\geq 1,\,|n|\geq 1} c_{mn}e^{i(mx+ny)},$$

the equality

$$\int_0^x \int_0^y f(t,\tau)\,dt\,d\tau = c_{00}xy + iy\sum_{|m|\geq 1} \frac{1}{m}\,c_{m0}(1-e^{imx}) + ix\sum_{|n|\geq 1}\frac{1}{n}\,c_{0n}(1-e^{iny}) - \sum_{|m|\geq 1,\,|n|\geq 1} \frac{1}{mn}\,c_{mn}(1-e^{imx})(1-e^{iny}),$$

is fulfilled uniformly on $[0,2\pi]^2$.

Moreover, the convergence of the series $\sum\limits_{|m|\geq 1,\,|n|\geq 1} \frac{c_{mn}}{mn}$ is obtained and its sum is found.

ფუნქციურ მწკრივთა თეორიის ერთი ტერმინის შესახებ

ომარ ძაგნიძე

ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის ანდრეა რაზმაძის მათემატიკის ინსტიტუტი, თბილისი, საქართველო

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ფუნქციურ მწკრივთა თეორიაში გაჩნდა ახალი სახის მწკრივები, რომელთა წევრები წარმოადგენენ მუდმივების ნამრავლებს სპეციალური სახის ფუნქციებზე და მათ ინგლისურად ეწოდებათ wavelet [‘weivlit] - პატარა ტალღა (небольшая волна - იხ. Англо-русский словарь под редакцией В.К. Мюллера, Москва, 1970, стр. 852).

ამასთან დაკავშირებით შესაძლებლად მიმაჩნია თანადობა wavelet - ტალღივა. ეს მსგავსად იმისა, რომ პატარა გუბეს ეძახიან გუბურას, პატარა კაცს კი კაცუნას და ა.შ.

რაიმე ფუნქციის ტალღივების მწკრივად წარმოდგენასთან დაკავშირებით შესაძლებლად მიმაჩნია გამოთქმა ‘‘ფუნქციის ტალღივა გამწკრივება’’.

თუკი ფუნქციის ტალღივა გამწკრივებაში საჭიროა ფურიეს მოხსენიება, მაშინ შეიძლება ვთქვათ ‘‘ფუნქციის ფურიესმიერი ტალღივა გამწკრივება’’.

ეს უკანასკნელი მივიღე არჩილ ხარაძის წიგნის ‘‘ორთოგონალურ პოლინომთა თეორიის ელემენტები’’ (თსუ, 1999წ) გვერდ 120-ზე მოწოდებული გამოთქმიდან ‘‘ლეჟანდრისეული გამწკრივება’’ იმ განსხვავებით, რომ ლეჟანდრისეული შეცვლილია ლეჟანდრისმიერით (ხომ გვაქვს გამოთქმა ‘‘ნებისმიერი’’ – ნებისგან გამომდინარე).

არჩევანი ‘‘ტალღივა’’ თუ ‘‘ტალღივი’’, დამოკიდებულია მასთან უშუალოდ დაკავშირებულ სიტყვაზე. ზემოხსენებულ გამოთქმაში სიტყვა ‘‘ტალღივა’’ მოითხოვა სიტყვამ ‘‘გამწკრივება’’. ზედაპირთან დაკავშირებით შეიძლება ვიხმაროთ ‘‘ტალღივი ზედაპირი’’. აქ ნასარგებლებია ჩვენთვის კარგად ცნობილი შესაბამისობა ‘‘высотный корпус‘‘ – ‘‘მაღლივი კორპუსი’’.

სიტყვამ მოიტანა და აღვნიშნავ, რომ უნდა ვთქვათ ‘‘შვარცისმიერი წარმოებული’’ და არა ‘‘შვარცის წარმოებული’’ (ეს რუსულის გავლენაა: გაწარმოება ხდება ფუნქციის და არა პიროვნების) – ინგლისურში იხმარება by და არა of (იხ. Англо-русский словарь - составители В.Д. Аракин, З.С. Выгодская, Н.Н. Ильина, четвёртое издание, Москва, 1962, стр. 124: by II [bai] 4) указывает на авторство произведения, пьесы т.п. – например “The Young Guard” by Fadeyev; the 6th symphony by Tchaikovsky”; “Moscow was founded by Yury Dolgoroocky in 1147”).

Synergetics and Higher Education

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This work is devoted to the synergy as a new approach to the functioning of the modern higher educational system. The emergence of such an approach is due primarily to the strong development of synergistic principles in applied science, in particular, in mathematical modeling. We consider different points of view on the synergy as a methodology of modern scientific research. To change the educational strategy the new methodology already developed - is an interdisciplinary branch of science - Synergetics or the theory of self-organization. Offered some examples of synergistic action principles.

On Regular Cohomologies of Biparabolic Subalgebras of $sl(n)$

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A Lie biparabolic subalgebra (firstly named a “seaweed algebra”) of a semisimple Lie algebra is relatively new object in Lie theory; it generalizes a note of a parabolic subalgebra. There are many articles about cohomologies of parabolic subalgebras and some its subalgebras, but cohomologies of biparabolic subalgebras are not investigate yet. In this paper we investigate regular cohomologies of biparabolic subalgebras of a simple Lie algebra $sl(n)$.

In 1972 Leger and Luks [1] have proved that regular cohomologies of Borel algebras with coefficients in himself (i.e. regular cohomologies) are equal to zero in any dimensions. In the same year Tolpygo [2] proved that this result is true in more general case, for parabolic subalgebras. We prove that the foresaid result is true for biparabolic subalgebras too, but we consider biparabolic subalgebras only of $sl(n)$, which definition is based on pair of partitions of n.

Our mane results are:

Theorem 1. *If P is a biparabolic subalgebra of $sl(n)$ and $Z(P)$ is it's center, then regular cohomologies of P are isomorphic in any dimensions to cohomologies of P with coefficients in $Z(P)$.*

Theorem 2. *If pair of partitions of n is indecomposable, then all regular cohomologies of corresponding biparabolic subalgebra of $sl(n)$ are equal to zero.*

We hope, that analogous theorems are true also for biparabolic subalgebras of all semisimple Lie algebras.

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On Martingales and the End of Life Problem in Inventory Control

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We consider an inventory problem of controlling the inventory of spare parts in the final phase of the service life cycle. The final phase starts when the production of a product is terminated and it continues until the last service contract or warranty period expires. Placing final orders for service parts at the end of the production cycle of a product is considered to be a popular tactic to satisfy demand during this period and to mitigate the effect of part obsolescence at the end of the service life cycle. Previous research focuses on repairing defective products by replacing the defective parts with properly functioning spare ones. However, for most of the inventory problems with the product in a no production phase there is typically a price erosion for the new type of product presently in production while repair cost for a defective product of a previous generation stays steady over time. As a consequence, there might be a point in time at which the unit price of a new generation product drops below the repair costs. If so, it is more cost effective to adopt an alternative policy to meet service demands toward the end of the final phase, such as offering customers the new product of a similar type or a discount on a next generation product. As an example we mention the handling of old generation iPhones after the introduction of a new type of iPhone. This study examines the cost trade-offs of implementing alternative policies for the repair policy and develops an exact expression for the expected total cost function under the assumption that the arrival process of demands for repairing defective products of an old type is given by a nonhomogeneous Poisson process with a given arrival rate function. This problem is know as the end of life problem and to derive this expression under very general cost assumptions we use well known results from martingale theory. As such this talk focuses on ongoing research and the use of more sophisticated techniques well known within the theory of stochastic processes contrary to the Markovian techniques used within inventory theory.

The talk is based on joint work with co-author Semih Onur Sezer, Sabanci University, Faculty of Engineering and Natural Sciences, Istanbul.

A Development of the Monotonicity Method for Unilateral and Bilateral Quasi-variational Inequalities

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We consider the variational inequalities with unilateral and bilateral obstacles for second order linear elliptic operator. The domain is bounded and the obstacles may appear in domain and on the boundary as well. We prove some monotone dependence between the solutions and the data of the variational inequalities. It gives an opportunity to construct the monotonicity method for quasi-variational inequalities when the obstacle operator is not monotone in L_2 sense. As an example we consider Implicit Signorini problem, the quasi-variational inequality with unilateral implicit obstacle on the boundary. We show the unique solvability of the problem and construct the iteration schemes for the solution. Then we consider the several statements of the mentioned problem; we consider this problem for the double boundary obstacles, also for the obstacles in domain, obtaining the similar results as for the classical statement of the problem. Some of the results can be generalized for the evolutionary variational inequalities.

Numerical Implementation for One System of Nonlinear Three-Dimensional Partial Differential Equations

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In the cylinder $\bar{\Omega} \times [0, T]$ the following nonlinear three-dimensional initial-boundary value problem is considered:

$$
\begin{gathered}
\frac{\partial U}{\partial t} = \frac{\partial}{\partial x_1}\left(V_1 \frac{\partial U}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(V_2 \frac{\partial U}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(V_3 \frac{\partial U}{\partial x_3}\right), \\
\frac{\partial V_\alpha}{\partial t} = -V_\alpha + g_\alpha\left(V_\alpha \frac{\partial U}{\partial x_\alpha}\right), \\
U(x,0) = U_0(x), \quad V_\alpha(x,0) = V_{\alpha\,0}(x), \quad \alpha = 1,2,3, \quad x \in \bar{\Omega}, \\
U(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,T],
\end{gathered} \tag{1}
$$

where $\Omega = \{x = (x_1, x_2, x_3) :\ 0 < x_1 < 1,\ 0 < x_2 < 1,\ 0 < x_3 < 1\}$, $\partial\Omega$ is the boundary of the domain Ω, T is some fixed positive number, U_0, $V_{\alpha\,0}$, g_α are given sufficiently smooth functions, such that:

$$
\begin{gathered}
V_{\alpha 0}(x) \geq \delta_0, \quad x \in \bar{\Omega}, \\
\gamma_0 \leq g_\alpha(\xi_\alpha) \leq G_0, \quad |g'_\alpha(\xi_\alpha)| \leq G_1, \quad \xi_\alpha \in R, \quad \alpha = 1,2,3,
\end{gathered}
$$

where δ_0, γ_0, G_0, G_1 are some positive constants.

One must note that in two-dimensional case system (1) describes the process of vein formation in meristematic tissues of young leaves.

For the numerical solution of problem (1) the following variable directions type difference scheme is considered:

$$
\begin{gathered}
u_{1t} = (\hat{v}_\beta \hat{u}_{1\bar{x}_1})_{x_1} + (v_2 u_{2\bar{x}_2})_{x_2} + (v_3 u_{3\bar{x}_3})_{x_3}, \\
u_{2t} = (\hat{v}_1 \hat{u}_{1\bar{x}_1})_{x_1} + (\hat{v}_2 \hat{u}_{2\bar{x}_2})_{x_2} + (v_3 u_{3\bar{x}_3})_{x_3}, \\
u_{3t} = (\hat{v}_1 \hat{u}_{1\bar{x}_1})_{x_1} + (\hat{v}_2 \hat{u}_{2\bar{x}_2})_{x_2} + (\hat{v}_3 \hat{u}_{3\bar{x}_3})_{x_3}, \\
v_{\alpha t} = -\hat{v}_\alpha + g_\alpha(v_\alpha u_{\alpha\bar{x}_\alpha}),
\end{gathered} \tag{2}
$$

with corresponding initial and boundary conditions. Here the well known notations for grid functions are used. In (2) the discrete functions u_α are defined on whole meshes while v_α are defined on central meshes.

Various numerical test experiments are carried out.

Numerical Modelling of Some Kinds of Humidity Processes

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On the basis of developed by us of numerical model of a mesoscale boundary layer of atmosphere (MBLA) it is simulated and investigated: a full cycle of clouds and fogs (origin, development and dissipation); simultaneous existence of fogs and clouds; the role of turbulent regime in formation of fog-cloudy ensembles.

Genesis of Foehns is studied and their new classification (dryadiabatic, mostadiabatic and most-dryadiabatic Foehns) is given; the contribution of the latent heat of condensation in formation of such kind of local winds is investigated. The role of Foehns from the point of view of ecology and different branches of a national economy is considered.

Possibility of active influence on some humidity processes (a radiating fog, regulation of an atmospheric precipitation, Foehns) is studied.

The problem about MBLA in a case of humidity and water nonhomogeneous of underlying surface is put and is at a stage of computer realisation (earlier we considered only temperature nonhomogeneous of underlying surface) that will certainly enrich and will improve our initial mesometeorological model.

Interaction of Elastic and Scalar Fields

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In this work we consider the problem of interaction of elastic body with scalar field. The general solution of uniforms system of equations (of elasticity theory) for static case is solved using Papkovich representation method. The contact problem is solved using special boundary-contact condition, in case when the contact surface is a stretched spheroid. The uniqueness theorem for the solution is also proved. Solutions are obtained in terms of absolutely and uniformly convengent series.

Geometrical Applications of Split Octonions

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Physical signals and space-time intervals are described in terms of the algebra of real split octonions. Geometrical symmetries are represented by its automorphism group - the real non-compact form of Cartan's smallest exceptional group G2. This group generates specific rotations of (3+4)-vector parts of split octonions with three extra timelike coordinates and in certain limits represents standard Lorentz transformations. In this picture several physical characteristics of ordinary (3+1)-dimensional theory (such as: number of dimensions, existence of maximal velocities, the uncertainty principle, some quantum numbers, etc.) are naturally emerge from the properties of the algebra.

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სასწავლო პროცესში მათემატიკური სიმბოლიკის გამოყენების შესახებ

გურამ გოგიშვილი

საქართველოს საპატრიარქოს წმიდა ანდრია პირველწოდებულის სახელობის ქართული უნივერსიტეტი, ინფორმატიკის, მათემატიკისა და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი, საქართველო

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მათემატიკის განვითარების ისტორია განუყოფელია მათემატიკური ნოტაციისა და სიმბოლიკის განვითარების ისტორიისგან. საუკუნეების მანძილზე არ შეწყვეტილა ტრადიციებით დამკვიდრებული სიმბოლიკის შემოქმედებითი გააზრება, მისი დახვეწა და მიზანმიმართული გამოყენება. ეს ამარტივებდა მათემატიკური თეორიების ჩამოყალიბებასა და მათ აღქმას, ხვეწდა აზროვნების სტილს.

ცოდნის გადაცემისას ბუნდოვანების გამოსარიცხად, თითოეული სიდიდის აღსანიშნავად უნდა გამოვიყენოთ ერთადერთი სიმბოლო, მოსწავლეს (სტუდენტს) კი მივაწოდოთ ინფორმაცია სხვაგვარი აღნიშვნების შესახებაც. მაგალითად, tg და $\tan$ (ტანგენსის აღნიშვნები); $\ln$ და $\log$ (ნატურალური, e-ფუძიანი ლოგარითმის აღნიშვნები); C_n^k და $\binom{n}{k}$ (ჯუფთებათა რიცხვის აღნიშვნები); $\{a_n\}$ და (a_n) (მიმდევრობის აღნიშვნები); e^x, $\exp(x)$, $e\hat{\ }x$ (მაჩვენებლიანი ფუნქციის აღნიშვნები) და სხვა. სადღეისოდ ამ აღნიშვნათა უნიფიცირება შეუძლებელია, რადგან სხვადასხვა სახელმძღვანელოში შესაძლოა სხვადასხვა აღნიშვნებს შეხვდეთ.

მათემატიკური ნოტაციის ლაკონურობასა და გამარტივებას ხელს უწყობს სპეციალურ ობიექტთა სტანდარტული სიმბოლოებით აღნიშვნა. ასეთია, მაგალითად, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, რომლებიც ასეც აღინიშნება $\mathbf{N}$, $\mathbf{Z}$, $\mathbf{R}$ (ნატურალურ, მთელ და ნამდვილ რიცხვთა სიმრავლეები). თუმცა, ზოგჯერ, ლექციაზე (გაკვეთილზე) გაკეთებულ ჩანაწერებში ეს სიმბოლოები წარმოგვიდგება, როგორც N, Z, R. თუ, ამასთანავე, რაიმე ნატურალური რიცხვი N-ით არის აღნიშნული, შესაძლებელია დაფაზე გაჩნდეს კურიოზული ჩანაწერი $N \in N$. ასეთი ჩანაწერები, ცხადია, უნდა გამოირიცხოს.

წრეწირის სიგრძის მისივე დიამეტრის სიგრძესთან შეფარდება სტანდარტულად π ასოთი აღინიშნება. თუმცა, ეს არ ნიშნავს ამ ასო-ბგერის წრეწირის სიგრძესთან აუცილებელ "მიბმას". მართლაც, x ნამდვილ რიცხვამდე მარტივ რიცხვთა ოდენობის სტანდარტული აღნიშვნაა $\pi(x)$. წარმოსახვით ერთეულს i-თი აღვნიშნავთ, მაგრამ, თუ i მხოლოდ კომპლექსურ რიცხვებს დავუკავშირეთ, მივალთ ისეთ აბსურდამდე, რომელიც $a_i = 1$ ჩანაწერში i-ს წარმოსახვით ერთეულად წარმოგვიდგენს.

მოხსენებაში განხილულია შემთხვევები, როცა მოუქნელი და შეუსაბამო სიმბოლიკა დაბრკოლებად იქცევა ხოლმე ნათელი მათემატიკური წარმოდგენების ფორმირებაში, განსაკუთრებით - მათემატიკის სწავლების პროცესში. წარმოდგენილია ამ პრობლემათა დაძლევის გზები.

Neural Network Software Library for Natural Language Understanding

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Neural Networks are the powerful tool in investigation and understanding of natural languages [1]. There are several types of neural networks that are especially effective in this field.

It is very important to have possibility to do quick testing of new approaches in order to find good solutions for particular tasks. Software library is an effective solution for creating and testing of custom neural networks.

We created software library which has ready components for creating and testing of custom Deep Neural Networks. Our goal was to develop components that are usually used in natural language processing. Particularly we have components for Feed-Forward Convolutional Neural Networks [2] and Recurrent Neural Networks [3].

Among ready components for neural networks, our software library provides necessary objects and functions for constructing new custom components. For the creation of custom components we provide linear algebra formulas, vector operations, matrix operations, etc..

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Nonlocal Contact Problems for Two Dimensional Stationary Equations of Mathematical Physics

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Nonlocal problems represent quite interesting generalization of classical problems of mathematical physics and at the same time they are naturally raised at construction of mathematical models of real processes and the phenomenon.

The present report is devoted to statement and the analysis of nonlocal contact boundary problems for linear elliptic equations of second order in two-dimensional domains. The existence and uniqueness of a regular solution is proved. The iteration process is constructed, which allows one to reduce the solution of the initial problem to the solution of a sequence of classical Dirichlet problems.

In the report the results of numerical calculations of nonlocal contact problem for Poisson's equation in two-dimensional domain are given.

Eigenvalues of Hermitian Toeplitz Matrices with Smooth Simple-Loop Symbols

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The report presents higher-order asymptotic formulas for the eigenvalues and eigenvectors of large Hermitian Toeplitz matrices with moderately smooth symbols which trace out a simple loop on the real line. The formulas are established not only for the extreme eigenvalues, but also for the inner eigenvalues. The results extend and make more precise existing results, which so far pertain to banded matrices or to matrices with infinitely differentiable symbols. Also given is a fixed-point equation for the eigenvalues which may be solved numerically by an iteration method.

Boundary-Domain Integral Formulation for Boundary Value Problem Involving the Laplace–Beltrami Operator

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A boundary value problem for the Laplace-Beltrami operator on a smooth two-dimensional surface embedded in $\mathbb{R}^3$ is considered. As in the case of an inhomogeneous heat transfer [1], a suitable parametrix (Levi function) is found and an integral formulation of the problem is derived. This formulation involves geometrical properties of the surface. Furthermore, besides the usual boundary integrals the integration along the surface is present.

A numerical method of finding the approximate solution is derived similarly to the corresponding case in $\mathbb{R}^3$ [2]. Several key differences and similarities to the popular finite element methods are discussed. Some aspects of implementation are commented on and several numerical examples are presented.

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A BEM-RBF Coupled Method for a Damage Model in Linear Elasticity

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For several years, the industry has brought the use of composite materials into focus, e.g. by using them for the construction of wind turbines, aircraft, or in the automotive industry. There exists a wide variety of possible applications due to the unbeatable advantages over conventional materials such as steel or aluminum; these are mainly the lower weight and an often significantly higher mechanical strength. In contrast to homogeneous materials, the modeling of composites is significantly more complex because of the fine geometrical features. We use a non linear strain- and stress-based continuum damage model, which was first introduced by Simon and Ju [2], and is well accepted throughout the engineering community [4]. The stress tensor σ is defined by $\sigma(x) = (1 - d(\epsilon, x))\mathbb{C}(x) : \epsilon(x)$, where ϵ is the strain tensor, d the internal damage variable and $\mathbb{C}$ the stiffness tensor.

Due to the model we make use of a multi domain Galerkin boundary element method for elasticity [1, 3, 5] coupled with a specific matrix valued radial basis function part to treat the non linear term. To reduce memory requirements of the fully populated matrices we apply a low rank approximation for the matrices generated by the BEM and RBF parts. The resulting linear system is then solved with the help of specially developed preconditioner technique.

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Blow-up Solutions Some Classes of the Nonlinear Parabolic Equations

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In this paper the unbounded increasing solutions of the nonlinear parabolic equations for the finite time is investigated. Before we considered Dirichlet boundary condition. In this paper Neuman boundary problem is investigated.

The sufficient condition for nonlinearity is established. Under this condition every solution of the investigated problem is blown-up. I.e., there is number $T > 0$ such that

$$\|u\,(x;t)\|_{L_2(R^n)} \to \infty, \quad t \to T < \infty.$$

The existence of the solution is proved by smallness of the initial function.

These type of nonlinear equations describe the processes of electron and ionic heat conductivity in plasma, fusion of neutrons and etc. One of the essential ideas in theory of evolutional equations is known as method of eigenfunctions. In this paper we apply such method. We different boundary problem is considered.

In [1] the existence of unbounded solution for finite time with a simple nonlinearity have been proved. In [2] has been shown that, under the critical exponent any nonnegative solution is unbounded increasing for the finite time. Similar results were obtained in [3] and corresponding theorems are called Fujita–Hayakawa's theorems. More detailed reviews can be found in [4], [5] and [6].

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Cauchy Problem of the Dynamical Equations of the Theory of the Thermo-Electro-Magneto Elasticity

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In the paper we consider uniqueness and existence of solutions of Cauchy problem of the dynamical equations of the theory of thermo-electro-magneto elasticity. Applying the Fourier transform we construct the solution of the problem explicitly.

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საინფორმაციო სივრცის საფრთხეების მოდელებისა და ალგორითმების შესახებ

გიორგი იაშვილი

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საინფორმაციო სივრცეს მრავალი საფრთხე ემუქრება. ზოგადად ინფორმაციული მუქარა შეიძლება განისაზღვროს როგორც საინფორმაციო-საკომუნიკაციო სისტემაზე გარკვეულ დროსა და ეტაპზე ისეთი მოვლენის წარმოშობა, რომლის შედეგია არსებულ ინფორმაციაზე გარკვეული არასასურველი ზემოქმედება. საფრთხეები განირჩევიან მათი წარმოშობის, ადგილის, ბუნების, ფორმის და კიდევ ბევრი სხვა კრიტერიუმის მიხედვით.

საინფორმაციო-საკომუნიკაციო ქსელების უსაფრთხოება გლობალური პრობლემაა და ქვეყანა, რომელიც ამ საკითხს სათანადო ყურადღებას არ აქცევს შესაძლოა მეტად სერიოზული პრობლემის წინაშე აღმოჩნდეს.

ზოგადად, ყველა ქვეყნის და შესაბამისად ჩვენი ქვეყნის მთავრობის, სახელმწიფო დაწესებულებების და მსხვილი კომერციული სტრუქტურებისათვის მოსალოდნელ მთავარ კიბერსაშიშროებებს წარმოადგენენ: მონაცემთა ბაზების ჰაკირება; ვებ-საიტებისა და ინტერნეტის მისამართების ფალსიფიცირება; დესტრუქციული ვირუსები; დაცული ინფორმაციების მოპარვა; პაროლების, კოდებისა და გასაღებების მოპარვა; ქსელებისა და კომპიუტერების სერვერების ფიზიკური განადგურება; საერთაშორისო ინტერნეტ-კავშირების დაკარგვა; მტრების და ტერორისტების მიერ ინფორმაციების არაკანონიერი გზით მოპოვება-მიღება და სხვა.

ცხადი გახდა, რომ ქვეყნის საინფორმაციო სივრცე და ამ სივრცის შემადგენელი ქსელები ძალზე დაუცველი და არამდგრადია, ხოლო კიბერუსაფრთხოების არსებული ხარისხი, როგორც სამთავრობო, ასევე სხვა ორგანიზაციებში ძალზედ არაადეკვატურია და მნიშვნელოვნად ჩამორჩება საერთაშორისო სტანდარტებს.

ქვეყნის საინფორმაციო სივრცის საფრთხეების კლასიფიკაცია წარმოადგენს საფუძველს, რომელიც განსაზღვრავს ინფორმაციების დაცვის მიზნობრივ მიმართულებებს. დღეისათვის არსებობს საფრთხეების განსაზღვრის სხვადასხვა მიდგომა. წარმოშობის ბუნების მიხედვით საფრთხეები შეიძლება იყოს: შემთხვევითი და წინასწარგამიზნული, ხოლო საფრთხეების გამოჩენის პირობები იყოფა ორ ჯგუფად: ობიექტური და სუბიექტური საფრთხეები.

საფრთხეების წარმოქმნის წყარო შეიძლება იყოს: ადამიანები; ტექნიკური მოწყობილობები; პროგრამები, ალგორითმები, მოდელები; ინფორმაციის დამუშავების ტექნოლოგიური სქემები და გარემო. სახეების მიხედვით გამოყოფენ ინფორმაციის ფიზიკური

მთლიანობის, მისი ლოგიკური სტრუქტურის, შინაარსის, კონფიდენციალობისა და საკუთრების რღვევას.

საერთოდ უნდა ითქვას, რომ საფრთხეების გარჩევა, კლასიფიცირება, მათი ფორმალიზებული აღწერა, მოდელების შექმნა და მათი წარმოშობის მიზეზების შესწავლა არამარტო აქტუალურია, არამედ ჩვენის აზრით წარმოადგენს აუცილებელ პირობას საინფორმაციო სივრცის დამცავი ღონისძიებების, მოწყობილობების და სისტემების შემუშავებისა და აგებისათვის.

გამოჭრისა და შეფუთვის ამოცანებში კომბინატორული ოპტიმიზაციის მეთოდების კლასიფიკაცია

გიორგი იაშვილი, ნუგზარ იაშვილი, გენადი ფედულოვი

საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

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რთულ გამოთვლათა თეორიაში კონტეინერებში შეფუთვის ცნობილი ამოცანა განიხილება როგორც კომბინატორულ ამოცანათა ერთი ტიპის მეტად რთული და საინტერესო ამოცანა. არსებობს ამ ამოცანის მრავალი სახეობა, რომლებიც გამოიყენებიან სხვადასხვა სფეროში და მათი ამოხსნისთვის მიმართავენ ევრისტიკულ და მეტაევრისტიკულ ალგორითმებს. გარდა ამისა, აქტიურად გამოიყენება ხელოვნული ინტელექტის მეთოდები (მაგ. ნეირონული ქსელები).

რაციონალურად გამოჭრის ამოცანა პირველად ფორმალიზებულად აღწერა ცნობილმა მეცნიერმა ლ. კანტოროვიჩმა. მან ჯერ კიდევ 1939 წელს თავის შრომაში მასალების ეკონომიურად გამოყენებისათვის განიხილა წრფივი პროგრამირების მეთოდების გამოყენება. იგივე შეიძლება ითქვას კონტეინერებში შეფუთვის ამოცანისთვისაც.

გამოჭრისა და კონტეინერებში შეფუთვის ამოცანებში გამოყენებული კომბინატორული ოპტიმიზაციის მეთოდების კლასიფიკაციისას ცალკე უნდა გამოიყოს სწორედ NP-რთული ისეთი ამოცანები, როგორებიცაა: ქვესიმრავლეთა ჯამის, მასალების გამოჭრის, კონტეინერებში შეფუთვის, „ზურგჩანთის" ამოცანები და სხვა მსგავსი. თავის მხრივ კონტეინერებში შეფუთვის ამოცანა შეიძლება იყოს ერთ, ორ და სამ განზომილებიანი, შეფუთვა წონისა და ფასის მიხედვით და სხვ.

ცნობილია, რომ კონტეინერებში შეფუთვისა და მასალების გამოჭრის ამოცანები მიეკუთვნებიან კომბინატორიკული ოპტიმიზაციის ე.წ. NP- რთულ ამოცანებს. მათი მოდელებისა და ალგორითმების კვლევა და ამ ბაზაზე ინტერაქტიული სისტემის შექმნა მეტად აქტუალური და მნიშვნელოვანია, რადგან მათ გააჩნია მრავალ სხვადასხვა დარგში ფართო პრაქტიკული გამოყენება: ნებისმიერი მასალების (მეტალი, ხე, მინა, პლასტმასი,

ქაღალდი და ა.შ.) გამოჭრა; სატვირთო საავტომობილო გადაზიდვების ორგანიზება; ინფორმაციის მატარებლების სარეზერვო ასლების შექმნა; ფაილების განლაგება CD-დისკზე; ტვირთების განაწილება კონტეინერებში და სხვა; ჩვენს წინაშე დასმული ამოცანების გადასაჭრელად არსებული სხვადასხვა მეთოდებიდან გამოიყენება ამოცანების მოდელირებისა და მათ ბაზაზე ოპტიმიზაციის ალგორითმების დამუშავების მეთოდები, კონკრეტულად კონტეინერებში შეფუთვის ამოცანებისათვის ოპტიმიზაციის მეტაევრისტიკული მეთოდები. ასევე განზრახულია კომბინირებული მეთოდების გამოყენება.

ჩვენს მიერ კონტეინერებში შეფუთვის ამოცანის კვლევისას შექმნილ იქნა ამოცანათა სხვადასხვა მოდელი (სულ 22), რომლებიც დაყოფილ იქნა ორ ჯგუფად. თითოეული მოდელისთვის ნაპოვნი იქნა ამონახსნი და მოხდა მათი ოპტიმალურობის შეფასება.

Solution of a Nonclassical Problems of Statics of Microstretch Materials with Microtemperatures

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The paper considers the static of the theory of linear thermoelasticity of microstretch materials with microtemperatures. The boundary value problem of statics is investigated when the normal components of displacement and the microtemperature vectors and tangent components of rotation vectors are given on the spherical surfaces. Uniqueness theorems are proved. Explicit solutions are constructed in the form of absolutely and uniformly convergent series.

Asymptotic Behavior of Solution and Semi-Discrete Scheme for One Nonlinear Integro-Differential Equation with Source Term

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The following nonlinear integro-differential equation with source term is considered:

$$\frac{\partial U}{\partial t} - a\left(\int_0^t \int_0^1 \left(\frac{\partial U}{\partial x}\right)^2 dx d\tau\right)\frac{\partial^2 U}{\partial x^2} + f(U) = 0, \tag{1}$$

where $a = a(S) \geq a_0 = Const > 0$, $a'(S) \geq 0$, $a''(S) \leq 0$, and $f = f(U)$ is nonnegative increasing function.

In the $[0,1] \times [0,\infty)$ let us consider the following initial-boundary value problem for equation (1)

$$\begin{gathered} U(0,t) = \left.\frac{\partial U(x,t)}{\partial x}\right|_{x=1} = 0, \\ U(x,0) = U_0(x), \end{gathered} \tag{2}$$

where $U_0 = U_0(x)$ is given function.

On the uniform mesh correspond to problem (1), (2) the following semi-discrete scheme:

$$\frac{du_i}{dt} - a\left(h\sum_{i=1}^{M}\int_0^t (u_{\bar{x},i})^2 \, d\tau\right) u_{\bar{x}x,i} + f(u_i) = 0, \quad i = 1,2,\ldots,M-1, \tag{3}$$

$$\begin{gathered} u_0(t) = u_{\bar{x},M}(t) = 0, \\ u_i(0) = U_{0,i}, \quad i = 0,1,\ldots,M. \end{gathered} \tag{4}$$

Here we used the well known notations for grid functions.

Asymptotic behavior as $t \to \infty$ of solution of problem (1), (2) is investigated. The stability and convergence of the scheme (3), (4) is proven.

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Interpolation Method of Shalva Mikeladze for Solving Partial Differential Equations

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This report presents an interpolation method of Shalva Mikeladze. Accuracy of the method depends on the number of interpolation points. This is a method without saturation. The method was designed to find a numerical solution of ordinary differential equations and was constructed on the basis of Shalva Mikeladze interpolation formula to solve numerically linear and nonlinear ordinary differential equations of any order as well as systems of such equations. Using different versions of interpolation formula it became possible to solve boundary value problems, eigenvalue problems and Cauchy problem. Later this method in combination with the method of lines was applied to solve boundary value problem for partial differential equations of elliptic type. The Dirichlet problem for the Poisson equation in the symmetric rectangle was considered as a model for application. This application created a semi-discrete difference scheme with matrices of central symmetry having certain properties.

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Commutators of Convolution Type Operators on Some Banach Function Spaces

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We study the boundedness of Fourier convolution operators $W^0(b)$ and the compactness of commutators of $W^0(b)$ with multiplication operators aI on some Banach function spaces $X(\mathbb{R})$ for certain classes of piecewise quasicontinuous functions $a \in PQC$ and piecewise slowly oscillating Fourier multipliers $b \in PSO^{\diamond}_{X,1}$. We suppose that $X(\mathbb{R})$ is a separable rearrangement-invariant space with nontrivial Boyd indices or a reflexive variable Lebesgue space, in which the Hardy-Littlewood maximal operator is bounded. Our results complement those of Isaac De La Cruz-Rodríguez, Yuri Karlovich, and Iván Loreto Hernández obtained for Lebesgue spaces with Muckenhoupt weights.

On the Necessary and Sufficient Conditions for the Stability of Linear Difference Systems

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We present the necessary and sufficient conditions for the stability (in the Lyapunov sense) of the solutions of the linear difference system

$$\Delta y(k-1) = G_1(k-1)y(k-1) + G_2(k)y(k) + g(k), \quad k = 1, 2, \dots, \tag{1}$$

with respect to small perturbations of the system, where $G_j \in E(N_0; R^{n\times n})$ $(j = 1, 2)$ and $g \in E(N_0; R^n)$, $N_0 = \{0, 1, \dots\}$, and $E(N_0; R^{n\times m})$ is the set of all matrix-functions from N_0 into $R^{n\times m}$.

Let $G \in E(N_0; R^{n\times n})$ be an arbitrary matrix-function. A solution $y_0 \in E(N_0; R^n)$ of the system (1) is called G-stable if for every $\varepsilon > 0$ and $k_0 \in N_0$ there exists $\delta = \delta(\varepsilon, k_0)$ such that for every solution y of the system satisfying the inequality $\big\|\big(I_n + G(k_0)\big) \cdot \big(y(k_0) - y_0(k_0)\big)\big\| < \delta$, the following estimate $\big\|\big(I_n + G(k)\big) \cdot \big(y(k) - y_0(k)\big)\big\| < \varepsilon$ holds for $k > k_0$.

The pair (G_1, G_2) is G-stable if each solution of the system (1) is G-stable.

Let $\mathscr{G}_j$ $(j = 1, 2)$ be operators defined by $\mathscr{G}_j(X)(k) \equiv (-1)^{j+1}X(k)$, and $\mathscr{P}_j$ $(j = 1, 2)$ be operators defined by $\mathscr{P}_j(X_1, X_2)(k) \equiv \big(X_1(k-j+1)+X_2(k-j+1)\big)\times\big(I_n-(-1)^j X_j(k-j+1)\big)^{-1}$ for corresponding matrix-functions X, X_1 and X_2 from $E(N_0; R^{n\times n})$.

Theorem. *Let* $\det\big(I_n - (-1)^i G_i(k)\big) \neq 0$ $(i = 1, 2, \quad k = 0, 1, \dots)$ *and let* $j \in \{1, 2\}$ *be fixed. Then the pair* (G_1, G_2) *is* $\mathscr{G}_j(G_j)$*-stable if and only if there exists a nonsingular matrix-function* $H \in E(N_0; R^{n\times n})$ *such that*

$$\sup\{\, \|H^{-1}(k)\| : k = 0, 1, \dots\} < +\infty$$

and

$$\sum_{k=1}^{+\infty} \|\Delta H(k-1) + H(k+j-2)\cdot \mathscr{P}_j(G_1,G_2)(k)\| < +\infty.$$

The analogous results are valid for another type of stability, as well.

To the Question of Full Transitivity of a Co-Torsion Hull

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A separable primary group T, which is the infinite direct sum of separable primary groups T_i, $T = \bigoplus\limits_{i\in I} T_i$, is considered. Therefore there exists a summand T_1 with a base subgroup $B = \bigoplus\limits_{n=1}^{\infty} B_n$, $B_n = \bigoplus Z(p^n)$, which satisfies the following condition: if $M = \{n_1, n_2, \dots\}$ is an infinite set of indexes, then in the socle of the group T_1 there exists an element with a carrier from an infinite subset of the set M. It is proved that the co-torsion hull $T^\bullet$ of the group T is not fully transitive.

The Chilker Task in Mathematical and Computer Models of Information Warfare

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The paper gives a formulation of the problem *Chilker* in mathematical and computer models of information warfare. We consider the controllability of this problem. We derive necessary and sufficient conditions for the existence of solutions of the problem *Chilker*.

2000 MSC: 34H05, 49J15, 49K15, 65C20, 68U20, 93C51, 93C81.

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About One Aspect of the Information Security

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It is proposed opportunity of increase security in information systems. Using the analogy of event logs on different operating systems [1], it is proposed the creation of a visual identification of the log - VIL. We establish necessary hardware components of VIL and requirements to the software.

2010 Mathematics Subject Classification. 68U35, 68N25

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სამგანზომილებიან ბადეთა შესახებ ოთხგანზომილებიან გაფართოებულ აფინურ სივრცეში

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განიხილება სამგანზომილებიანი V_3 ზედაპირი ოთხგანზომილებიან გაფართოებულ აფინურ $\overline{A}_4 = A_4 \cup A_3^*$ სივრცეში, სადაც A_3^* არის პროექციული P_3 სტრუქტურის მქონე არასაკუთრივი ჰიპერსიბრტყე. V_3 ზედაპირს მიუერთდება მოძრავი რეპერი $R = (A, A_i, A_4)$ $(i = 1, 2, 3)$, სადაც $A \in V_3$, $A_i \in T_3(A)$ ($T_3(A)$ არის მხები სიბრტყე V_3 ზედაპირისადმი A წერტილში), $\{A_i, A_4\} \subset A_3^*$. V_3 ზედაპირზე განიხილება წირთა ბადე $(\omega^1, \omega^2, \omega^3)$, ამასთან (AA_i) წრფეები წარმოადგენენ ω^i წირებისადმი მხებებს A წერტილში. როცა A წერტილი გადაადგილდება V_3 ზედაპირზე, მაშინ თითოეული A_i წერტილი A_3^* სივრცეში, ზოგად შემთხვევაში აღწერს სამგანზომილებიან (A_i) ზედაპირს, რომლებზეც ბუნებრივად აღმოცენდებიან $(\omega^1, \omega^2, \omega^3)$ ბრტყელი ბადეები. შეისწავლება ეს ბადეები, როცა: ა) V_3 ზედაპირზე წირთა ბადე $(\omega^1, \omega^2, \omega^3)$ შეუღლებულია, ბ) V_3 ზედაპირი $(\omega^1, \omega^2, \omega^3)$ ბადის მიმართ წარმოადგენს 3-შეუღლებულ სისტემას.

Cancer Proteins and the Blood Flow

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In the paper the connection of the blood flow with the amount of the cancer proteins in the blood at the small arteriole level is investigated. Blood supply all cells by nutrients, but cancer cells grow faster [1, 2]. When the volume of cancer reaches the critical size it becomes dangerous. At this stage cancer cells begin to circulate in the blood and the viscosity and density of blood flow changes dramatically. Consequently, changes metabolism, especially oxygen consumption by normal cells [3, 4]. Oxygen consumption depends on the velocity of a single erythrocyte at the capillary level and the blood plasma flow V [1,2]. In a cylindrical coordinates the velocity of plasma $V(V_x, V_r)$ satisfies the Stokes axi-symmetric system with the equation of continuity and specific boundary conditions. The solution of this problem is given by [5, 6]

$$V_x = q((x+a)((x+a)^2+r^2)^{-3/2} - (x-a)((x-a)^2+r^2)^{-3/2}) + r^2 C/(\rho\nu) - C_1,$$

$$V_r = qr((x+a)^2+r^2)^{-3/2} - ((x-a)^2+r^2)^{-3/2}),$$

where $V^0$is the velocity of erythrocyte, S is a boundary of the erythrocyte, q, C, C_0, C_1 are the definite constants, ρ is a density of plasma, ν is a viscosity. The pressure P at the capillary is given by the formula $P = Cx + C_0$. The profile of velocity is constructed by using experimental data and Maple.

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Indefinite Metric Spaces and Operator Linear Fractional Relations

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This is a natural continuation of our previous work in the considered area.

We consider strict and bistrict plus-operators between spaces with indefinite metrics, in particular, Krein spaces (or J-spaces). We call a plus-operator T in a Krein space strict if $T = dA$, where $d > 0$ is constant and A is a J-expansion, and we call T bistrict if both T and its conjugate operator T^* are strict plus-operators.

We consider operator and geometric sufficient and necessary conditions for a given strict plus-operator T in a Krein space H to be bistrict as an operator between H and $\operatorname{Im} T$ with the induced indefinite metric.

It is well known that a plus-operator T defines an operator linear fractional relation. In particular, we consider the special case of linear-fractional transformations. In the case of Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$, each linear-fractional transformation of the closed unit ball $\mathfrak{K}$ of the space $\mathfrak{L}(\mathfrak{H}_1, \mathfrak{H}_2)$ is of the form

$$\mathfrak{F}_T(K) = (T_{21} + T_{22}K)(T_{11} + T_{12}K)^{-1}$$

and is generated by the bistrict plus-operator T.

The theory of bistrict plus-operators and generated linear fractional transformations forms a significant part of the theory of spaces with indefinite metrics, in particular, Krein spaces. But the much more wider class of strict plus-operators is an open area for investigations. We hope that our new results on "similarity" of some subclass of strict plus-operators, namely the set of all strict plus-operators A, for which the so-called "plus-characteristic" $D(A)$ is non-negative operator, to the subclass of bistrict plus-operators, will allow to develop the theory of strict plus-operators and to obtain new results on generated linear fractional relations.

We consider applications of our results to the well-known Krein–Phillips problem of invariant subspaces of special type for sets of plus-operators acting in Krein spaces, to the so-called Koenigs embedding problem and some other fields of the Operator Theory.

Keywords. Strict and bistrict plus-operators, Krein space, linear fractional relation, operator ball.

Financial Market with Gaussian Martingale and Hedging of European Contingent Claim

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On the filtered probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0 \le t \le T}, P)$ consider stochastic process in discrete time as evolution of risky asset price

$$S_t = S_{t-1} \exp\{I(\tau \ge t)\Delta N_t^{(1)} + I(\tau < t)\Delta N_t^{(2)}\}, \ \ t = 1, \dots, T,$$

where $S_0 > 0$ is a constant, $(N_t^{(1)}, \mathscr{F}_t), N_0^{(1)} = 0$ and $(N_t^{(2)}, \mathscr{F}_t), N_0^{(2)} = 0$ are independent Gaussian martingales. τ is a random variable which takes values $1, 2, \dots, T$, with probabilities $p_i = P(\tau = i), i = \overline{1, T}$. The vector $(N^{(1)}, N^{(2)})$ is independent of τ and $I(A)$ is the indicator of $A \in \mathscr{F}$.

For this model we investigate the problem of European option hedging and consider special class of strategies $\pi_t = (\gamma_t, \beta_t), t \in \overline{1, T}$ which satisfy the condition

$$\Delta\beta_t + \Delta\gamma_t S_{t-1} = -\Delta G_t,$$

where $G_t, t \in \overline{1, T}$ is some stochastic sequence.

In this set of strategies we have obtained optimal in the following sense

$$E[f(S_T) - X_T^\pi]^2,$$

where X_T^π is the final capital and $f(S_T)$ is an European contingent claim.

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On Some Mathematical Method of Calculating Implied Volatility and Prices of Options

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In our report we will tell about application of the Newton–Rapson method for the calculation of implied volatility and application of the Monte Carlo method for the calculation of prices of options. Some examples will be exposed by using the program Excel.

Monte-Carlo Algorithms for Computations of Infinite-Dimensional Riemann Integrals with respect to Product Measures in R^∞

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In [1], the concept of increasing families of finite subsets uniformly distributed in infinite-dimensional rectangles has been introduced and a certain infinite generalization of the Weyl's famous result (cf. [2], Theorem 1.1, p. 2) has been obtained as follows.

Theorem ([1], Theorem 3.5, p. 339). *Let $(Y_n)_{n\in N}$ be an increasing family of finite subsets of $[0,1]^\infty$. Then $(Y_n)_{n\in N}$ is uniformly distributed in the infinite-dimensional rectangle $[0,1]^\infty$ if and only if for every Riemann integrable function f on $[0,1]^\infty$ the following equality*

$$\lim_{n\to\infty}\frac{\sum_{y\in Y_n} f(y)}{\#(Y_n)}=\int_{[0,1]^\infty} f(x)\,d\lambda(x)$$

holds true, where λ denotes the infinite-dimensional "Lebesgue measure" [3].

In this talk we introduce Riemann integrability with respect to product measures for real-valued functions in R^∞ and give some sufficient conditions under which a real-valued function of infinitely many real variables is Riemann integrable. We describe also Monte-Carlo algorithm for computing of such infinite-dimensional Riemann integrals. In addition, by using the properties of the uniformly distributed sequences of real numbers on $(0,1)$, we present a short proof of a certain version of Kolmogorov strong law of large numbers which essentially differs from Kolmogorov's original proof (cf. [4], Theorem 3 (Kolmogorov), p. 379).

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The Computational Implementation of the Conjugate Gradient Method

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In the paper, some aspects of the numerical implementation of the conjugate gradient method (CGM) for systems of linear algebraic equations with symmetric positive definite matrix in the presence of round-off errors are discussed. With exact calculations, CGM provides an exact solution in a finite number of iteration steps. But in fact CGM is an iterative process and the weak point in an iterative process is in a stopping criterion. It is required to determine the number of the iteration step, after which the accuracy of an approximation to a solution of a system of linear equations may not be considerably improved with a particular computer. Hence, the construction of inexpensive stopping criteria for CGM being the aim of this paper is an urgent problem. For four popular versions of CGM, the step-by-step behavior as well as stopping criteria for an iterative process are considered. Numerical results show that the most accurate approximation is achieved by the CGM-version where descent directions and residual vectors are orthogonal in the energy and Euclidean metrics, respectively, at each iteration step. A practical stopping criteria for CGM is proposed as a formula that enables one to determine the number of the CGM iteration step, starting with which the progress is no longer being made. The application of the constructed criteria to the solution of specific systems of linear algebraic equations with ill-conditioned matrices is demonstrated.

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On Infinite Sample Consistent Estimates of an Unknown Average Quadratic Deviation Defined by the Law of the Iterated Logarithm

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In this talk we study the structure of an infinite sample consistent estimate of an unknown average quadratic deviation defined by the law of the iterated logarithm (cf. [1], Theorem 1, p. 385). First we investigate the domain of this estimate in the sense of shyness [2]. More precisely, we prove the following lemma.

Lemma. *A set S, defined by*

$$\left\{(x_k)_{k\in N} : (x_k)_{k\in N} \in R^N \ \&\ \limsup_{n\to\infty} \frac{|\sum_{k=1}^n x_k|}{\sqrt{2n\log\log n}} \text{ exists and is finite}\right\}$$

is Borel shy set in R^N.

By using this lemma we prove the following statement.

Theorem. *Let μ_θ a Borel probability measure in R defined by the distribution function of the random variable Y with means zero and θ^2 variance. For $(x_k)_{k\in N} \in R^N$ we put $T_1((x_k)_{k\in N}) = \limsup_{n\to\infty} \frac{|\sum_{k=1}^n x_k|}{\sqrt{2n\log\log n}}$ if $\limsup_{n\to\infty} \frac{|\sum_{k=1}^n x_k|}{\sqrt{2n\log\log n}}$ exists and is finite, and $T_1((x_k)_{k\in N}) = 1$, otherwise. Then T_1 is a subjective infinite sample consistent estimate of the parameter $\theta \in (0,\infty)$.*

By using the main approach introduced in [3], we construct certain modifications of the estimate T_1 such that they stand objective or strong objective infinite sample consistent estimates of an unknown average quadratic deviation.

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Invariant Subspaces in the Polydisc

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In this talk we describe invariant subspaces of the Hardy space over the polydisc under the multiplication operators by the independent variables generated by a single function.

Using of the Genetic Algorithm Like "Island Model" for Cryptanalysis of the Merkli–Hellman's Cryptosystem

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This work is devoted to the application of genetic algorithms to cryptanalysis known system Merkle–Hellman's cryptosystem. Selection cryptosystem is defined so that the system is known to be compromised, and there is a known Shamir algorithm by which the system is broken. This makes it possible to compare how much better can handle genetic algorithm. The article describes a general approach to solving the problem using genetic algorithms, and proposes a model of the island parallelize computations. Conducted case studies confirm that this model can significantly reduce the time for solving the problem without loss of quality of the result.

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მათემატიკის სტანდარტის ოპტიმალური სტრუქტურა და მისი გავლენა განათლების ხარისხზე

ე. კორძაძე

განათლებისა და მეცნიერების სამინისტრო, ილიას სახელმწიფო უნივერსიტეტი, თბილისი, საქართველო

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მათემატიკის ეროვნული სასწავლო გეგმა (სტანდარტი) ძირითადი ნორმატიული დოკუმენტია, რომელიც განსაზღვრავს სასკოლო მათემატიკური განათლების მიზნებსა და ამოცანებს, სწავლების შინაარსს და ძირითად პრიორიტეტებს. უაღრესად დიდი მნიშვნელობა აქვს იმას, რომ აღნიშნული დოკუმენტი იქნას გაზიარებული და მიღებული მასწავლებელთა აბსოლუტური უმრავლესობის მიერ. მოხსენებაში განხილულია პრინციპები, რომლებიც უნდა ედოს საფუძვლად მათემატიკის სტანდარტის განვითარებასა და დახვეწას; მიმოხილულია საერთაშორისო კვლევებში წარმატებული შედეგის მქონე ქვეყნების სტანდარტების სტრუქტურა და შინაარსი; გაანალიზებულია ქართველ მასწავლებელთა საჭიროებები და მოთხოვნები ეროვნული სასწავლო გეგმის მიმართ.

A Generalization of Generalized $\oplus$-Supplemented Modules

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In this work, we define tg-supplemented modules and investigate some properties of these modules. These modules generalize generalized $\oplus-$supplemented modules. We prove that the finite t-sum of tg-supplemented modules is tg-supplemented. We also prove that the homomorphic image of a distributive tg-supplemented module is tg-supplemented.

Definition 1. Let M be an $R-$module. M is called a *tg-supplemented* module if every submodule of M has a generalized supplement that is a t-summand in M. Clearly, generalized $\oplus$-supplemented modules are tg-supplemented. But the converse is not true in general.

Lemma 2. *Let M be a distributive tg-supplemented $R-$module. Then every factor module of M is tg-supplemented.*

Corollary 3. *Let M be a distributive tg-supplemented $R-$module. Then every homomorphic image of M is tg-supplemented.*

Lemma 4. *Let M be a t-sum of M_1 and M_2. If M_1 and M_2 are tg-supplemented, then M is tg-supplemented.*

Corollary 5. *Let M be a t-sum of $M_1, M_2, \ldots, M_n$. If M_i is tg-supplemented ($i = 1, 2, \ldots, n$), then M is tg-supplemented.*

Key words and Phrases. Small submodules, radical, supplemented modules, generalized supplemented modules.

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Spectral Asymptotics for 2×2 Canonical Systems

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Based on continuity properties of the de Branges correspondence, we develop a new approach to study the high-energy behavior of m-functions and spectral functions of 2×2 first order canonical systems. Our main objective is to provide one-term asymptotic formulas for m-functions, as well as spectral functions of canonical systems. In particular, we characterize Hamiltonians such that the corresponding m-functions behave asymptotically at infinity as

$$m_\nu(z) = C\mathrm{e}^{\mathrm{i}\pi\frac{1-\nu}{2}} z^\nu, \quad z \in \mathbb{C}_+,$$

with some constants $C > 0$ and $\nu \in (-1, 1)$. In the case $\nu = 0$, this enables us to solve a problem posed by W. N. Everitt, D. B. Hinton and J. K. Shaw in [2]. Furthermore, we apply these results to radial Dirac and radial Schrödinger operators as well as to Krein strings and generalized indefinite strings.

The talk is based on joint work with J. Eckhardt and G. Teschl [1].

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Nonlinear Optimization and Hemi-Variational Inequalities for Unilateral Crack Problems

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Macro and micro cracks appear in material science, geophysical and biomedical applications. The modern problems of tribology and fracture require nonlinear modeling of cracking phenomena taking into account contact interaction between the crack faces [1, 3]. This results in quasi-brittle models of fracture.

From a mathematical viewpoint, modeling of dissipative and interaction phenomena due to contact with cohesion or friction results in hemi-variational inequalities within set valued and non-convex optimization context.

In the framework of nonlinear optimization, we suggest and investigate a class of hemi-variational inequalities (hemVI) supported by primal-dual active set algorithms which are of a generalized Newton type. The analysis of local as well as global convergence properties is provided and numerical tests are presented [2, 4].

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On Some Integral Formulae for Continued Fractions

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The following integral representation of a continued fraction was obtained by Euler (see, for example, [1]). Let R and P be two positive functions on $(0,1)$ such that, for $n = 0, 1, 2, \ldots$ and some positive $\alpha,\ \beta,\ \gamma$

$$(a+n\alpha)\int_0^1 PR^n dx = (b+n\beta)\int_0^1 PR^{n+1} dx + (c+n\gamma)\int_0^1 PR^{n+2} dx;$$

then

$$\frac{\int_0^1 PRdx}{\int_0^1 Pdx} = \frac{a}{b+\frac{(a+\alpha)c}{b+\beta+\frac{(a+2\alpha)(c+\gamma)}{\cdots}}}.$$

We generalize the above formula and obtain some integral representations for other types of continued fractions.

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Gaussian Approximation of Multi-Channel Networks with Phase-Type Service in Heavy Traffic

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The main objective of this paper is to study the service process in multi-channel stochastic networks with nodes in a heavy traffic regime. We consider the models of networks in which an input flow for each node is a non–homogeneous Poisson flow with rate depending on time. Service times of calls are independent random values. At first we consider the networks with exponentially distributed service times in each node. For service process in such a network a limit in uniform topology is found. We construct the Markov Gaussian approximate process with its correlation characteristics in explicit form.

In real queueing systems and networks there is a typical situation when service time of a call consists of a certain number of exponential phases with the same parameters. It means that the total service time of a call is distributed by Erlang law. So, further the networks with service times of Erlang type was studied. Obviously, service of calls in the $[\overline{M}_t|E_m|\infty]^r$-network is different from the service in networks of the $[\overline{M}_t|M|\infty]^r$-type only by replacement of the exponential distribution of service time to the Erlang distribution. Service time distributed by the Erlang low has the following interpretation. If a call arrives to service at a node of the network then the service process is split into some service phases that the call passes consistently one by one. Starting with the phase 1 it occupies each stage during exponentially distributed time. Times of phases occupying are independent random variables.

The idea to introduce additional service phases belongs to Erlang who used it for markovisation of the service process in stochastic systems. Following the idea to simplify the analysis of the $[\overline{M}_t|E_m|\infty]^r$-network we introduce "new" nodes for modeling of the each functioning node. These nodes are multi-channel stochastic systems of Markov type. It is shown that the $[\overline{M}_t|E_m|\infty]^r$-network reduces to Markov queueing $[\overline{M}_t|M|\infty]^r$-network by increasing the dimension of the phase set. In this case the approximative process in heavy traffic can be represented in terms of the Markov Gaussian process. The limit is, of course, a non-Markov Gaussian process. Components of the limit is a sum of some components of the many-dimensional Markov process.

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Binomial Option Pricing: One Time and Multiple Time Periods

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The binomial option pricing model is based on the assumption that the underlying stock follows a binomial return generating process. This means that for any period during the life of the option, the stock's value will change by one of two potential constant values.

Our first step in determining the value of the call might be to determine α, the hedge ratio. The hedge ratio defines the number of shares of stock that must be sold (or short sold) in order to maintain a riskless portfolio [1].

$$c_u - \alpha u S_0 = c_d - \alpha f S_0. \tag{1}$$

Thus, our first step in extending the model to two time periods is to substitute for the hedge ratio based on equation (1):

$$c_0 = \frac{(1+r_f)\left(\frac{c_u-c_d}{S_0(u-d)}\right) S_0 + c_d - \left(\frac{c_u-c_d}{S_0(u-d)}\right) dS_0}{1+r_f}. \tag{2}$$

The next two steps of our development are to simplify equation (2):

$$c_0 = \frac{c_u\left(\frac{(1+r_f)-d}{u-d}\right) + c_d\left(\frac{u-(1+r_f)}{u-d}\right)}{1+r_f}. \tag{3}$$

This expression (3) is quite convenient because of the arrangement of potential cash flows in its numerator. Assume for the moment that investors will discount cash flows derived from the call based on the riskless rate. This assumption is reasonable if investors investing in options behave as though they are risk neutral; in fact, they will evaluate options as though they are risk neutral because they can eliminate risk by setting appropriate hedge ratios.

Using the binomial distribution function, this model is easily extended to n time periods as follows:

$$c_0 = \frac{\sum \frac{n!}{j!(n-j)!}\pi!(1-\pi)^{(n-j)} MAX\left[0, (u^j d^{(n-j)} S_0) - X\right]}{(1+r_f)^n}.$$

One apparent difficulty in applying the binomial model is in obtaining estimates for u and d that are required for π. However, if we assume that stock returns are to follow a binomial distribution, we can relate u and d to standard deviation estimates as follows:

$$u = \exp(\sigma\sqrt{1/n}), \quad d = 1/u.$$

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A Variant of φ-Amenability for Dual Banach Algebras

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Let $\mathscr{A}$ be a dual Banach algebra and let φ be a w^*-continuous homomorphism from $\mathscr{A}$ to $\mathbb{C}$. We study the notion of φ-Connes amenability for $\mathscr{A}$. We prove that the existence of a certain diagonal for $\mathscr{A}$ is equivalent to its φ-Connes amenability. we also show that φ-Connes amenability is equivalent to so-called φ-splitting of a certain short exact sequence.

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Keywords. Dual Banach algebra, φ-amenability, Connes amenability.

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On Poincare's Type Inequality with General Weights

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In this note we concern the Poincare's type gradient inequality in the convex domains Ω and positive weight functions $v, \omega^{1-p'} \in L^{1,loc}$. The survey on topic see, e.g. in [1, 2]. The Sobolev's type trace inequality see, in [3].

We suppose also that there exists an $\varepsilon > 0$ such that for any ball $Q = Q(x, \rho)$ with $0 < \rho < \operatorname{diam} \Omega$, $x \in \Omega$, it follows

$$\int_{Q\cap\Omega} v(x)dx \geq \varepsilon \int_{Q} v(x)dx. \tag{1}$$

We assert the following

Theorem. *Let* $1 < p \le q < \infty$, $v, \omega^{1-p'} \in L^{1,loc}$ *and the domain* Ω *satisfy the above condition. Then for the inequality*

$$\Big(\int_\Omega v|f|^q dx\Big)^{\frac{1}{q}} \le C\Big(\int_\Omega \omega|\nabla f|^p dx\Big)^{\frac{1}{p}}$$

to hold for any Lipshitsz continuous function f *such that*

$$\int_\Omega v(x)f(x)dx = 0 \quad or \quad \int_\Omega f(x)dx = 0$$

it suffices that

$$\int_\Omega \sigma(x)\left(\int_Q \frac{v(y)dy}{|x-y|^{n-1}}\right)^{p'} dx \le C\left(\int_Q v(x)dx\right)^{\frac{p'}{q'}}$$

for all balls $Q = Q(x,\rho)$; $x \in \Omega$, $0 < \rho < \mathrm{diam}_\Omega$, *where the constant* $C > 0$ *and does not depend on* f.

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A Compactness Criterion for the Weighted Hardy Operator in $L^{p(x)}$

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We claim a necessary and sufficient condition on $v(\cdot)$, $\omega(\cdot)$ and exponent functions $p(\cdot), q(\cdot)$ governing the compactness of the weighted Hardy operator $H_{v,\omega}f(x) = v(x)\int\limits_0^x f(t)\omega(t)dt$ from $L^{p(x)}(0,\infty)$ to $L^{q(x)}(0,\infty)$. Set $V(x) = \int\limits_x^\infty v(y)^{q(y)}dy$, $W(x) = \int\limits_0^x \omega(y)^{p'(y)}dy$. Denote by Λ_0 and Λ_∞ the class of measurable functions such that $\limsup\limits_{x\to 0} |y(x)-y(0)| \ln \frac{1}{W(x)} < \infty$ and $\limsup\limits_{x\to\infty} |y(x)-y(\infty)| \ln \dfrac{1}{V(x)} < \infty$ and assume that $\lim\limits_{x\to +0} V(x) = \lim\limits_{x\to+\infty} W(x) = \infty$;

Theorem. *Let $p, q \in \Lambda_0 \cap \Lambda_\infty$ and $f(x) \ge 0$ be measurable functions such that $p^- > 1$, $q(0) \ge p(0) > 1$, $q(\infty) \ge p(\infty) > 1$ Then operator $H_{v,\omega}$ is compact from $L^{p(\cdot)}(0,\infty)$ to $L^{q(\cdot)}(0,\infty)$ iff*

$$\lim_{a\to 0} B_a = 0 \quad \textit{where} \quad B_a = \sup_{0<t<a} V(t)^{\frac{1}{q(0)}} W(t)^{\frac{1}{p'(0)}}$$

$$\textit{and} \quad \lim_{b\to\infty} C_b = 0 \quad \textit{where} \quad C_b = \sup_{b<t<\infty} V(t)^{\frac{1}{q(\infty)}} W(t)^{\frac{1}{p'(\infty)}}.$$

Key Words. weights, Hardy's inequality, fractional Hardy inequality, inequality for differences.

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Direct Boundary Integral Equations Method for Acoustic Problems in Unbounded Domains

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We investigate some aspects of the so called direct boundary integral equation method in acoustic scattering theory. It is well known that by the direct approach the uniquely solvable exterior boundary value problems for the Helmholtz equation can not be reduced to the boundary integral equations which are uniquely solvable for arbitrary value of the frequency parameter. This implies that for such resonant frequencies the corresponding integral operators are not invertible and consequently solutions to the nonhomogeneous integral equations are not defined uniquely. They are defined modulo a linear combination of the elements of the null spaces of the corresponding integral operators. In the paper, it is shown that among the infinitely many solutions of the corresponding integral equations there is only one solution which has a physical meaning and corresponds ether to the boundary trace of the unique solution to the exterior problem or to the boundary trace of its normal derivative. We analyze also modified direct boundary integral equation approaches which reduce the Dirichlet and Neumann boundary value problems to the equivalent uniquely solvable integral or singular integro-differential equations.

Riemann-Hilbert Type Boundary Value Problems on a Plane

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Certain boundary value problems of Riemann-Hilbert type on a plane are investigated, conditions of normal solvability are found, connections with the theory of Fredholm operators are established. Some applications of the main results are presented.

The Adaptation of Heuristics Used for Programming Non-Deterministic Games to the Problems of Discrete Optimization

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We use the terminology, notation and heuristics of [1, 2]. We consider in this paper the adaptation of heuristics used for programming non-deterministic games to the problems of discrete optimization, in particular, some heuristic methods of decision-making in various discrete optimization problems. The object of each of these problems is programming anytime algorithms. Among the problems solved in this paper, there are the classical traveling salesman problem and some connected problems of minimization for nondeterministic finite automata. Considered methods for solving these problems are constructed on the basis of special combination of some heuristics, which belong to some different areas of the theory of artificial intelligence. More precisely, we shall use some modifications of unfinished branch-and-bound method; for the selecting immediate step using some heuristics, we apply dynamic risk functions; simultaneously for the selection of coefficients of the averaging-out, we also use genetic algorithms; and the reductive self-learning by the same genetic methods is also used for the start of unfinished branch-and-bound method. This combination of heuristics represents a special approach to construction of anytime-algorithms for the discrete optimization problems. This approach can be considered as an alternative to application of methods of linear programming, and to methods of multi-agent optimization, and also to neuronets.

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The Boundedness of Integral Operators in Grand Variable Exponent Lebesgue Spaces

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Recently grand variable exponent Lebesgue spaces (GVELS) were introduced by the speaker jointly with V. Kokilashvili. We present some structural properties of these spaces. The boundedness results of maximal, Calderón-Zygmund and potential operators under the log-Hölder continuity condition on exponents of spaces will be also discussed. Appropriate results are derived for one-sided integral operators in GVELS for exponents of spaces satisfying the condition weaker than the log-Hölder continuity condition.

მასწავლებელთა კვალიფიკაციის ამაღლების და სასერტიფიკაციო გამოცდების შესახებ

რუსუდან მესხია

ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი
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მასწავლებელთა კვალიფიკაციის ამაღლებაში სასერტიფიკაციო გამოცდებისთვის მზადება და წარმატებით ჩაბარება ერთ-ერთი მნიშვნელოვანი კომპონენტია.

მათემატიკაში მასწავლებელთა სასერტიფიკაციო გამოცდების პროგრამის პროექტში წარმოდგენილია დიფერენციალური და ინტეგრალური აღრიცხვის ელემენტები, რომლებიც ეროვნული სასწავლო პროგრამით სასკოლო კურსში არ არის გათვალისწინებული. ამიტომ, მიგვაჩნია, რომ პედაგოგებისთვის განსაკუთრებით საინტერესო და სასარგებლო იქნება გამოიცეს დამხმარე მასალა: "მათემატიკური ანალიზის ელემენტები მასწავლებელთა სასერტიფიკაციო გამოცდებისთვის", სადაც წარმოდგენილი იქნება თეორიული საკითხების მიმოხილვა საგამოცდო პროგრამის პროექტის ფარგლებში და აგრეთვე შესაბამისი სავარჯიშოები და განვლილი საგამოცდო ტესტებიდან ამოცანები ამოხსნითურთ.

Problems of Statics of Linear Thermoelasticity for a Half-Space

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We consider the statics case of the theory of two-temperature elastic mixtures, when partial displacements of the elastic components of which the mixture consists are equal to each other. We consider boundary value problems for a half-space, when limiting values of the normal components of displacement vectors and tangential components of rotation vectors are given on the boundary. Also the difference between boundary limit of temperatures or of temperature flows are given. The uniqueness theorem in proved. Solutions are represented in quadratures.

On the Construction of Statistical Structures Parabolic Equations

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Let (E, S) be measurable space. Remember some definitions (see [1], [2]).

Definition 1. A statistical structure is object $\{E, S, \mu_i, i \in I\}$, where $\{\mu_i, i \in I\}$, a family of probability measures which are defined on σ-algebra S. A generalized statistical structure is object $\{E, S, \mu_i, i \in I\}$, where $\{\mu_i, i \in I\}$ a family of σ-finite measures on S.

Definition 2. Consider two statistical structures: $\{E, S, \mu_i, i \in I\}$, $\{E, S, \nu_\alpha, \alpha \in A\}$. We say, that the first statistical subordinated to second statistical structure if for $\forall i \in A$ there exist such sequense $\{\alpha_k\}, \alpha_k \in A$, that measure μ_i absolutely continue with respect $\sum_{k\in A} \rho_k \nu_{\alpha_k}$, where $\rho_k \geq 0$ and $\sum_{k\in A} \rho_k < \infty$.

There is proved next theorems:

Theorem 1. *Any generalized statistical structure subordinated to some statistical structure.*

Theorem 2. *Any statistical structure $\{E,S,\mu_i,i\in I\}$, where I have more tan first uncountable power, orthogonal or dominated by statistical structure $\{E,S,\mu\}$, which contained for only wan probability measures.*

Theorem 3. *If statistical structure $\{E,S,\mu_i,i\in I\}$, where I have more than first uncountable power, dominated by statistical structure $\{E,S,\mu\}$, then there exists countable subfamily $\{\mu_{i_n},n\in N\}$, such that statistical structures $\{E,S,\mu_i,i\in I\}$ and $\{E,S,\mu_{i_n},n\in N\}$ are equivalent.*

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The Limiting Distribution of an Integral Square Deviation of Two Kernel Estimators of Bernoulli Regression Function

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Let random variables $Y^{(i)}$, $i=1,2$, take two values 1 and 0 with probabilities p_i (success) and $1-p_i$, $i=1,2$ (failure), respectively. Assume that the probability of success p_i is the function of an independent variable $x\in[0,1]$, i.e. $p_i=p_i(x)=\mathrm{P}\{Y^{(i)}=1\mid x\}$ $(i=1,2)$ ([1]–[3]). Let t_j, $j=1,\dots,n$, be the division points of the interval $[0,1]$: $t_j=\frac{2j-1}{2n}$, $j=1,\dots,n$.

Let further $Y_i^{(1)}$ and $Y_i^{(2)}$, $i=1,\dots,n$, be mutually independent random Bernoulli variables with $\mathrm{P}\{Y_i^{(k)}=1\mid t_i\}=p_k(t_i)$, $\mathrm{P}\{Y_i^{(k)}=0\mid t_i\}=1-p_k(t_i)$, $i=1,\dots,n$, $k=1,2$. Using the samples $Y_1^{(1)},\dots,Y_n^{(1)}$ and $Y_1^{(2)},\dots,Y_n^{(2)}$ we want to check the hypothesis

$$H_0:\ p_1(x)=p_2(x)=p(x),\ \ x\in[0,1],$$

against the sequence of "close" alternatives of the form

$$H_{1n}:\ p_k(x)=p(x)+\alpha_n u_k(x)+o(\alpha_n),\ \ k=1,2,$$

where $\alpha_n\to 0$ relevantly, $u_1(x)\neq u_2(x)$, $x\in[0,1]$ and $o(\alpha_n)$ uniformly in $x\in[0,1]$.

The problem of comparing two Bernoulli regression functions arises in some applications, for example in quantal bios says in pharmacology. There x denotes the dose of a drug and $p(x)$ the probability of response to the dose x.

We consider the criterion of testing the hypothesis H_0 based on the statistic function

$$\begin{aligned}T_n&=\frac{1}{2}\,nb_n\int\limits_{\Omega_n(\tau)}\big[\widehat{p}_{1n}(x)-\widehat{p}_{2n}(x)\big]^2p_n^2(x)\,dx\\&=\frac{1}{2}\,nb_n\int\limits_{\Omega_n(\tau)}\big[p_{1n}(x)-p_{2n}(x)\big]^2\,dx,\ \ \Omega_n(\tau)=\big[\tau b_n,(1-\tau)b_n\big],\ \ \tau>0,\end{aligned}$$

where

$$\widehat{p}_{in}(x)=p_{in}(x)p_n^{-1}(x),\quad p_{in}(x)=\frac{1}{nb_n}\sum_{j=1}^{n}K\Big(\frac{x-t_j}{b_n}\Big)Y_j^{(i)},\ \ i=1,2,$$
$$p_n(x)=\frac{1}{nb_n}\sum_{i=1}^{n}K\Big(\frac{x-t_i}{b_n}\Big),$$

$K(x)$ is some distribution density and $b_n\to 0$ is a sequence of positive numbers, $\widehat{p}_{in}(x)$ is the kernel estimator of the regression function ([4], [5]).

Theorem. *Let $K(x)\in H(\tau)$ and $p(x),u_1(x),u_2(x)\in C^1[0,1]$. If $nb_n^2\to\infty$, $\alpha_n b_n^{-1/2}\to 0$ and $nb_n^{1/2}\alpha_n^2\to c_0$, $0<c_0<\infty$, then for the hypothesis H_{1n}*

$$b_n^{-1/2}(T_n-\Delta(p))\sigma^{-1}(p)\xrightarrow{d}N(a,1),$$

$\xrightarrow{d}$ denotes convergence in distribution and $N(a,1)$is a random variable having the standard normal distribution with parameters $(a,1)$,

$$a=\frac{c_0}{2\sigma(p)}\int\limits_0^1(u_1(x)-u_2(x))^2\,dx,$$

where

$$H(\tau)=\left\{K(x)\geq 0,\ K(x)=0\ \text{ for } |x|\geq\tau>0,\ \int K(x)\,dx=1\right\},$$

$$\Delta(p)=\int_0^1 p(x)(1-p(x))\int_{|x|\leq\tau} K^2(u)\,du,$$

$$\sigma^2(p)=2\int p^2(x)(1-p(x))^2\,dx\int_{|x|\leq 2\tau} K_0^2(x)\,dx,\quad K_0=K*K.$$

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The General Solution of the Homogeneous Problem

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Consider the following homogenous Riemann problem in $H^+_{p(\cdot),\rho}\times{}_{m_0}H^-_{p(\cdot),\rho}$ classes:

$$F^+(\tau)-G(\tau)F^-(\tau)=0,\ \tau\in\partial\omega. \tag{1}$$

By the solution of problem (1) we mean a pair of analytic functions

$$(F^+(z);F^-(z))\in H^+_{p(\cdot),\rho}\times{}_{m_0}H^-_{p(\cdot),\rho},$$

boundary values of which satisfy the relation (1) almost everywhere. Introduce the following functions $X_i(z)$, which are analytic inside (with the sign $+$) and outside (with the sign $-$) the unit circle, respectively:

$$X_1(z) \equiv \exp\left\{\frac{1}{4\pi}\int_{-\pi}^{\pi} \ln\left|G\left(e^{it}\right)\right| \frac{e^{it}+z}{e^{it}-z}dt\right\},$$

$$X_2(z) \equiv \exp\left\{\frac{i}{4\pi}\int_{-\pi}^{\pi} \theta(t)\,\frac{e^{it}+z}{e^{it}-z}dt\right\},$$

where $\theta(t) \equiv \arg G\left(e^{it}\right)$. Define

$$Z_i(z) \equiv \begin{cases} X_i(z), & |z|<1, \\ \left[X_i(z)\right]^{-1}, & |z|>1, \quad i=1,2. \end{cases}$$

Assume

$$Z^{\pm}(z) \equiv Z_1^{\pm}(z)\,Z_2^{\pm}(z).$$

The following theorem is true.

Theorem. *Let $\{\beta_k\}_1^r$, be defined by $\beta_k = -\sum_{i=1}^{m}\alpha_i\chi_{\{t_k\}}(\tau_i)+\frac{1}{2\pi}\sum_{i=0}^{r}h_i\chi_{\{t_k\}}(s_i)$, $k=\overline{0,l}$, and the inequality $-\frac{1}{q(t_k)}<\beta_k<\frac{1}{p(t_k)}$, $k=\overline{0,r}$, be satisfied. If the inequality*

$$-\frac{1}{p(\tau_k)}<\alpha_k<\frac{1}{q(\tau_k)}, \quad k=\overline{1,m},$$

is fulfilled, then the general solution of the homogeneous Riemann problem (1) *in classes $H^{+}_{p(\cdot),\,\rho}\times_{m_0} H^{-}_{p(\cdot),\,\rho}$ can be represented as*

$$F(z)=P_{m_0}(z)\,Z(z),$$

where $Z(\cdot)$ is the canonical solution of homogeneous problem, $P_{m_0}(\cdot)$ is a polynomial of order $k\le m_0$.

On g-Supplement Submodules

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In this work, some properties of g-supplement submodules are investigated. Let V be a g-supplement of an essential submodule U in M. Then it is possible to define a bijective map between essential maximal submodules of V and essential maximal submodules of M which contain U. It is also proved that $Rad_g V = V \cap Rad_g M$.

Lemma 1. *Let M be an $R-$module, $K \leq V \leq M$ and V be a g-supplement of an essential submodule of M. Then $K \ll_g V$ if and only if $K \ll_g M$.*

Theorem *Let M be an $R-$module, $V \leq M$ and V be a g-supplement of an essential submodule of M. Then $Rad_g V = V \cap Rad_g M$.*

Lemma 3. *Let V be a g-supplement of U in M, $T \leq V$ and $K \trianglelefteq V$. Then T is a g-supplement of K in V if and only if T is a g-supplement of $U + K$ in M.*

Corollary 4. *Let V be a supplement of U in M, $T \leq V$ and $K \trianglelefteq V$. Then T is a g-supplement of K in V if and only if T is a g-supplement of $U + K$ in M.*

Proposition 5. *Let M be an $R-$module, $V \leq M$, $U \trianglelefteq M$ and V be a g-supplement of U in M. Then it is possible to define a bijective map between essential maximal submodules of V and essential maximal submodules of M which contain U.*

Keywords. Small submodules, radical, supplement submodules, supplemented modules.

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Parallel Algorithm for Timoshenko Non-linear Problem

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The plate problem described by Timoshenko system is considered [1]. The system of equations is reduced to one non-linear integro-differential equation [2]. Using the projection method the infinite-dimensional task is replaced by finite-dimensional one [3]. Existence of generalized solution and convergence of Galerkin method are proved [4]. Resulting system of cubic equations is solved by iterative method [5]. Parallel computing system is used for getting numerical solution [6].

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Modeling of Wing Panel Manufacture Processes

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Problems of inelastic straining of three-dimensional bodies with large strains are considered. The type of finite element representation for simulation of the forming process is optimized. The process of high-endurance riveted jointing of panels to stiffener ribs is modelled.

Four 3D finite element models with different types of finite elements (tetrahedral and hexahedral, with trilinear and triquadratic interpolation functions representing coordinates and displacements) are considered. It is shown that application of tetrahedral finite elements of constant deformation does not allow us to calculate the shape of a formed panel correctly.

Spatial discretization of the equations is combined with stepping procedure of time integration of the quasi-static deformation equations with iterated correction of the solving on each discrete instant in time. Convergence of the numerical solution to the exact solution is analyzed. It is including in a case the solution does not to belong to regular Sobolev space.

An algorithm is offered for definition of the pre-shaped curvature of ribs needed to ensure the aimed geometry parameters of riveted panels.

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Description of the Structure of Uniformly Distributed Sequences on $[\ 1/2, 1/2]$ from the Point of View of Shyness

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Let $\mathbf{R}^\infty$ be an infinite-dimensional Polish topological vector space equipped with product topology. Recall that a sequence of real numbers $(x_k)_{k\in\mathbb{N}} \in \mathbf{R}^\infty$ is called uniformly distributed on $[a,b]$ if for each c,d with $a \leq c < d \leq b$ we have

$$\lim_{n\to\infty} \frac{\#(\{x_k : 1 \leq k \leq n\} \cap [c,d])}{n} = d - c, \tag{1}$$

where $\#(\cdot)$ denotes the counter measure of a set.

In [3] has been obtained the validity of the following statement.

Theorem 1 ([3], Theorem 3.1, p. 26). *Let μ be Yamasaki–Kharazishvili measure on $\mathbf{R}^\infty$ which gets a numerical value one on the set $[-\frac{1}{2},\frac{1}{2}]^\infty$ (cf.* [1], [2]*). Then μ-almost every element of $\mathbf{R}^\infty$ is uniformly distributed on $[-\frac{1}{2},\frac{1}{2}]$.*

In this talk we study the structure of the set of all uniformly distributed sequences on $[-\frac{1}{2},\frac{1}{2}]$ from the point of view of shyness [4]. More precisely, we establish the validity of the following statement.

Theorem 2. *The set of all uniformly distributed sequences on $[-\frac{1}{2},\frac{1}{2}]$ is shy.*

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On One Method of Approximate Solution of Antiplane Problem of Elasticity Theory for Two-Dimensional Body Having Cross Form

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New algorithms of the approached decision of antiplane problems of elasticity theory (Poisson's equation) for a two-dimensional crosswise body by means of Schwartz iterative method [1] are considered.

Let us solve a problem Dirichlet for the Poisson equation by an

$$\Delta u(x,y) = f(x,y), \quad (x,y) \in \Omega, \tag{1}$$

$$u(x,y) = g(x,y), \quad (x,y) \in \partial\Omega, \tag{2}$$

where $u(x,y) \in C^2(\Omega)$ is unknown, $f(x,y) \in C(\Omega)$, $g(x,y) \in C(\partial\Omega)$ are given functions,
$\Omega = \Omega_1 \cup \Omega_2$ is a given body, $\partial\Omega = \Gamma = \Gamma_1 \cup \Gamma_2$ is a boundary of the given body,
$\Omega_1 = \{(x,y) : -2 \le x \le 2,\ -1 \le y \le 1\}$, $\Omega_2 = \{(x,y) : -2 \le y \le 2,\ -1 \le x \le 1\}$,
$\Gamma_1 = \{(x,y) : y = \pm 1,\ -2 \le x \le -1 \text{ or } 1 \le x \le 2;\ x = \pm 2,\ -1 \le y \le 1\}$,

$\Gamma_2 = \{(x, y) : x = \pm 1,\ -2 \le y \le -1 \text{ or } 1 \le y \le 2;\ y = \pm 2,\ -1 \le x \le 1\}$.

The algorithm consists of two parts: the Schwartz method and the Galerkin method. Unknown function expands in row Fourior–Legendre. Differences of polynoms Legendre are used as basic functions. It is received the five-dot linear system of the algebraic equations concerning unknown coefficients (see [2]). A count process is stable, as corresponding matrix of algebraic equation system has diagonal dominating property relative to rows. It is created the program code (on the basis of Maple 16) for the approached decision of the consider problem (1), (2).

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Statistical Modeling of Random Fields for Solving Boundary Values Problems

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In work [1] random fields for various problems of structural mechanics were examined. To assess the statistical characteristics of the solution of boundary values problem random processes and fields with next spectral densities are recommended:

$$S_1(\overrightarrow{\lambda}) = \frac{\alpha\sigma^2}{\pi(a^2 + |\overrightarrow{\lambda}|^2)}, \quad S_2(\overrightarrow{\lambda}) = \frac{\alpha\sigma^2(a^2 + |\overrightarrow{\lambda}|^2)}{\pi(4\alpha^2|\overrightarrow{\lambda}|^2 + (|\overrightarrow{\lambda}|^2 - a^2)^2)}.$$

Statistical simulation methods using for solving these problems has been studied in [2] before. Solution of the problem leads to: formulation of boundary values problem, in the form of boundary integral equations, and usage of statistical simulation method, to solve these equations. The main difficulty in solving boundary value problems by the statistical simulation is how to obtain the required integral equation equivalent to the problem and how to choose the Markov chain, which is based on the trajectories of unbiased estimates.

For the simulation of random fields, we use methods described in[3]. Let the area represents e square with side T and the origin at the top of the square. Domain of definition field will be considered in the form of the square $A = \{\overrightarrow{\lambda} : |\overrightarrow{\lambda}| \leq \Lambda\}$, $D = \{d_i\}$ – a partition of the field A, $\overrightarrow{\lambda} \in d_i$. Realization of a random field built by the formula:

$$X(\overrightarrow{u}) = \sum_{i=1}^{M}(\cos(\overrightarrow{u}, \overrightarrow{\lambda_i})\xi_{1i} + \sin(\overrightarrow{u}, \overrightarrow{\lambda_i})\xi_{2i}),$$

where $\{\xi_{1i}, \xi_{2i}\}$ – Strictly independent subGaussian random variables with $E\xi_{1i} = E\xi_{2i} = 0$, $E\xi_{1i}^2 = E\xi_{2i}^2 = \upsilon(d_i)$. Measure $\upsilon(\cdot)$ is built by formula $\upsilon(\overrightarrow{\lambda}) = \int\int_{d_i} S(\overrightarrow{\lambda})d\overrightarrow{\lambda}$, where $S(\overrightarrow{\lambda})$ -spectral density fields.

For a given accuracy $\delta > 0$ and reliability ε modeling in space L_2 value Λ and M are chosen such that the inequalities[3]:

$$B(M,\Lambda) < \delta^2, \quad \frac{\delta}{\sqrt{B(M,\Lambda)}}\exp\left\{\frac{1}{2} - \frac{\delta^2}{2B(M,\Lambda)}\right\} \leq 1 - \varepsilon,$$

where $B(M,\Lambda) = \frac{16}{3M^2}\upsilon(A) + \upsilon(R^2\backslash A)$.

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An Equation for the Transverse Displacement of a Nonlinear Static Shell

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Let the static behaviour of a slopping shell be described by the system of equations [1]

$$\frac{\partial N_i}{\partial x_i}+\frac{\partial N_{12}}{\partial x_j}+p_i=0,\ \ i,j=1,2,\ \ i\neq j,\ \ D\Delta^2 w=\frac{\partial}{\partial x_1}\Big(N_1\,\frac{\partial w}{\partial x_1}\Big)+\frac{\partial}{\partial x_2}\Big(N_{12}\,\frac{\partial w}{\partial x_1}\Big)$$
$$+\frac{\partial}{\partial x_2}\Big(N_2\,\frac{\partial w}{\partial x_2}\Big)+\frac{\partial}{\partial x_1}\Big(N_{12}\,\frac{\partial w}{\partial x_2}\Big)+k_1N_1+k_2N_2+q,\quad (x_1,x_2)\in\Omega, \tag{1}$$

where

$$N_i=\frac{Eh}{1-\nu^2}\Big\{\frac{\partial u_i}{\partial x_i}-k_iw+\frac{1}{2}\Big(\frac{\partial w}{\partial x_i}\Big)^2+\nu\Big[\frac{\partial u_j}{\partial x_j}-k_jw+\frac{1}{2}\Big(\frac{\partial w}{\partial x_j}\Big)^2\Big]\Big\},\ \ i,j=1,2,\ \ i\neq j,$$
$$N_{12}=\frac{Eh}{2(1+\nu)}\Big(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}+\frac{\partial w}{\partial x_1}\,\frac{\partial w}{\partial x_2}\Big).$$

Here $u_i=u_i(x_1,x_2)$ are longitudinal, $i=1,2$, and $w=w(x_1,x_2)$ transverse displacements of points of the shell midsurface Ω, $p_i=p_i(x_1,x_2)$, $i=1,2$, $q=q(x_1,x_2)$ are external force components, $k_i=k_i(x_1,x_2)$ the shell curvature components, $i=1,2$, Δ is the Laplace operator, E and $0<\nu<\frac{1}{2}$ are respectively Young's modulus and Poisson's ratio, D is the shell flexural rigidity, h is the thickness.

Assuming that Ω is the rectangle and for $u_i(x_1,x_2)$, $i=1,2$, the first and second kind conditions are fulfilled on the boundary $\partial\Omega$ of Ω, from (1) we obtain the following nonlinear equation for the function $w(x_1,x_2)$

$$D\Delta^2 w-\sum_{i=1}^{2}\sum_{j=1}^{2}\bigg\{\int_\Omega\bigg[A_{ij}\bigg(\frac{1}{2}\bigg(\frac{\partial w}{\partial\xi_1}\bigg)^2-k_1w\bigg)+C_{ij}\,\frac{\partial w}{\partial\xi_1}\,\frac{\partial w}{\partial\xi_2}$$
$$+B_{ij}\bigg(\frac{1}{2}\bigg(\frac{\partial w}{\partial\xi_2}\bigg)^2-k_2w\bigg)+d_{1ij}p_1+d_{2ij}p_2\bigg]d\xi_1\,d\xi_2+\int_{\partial\Omega}\bigg[a_{ij}\bigg(\frac{1}{2}\bigg(\frac{\partial w}{\partial\xi_1}\bigg)^2-k_1w\bigg)$$
$$+c_{ij}\,\frac{\partial w}{\partial\xi_1}\,\frac{\partial w}{\partial\xi_2}+b_{ij}\bigg(\frac{1}{2}\bigg(\frac{\partial w}{\partial\xi_2}\bigg)^2-k_2w\bigg)\bigg]ds\bigg\}\bigg(\delta_{ij}k_i+\frac{\partial^2 w}{\partial x_i\partial x_j}\bigg)$$
$$+p_1\,\frac{\partial w}{\partial x_1}+p_2\,\frac{\partial w}{\partial x_2}=q,\quad (x_1,x_2)\in\Omega,$$

where the integrand coefficients A_{ij}, B_{ij}, C_{ij}, d_{1ij}, d_{2ij} and a_{ij}, b_{ij}, c_{ij} depend on x_1,x_2 and ξ_1,ξ_2, ds is an element of the boundary $\partial\Omega$, δ_{ij} is the Kronecker symbol, $i,j=1,2$.

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Mathematical Modeling of hydraulic Fractures: Shear-Thinning Fluids

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Hydraulic fracture (HF) is a physical process of a hydraulically induced crack propagating in a brittle material. It can be found in nature, e.g. magma driven dykes or subglacial drainage of water. Moreover, it has many technological applications, for example exploitation of geothermal reservoirs or methane extraction from coal seams, but is now mainly associated with stimulation of hydrocarbon reservoirs. Understanding and control of the process is also crucial in cases like CO2 sequestration and storage of dangerous waste underground.

Alongside the development of modern stimulation techniques, the need for more efficient and accurate numerical modeling of the problem has emerged. However, mathematical modeling of hydraulically induced fracture is very challenging, due to its complexity. Even in the simplest formulation we need to take into account interaction between solid and fluid phases, non-local elasticity operator, leak-off into rock formation or fracture mechanics criteria. Main mathematical difficulties stem from: i) strong non-linearity of the system, ii) singularities occurring in the fracture front region, iii) moving boundaries, iv) degeneration of the governing equations near the crack-tip, v) multiscaling, and others. Any efficient and accurate numerical solver for HF needs to address these challenges.

Although a number of dedicated software packages are available on the market, there is still a great need to improve their performance.

In the talk, recently developed universal numerical scheme for simulation of hydraulic fractures for Newtonian fluids will be extended for the case of shear-thinning fluids. The adaptation of the algorithm addresses new challenges resulting from rheological properties of the fluid: i) order of non-linearity of Poiseulle equation, ii) crack-tip asymptotics dependent on fluid behaviour index, iii) degeneration of the equations for perfectly plastic fluid. Additional non-linearity necessitates modification of regularization techniques (such us utilization of proper independent and dependent variables or the so called ε-regularization) used to stabilize the computations. Numerical techniques employed in the solver will be discussed and its advantages will be illustrated by computational results.

ქართული ხმიდან-ხმაზე და ტექსტიდან-ტექსტზე მთარგნელი სისტემის საცდელი ვერსიები

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2015 წლიდან სტუ ქართული ენის ტექნოლოგიზების ცენტრში შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის საგრანტო მხარდაჭერით ამოქმედდა პროექტი „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“.[1]

ტექსტიდან-ტექსტზე მთარგნელი სისტემის საცდელი ვერსია ჩვენ გვქონდა შემუშავებული ჯერ კიდევ 2009 წელს. ამ საცდელი სისტემის გაუმჯობესებული საინტერნეტო ვერსიები და ხმიდან-ხმაზე მთარგნელი პირველი ქართული საცდელი საინტერნეტო სისტემა ჩვენ შევიმუშავეთ 2013 წელს გამოყენებითი კვლევებისთვის სტუ N048 და შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის N31/70 გრანტების მხარდაჭერით [1], [2].

მოხსენებისას წარმოვადეგნთ ამ სისტემების ახალ - გაუმჯობესებულ ვერსიებს, რომლებიც სტუ ქართული ენის ტექნოლოგიზების ცენტრში დღეს მოქმედი პროექტით „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ წარმოებული კვლევების ფარგლებში შემუშავდა.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ AR/122/4-105/14 პროექტზე გაღებული საგრანტო მხარდაჭერით.

ლიტერატურა

[1] K. Pkhakadze, G. Chichua, M. Chikvinidze, A. Maskharashvili, I. Beriashvili, An overview of the trial version of the georgian self-developing intellectual corpus necessary for creating georgian texts analyzer, speech processing, and automatic translation systems. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **28** (2014), 70–75.

[2] კ. ფხაკაძე, ქართული ენის ტექნოლოგიური ანბანი — XXI საუკუნის ერთ-ერთი უმთავრეს ქართული გამოწვევა. საპარლამენტო კონფერენციის „ქართული ენა — 21-ე საუკუნის გამოწვევები“ შრომები, 95-102, 2013.

[1] ეს ორწლიანი პროექტი სტუ ქართული ენის ტექნოლოგიზების ცენტრში კ.ფხაკაძის ხელმძვანელობით მოქმედი ხანგრძლივვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [3] ერთ-ერთი სასაფუძვლოდ მნიშვნელოვანი ქვეპროექტია.

[3] კ. ფხაკაძე, მ. ჩიქვინიძე, გ. ჩიჩუა, ქართული თვითგანვითარებადი ინტელექტუალური კორპუსში ჩადგმული ქართული მეტყველების ამომცნობისა და ქართული ხმიდან-ხმაზე მთარგმნელი სისტემების საცდელი ვერსიები. საქართველოს მათემატიკოსთა კავშირის V საერთაშორისო კონფერენციის თეზისები, გვ. 139-140, 2014.

ქართული ვებ-გვერდების ხმით მართვადი მკითხველი სისტემა

კონსტანტინე ფხაკაძე, გიორგი ჩიჩუა, მერაბ ჩიქვინიძე, დავით კურცხალია

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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ვებ-გვერდების მკითხველი პირველი საცდელი სისტემა ჩვენ (კ.ფხაკაძე, გ.ჩიჩუა, მ.ჩიქვინიძე) შევიმუშავეთ 2013 წელს საქართველოს ტექნიკური უნივერსიტეტის N048 და შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის No31/70 გრანტების მხარდაჭერით წარმოებული კვლევების ფარგლებში [1], [2].

მოხსენებისას მიმოვიხილავთ ამ სისტემის ახალ - გაუმჯობესებულ ვერსიას, რომელიც ხმოვანი მართვითაა აღჭურვილი და რომელიც სტუ ქართული ენის ტექნოლოგიზების ცენტრში დღეს მოქმედი პროექტით „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ წარმოებული კვლევების ფარგლებში შემუშავდა.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ AR/122/4-105/14 პროექტზე გაღებული საგრანტო მხარდაჭერით.

ლიტერატურა

[1] K. Pkhakadze, G. Chichua, M. Chikvinidze, A. Maskharashvili, I. Beriashvili, An overview of the trial version of the georgian self-developing intellectual corpus necessary for creating georgian texts analyzer, speech processing, and automatic trans-

[1] ეს ორწლიანი პროექტი სტუ ქართული ენის ტექნოლოგიზების ცენტრში კ.ფხაკაძის ხელმძვანელობით მოქმედი ხანგრძლივვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [3] ერთ-ერთი სასაფუძვლოდ მნიშვნელოვანი ქვეპროექტია.

lation systems. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **28** (2014), 70–75.

[2] კ. ფხაკაძე, ქართული ენის ტექნოლოგიური ანბანი – XXI საუკუნის ერთ-ერთი უმთავრეს ქართული გამოწვევა. საპარლამენტო კონფერენციის „ქართული ენა – 21-ე საუკუნის გამოწვევები" შრომები, 95-102, 2013.

[3] კ. ფხაკაძე, გ. ჩიჩუა, მ. ჩიქვინიძე, ქართული არაშინაარსულად მკითხველი სისტემის ინტერნეტ ვერსია და პირველი ნაბიჯები ქართული შინაარსობრივად მკითხველი სისტემის აგების მიმართულებით. საქართველოს მათემატიკოსთა კავშირის IV საერთაშორისო კონფერენციის თეზისები, გვ. 149-150, 2013.

ქართული ენით ევროკავშირში ანუ პროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ" მიზნებისა და მეთოდების მოკლე მიმოხილვა

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა,
ინეზა ბერიაშვილი, დავით კურცხალია

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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პროექტის მიზანია მოსაუბრე ქართული ინტელექტუალური ვებ-კორპუსის [1] აგება, რაც მეტა-ქსელის ანუ მრავალენოვანი ევროპის ტექნოლოგიური დაფუძნების ქსელის ნაშრომით „სტრატეგიული კვლევითი გეგმა 2020 წლის მრავალენოვანი ევროპისათვის" დაგეგმილი მრავალენოვანი ევროპის ტექნოლოგიური დაფუძნების პროცესში ქართული ენის დროული ჩართულობის უზრუნველსაყოფად სრულიად ცხად აუცილებლობას წარმოადგენს.

[3]ეს ორწლიანი პროექტი სტუ ქართული ენის ტექნოლოგიზების ცენტრში კ.ფხაკაძის ხელმძღვანელობით მოქმედი ხანგრძლივვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი" [3] ერთ-ერთი სასაფუძვლოდ მნიშვნელოვანი ქვეპროექტია.

ამასთან, მიზნის მიღწევის გზად დასახულია კ.ფხაკაძის ქართული ენის ლოგიკური გრამატიკაზე [2] დაყრდნობით კორპუსში ჩადგმული მოსაუბრე ქართული ინტელექტუალური სისტემის ანუ ქართული ენის ტექნოლოგიური ანბანის აგება [3] და მისი გაფართოება როგორც ხმიდან-ხმაზე, ისე ტექსტიდან-ტექსტზე მთარგნელობითი უნარებით.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ AR/122/4-105/14 პროექტზე გაღებული საგრანტო მხარდაჭერით.

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[1] K. Pkhakadze, G. Chichua, M. Chikvinidze, A. Maskharashvili, I. Beriashvili, An overview of the trial version of the georgian self-developing intellectual corpus necessary for creating georgian texts analyzer, speech processing, and automatic translation systems. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **28** (2014), 70–75.

[2] კ. ფხაკაძე, მ. ჩიქვინიძე, გ. ჩიჩუა, ა. მასხარაშვილი, ქართული ენის ლოგიკური გრამატიკის საფუძვლები და მისი გამოყენებანი, (იბეჭდება) 2015.

[3] კ. ფხაკაძე, ქართული ენის ტექნოლოგიური ანბანი — XXI საუკუნის ერთ-ერთი უმთავრეს ქართული გამოწვევა. საპარლამენტო კონფერენციის „ქართული ენა — 21-ე საუკუნის გამოწვევები“ შრომები, 95-102, 2013.

პროექტის „ქართული ენის ლოგიკური გრამატიკის საფუძვლები და მისი გამოყენება საინფორმაციო ტექნოლოგიებში“ ფარგლებში აგებული ხმით მართვადი ხმოვანი ქართული ინტელექტუალური კომპიუტერული სისტემის საცდელი საინტერნეტო ვერსიის მოკლე მიმოხილვა

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, ინეზა ბერიაშვილი, დავით კურცხალია

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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2013 წლიდან სტუ ქართული ენის ტექნოლოგიზების ცენტრში შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის N31/70 გრანტის მხარდაჭერით ამოქმედდა პროექტი

„ქართული ენის ლოგიკური გრამატიკის საფუძვლები და მისი გამოყენება საინფორმაციო ტექნოლოგიებში“ [1].[1]

პროექტით დაგეგმილი იყო ხმით მართვადი ხმოვანი ქართული ინტელექტუალური კომპიუტერული სისტემის [2] ანუ ქართული ენის ტექნოლოგიური ანბანის [3] საცდელი დესკტოპ ვერსიის აგება. ჯგუფმა ნაცვლად დაგეგმილისა შეიმუშვა იმავე ტიპის ბევრად უფრო მრავალფუნქციური საინტერნეტო სისტემა. მოხსენებისას მიმოვიხილავთ ამ სისტემასა და მის იმ ქვესისტემებს, რომლებიც ქართული ენის ლოგიკურ გრამატიკაზე [4] დაყრდნობითაა აგებული და რომელთაგან უმეტესობა უნიკალურია იმ გაგებით, რომ მათი სხვა ქართული ანალოგები არ არსებობს.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ FR/362/4-105/12 პროექტზე გაღებული N31/70 საგრანტო მხარდაჭერით.

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ქართული ვებ-გვერდების ხმოვანი მართველის საცდელი ვერსია

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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2015 წლიდან სტუ ქართული ენის ტექნოლოგიზების ცენტრში შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის საგრანტო მხარდაჭერით ამოქმედდა პროექტი ,,კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“.[1]

ქართული ვებ-გვედრების ხმოვანი მართვის სისტემის პირველი საცდელი ვერსია ჩვენ (კ.ფხაკაძე, გ.ჩიჩუა, მ.ჩიქვინიძე) გვქონდა შემუშავებული ჯერ კიდევ 2009 წელს (იხ. `https://www.facebook.com/qartulienisteqnologizebiscentri?ref=hl` მისამართზე). სისტემის გაუმჯობესებული ვერსია შემუშავდა 2015 წელს შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის N31/70 გრანტის მხარდაჭერით მოქმედი ორწლიანი პროექტით „ქართული ენის ლოგიკური გრამატიკის საფუძვლები და მისი გამოყენება საინფორმაციო ტექნოლოგიებში“ წარმოებული კვლევების ფარგლებში.

მოხსენებისას მიმოვიხილავთ ამ სისტემის ახალ - გაუმჯობესებულ ვერსიას, რომელიც სტუ ქართული ენის ტექნოლოგიზების ცენტრში დღეს მოქმედი პროექტით ,,კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ წარმოებული კვლევების ფარგლებში შემუშავდა.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ AR/122/4-105/14 პროექტზე გაღებული საგრანტო მხარდაჭერით.

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Multiple Walsh Series and Sets of Uniqueness

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The aim of our research is to study uniqueness problems for multiple series on the Walsh system of functions $\{W_n\}_{n=0}^{\infty}$. We will write MWS for those series.

Let $\{f_n\}$ be a system of function defined on some set X. Recall that a set $A \subset X$ is said to be a *set of uniqueness* (or *$\mathscr{U}$-set*) for series $\sum_n a_n f_n(x)$, $a_n \in \mathbb{R}$ or $\mathbb{C}$, if the only series converging to zero on $X \setminus A$ is the trivial series.

Denote by $\mathscr{U}_{\mathrm{MWS,\,rect}}$ the class of $\mathscr{U}$-sets for rectangularly converging MWS and by $\mathscr{U}_{\mathrm{MWS},\,\lambda}$, where $\lambda \geq 1$, the one for λ-converging MWS. We notice that $\mathscr{U}_{\mathrm{MWS},\,\lambda} \subsetneqq \mathscr{U}_{\mathrm{MWS,\,rect}}$ whenever $\lambda \geq 1$ (see [1]).

Subclasses of $\mathscr{U}_{\mathrm{MWS,\,rect}}$ have constructed in numerous works (Skvortsov, Movsisyan, Lukomskii, Kholshchevnikova, Gogoladze, Zhereb'eva). For example, every countable union of hyperplanes is a $\mathscr{U}_{\mathrm{MWS,\,rect}}$-set. The widest known subclasses of $\mathscr{U}_{\mathrm{MWS,\,rect}}$ are contained in [2] and [3].

In some papers of the author (see, for instance, [4], [5]) subclasses of $\mathscr{U}_{\mathrm{MWS},\,\lambda}$ were constructed. In particular, every countable intersection of various "chessboard" sets $R_k \stackrel{\mathrm{def}}{=} \{\mathbf{t} \in \mathbb{G}^d : W_{2^k,\dots,2^k}(\mathbf{t}) = 1\}$ belongs to $\mathscr{U}_{\mathrm{MWS},\,\lambda}$, for each $\lambda \geq 1$. The mentioned results were obtained for MWS defined on the dyadic product groups $\mathbb{G}^d$. The problem whether $\emptyset \in \mathscr{U}_{\mathrm{MWS},\,\lambda}$ is still open if we consider MWS on the unit cube $[0,1]^d$.

We intend to discuss new results concerning U-sets for λ-converging MWS.

Acknowledgement. This research was supported by RFBR (grant no. 14-01-00417) and the program "Leading Scientific Schools" (grant no. NSh-3682.2014.1).

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Uniqueness for Rearranged Multiple Haar Series

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In 1870 Cantor proved the following theorem (see, for example, [1, vol. 1, Ch. 9]). *Let $A \subset [0, 2\pi]$ be some finite set; if a trigonometric series (TS) converges to zero everywhere on $[0, 2\pi] \setminus A$, then (TS) is the trivial series, i.e. all coefficients of (TS) are equal to zero.* Further investigations show that the statement of the Cantor theorem remains true if any countable set or even some uncountable set instead of finite set is considered.

Unlike trigonometric series, the Cantor type theorem for Haar ones holds only for everywhere convergence. Uniqueness for multiply Haar series everywhere converging over rectangles has proved in 1970s independently by Skvortsov, Ebralidze, and Movsisyan.

Uniqueness for multiple Haar series also holds if we consider λ-convergence, where $\lambda \geq 2$ (see [2]). However, uniqueness is violated for λ close to 1: for every $\lambda \in [1, \sqrt{2})$ there exists a non-trivial double Haar series (MHS) such that (MHS) λ-converges to zero everywhere on $[0, 1]^2$ (see [2]). This fact was quite unexpected. Notice that the constant $\sqrt{2}$ above is sharp (see [3]).

We intend to present results about uniqueness for rearranged multiple Haar series. In papers [2]–[4] the fact that the Haar system has the natural order played an important role.

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Convergence Rate of Stationary Distribution of Retrial Queueing Systems

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A significant part of the queueing theory is the theory of systems with repeated calls. The detailed overviews of the related references with retrial queues can be found in [1], [2]. These systems are widely used as mathematical models for the real systems in economics, transport, computer network designing as well as in modern mobile communication systems etc.

In this work, we deal with retrial queues of the type $M_Q/M/m/\infty$ and $M_Q/M/m/N$, $m = 1, 2$ in which intensity of primary call flow λ_j depends on the number j - of customers in the orbit. The intensities of repeated calls ν and the service process μ are supposed to be constant. Variable character of the input flow rate allows to consider the system under threshold strategy which realizes the following algorithm of the service process control: we set $\lambda_j = \lambda_1$ if $j = 0, 1, \dots, H$ and $\lambda_j = \lambda_2$ if $j = H + 1, \dots$. H is a threshold related to by an jump-like way. Formulas for stationary distribution and conditions of their existence for these systems were found in [3]. Changing intensity of input flow in models of this type allows us to resolve optimization problems for them.

In this work we evaluate convergence rates of stationary distribution $\pi_{ij}^{(N)}$ of systems $M_Q/M/m/N$ to relevant distribution π_{ij} of systems $M_Q/M/m/\infty$, $m = 1, 2$ with repeated calls.

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Clark–Ocone Representation of Nonsmooth Wiener Functionals

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We have developed some methods of obtaining the stochastic integral representation of nonsmooth (in the Malliavin sense) Wiener functionals. On the one hand, for receiving obvious integral expressions, we use the result of stochastic integral representation proved by us earlier, which demands smoothness only a conditional mathematical expectation of the considered functional, instead of the usual requirement of smoothness of the functional (as it was in the well-known Clark–Ocone formula). The second method is based on the notion of semimartingale local time and the well-known theorem of Trotter–Mayer which establishes the connection between a predictable square variation of a semimartingale and its local time. The offered methods allow remove integral representation for the indicator $I_{\{K_1 \le f(w_T) \le K_2\}}$ (which it is known that is not differentiable in Malliavin sense), for the functional of integral type $\int_0^T f(w_t) I_{\{K_1 \le g(w_t) \le K_2\}} dt$ (which it is proved that is also not differentiable in Malliavin sense) and others.

Theorem 1. Let f be a nondecreasing function. Then we have

$$I_{\{K_1 \le f(w_T) \le K_2\}} = \Phi\left[\frac{f^{-1}(K_2)}{\sqrt{T}}\right] - \Phi\left[\frac{f^{-1}(K_1)}{\sqrt{T}}\right]$$

$$-\int_0^T \left\{\varphi\left[\frac{f^{-1}(K_2) - w_t}{\sqrt{T-t}}\right] - \varphi\left[\frac{f^{-1}(K_1) - w_t}{\sqrt{T-t}}\right]\right\} dw_t,$$

where Φ is the standard normal distribution function and φ is its density.

Consider the Black-Scholes financial market model with $B_t \equiv 1$ and $S_t = \exp\{\sigma w_t + (\mu - \sigma^2/2)t\}$. Let $dZ_t = -(\mu/\sigma)Z_t dw_t$, $\widetilde{w}_t = w_t + \mu t/\sigma$, $d\widetilde{P} = Z_T dP$.

Theorem 2. For the Wiener functional $F = \int_0^T I_{\{K_1 \le S_t \le K_2\}} S_t^2 dt$ the following stochastic integral representation is valid

$$F = \frac{1}{\sigma^2} \int_{K_1}^{K_2} [\widetilde{E}(|S_T - x|) - |1 - x|] dx + \int_0^T \frac{1}{\sigma} S_t V_t d\widetilde{w}_t,$$

where

$$V_t = \int_{K_1}^{K_2} \left\{ 1 - 2\Phi \left[\frac{lnx - \sigma\widetilde{w}_t - \sigma^2(T/2 - t)}{\sigma\sqrt{T - t}} \right] - \operatorname{sgn}(S_t - x) \right\} dx.$$

Acknowledgement. Research partially supported by Shota Rustaveli National Scientific Grant No FR/308/5-104/12.

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On a Question of A. Hinrichs and A. Pietsch

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We discuss the problems around a question, posed by A. Hinrichs and A. Pietsch [1]: Suppose T is an operator acting between Banach spaces X and Y, and let $s \in (0,1)$. Is it true that if T^* is s-nuclear then T is s-nuclear too?

As is well known, for $s = 1$, a negative answer was obtained already by T. Figiel and W.B. Johnson in 1973. The following result (which is sharp in the scale of s-nuclear operators in the sense of Theorem 2 below) gives one of the possible positive answers in this direction. To formulate the theorem, we need a definition: Let $0 < q \le \infty$ and $1/s = 1/q + 1$. We say that X has the approximation property of order s, if for every $(x_n) \in l_q(X)$ (where $l_q(X)$ means $c_0(X)$ for $q = \infty$) and for every $\varepsilon > 0$ there exists a finite rank operator R in X such that $\sup_n ||Rx_n - x_n|| \le \varepsilon$.

Theorem 1. *If $s \in [2/3, 1]$ and T is a linear operator with s-nuclear adjoint from a Banach space X to a Banach space Y and if one of the spaces X^* or Y^{***} has the approximation property of order s, then the operator T is nuclear.*

Remark. In the case where $s = 2/3$, a famous theorem due to A. Grothendieck says that every Banach space has the corresponding approximation property and the result is trivial. The case where $s = 1$ (Grothendieck's AP) was firstly investigated in the paper [8] by Eve Oja and the author [2].

The examples in the following result show that the condition "X^* or Y^{***} has the approximation property of order s" is essential.

Theorem 2. *For each $s \in (2/3, 1]$ there exist a Banach space Z_s and a non-nuclear operator $T_s : Z_s^{**} \to Z_s$ so that Z_s^{**} has the metric approximation property, Z_s^{***} has the AP_r for every $r \in (0, s)$ and T_s^* is s-nuclear.*

Remark. The space Z_1^{***} is isomorphic to a space of type $Z_1^* \oplus E$, where E is an asymptotically Hilbertian space. This gives us one more example of an asymptotically Hilbertian space which fails the approximation property.

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The Estimation of Large Deviation for the Response Function

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A time-invariant casual continuous linear Volterra system with response function $H(\tau)$, $\tau \in \mathbb{R}$, is considered. It means that the real-valued function $H(\tau) = 0$ as $\tau < 0$, and the response of the system to an input process $X(t)$, $t \in \mathbb{R}$ has such form

$$Y(t) = \int_0^\infty H(\tau)X(t-\tau)d\tau. \tag{1}$$

One of the problems arising in the theory of such systems is to estimate or identify the function H by observations of the responses of the system. Here we use a method of correlograms for the estimation of the response function H.

Assume that $X = (X(t), t \in \mathbb{R})$ is a measurable real-valued stationary zero-mean Gaussian process with known spectral density. The reaction of the system on an input signal X is represented by (1).

The so-called cross-correlogram (or sample cross-correlogram function)

$$\hat{H}_T(\tau) = \frac{1}{T}\int_0^T Y(t+\tau)X(t)dt, \quad \tau > 0,$$

will be used as an estimator for H. Here T is the length of the averaging interval.

The inequality of large deviation probability for $\hat{H}_T(\tau) - H(\tau)$ in uniform norm is founded. The theory of Square-Gaussian Processes is used.

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The τSR-Analog of the Herbrand Method of Automatic Theorem Proving

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We defined τSR-analog of Herbrand universe, Herbrand τSR-base,Herbrand τSR-interpretation and the following theorems are proved.

Theorem 1. *If an interpretation I over some domain D satisfies a formula A of τSR-logic, then any one of the τSR-interpretations $I^\star$ corresponding to I also satisfies a formula A of τSR-logic.*

Theorem 2. *A formula A of τSR-logic is unsatisfiable if and only if A is false under all the τSR-interpretation.*

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The General Solution of the Homogeneous Riemann Problem in the Weighted Smirnov Classes

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Let $A(\xi) \equiv |A(\xi)| e^{i\alpha(\xi)}$, $B(\xi) \equiv |B(\xi)| e^{i\beta(\xi)}$ be complex-valued functions given on the curve Γ. We'll assume that they satisfy the following basic conditions:

i) $|A|^{\pm 1}, |B|^{\pm 1} \in L_\infty(\Gamma)$;

ii) $\alpha(\xi)$, $\beta(\xi)$ *are piecewise-continuous on* Γ *and let* $\left\{ \xi_k, \, k = \overline{1,r} \right\} \subset \Gamma$ *be discontinuity points of the function* $\theta(\xi) \equiv \beta(\xi) - \alpha(\xi)$.

For the curve Γ we require the following conditions be fulfilled:

iii) Γ *is any Lyapunov or Radon curve* (*i.e. it is a curve of bounded rotation without cusps*). *We'll assume that the direction along* Γ *is positive, i.e. while moving in this direction, the domain* D *remains in the left side. Let* $a \in \Gamma$ *be a start point* (*also an end point*) *of the curve* Γ. *We'll assume that* $\xi \in \Gamma$ *follows the point* $\tau \in \Gamma$, *i.e.* $\tau \prec \xi$, *if* ξ *follows* τ *while moving in positive direction along* $\Gamma \backslash a$, *where* $a \in \Gamma$ *is the junction of two points* $a^+ = a^-$, a^+ *is the start point and* a^- *is the end point of the curve* Γ.

Consider the following homogeneous Riemann problem in the weighted classes $E_{p,\rho}(D^+) \times_m E_{p,\rho}(D^-)$:

$$A(\xi) F^+(\xi) + B(\xi) F^-(\xi) = 0, \ a.e. \ \xi \in \Gamma. \tag{1}$$

The following theorem is true.

Theorem. *Let the conditions* i)–iii) *be fulfilled with respect to the complex-valued functions* $A(\xi)$, $B(\xi)$ *and the curve* Γ. *Assume that with respect to jumps* $\{h_k\}$ *and the weight function* $\rho(\xi)$ *the conditions*

$$\frac{h_k}{2\pi} < 1\,, \quad k = \overline{0,r},$$
$$\int_0^S \sigma^{pp_1}(s)\, \rho^{p_1}(z(s))\, ds < +\infty,$$
$$\int_0^S \sigma^{-qp_2}(s)\, \rho^{-\frac{q}{p}p_2}(z(s))\, ds < +\infty$$

are fulfilled. Then the general solution of the homogenous problem (1) *has a representation*

$$F(z) \equiv Z(z)\, P_m(z)$$

in classes $E_{p,\rho}(D^+) \times_m E_{p,\rho}(D^-)$, *where* $Z(z)$ *is a canonical solution, and* $P_m(z)$ *is an arbitrary polynomial of order* $k \leq m$.

Bohr Radii of Elliptic Regions

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In this talk we use Faber series to define the Bohr radius for a simply connected planar domain bounded by an analytic Jordan curve. We estimate the value of the Bohr radius for elliptic domains of small eccentricity and show that these domains do not exhibit Bohr phenomenon when the eccentricity is large. We obtain the classical Bohr radius as the eccentricity tends to 0.

Mathematical Modeling of the Dynamics of a Blocky Medium Taking into account the Nonlinear Deformation of Interlayers

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Many natural materials, such as rocks, are characterized by inhomogeneous blocky-hierarchical structure. The blocky structure is observed at different levels of scale: from the size of crystal grains to large blocks of a rock body, separated by faults. The blocks are connected to each other by means of interlayers of a rock with substantially more compliant mechanical properties. The blocky structure is also found in many artificial materials, especially in a masonry. The presence of such a structure has a significant influence on the dynamic processes, occurring under the action of external perturbations. In this case, classical models of the mechanics of deformable media are not applicable for the description of wave motions.

In this report parallel computational algorithms, based on mathematical models of the inhomogeneous elasticity theory taking into account linear and nonlinear behaviour of interlayers and the Cosserat elasticity theory [1], are applied to the analysis of propagation of elastic waves in materials with layered and blocky microstructure. These algorithms are realized as parallel program systems for multiprocessor computers of the cluster type using the MPI library. Monotone grid-characteristic schemes with a balanced number of time steps in elastic layers or blocks and in viscoelastic interlayers are used.

The next rheological schemes of mathematical models of the interaction of elastic blocks through compliant interlayers are considered: the simplest scheme of an elastic interlayer, the scheme taking into account viscous deformations, and the scheme of nonlinear contact interaction taking into account different resistance of the interlayer material to tension and compression. Governing equations of the models are obtained using the generalized rheological method [1]. The algorithms of numerical realization of these equations, which play the role of internal boundary conditions for the equations of the linear elasticity theory, recorded in each of the blocks, are developed.

In 1D problem about the propagation of short-time impulses of pressure through a layered medium with thin viscoelastic interlayers the dependence of frequency of the pendulum wave on the compliance of a material of interlayers was analyzed. 2D computations for the Lamb problem about a localized impulse action on a homogeneous elastic half-plane and on a half-plane, filled with a micro-inhomogeneous medium with rotational degrees of freedom, were carried out. The main qualitative distinction is that specific waves of pendulum type appear behind the front of transverse wave in the Cosserat medium.

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Analysis of Resonant Excitation of a Blocky Media Based on Discrete Models

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To analyze a wave motion in an inhomogeneous deformable medium, discrete and continuous models are proposed. In the simplest discrete model of multilayered media with compliant interlayers, a linear chain of particles (material points) is considered. Particles are successively connected among themselves by elastic springs. In monoatomic chain masses of all particles and springs stiffnesses are equal. Such approximation is possible in the case of thin interlayers, so their masses can be neglected. In diatomic chain two different masses alternate, so one may consider them as masses of layers and interlayers, respectively. Wave processes (in particular, resonances caused by external periodic perturbations) were investigated using linear and non-linear approaches. It was shown that, allowing for viscosity forces in the chain, resonance amplitudes become finite; resonant frequency spectrum rearranges if defects of constraints appear.

The simplest continuous model of a deformable medium with microstructure is formulated in terms of one-dimensional wave equation. This equation may be derived from the discrete chain model, when the number of particles tending to infinity, suggesting that density and elastic wave velocity are constant. More complicated model that accounts rotational degrees of freedom of the chain leads to equations of the Cosserat continuum. Resonance properties of the Cosserat continuum were studied in [1, 2] within the framework of the spatial stress–strain state model. It has been found that there exists a resonance frequency associated with rotational motion of particles, which is independent on the size of a specimen and on the type of boundary conditions.

In this report the resonance processes (in the context of discrete models with different complexity level) in inhomogeneous materials with blocky and layered microstructure are analyzed. Eigenfrequencies of longitudinal particle motion in the linear monoatomic chain, that imitates a blocky medium, are calculated with different boundary conditions. To analyze a behaviour of the chain in resonant frequencies neighbourhood, spectral portraits were built. It was shown that in the passage to the limit from the model of monoatomic chain with elastic constraints that accounts particles rotation resistance to the Cosserat continuum model, a specific resonant frequency exudes. This frequency does not depend on the chain length.

This work was supported by the Russian Foundation for Basic Research (grant no. 14-01-00130).

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Commutative C^*-Algebra of Toeplitz Operators on the Superball

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In this talk we study Toeplitz operators acting on the super Bergman space on the superball. We consider five different types of commutative super subgroups of the biholomorphisms of the superball or the super Siegel domain and we prove that the C^*-algebras generated by Toeplitz operators whose symbols are invariant under the action of these groups are commutative.

The talk is based on joint work with R. Quiroga-Barranco.

Quadrature Formulas of High Accuracy for Cauchy Type Singular Integrals and Some of Their Applications

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Quadrature formulas with Cauchy type singularity whose accuracy are higher than that of interpolational type formulas are considered.

Questions of their application to various problems of harmonic functions and theory of elasticity are studied.

Stability and Accuracy of RBF Direct Method for Solving a Dynamic Investment Model

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In this paper we consider a Dynamic investment model. In the model, firm's objective is maximizaing discounted sum of profits over an interval of time. The model assumes that firm's capital in time t increases with investment and decreases with depreciation rate that can be expressed by means of differential equation.

We propose a direct method for solving the problem based on Radial Basis Functions(RBFs). The authors describe operational matrices of RBFs and use them to reduce the variational problem to a static optimization problem which can be solved via some optimization techniques. Next, we describe some economic interpretation of the solution. Finally, the accuracy and stability of the Multiquadric (MQ), Inverse Multiquadric (IMQ) RBFs are illustrated by conducting some numerical experiments.

Keywords: RBFs, accuracy, stability, variational problems, Dynamic Investment problem.

2010 Mathematics Subject Classification. 49Mxx; Secondary 37Mxx.

მათემატიკური ფიზიკის განტოლებების ავტომოდელური ამონახსნების მოძებნის ერთი ალგორითმის შესახებ

ნუგზარ სხირტლაძე

კავკასიის უნივერსიტეტი, თბილისი, საქართველო

ფიზიკურ სიდიდეთა განზომილების თეორიის საფუძველზე მათემატიკური ფიზიკის განტოლებების ავტომოდელური ამონახსნების აგება საკვლევი ობიექტის თვისობრივი თავისებურებების დადგენის შესაძლებლობის გარდა, მნიშვნელოვანია როგორც შესაბამისი მათემატიკური მოდელის რიცხვითი ამოხსნის ალგორითმის ტესტირების საშუალება.

მოხსენება ეძღვნება მათემატიკური ფიზიკის ევოლუციური განტოლებების ავტომოდელური ამონახსნების აგების ერთ ალგორითმს, რომლის მიხედვითაც თანმიმდევრობით არის გაწერილი თითოეული ნაბიჯის არსი და პროცედურა.

Solution of the Nonclassical Problems of Stationary Thermoelastic Oscillation

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We consider the stationary oscillation case of the theory of linear thermoelasticity with microtemperatures of materials. The boundary value problem of oscillation is investigated when the normal components of displacement and the microtemperature vectors and tangent components of rotation vectors are given on the spherical surfaces. Uniqueness theorems are proved. We construct an explicit solutions in the form of absolutely and uniformly convergent series.

Some Properties of the Fundamental Solution to the Generalized Maxwell's Body Movement Equation

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For the generalized Maxwell's body the constitutive relationship has a form:

$$\sigma(t) = E\Big[1 - \frac{1}{\eta^\beta}\mathscr{E}^*_{\beta-1}\left(-\frac{1}{\eta^\beta}\right)\Big]e(t),$$

where $\mathscr{E}^*_{\beta-1}\big(-\frac{1}{\eta^\beta}\big)$ is the operator the kernel of which is the fractional-exponential function.

The equation of movement in displacements has form

$$P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)u(t,x) = \frac{1}{\rho}f(t,x),$$

where $u(t,x)$ is the material element movement, $f(t,x)$ external loading, operator $P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$ has a form $P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + \Phi(t) * \frac{\partial^2}{\partial x^2}$ $\Phi(t) = \mathscr{E}_{\beta-1}\big(-\frac{1}{\eta^\beta}\big)$.

The operation of convolution concerning a variable t, is denoted by "$*$".

Let us denote by $\Upsilon(t,x)$ the fundamental solution of the operator $P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$.

The following theorems are true.

Theorem 1. *If there exists a small value, $\varepsilon > 0$ such as $\frac{3}{4} + \varepsilon \le \beta < 1$, then*

$$\lim_{t\to|x|+0}\Upsilon(t,x) = \begin{cases} 0, & x \neq 0, \\ \frac{1}{2}, & x = 0. \end{cases}$$

Theorem 2. *Let $\exists\delta > 0$ such that $\beta \ge 1-\delta$, then the fundamental solution $\Upsilon(t,x)$ of the operator $P\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$ is the smooth function for $t > |x|$, $x \neq 0$, and*

$$\lim_{x\to\pm0}\frac{\partial}{\partial x}\Upsilon(t,x) = \mp\frac{1}{2\eta^\beta\Gamma(\beta)}t^{\beta-1}, \quad t > 0.$$

The Plane Problem of the Theory of Elastic Mixture for a Polygonal Domain with a Rectilinear Cut

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The plane problem of statics of the linear theory of elastic mixture for a polygonal domain with a rectilinear cut is considered under the condition that uniformly distributed stretching forces or normal displacements are prescribed on the external boundary of the domain, while the cut edges are free from external forces.

To solve the problem we use the generalized formulas due to Kolosov–Muskhelishvili and the method described in [1].

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მათემატიკის სწავლების თანამედროვე მეთოდები STEM სპეციალობებზე

კახაბერ თავზარაშვილი, ქეთევან კუთხაშვილი

ინფორმატიკის, ინჟინერიისა და მათემატიკის სკოლა, საქართველოს უნივერსიტეტი
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ქვეყნის ტექნოლოგიური და ეკონომიკური განვითარებისთვის მნიშვნელოვანია მათემატიკური განათლების ხარისხის გაუმჯობესება ისეთ სპეციალობებზე, როგორიცაა მეცნიერება, ტექნოლოგია, ინჟინერია და მათემატიკა (STEM).

თანამედროვე მოთხოვნების შესაბამისი კომპეტენციების მქონე სპეციალისტის მომზადება ზოგადად ინჟინერიაში და კერძოდ კომპიუტერულ მეცნიერებებში გულისხმობს სწავლების ახალი ტექნოლოგიების დანერგვას. საინჟინრო და კომპიუტერულ მეცნიერებებში მათემატიკური საგნები ქმნიან იმ ფუნდამენტს, რაზეც შემდეგ აგებულია სპეციალიზაციით

გათვალისწინებული მთელი რიგი დისციპლინები. საინჟინრო დარგებში მაღალკვალიფიციური და კონკურენტუნარიანი სპეციალისტის მომზადება მოითხოვს მათემატიკურ განათლებას. აქედან გამომდინარე, აუცილებელია მათემატიკური საგნების შინაარსისა და სწავლების მეთოდების გაუმჯობესება თანამედროვე მოთხოვნების შესაბამისად. მათემატიკური განათლების გაუმჯობესება გულისხმობს მათემატიკის საგნების კონკრეტულ დარგზე მორგებულ სწავლებას და თანამედროვე საგანმანათლებლო ტექნოლოგიებისა და ელექტრონული სწავლების მეთოდების დანერგვას სწავლების პროცესში.

საქართველოს უნივერსიტეტი ჩართულია TEMPUS-ის პროექტში (MATHGEAR), რომლის ძირითადი მიზანია STEM სპეციალობების სტუდენტებისათვის მათემატიკური საგნების სწავლების ახალი მეთოდოლოგიების დანერგვა საქართველოს რიგ უმაღლეს სასწავლებლებში. პროექტში დაგეგმილია სწავლების პროცესში სხვადასხვა კომპიუტერული პროგრამების (Math-Bridge, MatLab, GeoGebra და სხვა) გამოყენება.

მოხსენებაში წარმოდგენილი იქნება STEM სპეციალობებზე მათემატიკურ საგნებში თანამედროვე კომპიუტერული ტექნოლოგიების გამოყენების შესაძლებლობები, მათემატიკის სწავლების პროგრამის ძირითადი სტრუქტურა და პროექტის ფარგლებში მიღწეული შედეგები.

On the Partial Sums of Vilenkin-Fourier Series on the Martingale Hardy Spaces

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In [2] (see also [3]) it was proved that the maximal operator of partial sums $S^* := \sup_{n\in\mathbb{N}} |S_n f|$, with respect to Vilenkin systems is not bounded from the maringale Hardy space H_p to the space L_p, when $0 < p \leq 1$.

On the other hand, it is well known (for details see e.g. [1], [4] and [5]) that the restricted maximal operator $S^{\#} f = \sup_{n\in\mathbb{N}} |S_{2^n} f|$ with respect to Walsh system (Walsh system is an example of Vilenkin systems) is bounded from the martingale Hardy space H_p to the space L_p for $p > 0$.

This lecture is devoted to review the boundedness of subsequences of partial sums with respect to Vilenkin systems on the Hardy spaces, when $0 < p \leq 1$. In particular, we characterise a maximal subspace Q of natural numbers $\mathbb{N}$, such that the restricted maximal operator $S^{\#,*} f = \sup_{n_k\in Q\subset\mathbb{N}} |S_{n_k} f|$ is bounded from the martingale Hardy spaces H_p to the space L_p for $0 < p \leq 1$.

In the talk will be also review boundedness (or even the ratio of divergence of the norm) of the subsequence of partial sums of the Vilenkin-Fourier series of H_p martingales in terms of measurable properties of a Dirichlet kernel corresponding to partial summing. As a consequence we get the corollaries about some convergence and divergence of some specific subsequences of partial sums. For $p = 1$ the simple numerical criterion for the index of partial sum in terms of its dyadic expansion is given which governs the convergence (or the ratio of divergence). Moreover we find a relationship of the ratio of convergence of the partial sum of the Vilenkin series with the modulus of continuity of a martingale. The conditions are in a sense necessary and sufficient.

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ერთეულოვან წრეში ბლიაშკეს ტიპის ნამრავლის სასაზღვრო თვისებების შესახებ

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ცნობილია, რომ თუ $\lambda+1$ ნატურალური რიცხვია, $0 < |a_n| \le |a_{n+1}| < 1$, $\lim\limits_{n\to\infty} |a_n| = 1$, $|z| < 1$, მაშინ ბლიაშკეს ტიპის ნამრავლი

$$B(z,\lambda,(a_n)) = z^\lambda \prod_{n=1}^{\infty} \Big(1 - \frac{1-|a_n|^2}{a-\overline{a}_n z}\Big) \exp\left(\sum_{k=1}^{n} \frac{1}{k}\Big(\frac{1-|a_n|^2}{a-\overline{a}_n z}\Big)\right)$$

ანალიზური ფუნქციაა ერთეულოვან წრეში, ნულებით

$$\underbrace{0,0,\dots,0}_{\lambda},a_1,a_2,\dots,a_n,\dots\,.$$

თეორემა. თუ $B(z,\lambda,(a_n))$ ბლიაშკეს ტიპის ნამრავლია ერთეულოვან წრეში და (a_{n_k}) ქვემიმდევრობა აკმაყოფილებს პირობებს

$$\lim_{k\to\infty} a_{n_k}=e^{i\theta},\quad \lim_{k\to\infty}\frac{1-|a_{n_k}|}{|e^{i\theta}-a_{n_k}|}\geq\frac{1}{2}\,,\quad \lim_{k\to\infty}\frac{|a_{n_k}-a_{n_{k+1}}|}{|e^{i\theta}-a_{n_k}|}=0,$$

მაშინ

$$\lim_{r\to 1} B(re^{i\theta},\lambda,(a_n))=0.$$

On the Solvability of General Boundary Value Problems for Nonlinear Difference Systems

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We consider the problem on the solvability of the system of nonlinear discrete equations

$$\Delta y(l-1)=g(l,y(l),y(l-1)) \ \text{ for } \ l\in N_{m_0} \tag{1}$$

under the boundary value condition

$$\zeta(y)=0, \tag{2}$$

where $m_0\geq 2$ is a fixed natural number, $N_{m_0}=\{1,\dots,m_0\}$, the vector-function g belongs to discrete Carathéodory class $Car(N_{m_0}\times R^n,R^n)$, and $\zeta : E(\widetilde{N}_{m_0},R^n)\to R^n$, $\widetilde{N}_{m_0}=\{0,1,\dots,m_0\}$, is a continuous, nonlinear in general, vector-functional.

There are given the sufficient, among them effective, conditions for solvability and unique solvability of the general nonlinear discrete boundary value problem (1), (2) in the case when the right part is quasi-linear with respect to the phase variables.

Analogous problems are investigated in [1]–[4] (see also the references therein) for the general nonlinear boundary value problems for ordinary differential and functional-differential systems.

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About Solving of Large Scale Electromagnetic Problem

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The given report describes some ways for acceleration of solution of large scale and complex electromagnetic (EM) problems by Method of Moments (MoM), which is generally used technique and finally lead to the solution of linear system of equations with complex coefficients [1]. Solution of such system when number of unknowns is very large (100,000 and more) requires big computational time and large amount of computer memory. For reducing required memory and speed up calculation time we use ACA (Adaptive Cross Approximation) algorithm [2]. This method divides a matrix into blocks and the most part of them are decomposed via ACA, requiring significantly less memory. Compressible matrices and their low rank approximations fundamentally mean that most of the blocked MoM system matrix equation elements, before compression, contain very little physical information. After such decomposition system may be solved iteratively or directly. As an iterative solver BICGSTAB (BiConjugate Gradient Stabilized) method is used. In order to improve convergence of iterative process SPAI (Sparse Approximation Inverse) preconditioner is applied [3]. In some cases many right-hand sides of system are obtained and efficiency of iterative solver can fall. In such cases direct methods may be more effective. After ACA decomposition LU-factorization and LU-solve may be applied [4].

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არასაკუთრივი ინტეგრალის განშლადობის ზოგიერთი კრიტერიუმის შესახებ

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ნაშრომში მოყვანილია არასაკუთრივი ინტეგრალის განშლადობის ზოგიერთი კრიტერიუმი. ეს კრიტერიუმები, საშუალებას იძლევა ვაჩვენოთ საკმაოდ რთული ინტეგრალების განშლადობა. აღნიშნული კრიტერიუმები სავარჯიშოების სახით არის წარმოდგენილი დავიდოვის, კოროვკინისა და ნიკოლსკის მათემატიკური ანალიზის ამოცანათა კრებულში.

About Chippot Method of Solution of Different Dimensional Kirchhoff Static Equations

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Kirchhoff equations of one and two dimensions have been considered. Chippot algorithm has been used. Computer calculations have been made.

On the Absolute Convergence of the Fourier Series of an Indefinite Double Integral

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It is well known that the Fourier series $S[\lambda]$ of every absolutely continuous and 2π periodic function $\lambda(x)$ is uniformly converging on $[0,2\pi]$ (see [1, Ch. I, Section 39]). At the same time, we can choose a absolutely continuous function λ such that the series $S[\lambda]$ has no point of absolute convergence (see [1, Ch. IX, Section 3]).

If, however, the derivative $\lambda'(x)$ belongs to the class $L^2[0,2\pi]$, then $S[\lambda]$ is a uniformly and absolutely converging series on $[0,2\pi]$ (see [1, Ch. I, Section 26]).

Let a 2π periodic function f with respect to each variable is summable an $[0,2\pi]^2$ and

$$f \sim \frac{1}{4}\,a_{00} + \frac{1}{2}\sum_{m=1}^{\infty}(a_{m0}\cos mx + d_{m0}\sin mx) + \frac{1}{2}\sum_{n=1}^{\infty}(a_{0n}\cos ny + c_{0n}\sin ny)$$
$$+ \sum_{m=1,\,n=1}^{\infty}\Big(a_{mn}\cos mx\cos ny + b_{mn}\sin mx\sin ny + c_{mn}\cos mx\sin ny + d_{mn}\sin mx\cos ny\Big).$$

For a double Fourier series we have

Theorem 1. *If the function f belonging to the class $L^2[0,2\pi]^2$ and the indefinite double integral*

$$F_f(x,y) = \int_0^x\int_0^y f(t,\tau)\,dt\,d\tau$$

is 2π periodic in each variable, then the equality

$$\int_0^x \int_0^y f(t,\tau)\,dt\,d\tau$$

$$= \sum_{m,n=1}^{\infty} \frac{1}{mn} \Big[b_{mn}\cos mx\cos ny + a_{mn}\sin mx\sin y - d_{mn}\cos mx\sin y - c_{mn}\sin mx\cos ny\Big]$$

is fulfilled uniformly on $[0,2\pi]^2$ and

$$\sum_{m,n=1}^{\infty} \frac{1}{mn} \big(|a_{mn}| + |b_{mn}| + |c_{mn}| + |d_{mn}|\big) < +\infty.$$

Theorem 2. *If a 2π periodic function f with respect to each variable belongs to the class $L^2[0,2\pi]^2$, then*

$$\sum_{m=1}^{\infty} \frac{1}{m} \big(|a_{m0}| + |d_{m0}|\big) < +\infty, \quad \sum_{n=1}^{\infty} \frac{1}{n} \big(|a_{0n}| + |c_{0n}|\big) < +\infty.$$

Lemma 3 (L^2 variant of Fubini's theorem). *If a function $f(x,y)$ belongs to the class $L^2[0,2\pi]^2$, then the functions*

$$\varphi(t) = \frac{1}{2\pi} \int_0^{2\pi} f(t,y)\,dt, \quad \psi(\tau) = \frac{1}{2\pi} \int_0^{2\pi} f(x,\tau)\,dx$$

belong to the class $L^2[0,2\pi]$.

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On the Unsteady Motion of a Viscous Hydromagnetic Fluid Contained between Rotating Coaxial Cylinders of Finite Length

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The problem of unsteady rotational motion of electrically conducting viscous incompressible fluid, contained within two axially concentric cylinders of finite length in the presence of an axial symmetric magnetic field of constant strength, has been solved exactly using finite Hankel transform in combination with a technique presented in this paper. This paper presents a complete of the problem under consideration, which has been of interest for many years; moreover the Pneuman-Lykoudis solution in Magnetohydrodynamics and Childyal solution in hydrodynamics appears as a special case of this study. The analysis shows that the disturbance in the fluid disappears by increasing the magnetic field.

The Riesz Potential Operator in Generalized Grand Lebesgue Spaces

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We denote $L_a^{p),\theta}(\Omega)$ the generalized grand Lebesgue space (see [1, 2]) on set $\Omega \subseteq \mathbb{R}^n$:

$$L_a^{p),\theta}(\Omega) := \left\{ f : \sup_{0<\varepsilon<p-1} \left(\varepsilon^\theta \int_\Omega |f(x)|^{p-\varepsilon} [a(x)]^\varepsilon \, dx \right)^{\frac{1}{p-\varepsilon}} < \infty \right\},$$

where $p > 1$, $\theta > 0$ and a – some weighting function.

Theorem. *Let* $0 < \alpha < n$, $1 < p < \frac{n}{\alpha}$, $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$, $\theta > 0$ *and* a *– weight from* $L^p(\mathbb{R}^n)$. *The Riesz potential operator*

$$I^\alpha f = \int_{\mathbb{R}^n} \frac{\varphi(t)}{|x-t|^{n-\alpha}} \, dt, \quad 0 < \alpha < n,$$

is bounded from $L_a^{p),\theta}(\mathbb{R}^n)$ *to* $L_{a^{\frac{p}{q}}}^{q),\frac{q}{p}\theta}(\mathbb{R}^n)$ *if and only if exist number* $\delta \in (0, \frac{p}{q'})$ *such that*

$$a^\delta \in A_{\frac{p-\delta}{p}\left(1+\frac{q}{p'}\right)}.$$

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Corteges of Objects

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In [1] introduces the concept of full, simple and non-simple ordered sequences of objects of an arbitrary nature. They are called the corteges of objects. The purpose of the introduction of these and other concept it was their application in the synthesis and recognition of speech. We describe the overall diagram of their application.

Assume that *O, L, R* are a tree finite sets of objects. It is built new set of objects $T_O = L \times O \times R$ the set of contextual objects, and P_{T_O} is its space of the properties. Let $P', P'' \subseteq P_{T_O}$ are two non-degenerate sets of properties and $\langle P' \rangle$ and $\langle P'' \rangle$ their full corteges, correspondingly. They determine two full corteges of the objects $\langle T_O \rangle_{\langle P' \rangle}$ and $\langle T_O \rangle_{\langle P'' \rangle}$. They are the elements of the symmetric group of permutations - $S(T_O)$ [1].

To each symbol $u \in O$ of alphabet of internal formal grammar *G(O)* of synthesizer, corresponds the contextual unit $cu \in T_O$, depending on the position of the symbol in the chain of internal language *L(G)*, and uniquely identifies it (symbol-unit). By the internal grammar of synthesizer the contextual unit $cu \in T_O$ is defined as the object of structure *CUNIT* with three fields:

(1) $cu = \alpha(cu) \times \beta(cu) \times \gamma(cu), \alpha(cu) \in L, \beta(cu) \in O, \gamma(cu) \in R, cu \in T_O$;

(2) $indcu = \varphi_k(indlw, indu), indu, indlw \in N$;

(3) $indclw = max(indlw, 2 \cdot k + 1)$.

This prompts, that during the construction the cortege of text properties $\langle P' \rangle \in S(P')$ of the unit-symbol, we must consider five functions.

1. $p_1' = \frac{pos[\beta(cu), A^R{}_{HX}]}{|A^R{}_{HX} - 1|} \leq 1$;
2. $p_2' = \frac{pos[\alpha(cu), A^R{}_{HX}]}{|A^R{}_{HX} - 1|} \leq 1$;
3. $p_3' = \frac{pos[\gamma(cu), A^R{}_{HX}]}{|A^R{}_{HX} - 1|} \leq 1$;
4. $p_4' = \frac{\vartheta_k(w,\varphi) - \varphi_k(indlw, indu) + 1}{\vartheta_k(w,\varphi)} \leq 1$;
5. $p_5' = \frac{\vartheta_k(w,\varphi)}{2 \cdot k + 1} \leq 1$;

The cortege of voice properties $\langle P'' \rangle \in S(P'')$, for the same symbol is:

1. $p_1'' = h_{\langle P'' \rangle}[\beta(cu)]$;
2. $p_2'' = h_{\langle P'' \rangle}[\alpha(cu)]$;
3. $p_3'' = h_{\langle P'' \rangle}[\gamma(cu)]$;

Two corteges of the objects are built

$\langle T_O \rangle_{\langle P' \rangle} \longleftrightarrow O \longleftrightarrow \langle T_O \rangle_{\langle P'' \rangle}$

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Commutative Algebras of Toeplitz Operators on the Unit Ball

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Let $\mathbb{B}^n$ be the unit ball in $\mathbb{C}^n$. Denote by $\mathscr{A}^2_\lambda(\mathbb{B}^n)$, $\lambda \in (-1, \infty)$, the standard weighted Bergman space, which is the closed subspace of $L^2_\lambda(\mathbb{B}^n)$ consisting of analytic functions. The Toeplitz operator T_a with symbol $a \in L_\infty(\mathbb{B}^n)$ and acting on $\mathscr{A}^2_\lambda(\mathbb{B}^n)$ is defined as the compression of a multiplication operator on $L^2_\lambda(\mathbb{B}^n)$ onto the Bergman space, i.e., $T_a f = B_\lambda(af)$, where B_λ is the Bergman (orthogonal) projection of $L^2_\lambda(\mathbb{B}^n)$ onto $\mathscr{A}^2_\lambda(\mathbb{B}^n)$.

Note that for a generic subclass $S \subset L_\infty(\mathbb{B}^n)$ of symbols the algebra $\mathscr{T}(S)$ generated by Toeplitz operators T_a with $a \in S$ is non-commutative and practically nothing can be said on its structure. However, if $S \subset L_\infty(\mathbb{B}^n)$ has a more specific structure (e.g.

induced by the geometry of $\mathbb{B}^n$, invariance under a certain group action, or with a specific smoothness properties) the study of operator algebras $\mathscr{T}(S)$ is quite important and has attracted lots of interest during the last decades.

It was observed recently that there exist many non-trivial algebras $\mathscr{T}(S)$ (both C^* and Banach) that are commutative on each standard weighted Bergman space. We present the description, classification, and the structural analysis of these commutative algebras. In particular, we characterize the majority of the essential properties of the corresponding Toeplitz operators, such as compactness, boundedness, spectral properties, invariant subspaces, etc.

On the Number of Representations of Positive Integers by the Gaussian Binary Quadratic Forms

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The modular properties of generalized theta-functions with characteristics and spherical polynomials are used to build cusp forms corresponding to the binary quadratic forms. It gives the opportunity of obtaining formulas for the number of representations of positive integers by all binary quadratic forms with the discriminants -80, -128, and -140.

Mathematical Modeling of Hydraulic Fractures: Particle Velocity Based Simulation

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The notion of hydraulic fracture refers to a hydraulically induced crack propagating in a brittle material. It can be observed in many natural phenomena, but recently it has been associated mostly with the method of hydrocarbon reservoirs stimulation. Without any doubt, hydrofracturing has revolutionized the exploitation of shale oil and gas and has become a key technology allowing to exploit the non-conventional reservoirs.

Mathematical modelling of this multiphysics process is a challenging task. It goes far beyond the classical theory of fracture mechanics and should account for various mechanisms of interaction between the fracturing fluid and the surrounding rock. The main computational problems stem from: (a) strong non-linearities being a result of interaction between the solid and fluid phases, (b) singularities of the physical fields and corresponding degeneration of the governing equations, (c) moving boundaries, (d) multiscaling and others. The first mathematical models were proposed in 1940s and 1950s and although immense progress has been made since then, there is still a demand for further improvements in efficiency and credibility of computations.

Recent advances in the area underline the importance of the multiscale character of the problem. In particular, it has been proved that the global behaviour of a fluid driven fracture depends critically on the features of local solution in the near-tip region. Thus, complying with correct asymptotic regime of the solution, and consequently its numerical implementation, becomes vital for accurate end efficient computations. Especially, the problem of fracture front tracing has been recognized prominent.

In the repot we discuss we discuss the concept of numerical simulation of hydraulic fractures employing the particle velocity. Special attention is paid to the computation of the crack propagation speed from the local relation based on the Stefan condition (speed equation). The advantages of such approach are itemized with relation to the main computational difficulties. A universal numerical scheme containing the latest discoveries in the area is presented. The analysis of algorithm performance is shown on the examples of classical 1D PKN and KGD models under various propagation regimes.

Convergence of Bi-shift Localized Szász-Mirakjan Operators

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Let $\{\delta_n\}_{n=1}^{\infty}$ and $\{\delta'_n\}_{n=1}^{\infty}$ be two sequences of positive numbers, and

$$C_{n,x} = \{k : k \in N \cup \{0\} \text{ and } n(x - \delta'_n) \leq k \leq n(x + \delta_n)\}.$$

For any continuous function $f : [0, \infty) \to \mathbb{R}$, we define a new localized Szász-Mirakjan operator as follows:

$$S_{n,\delta_n,\delta'_n}(f, x) = e^{-nx} \sum_{k \in C_{n,x}} \frac{(nx)^k}{k!} f\left(\frac{k}{n}\right), \ x \geq 0.$$

We call this bi-shift localized Szász-Mirakjan operators. Certain new convergence theorems are obtained for such operators when the limits both $\lim_{n\to\infty} \delta_n\sqrt{n}$ and $\lim_{n\to\infty} \delta'_n\sqrt{n}$ exist. This is a joint work with Tingfan Xie.

Optimal Control of a Beam with Time-Delayed in Control Function

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In this paper, optimal time-delayed control of a damped smart beam is studied. Optimal control problem is defined with the performance index including a weighted quadratic functional of the displacement and velocity which is to be minimized at a given terminal time and a penalty term defined as the control voltage used in the control duration. Numerical results are presented to show the effectiveness and applicability of the piezoelectric control.

Numerical Solution of Some Boundary Problems Using Computer Modeling of Diffusion Processes

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Some questions connected with numerical solution of some boundary problems are studied.

Namely, connection of the mentioned problems with certain diffusion processes are established. On the basis of computer modeling of these processes, method of approximate solution of boundary problems is established.

Effectiveness of the used method is shown both for interior and exterior plane and spatial problems.

Concrete examples are given for domains with various configuration.

Application of Fourier Boundary Element Method to Solution of Some Problems Elasticity

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The research in recent several decades has established the boundary element methods (BEM) as a powerful tool in computational mechanics. One of the remaining drawbacks is that the BEM as previously used is based on an explicit knowledge of fundamental solutions. In many engineering problems we do not know these fundamental solutions. The overcome this drawback, an alternative BEM is presented here the method developed by means of the spatial Fourier transform generalizes the boundary element method to the so-called Fourier BEM [1]. Recent approach is available for all cases as long as the differential operator is linear and has constant coefficients and possible for all variants of the BEM. The basis of Fourier BEM are two well known theorems of the Fourier transformation: the theorem of Parseval and the convolution theorem. Parseval's theorem states the equivalence of energy or work terms in the original space and in the Fourier space, and the convolution theorem links a convolution in the original space to a simple multiplication in the transformed space. The idea is to avoid the inverse Fourier transform of the fundamental solution and to work directly with the Fourier transformed fundamental solution. The elements and shape functions also can be transformed to the Fourier domain. In this work, the method is presented and then applied to elasticity problem for demonstrate the equivalence between traditional BEM and Fourier BEM.

References

[1] F. M. E. Duddeck, *Fourier BEM. Generalization of Boundary Element Methods by Fourier Transform.* Lecture Notes in Applied Mechanics, 5. Springer-Verlag, Berlin, 2002.

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