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ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი
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საქართველოს უნივერსიტეტი

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პროფესორი გვანჯი მანია (1918 - 1985)

გვანჯი მიხეილის ძე მანია 1918 წლის 29 მაისს წალენჯიხის რაიონის სოფელ ეწერში დაიბადა. მამა, მიხეილ მურზაყანის ძე მანია რუსული ენის პედაგოგი გახლდათ, ხოლო მისი დედა - ფედოსი გეთია სამღვდელო ოჯახიდან იყო. აღსანიშნავია, რომ ბატონი გვანჯის ბაბუა დედის მხრიდან და მისი ორი ბიძა სოფელ ჯვარის წმინდა გიორგის სახელობის ეკლესიას ემსახურებოდნენ.

პირველდაწყებითი განათლება გვანჯი მანიამ ქალაქ ზუგდიდში მიიღო. 1932 წელს იგი ზუგდიდის პედაგოგიურ ტექნიკუმში შევიდა, რომელიც 1935 წელს დაამთავრა. იმავე წელს მან ჩააბარა თბილისის სახელმწიფო უნივერსიტეტში ფიზიკა-მათემატიკის ფაკულტეტზე, რომლის დამთავრების შემდეგ 1940 წლიდან 1945 წლამდე ზუგდიდის სამასწავლებლო ინსტიტუტში ასისტენტად მუშაობდა. პარალელურად 1943 წლიდან 1946 წლამდე ის თბილისის რკინიგზის ტრანსპორტის ინჟინერთა ინსტიტუტში ასისტენტად მუშაობს. 1945-1946 წლებში ის იყო საქართველოს განათლების სამინისტროს უმაღლესი სკოლის ინსპექტორი. 1945 წელს მას აჯილდოებენ მედლით: “შრომითი მამაცობისათვის 1941-1945 წლების დიდ სამამულო ომში”.

1946-1949 წლებში გვანჯი მანია სწავლობს მოსკოვის პოტიომკინის სახელობის პედაგოგიური ინსტიტუტის ასპირანტურაში. მისი სამეცნიერო ხელმძღვანელი ცნობილი მათემატიკოსი, პროფესორი სმირნოვი იყო. სმირნოვმა მას შესთავაზა განეხილა ამოცანები, რომლებიც ანალოგიური იყო იმ ამოცანების, რომლებსაც თავად სმირნოვი და აკადემიკოსი კოლმოგოროვი იხილავდნენ. მას უნდა შეედარებინა არა განაწილების მთელი ემპირიული

წირი თეორიულ განაწილების კანონთან, არამედ ამ წირის მხოლოდ გარკვეული, წინასწარ დაფიქსირებული ნაწილი თეორიული წირის შესაბამის ნაწილთან. საკითხის დასმის ასეთი აქტუალობა იმით იყო განპირობებული, რომ ხშირად ემპირიული მონაცემები შეიცავენ არასაიმედო დაკვირვებების გარკვეულ ნაწილს, რომლებიც, როგორც წესი, განაწილების წირის უკიდურეს ნაწილებში ხვდებიან და არღვევენ თანხმობას კიდეებზე. ვინაიდან ასეთი დაკვირვებები მოვლენას მთლიანობაში ვერ დაახასიათებენ, ამიტომ მიზანშეწონილია ისინი გადავაგდოთ ემპირიული და თეორიული განაწილებების შედარების დროს. 1949 წლის 3 ოქტომბერს გ. მანია პოტიომკინის სახელობის ინსტიტუტის ფიზიკა-მათემატიკის ფაკულტეტის სამეცნიერო საბჭოზე წარმატებით იცავს საკანდიდატო დისერტაციას თემაზე: "განაწილების კანონის სტატისტიკური შეფასება", რომელიც ოპონენტების მხრიდან უმაღლეს შეფასებას იმსახურებს.

დისერტაციის ოფიციალური ოპონენტები იყვნენ აკადემიკოსი ბორის გნედენკო და პროფესორი ლიაპუნოვი. დისერტანტის მოხსენების შემდეგ თავის გამოსვლაში აკადემიკოსმა გნედენკომ აღნიშნა: "გლივენკო, კოლმოგოროვი და სმირნოვი მუდმივად მიუთითებდნენ თავიანთი თეორემების ნაკლოვანებებზე. აქ საჭიროა უფრო ზუსტი ფაქტები, შეფასება უნდა ვაწარმოოთ არა მთელ რიცხვით ღერძზე, არამედ იქ, სადაც შეიძლება გვქონდეს დიდი გადახრები. გ. მანია სმირნოვის ხელმძღვანელობით მუშაობდა სწორედ ამ ძალიან საინტერესო და მნიშვნელოვან პრობლემაზე. დისერტაციაში მიღებული შედეგები პირველხარისხოვანი მნიშვნელობისაა (გნედენკო ხმარობს ფრაზას: первоклассного значения) და მე მგონია, რომ ამ თემატიკიდან შეიძლება გაკეთდეს სადოქტორო დისერტაცია. მიღებული შედეგები – ორი შესანიშნავი თეორემა – საჭიროა რაც შეიძლება სწრაფად გამოქვეყნდეს და შეტანილ იქნეს სტატისტიკის სახელმძღვანელოებში. ძალიან სასარგებლოა ეს დისერტაცია მომზადდეს გამოსაქვეყნებლად."

მეორე ოფიცილური ოპონენტი პროფ. ლიაპუნოვი კი აღნიშნავდა, რომ: "გამოთვლების შედეგად ავტორმა მიიღო განაწილების ზღვრული კანონები როგორც ერთი, ისე მეორე გადახრისათვის. სრულიად ნათელია, რომ ეს ორი თეორემა შევა მათემატიკური სტატისტიკის ოქროს ფონდში (эти две теоремы войдут в золотой фонд математической статистики, ხოლო ოფიციალურ გამოხმაურებაში იგი წერდა, რომ: "можно смело сказать, что решения этих задач, прочно войдут в золотой фонд математических методов статистики") და შემდეგ იგი აგრძელებს: მე ვეთანხმები აკადემიკოს გნედენკოს, რომ ამ თემატიკის შემდგომი განვითარება წარმოადგენს მტკიცე საფუძველს სადოქტორო დისერტაციის მოსამზადებლად."

აქვე მოვიყვანთ ერთ ნაწყვეტს გ. მანიას ხელმძღვანელის პროფესორ ნიკოლოზ სმირნოვის გამოსვლიდან: "მე ვიხსენებ 1946 წელს, როდესაც გამოჩნდა გვანჯი მანია. ის იყო მაშინ ძალიან ახალგაზრდა, მაგრამ შემოქმედებითი ენთუზიაზმი და მეცნიერებისადმი სიყვარული გამოსჭვიოდა მის ყოველ ნაბიჯში. მან ცუდად იცოდა რუსული ენა. ჩვენ მივეცით მას წასაკითხად ჰამერშტაინის ერთი მემუარი და ვიყავით უკიდურესად განცვიფრებულნი, როდესაც ამ ახალგაზრდა კაცმა, რომელსაც ძალზედ უჭირდა რუსულ ენაზე ფრაზების სწორად დალაგებაც კი, ძალიან კარგად და ზუსტად აღადგინა მძიმეწონიანი ფუნდამენტური გერმანული წინადადებები, ამასთანავე ყველა ძირითადი აზრი, ყველა დეტალი და ყველა დამტკიცება ბრწყინვალედ იყო გადმოცემული. მას შემდეგ, რაც გ. მანია

შეუდგა დამოუკიდებელ მუშაობას, მის მიერ შემოთავაზებულ იქნა ბევრი სხვადასხვა მიდგომა. მაგრამ ზოგიერთს მივყავდით ახალ უფრო რთულ პრობლემებამდე, ხოლო ზოგიერთი იძლეოდა პრობლემის ძალიან ვრცელ დასმას და მხოლოდ ინტუიციამ უკარნახა გ. მანიას ყველაზე უფრო მოხერხებული და სასარგებლო მეთოდი, რომელიც ნამდვილად წარმოადგენს სანიმუშოს მსგავსი ამოცანების დასმისა და ამოხსნის დროს."

რუსეთის ეპოპეის შემდეგ გვანჯი მანია ბრუნდება საქართველოში და 1949–1950 წლებში მუშაობს გორის ნიკოლოზ ბარათაშვილის სახელობის პედაგოგიურ ინსტიტუტში დოცენტის თანამდებობაზე. 1950–1953 წლებში იგი საქართველოს პოლიტექნიკური ინსტიტუტის დოცენტია, 1955, 1956 წლებში კი - ანდრია რაზმაძის სახელობის თბილისის მათემატიკის ინსტიტუტის უფროსი მეცნიერ-თანამშრომელი.

განუზომლად დიდია პროფესორ გვანჯი მანიას ღვაწლი საქართველოში ისეთი ახალი სამეცნიერო ცენტრების ჩამოყალიბების საქმეში, როგორებიცაა გამოთვლითი ცენტრი და გამოყენებითი მათემატიკის ინსტიტუტი. ამდენად, სრულიად ბუნებრივი და ნიშანდობლივია, რომ 2008 წელს, როცა აღინიშნებოდა გამოყენებითი მათემატიკის ინსტიტუტის (შემდგომში - აკადემიკოს ილია ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტის) დაარსებიდან 40 წელი და ამ თარიღს მიეძღვნა სამეცნიერო კონფერენცია, გადაწყდა, რომ პროფესორ გვანჯი მანიას 90 წლისთავისადმი მიძღვნილი საიუბილეო სხდომა საქართველოს მათემატიკოსთა კავშირის თაოსნობითა და ინიციატივით სწორედ ამ ინსტიტუტის კედლებში ჩატარებულიყო. 1956-1964 წლებში ბატონი გვანჯი მუშაობდა დირექტორის მოადგილედ სამეცნიერო დარგში საქართველოს მეცნიერებათა აკადემიის გამოთვლით ცენტრში (შემდგომში – აკადემიკოს ნიკოლოზ მუსხელიშვილის სახელობის გამოთვლითი მათემატიკის ინსტიტუტი), ხოლო 1966-1972 წლებში კი იყო თსუ გამოყენებით მათემატიკის ინსტიტუტის დირექტორის მოადგილე სამეცნიერო ნაწილში.

არანაკლებ წარმატებული იყო გ. მანიას სადოქტორო დისერტაცია თემაზე: "მათემატიკური სტატისტიკის ზოგიერთი მეთოდი", რომელიც წარმოადგენდა მისი ათწლიანი ნაყოფიერი მეცნიერული მოღვაწეობის შედეგს. გ. მანიამ სადოქტორო დისერტაცია 1963 წელს, ა. რაზმაძის სახელობის მათემატიკის ინსტიტუტში დაიცვა. მისი ოფიციალური ოპონენტები იყვნენ აკადემიკოსები: ხვედელიძე, პროხოროვი და სირაჟდინოვი. 1964 წელს მას თსუ პროფესორის თანამდებობაზე ირჩევენ.

1963 წელს გ. მანიას უდიდესი ძალისხმევით თბილისში ჩატარდა ძალიან დიდი და მნიშვნელოვანი კონფერენცია - საკავშირო კონფერენცია ალბათობის თეორიასა და მათემატიკურ სტატისტიკაში. საბჭოთა კავშირში, 1963 წელს, კონფერენციის ორგანიზატორებს უფლება მისცეს მოეწვიათ "კაპიტალისტურ ქვეყნებიდან 10 მონაწილე და სოციალისტური ქვეყნებიდან 15 მონაწილე" - რაც იმ დროისათვის წარმოუდგენელი ფუფუნება იყო. კონფერენციაში მონაწილეობდნენ: კრამერი, მარტინ ლოვი, პარზენი, ვოლფოვიცი, როზენბლატი, დევიდ კენდალი და სხვები.

კონფერენციის მონაწილეები იყვნენ არა მარტო ალბათობის თეორიისა და მათემატიკური სტატისტიკის სპეციალისტები, არამედ ფუნქციათა თეორიისა და ფუნქციონალური ანალიზის ზოგიერთი საუკეთესო არმომადგენელი, როგორიცაა მაგალითად პროფესორი ს. სტეჩკინი. პროფესორი ლინნიკი აქ იყო ზოგიერთ თავის მოსწავლესთან ერთად და ახალგაზრდა მ. სტრატანოვიჩი, რომელიც იმ დროს მუშაობდა ფიზიკაში კერძოწარმოებულე-

ბიანი დიფერენციალური განტოლებების აპროქსიმაციის მეთოდებში. ა. კოლმოგოროვი, თავის მოსწავლეებთან და თანამშრომლებთან ერთად: ბ. გნედენკო, ა. შირიაევი, ი. სინაი, ა. ბოროვკოვი და სხვები, ქმნიდნენ კონფერენციის სამეცნიერო ''ბირთვს'', მაგრამ პრობლემების უმრავლესობას, როგორც მცირეს, ისე დიდს, წარმატებით უმკლავდებოდა ერთ ახალგაზრდა კაცი, ჯერ კიდევ არა პროფესორი -- გვანჯი მანია.

20-25 წლის შემდეგ და უფრო მოგვიანებით, კოლეგები ყველგან იხსენებდნენ ამ კონფერენციას, როგორც უდიდეს მოვლენას და სასიამოვნო მოგონებას.

საქართველოში ალბათობის თეორიისა და მათემატიკური სტატისტიკის სწავლების ტრადიციას საფუძველი დაუდო პირველმა ქართველმა მკვლევარ-მათემატიკოსმა, თბილისის უნივერსიტეტის ერთ-ერთმა დამაარსებელმა, პროფ. ანდრია რაზმაძემ (1889-1929), რომელიც კითხულობდა ლექციების კურსს ახლად დაარსებულ თბილისის უნივერსიტეტში, ხოლო მათემატიკის ამ დარგის განვითარებას საფუძველი ჩაუყარა გვანჯი მანიამ (1918-1985). 1968 წელს პროფ. გ. მანიას თაოსნობით თბილისის სახელმწიფო უნივერსიტეტში ფუძნდება ალბათობის თეორიისა და მათემატიკური სტატისტიკის კათედრა, რომელსაც ის ხელმძღვანელობდა სიცოცხლის ბოლომდე. წელს ჩვენ აღვნიშნავთ როგორც ბატონი გვანჯის დაბადებიდან 100 წლისთავს, ისე მისი მშობლიური უნივერსიტეტის დაარსებიდან 100 წლის იუბილეს და, რა თქმა უნდა, მის მიერ დაფუძნებული კათედრის 50 წლის იუბილესაც (ამჟამად ამ კათედრას სათავეში უდგას საქართველოს მეცნიერებათა ეროვნული აკადემიის წ/კ, პროფესორი ელიზბარ ნადარაია). პარალელურად, 1973-1983 წლებში, პროფ. გ. მანია იყო საქართველოს მეცნიერებათა აკადემიის ეკონომიკისა და სამართლის ინსტიტუტის სექტორის გამგე, 1983 წლიდან კი - საქართველოს მეცნიერებათა აკადემიის ანდრია რაზმაძის სახელობის ინსტიტუტთან არსებული ალბათობის თეორიისა და მათემატიკური სტატისტიკის სექტორის გამგე.

პროფესორი გ. მანია ეწეოდა აქტიურ და ნაყოფიერ სამეცნიერო და პედაგოგიურ მოღვაწეობას. იგი არის ავტორი 50-ზე მეტი სამეცნიერო ნაშრომის. საქართველოში ალბათობის თეორიისა და მათემატიკური სტატისტიკის, როგორც მათემატიკის დარგის, განვითარებას საფუძველი სწორედ გ. მანიამ ჩაუყარა. მას ეკუთვნის პირველი ქართული სახელმძღვანელოები და მონოგრაფიები ამ დისციპლინებში. მისი მოღვაწეობის შედეგად გასული საუკუნის 50-იანი წლებიდან შეიქმნა მეცნიერთა კოლექტივები, რომლებიც იკვლევდნენ ალბათობის თეორიისა და მათემატიკური სტატისტიკის პრობლემებს და წყვეტდნენ როგორც თეორიულ, ისე პრაქტიკულ ამოცანებს ალბათურ-სტატისტიკური მეთოდების გამოყენებით. პროფესორ გ. მანიას ხელმძღვანელობით მომზადდა 10 საკანდიდატო დისერტაცია.

პროფესორი გ. მანია მრავალი საკავშირო და საერთაშორისო კონფერენციისა და სიმპოზიუმის მუშაობასა და მათ ორგანიზებაში მონაწილეობდა. 1969 წელს იგი მივლინებული იყო საერთაშორისო სტატისტიკური ინსტიტუტის 37-ე სესიის დელეგატად ლონდონში, ხოლო 1970 წელს ის იყო დელეგატი მათემატიკოსთა საერთაშორისო კონგრესის, რომელიც საფრანგეთის ქალაქ ნიცაში ჩატარდა.

გვანჯი მანიას თაოსნობით მთელი 20 წლის მანძილზე ყოველწლიურად საქართველოში (კერძოდ, ბაკურიანში) ტარდებოდა საკავშირო სკოლა-კოლოკვიუმი ალბათობის თეორიასა და მათემატიკურ სტატისტიკაში, რომელიც სულ მალე ფაქტიურად გახდა საერთაშორისო,

ვინაიდან მასში რეგულარულად მონაწილეობას იღებდა არაერთი გამოჩენილი უცხოელი მეცნიერი. 1982 წელს თბილისში მისი უშუალო ხელმძღვანელობით წარმატებით ჩატარდა საბჭოთა კავშირ-იაპონიის IV სიმპოზიუმი ალბათობის თეორიასა და მათემატიკურ სტატისტიკაში. ამ სიმპოზიუმიდან დაბრუნებული აკადემიკოსი კოლმოგოროვი ბატონ გ. მანიას წერდა: ‘‘ძალიან დიდ მადლობას გიხდით ჩემს მიმართ გაწეული ყოველგვარი მზრუნველობისათვის თბილისსა და სოხუმში. ამ მივლინების განმავლობაში მე მქონდა პირადი ბედნიერება საქართველოში ალბათობის თეორიისა და მათემატიკური სტატისტიკის განვითარებაში თქვენი როლისთვის მიმეცა უმაღლესი შეფასება‘‘.

პროფესორი გ. მანია იყო სხვადასხვა სამეცნიერო საბჭოსა და საზოგადოების წევრი, მათ შორის საქართველოს მათემატიკოსთა კავშირის პრეზიდიუმის წევრი, საერთაშორისო სტატისტიკური ინსტიტუტის, კერძოდ, ალბათობის თეორიასა და მათემატიკურ სტატისტიკაში სტატისტიკის გამოყენებების ბერნულის სახელობის საერთაშორისო საზოგადოების წევრი 1969 წლიდან, ამერიკის მათემატიკური საზოგადოების წევრი. საერთაშორისო ჟურნალის ‘‘Statistics’’ (რომელიც გამოდიოდა ბერლინში) სარედაქციო კოლეგიის წევრი. მას მიღებული ჰქონდა მთავრობის ორი ჯილდო და აკადემიკოს ივ. ჯავახიშვილის სახელობის მედალი.

1989 წელს, პროფესორ გ. მანიას დაბადებიდან 70 წელს მიეძღვნა შრომების კრებული სახელწოდებით: ‘‘ალბათობის თეორია და მათემატიკური სტატისტიკა’’, რომელსაც დაეთმო საქართველოს მეცნიერებათა აკადემიის ა. რაზმაძის სახელობის მათემატიკის ინსტიტუტის შრომების 92-ე ტომი, სადაც სხვა მრავალი გამოჩენილი მეცნიერის ნაშრომთან ერთად გამოქვეყნდა აკადემიკოს კარალიუკის სტატია სათაურით: ‘‘Асимптотика статистик Мания’’.

როგორც ზემოთ აღვნიშნეთ, გვანჯი მანიას პირველი გამოკვლევები შესრულებული იყო პროფესორ სმირნოვის ხელმძღვანელობით, სადაც მან შეისწავლა უწყვეტი განაწილების ფუნქციის გადახრის მაქსიმუმის ყოფაქცევა ემპირიული განაწილების ფუნქციისაგან, აღებული არა მთელს ღერძზე, არამედ მხოლოდ ფუნქციის მოცემულ ზრდის მონაკვეთზე. მის მიერ ნაპოვნი იყო შემდეგი სტატისტიკების ზღვარითი განაწილება:

$$\begin{aligned} D_n^+(\theta_1,\theta_2) &= \sup_{x:\theta_1\le F(x)\le\theta_2} \big(F_n(x)-F(x)\big), \\ D_n(\theta_1,\theta_2) &= \sup_{x:\theta_1\le F(x)\le\theta_2} \big|F_n(x)-F(x)\big|, \end{aligned} \tag{1}$$

სადაც θ_1 და θ_2 მოცემული რიცხვებია, $0 \le \theta_1 < \theta_2 \le 1$ (მოგვიანებით, მანვე მოახდინა (1) სტატისტიკების ზღვრული განაწილებების ტაბულირება, როცა $\theta_2 = 1-\theta_1$). აღნიშნული შედეგების გონებამახვილური დამტკიცება ემყარება გვანჯი მანიას მიერ დადგენილ ‘‘აბელისა’’ და ‘‘ტაუბერის’’ ტიპის თეორემებს, რომლებშიც აღმოფხვრილია ფელერის ანალოგიურ თეორემებში დაშვებული შეცდომები. ამ შედეგმა მაშინვე მიიპყრო სპეციალისტთა ყურადღება. მას არაერთხელ მიმართავდნენ სხვა მეცნიერები. (1) სტატისტიკები ლიტერატურაში ცნობილია მანიას სტატისტიკების (კრიტერიუმების) სახელით.

1961 წელს გ. მანიას მიერ შემოთავაზებული იყო ორი დამოუკიდებელი ნორმალური

შერჩევის ერთგვაროვნების კრიტერიუმი, დაფუძნებული სტატისტიკაზე:

$$L = \max_x \left|\Phi\Big(\frac{x-\overline{x}_1}{s_1}\Big) - \Phi\Big(\frac{x-\overline{x}_2}{s_2}\Big)\right|,$$

სადაც Φ სტანდარტული ნორმალური განაწილების ფუნქციაა, ხოლო $\overline{x}_i$ და s_i შესაბამისად n_i მოცულობის მქონე შერჩევის მიხედვით აგებული ემპირიული საშუალო და დისპერსიაა, $i=1,2$. მან აჩვენა, რომ

$$\frac{n_1\cdot n_2}{n_1+n_2}\cdot L$$

სტატისტიკის ზღვრული განაწილება (როცა $n_1,n_2\to\infty$) არ არის დამოკიდებული ნორმალური განაწილების პარამეტრებზე და ემთხვევა სმირნოვის მიერ დადგენილ

$$n_1 \max_x \left|\Phi\Big(\frac{x-\overline{x}_1}{s_1}\Big) - \Phi(x)\right|$$

სტატისტიკის ზღვრულ განაწილებას.

იმავე წელს გ. მანიამ შემოიღო ორგანზომილებიანი განაწილების სიმკვრივე

$$f_n(x,y) = \frac{\Delta_{h_1}\Delta_{h_2}F_n(x,y)}{4h_1h_2}\,.$$

მან მოძებნა h_1-სა და h_2-ის ოპტიმალური მნიშვნელობა ინტეგრალური კვადრატული გადახრის აზრით და აჩვენა, რომ

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\big(f_n(x,y)-f(x,y)\big)^2\,dx\,dy \approx cn^{-\frac{2}{3}},$$

სადაც c გარკვეული აზრით დამოკიდებულია $f(x,y)$-ის მეორე რიგის წარმოებულზე.

შემდგომ, გ. მანიამ გამოიკვლია ნორმალური განაწილების სიმკვრივის არაპარამეტრული შეფასების თვისებები. კერძოდ, k-განზომილებიანი ნორმალური განაწილების (საშუალოთი a და კოვარიაციის მატრიცით C) სიმკვრივის პარამეტრული შეფასების $n(x|a,C)$ კვადრატული ცდომილებისათვის

$$\Phi_n = \int_{R^k} \big[n(x|a,C) - n(x|\overline{a},\overline{C})\big]^2\,dx$$

მის მიერ მოძებნილ იქნა ზღვარითი განაწილება:

$$P\big\{n2^{k+3}\pi^{k/2}\sqrt{\det C}\,\Phi_n < u\big\} \longrightarrow G(u),$$

სადაც $G(u)$ ემთხვევა ნორმალური სიდიდეების გარკვეული სახის კვადრატული ფორმის განაწილებას. გამომდინარე გ. მანიას აღნიშნული შედეგიდან, შეიძლება ვამტკიცოთ (ისევე როგორც დევროის და დერფის წიგნში), რომ შეუძლებელია გაუმჯობესებულ იქნას ბოიდისა და სტილის ცნობილი თეორემა.

პროფესორ გ. მანიას ზემოთ ჩამოთვლილი შედეგები, და ასევე მისი მთელი რიგი ნაშრომები და გამოკვლევები შეფასებათა თეორიაში თავმოყრილია მონოგრაფიაში: ‘‘ალბათობათა განაწილების სტატისტიკური შეფასებები’’, რომელიც გამოიცა 1974 წელს და მას მიეცა სპეციალისტების მაღალი შეფასება. აღნიშნული მონოგრაფიის გამოქვეყნებამდე, ხელნაწერის წაკითხვის შემდეგ აკადემიკოსი გნედენკო თავის გამოხმაურებაში წერდა: ’’В математическом отношении рукопись выполнена безупречно. Она после опубликования несомненно оживит интерес к тому направлению исследований, которое представляет автор’’. მონოგრაფიის გამოქვეყნების შემდეგ კი 1979 წელს ჟურნალში ‘‘International Statistical Review’’ სიმკვრივის სტატისტიკური შეფასების შესახებ ვერცისა და შნაიდერის ბიბლიოგრაფიაში მითითებულია პროფ. გ. მანიას 17 ნაშრომი, მათ შორის ზემოთ აღნიშნული მონოგრაფია, რომლის შესახებაც ნათქვამია, რომ აღნიშნული მიმართულებით იგი წარმოადგენს ბრწყინვალე წიგნს (The excellent book, სწერს ‘‘International Statistical Review’’).

მონოგრაფია ‘‘მათემატიკური სტატისტიკის ზოგიერთი მეთოდი’’, რომელიც გამოვიდა 1963 წელს ქართულ ენაზე და მასთან ერთად წიგნი ‘‘მათემატიკური სტატისტიკა ტექნიკაში’’, რომელიც გამოიცა 1985 წელს, ქართული სამეცნიერო-ტექნიკური ინტელიგენციისათვის მნიშვნელოვანი შენაძენი იყო, რომელიც მათ საშუალებას აძლევდა მშობლიურ ენაზე გარკვეულიყვნენ ალბათურ-სტატისტიკური მეთოდოლოგიის გამოყენების ასპექტებში პრაქტიკული ამოცანების გადაწყვეტისას. წლების მანძილზე ბატონ გვანჯისთან კათედრაზე მოდიოდნენ სხვადასხვა სფეროს წარმომადგენლები, რათა მიეღოთ კონსულტაციები ალბათურ-სტატიტიკური მეთოდების პრაქტიკულ გამოყენებებზე. მათ შორის იყვნენ ექიმები, ბიოლოგები, ინჟინრები, რუსთავის მეტალურგიული ქარხნის ხელმძღვანელები და სხვები. ამ საქმიანობაში მე და ჩემი კოლეგებიც ვიღებდით მონაწილეობას და მახსოვს თუ როგორ კვალიფიციურ დახმარებას უწევდა ბატონი გვანჯი სხვა დარგების დაინტერესებულ სპეციალისტებს.

პროფესორ გ. მანიას უკანასკნელი გამოკვლევები ეძღვნება მდგრადი კანონების პარამეტრების შეფასებას და უსასრულოდ დაყოფადი და მდგრადი კანონების ანალოგების გამოკვლევას შესაკრებთა შემთხვევითი რაოდენობის შეჯამების სქემაში.

გვანჯი მანიას, როგორც შესანიშნავი პედაგოგისა და აღმზრდელის როლი ძალიან დიდია. მუდმივი ზრუნვა მის მოწაფეებსა და კოლეგებზე გვანჯი მანიას ცხოვრების მნიშვნელოვან ნაწილს შეადგენდა. ალბათობის თეორიისა და მათემატიკური სტატისტიკის ქართული სკოლის საყოველთაო წარმატება დიდ წილად, საქართველოში ამ დარგის დამფუძნებლის – გვანჯი მანიას სახელს უკავშირდება.

ბატონ გვანჯის ყავდა შესანიშნავი მეუღლე, ქალბატონი ირინე მიხეილის ასული ნოდია, გამოჩენილი მეცნიერის, პროფესორ მიხეილ ნოდიას ქალიშვილი. თვითონ ქალბატონი ირინე გახლდათ აღმოსავლეთმცოდნე – თავისი დარგის მაღალი რანგის პროფესიონალი. ბატონმა გვანჯიმ და ქალბატონმა ირინემ საქართველოს აღუზარდეს ორი ღირსეული შვილი – მიხეილი და მაია, თავიანთი სფეროების ნამდვილი პროფესიონალები. მიხეილ მანია არის ა. რაზმაძის მათემატიკის ინსტიტუტის ალბათობის თეორიისა და მათემატიკური სტატისტიკის განყოფილების გამგე, ხოლო მაია მანია - არქიტექტურის ისტორიკოსი, თბილისის სამხატვრო აკადემიის პროფესორი. ორივე დაოჯახებულია.

ბატონი გვანჯი გადრაიცვალა 67 წლის ასაკში, 1985 წლის 16 მარტს.

პროფესორ გ. მანიას დაბადებიდან 70 წელს მიეძღვნა ა. რაზმაძის ინსტიტუტის შრომების კრებული (1989 წ.) სახელწოდებით: „ალბათობის თეორია და მათემატიკური სტატისტიკა“.

2008 წელს საქართველოს მათემატიკოსთა კავშირის ეგიდით აღინიშნა პროფ. გ. მანიას დაბადებიდან 90 წლის იუბილე.

2013 წელს 95 წლის იუბილესთან დაკავშირებით, ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის XI კორპუსის #335 კაბინეტს ეწოდა პროფ. გვანჯი მანიას სახელობის კაბინეტი.

მიმდინარე, 2018 წელს, პროფ. გვანჯი მანიას დაბადებიდან 100 წლისთავთან დაკავშირებით, საქართველოს სტატისტიკური ასოციაციის გამგეობამ მიიღო გადაწყვეტილება ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტში დაარსდეს პროფ. გვანჯი მანიას სახელობის სტიპენდია.

დღეს ჩვენ ყველანი, პროფესორ გვანჯი მანიას მოსწავლეები და უმცროსი კოლეგები, დიდი სიყვარულით ვიხსენებთ ჩვენს კეთილგონიერ უფროს მეგობარს და უდიდეს მხარდამჭერს.

ე. ნადარაია, ო. ფურთუხია

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Professor Gvanji Mania (1918 – 1985)

Professor Gvanji Mania was born in the village of Etseri, Georgia on May 29, 1918. His father, Mikheil Mania, was a Russian language teacher and his mother, Fedosi was daughter of clergyman. It is noteworthy that Professor Mania's maternal grandfather and two of his uncles served at Saint George's church in the village of Jvari.

From 1932 to 1935 Mania studied at Zugdidi Pedagogical College and immediately after its graduation he entered the Department of Physics and Mathematics of Tbilisi State University. From 1940 to 1945 he worked as Assistant Professor at Zugdidi Pedagogical Institute and at the same time, from 1943 to 1946, as an Assistant Professor at Tbilisi Institute of Railway Engineers. In 1945–1946 he was a higher school inspector at the Ministry of Education of Georgia. In 1945 he was awarded the medal "For labor valor during the Great Patriotic War".

From 1946 till 1949 Gvanji Mania studied at the Moscow Potemkin Pedagogical Institute as a postgraduate student. His research supervisor was a well-known mathematician Professor Smirnov. Smirnov offered him to study problems similar to those Prof. Smirnov was working on together with Academician Kolmogorov. Mania had to compare not just the entire empirical line with the theoretical law of distribution, but only a certain a priori fixed part of this line – to the respective part of the theoretical line. The relevance of the stated problem was due to the fact that empirical data often contain unreliable observations, which, as a rule, are found at the extreme intervals of the distribution line and therefore break fitting on these intervals. Since such observations do not generally define the phenomenon, it is reasonable to omit them when empirical and theoretical distributions are compared. On October 3, 1949 G. Mania defended his thesis *Statistical Estimation of Distribution Law* for Candidate's degree at the Scientific Council of the Depart-ment of Physics and Mathematics at Potemkin Institute, his thesis gained a high appreciation of the opponents.

The official opponents of the thesis were Academician Boris Gnedenko from Kiev and Professor Liapunov. After the applicant's speech Academician Gnedenko said: "Glivenko, Kolmogorov and Smirnov always point out to the drawbacks of their theorems. More accurate facts are necessary here – we should perform estimation not on the whole numerical axis, but at points where large deviations can be observed. G. Mania, under Smirnov's guidance, investigated just these particularly interesting and important problems. The results obtained in the thesis are of primary importance (Gnedenko uses the phrase: "первоклассного значения"), and I think that this topic can make a subject of a doctoral dissertation. The obtained results – the two beautiful theorems – should be published as soon as possible and included in statistics manuals. It is advisable to prepare this thesis for publication".

The second official opponent, Professor Liapunov, noted: "As a result of calculations, the author obtained boundary laws of distribution both for the first and the second deviation. It is obvious that these two theorems will enter the gold fund of mathematical statistics (эти две теоремы войдут в золотой фонд математической статистики), while in a formal review he wrote: "можно смело сказать, что решения этих задач, прочно войдут в золотой фонд математических методов статистики" -– undoubtedly, these theorems will enter the gold fund of mathematical methods in statistics"). Then he added: "I agree with Academician Gnedenko that extension of this topic will make a firm basis for a doctoral dissertation".

We shall present here a fragment from Professor N. Smirnov's, G. Mania's thesis supervisor's, speech: "I remember the year of 1946 when Gvanji Mania first appeared. He was very young then, but creative enthusiasm and love for science characterized each step he took. His Russian was rather poor then. We gave him Hammerstein's memoirs to read and were greatly astonished when this young man, who could hardly arrange Russian words into sentences while speaking, managed to reproduce very precisely heavy German phrases and present not only all basic ideas, but also all details and proofs in a brilliant way. Since the time G. Mania started working independently he introduced a number of different approaches, but some of them led to more difficult problems while others resulted in very long statements, and it was his intuition that made him choose the most convenient and useful method that would become a model for statement and proof of similar problems".

After the Russian period of his activity G. Mania came back to Georgia and in 1949–1950 worked as Assistant Professor at Gori Nikoloz Baratashvili Pedagogical Institute. In 1950–1953 he was an Assistant Professor at Georgian Polytechnic Institute. In 1955–1956 he became a Senior Researcher at Tbilisi Andrea Razmadze Institute of Mathematics.

One cannot overestimate Professor G. Mania's share in the foundation of new scientific centers, such as Computational Centre and Institute of Applied Mathematics. Hence it is quite natural and noteworthy that when during the celebration of the 40-th anniversary of the Institute of Applied Mathematics (later – Academician I. Vekua Institute of Applied Mathematics) in 2009 a scientific conference dedicated to this event was held, it was decided that the session devoted to Professor G. Mania's 90-th anniversary, prepared on the initiative and under the leadership of Georgian Mathematical Society, would take place just within the walls of this Institute. In 1956–1964 Professor G. Mania was Deputy Director for Science at the Computational Center of Georgian Academy of Sciences (later – Academician Nikoloz Muskhelishvili Institute of Computational Mathematics) and in 1966–1972 he worked as Deputy Director for Science at The Institute of Applied Mathematics of TSU.

G. Mania's doctoral dissertation titled *Some Methods of Mathematical Statistics* was as successful as his Candidate's thesis. It was a result of his fruitful ten-year scientific studies. G. Mania defended his doctoral dissertation at A. Razmadze Institute of Mathematics in 1963. His official opponents were Academicians: Khvedelidze, Prokhorov and Sirazhdinov. In 1964 he was elected for the position of TSU Professor.

In 1963 G. Mania organised, basically singlehandedly, very large and important conference – All-Union Conference in Probability Theory and Mathematical Statistics. In Soviet Union, in 1963, the conference was allowed "10 participants from capitalist countries and 15 participants from socialist countries" – an unseen luxury for the times. H. Cramér, was a participant, and Martin Lóff, E. Parzen, J. Wolfowitz and M. Rosenblatt, David Kendall. Not just probabilists and statisticians, but some of the best specialists in the theory of functions and functional analysis, such as Prof. S. Stechkin, also participated. Prof. Yu. Linnik was there with some of his pupils, and young M. Stratonovich, who at that time, worked in approximation methods for PDE in Physics. A. Kolmogorov, along with his pupils and collaborators B. Gnedenko, A. Shityaev, Ya. Sinai, A. Borovkov, and others, formed the scientific "core", but myriads of problems, small and large, have been laid on shoulders of one young, not yet professor, person – Gvanji Mania.

Some 20–25 years later, and more, colleagues everywhere remembered the Conference as a great and joyful event of they experienced.

The foundations of studies in probability theory and mathematical statistics were laid by the first Georgian mathematician, one of the founders of Tbilisi University – Professor Andrea Razmadze (1889–1929). He was a lecturer at the newly established Tbilisi University, while Gvanji Mania (1918–1985) was his successor developing this field of mathematics in Georgia. In 1968 under the direction of Professor G. Mania Probability Theory and Mathematical Statistics Chair was founded at Tbilisi State University, the head of which he remained till the end of his life. This year we celebrate both Professor Mania's centenary and the 100-th anniversary of his native university and the 50-th anniversary of the chair he founded (today the Head of the Chair is Professor Elizbar Nadaraya, Member of Georgian Academy of Sciences). At the same time in the period, 1973–1983 Professor G. Mania was Head of the Sector at the Institute of Economics and Law of Georgian Academy of Sciences, while since 1983 he was Head of the Sector of Probability Theory and Mathematical statistics at Tbilisi A. Razmadze Institute of Mathematics.

Professor G. Mania was actively engaged in scientific and pedagogical work. He is an author of more than 50 scientific works. It was G. Mania who laid the foundations for the development of Probability Theory and Mathematical Statistics as a branch of mathematics in Georgia. He is the author of first Georgian manuals and a number of monographs in this field. As a result of his activity since the 50-th of the last century teams of scientists were formed studying problems of probability theory and mathematical statistics and solving both theoretical and practical problems using probabilistic and statistical methods. Under the direction of Professor Mania 10 Master's Theses were prepared.

Professor G. Mania took an active part in the work and organization of a number of All-Union and international conferences and symposia. In 1969 he participated in the work of the 37-th session of the International Institute of Statistics in London and in 1970 he was delegated to the International Congress of Mathematicians held in Nice, France.

Under G. Mania's leadership All-Union Winter School in Probability Theory and Mathematical Statistics was yearly held in Bakuriani, Georgia, in the course of 20 years. It soon became International since it was regularly attended by famous foreign scientists. In 1982 under Professor G. Mania's direct supervision Tbilisi hosted the VI USSR-Japan Symposium in Probability Theory and Mathematical Statistics. After coming back home Academician Kolmogorov in a letter to Professor G. Mania wrote: "Thank you very much for all your efforts in Tbilisi and Sukhumi. During my visit I was happy to witness and appreciate your major part in the progress of probability theory and mathematical statistics in Georgia".

Professor J. Mania was a member of various scientific societies and councils, including Georgian Mathematical Society, where he was a member of the Presidium, of the International Institute of Statistics, since 1969 – of International Bernoulli Society for Application of Statist-ics in Probability Theory and Mathematical Statist-ics, a member of American Mathematical Society, a member of the Editorial Board of the international "Statistics" Journal (published in Berlin). He got two government awards and Academician Iv. Javakhishvili Medal.

In 1989, to commemorate his 70-th anniversary, a book of his works was issued titled *Probability Theory and Mathematical Statistics*, which entered the 92-th volume of scientific articles published by A. Razmadze Institute of Mathematics of Georgian Academy of Sciences, where together with a number of other works by outstanding scientist, Academician Korolyuk's paper "Asymptotic Behavior of Mania's Statistics" was also published.

As we have noted earlier, Gvanji Mania's first works were written under Professor Smirnov's guidance, where Mania studied the maximum deviation behavior of the continuous distribution function $F(x)$ from the empirical distribution function $F_n(x)$ taken not on the entire axis, but only on the maximum growth interval of the function $F(x)$. He found the limit distribution of the following statistics:

$$D_n^+(\theta_1,\theta_2) = \sup_{x:\theta_1\le F(x)\le\theta_2} \big(F_n(x)-F(x)\big),$$
$$D_n(\theta_1,\theta_2) = \sup_{x:\theta_1\le F(x)\le\theta_2} \big|F_n(x)-F(x)\big|,$$

where θ_1 are θ_2 given numbers, $0\le\theta_1<\theta_2\le 1$ (later, he also tabulated the limit distribution of these statistics when $\theta_2=1-\theta_1$). The sharp-witted proof of the above-mentioned results is based on Abel and Tauber type theorems, where errors made in Feller's similar theorems were eliminated. This result brought about immediate attention of specialists and it was often used by other scientists. In scientific literature these statistics are referred to as Mania's statistics (criteria).

In 1961, G. Mania introduced two independent normal sample homogeneity criteria based on the statistics:

$$L = \max_x \Big|\Phi\Big(\frac{x-\overline{x}_1}{s_1}\Big) - \Phi\Big(\frac{x-\overline{x}_2}{s_2}\Big)\Big|,$$

where Φ is a standard normal distribution function while $\overline{x}_i$ and s_i are, respectively, empirical mean and variance constructed according to the n_i-size sample, $i=1,2$. He showed that the limit distribution of the statistics

$$\frac{n_1\cdot n_2}{n_1+n_2}\cdot L$$

(when $n_1,n_2\to\infty$) is independent of normal distribution parameters and is based on the limit distribution of Smirnov's statistics

$$n_1\max_x\Big|\Phi\Big(\frac{x-\overline{x}_1}{s_1}\Big)-\Phi(x)\Big|.$$

In the same year G. Mania introduced the two-dimensional distribution density

$$f_n(x,y) = \frac{\Delta_{h_1}\Delta_{h_2}F_n(x,y)}{4h_1h_2}\,.$$

He found the optimal value of h_1 and h_2 in the sense of integral square deviation and showed that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(f_n(x,y)-f(x,y))^2\,dx\,dy \approx cn^{-\frac{2}{3}},$$

where c, in a certain sense, depends on the second-order derivative of $f(x,y)$.

After that G. Mania studied the properties of normal distribution density nonparametric estimate. In particular, that of density parametric estimate $n(x|a,C)$ of the k-dimensional normal distribution (by the mean and covariance matrix C) for the square error

$$\Phi_n=\int\limits_{R^k}\big[n(x|a,C)-n(x|\overline{a},\overline{C})\big]^2\,dx.$$

He found the limit distribution:

$$P\big\{n2^{k+3}\pi^{k/2}\sqrt{\det C}\,\Phi_n<u\big\}\longrightarrow G(u),$$

where $G(u)$ coincides with a certain type of square distribution of normal values. Thus G. Mania's above-mentioned results imply (the same as the book by L. Devroye and L. Gyofhi) that it is impossible to improve the famous Boyd and Still's Theorem.

Professor G. Mania's above mentioned results and a number of his works and studies in Estimation Theory are published in the monograph *Statistical Estimates of Probability Distribution* issued in 1974, which deserved specialists' appreciation. Before the monograph was published Academician Gnedenko wrote in his review of the manuscript: "В математическом отношении рукопись выполнена безупречно. Она после опубликования, несомненно оживит интерес к тому направлению исследований, которое представляет автор" ("As far as mathematics is concerned the manuscript is perfect. After its publication it will undoubtedly enliven the interest in the field of mathematics the Author presents"). After the publication of the monograph on density statistical estimation in *International Statistical Review* in 1979 in Werz and Schneider Reference Book G. Mania's 17 works are mentioned and *International Statistical Review* calls the above-mentioned book "an excellent book in the given field".

The monograph *Some Methods of Mathematical Statistics*, which appeared in Georgian in 1963 together with the book *Mathematical Statistics in Technology*, issued in 1985 were of primary importance for Georgian scientists and engineers enabling them to become aware of certain probabilistic and statistical methods, described in their native language, and apply them for the solution of some practical problems. For years specialists in different fields used to come to Professor G. Mania's Chair to consult on the practical application of probabilistic and statistical methods for the solution of various problems. Among them there were doctors, biologists, engineers, members of the administration of Rustavi Metallurgical Factory and others. His younger collegues also participated in this activity, and they remember clearly Professor G. Mania's qualified help he rendered to those specialists in different fields.

Professor G. Mania's last studies were devoted to problem of estimation of sustainable distributions' parameters and also to the investigation of infinitely divisible and sustainable distribution analogues within the scope of models with random number of summands.

G. Mania's role as a teacher and a tutor cannot be overestimated. His students and younger colleagues always felt his constant support. Caring for them was an important part of his life. The success achieved by contemporary Georgian scientists in the field of probability theory and mathematical statistics largely owes to Professor G. Mania's dedication and help.

Professor G. Mania's wife, Mrs. Irina Nodia, was a scholar specializing in Byzantian Studies. Their son Michael Mania is Head of the Department of Probability Theory and Mathematical Statistics at the A.Razmadze Institute of Mathematics. Their daughter Maia Mania is an Architectural Historian and a Professor at the Tbilisi State Academy of Arts. Both are married.

Professor G. Mania died on March 16, 1985 at the age of 67.

A. Razmadze Institute issued a collection of articles (1989) titled *Theory of Probability and Mathematical Statistics* to celebrate G. Mania's 70-th Anniversary.

In 2008, under the auspices of Georgian Mathematical Society G. Mania's 90-th Anniversary was celebrated.

In 2013, for his 95-th Anniversary lecture hall number 335 of the XI Building of Iv. Javakhishvili Tbilisi State University was given Professor G. Mania's name.

In the current 2018, to celebrate G. Mania's centenary, Georgian Statistical Association Office decided to establish Professor G. Mania Scholarship at Iv. Javakhishvili Tbilisi State University.

E. Nadaraya, O. Purtukhia

Main Publications

(i) Monographs

1. Some methods of mathematical statistics. (Georgian) *Publishing House of Georgian Academy of Sciences, Tbilisi*, 1963, pp. 351.
2. Statistical estimation of probability distributions. (Russian) *Tbilisi University Press*, 1974, 240 pp.
3. Probability theory. (Georgian) *Publishing House of Ministry of Education, Tbilisi*, 1954, 240 pp.
4. Mathematical statistics in technics. (Georgian) *Sabchota Sakartvelo, Tbilisi*, 1958, 345 pp.
5. The course of probability theory. (Georgian) *Tbilisi University Press, Tbilisi*, 1962, 340 pp.
6. Linear programming. (Georgian) *"Ganatleba", Tbilisi*, 1967, 295 pp.
7. The course of high mathematic. (Georgian) *Tbilisi State University, Tbilisi*, 1967, 498 pp. (with P. Zeragia).

8. Ilia Vekua. (Georgian) *Publishing House of Tbilisi State University*, 1967, 75 pp. (with B. Hvedelidze).

9. Probability theory and mathematical statistic. (Georgian) *Publishing House of Tbilisi State University*, 1976, 350 pp.

10. A book of problems in probability theory and mathematical statistic. (Georgian) *Publishing House of Tbilisi State University*, 1976, 120 pp. (with A. Ediberidze and N. Anthelava).

(ii) Selected Publications

11. Generalization of A. N. Kolmogorov's criterion for the estimation of distribution laws by empirical data. (Russian) *Dokl. Akad. Nauk SSSR* **69** (1949), no. 4, 495–497.

12. Statistical estimation of distribution laws. (Russian) *Uchenie Zapiski MGPI imeni V. P. Potiomkina* **16** (1951), 17–63.

13. Practical applications of an estimation of a maximum of two-sided deviations of empirical distribution in a given interval of growth of a theoretical law. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **14** (1953), no. 9, 521–524.

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15. Square estimation of divergence of normal densities by empirical data. (Georgian) *Soobshch. Akad Nauk Gruzin. SSR* **17** (1956), no. 3, 201–204.

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17. Quadratic error of an estimation of twodimensional normal density by empirical data. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **20** (1958), no. 6, 655–658.

18. Quadratic error of the estimation of normal density by empirical data. (Russian) *Trudy Vychisl. Tsentra Akad. Nauk Gruzin. SSR* **1** (1960), 75–96.

19. On one method of constructing of confidence regions for two samples from general population. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **27** (1961), no. 2, 137–142.

20. Remark on non-parametric estimations of twodimensional densities. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **27** (1961), no. 4, 385–390.

21. Square estimation of divergence of twodimensional normal distribution densities by empirical data. (Russian) *Trudy BC Akad. Nauk Gruzin. SSR* **2** (1961), 153–211.

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23. Quadratic error of an estimation of densities of normal distributions by two samples. (Russian) *Trudy. BC Akad. Nauk SSSR* **4** (1963), 213–216.

24. Hypothesis testing of identity of distributions of two independent samples. (Russian) *Trudy Vichisl. Tsentra Akad. Nauk Gruzin. SSR* **7** (1966), 1–34.

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31. Quadratic error of an estimation of normal distribution densities by several samples. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **67** (1972), no. 2, 301–304.

32. One approximation of distributions of positive defined quadratic forms of normal random variables. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **107** (1982), no. 2, 241–244 (with E. Khmaladze and V. Felker).

33. On the estimation of parameters of type of stable laws. *Proceedings of the first International Tampere Seminar on linear Statistical Models and their Applications (1983) Tampere University*, 1985, pp. 202–223 (with L. Klebanov and I. Melamed).

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2. T. Shervashidze, Probability Theory and Mathematical Statistics (Russian) in 50-th anniversary of Tbilisi A. Razmadze Math. Institute, Metsniereba, Tbilisi, 1985.

Abstracts of Plenary and Invited Speakers

პლენარული და მოწვეული მომხსენებლების თეზისები

All Extensions of C_2 by $C_{2^n} \times C_{2^n}$ are Good for the Morava K-Theory

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This talk is concerned with analyzing the 2-primary Morava K-theory of the classifying spaces BG of the groups G in the title. In particular it answers affirmatively the question whether transfers of Euler classes of complex representations of subgroups of G suffice to generate $K(s)^*(BG)$. Here $K(s)$ denotes Morava K-theory at prime $p = 2$ and natural number $s > 1$. The coefficient ring $K(s)^*(pt)$ is the Laurent polynomial ring in one variable, $\mathbb{F}_2[v_s, v_s^{-1}]$, where $\mathbb{F}_2$ is the field of 2 elements and $\deg(v_s) = -2(2^s - 1)$.

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Metric Spaces, Lattices, Atoms, and Models

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Let Ω be a metric space, A^t the metric neighborhood of $A \subset \Omega$ of radius t, $A^0 := A$; $\mathfrak{O}$ the lattice of the open sets in Ω with the partial order $\subseteq$ and the order convergence topology. The lattice of the $\mathfrak{O}$-valued functions of $t \in [0, \infty)$ with the point-wise order and topology contains the family $I\mathfrak{O} = \{A(\cdot) \mid A(t) = A^t,\ A \in \mathfrak{O}\}$. Let $\tilde{\Omega}$ be the set of the atoms of $\overline{I\mathfrak{O}}$. We describe a class of spaces such that the set $\tilde{\Omega}$ endowed with the relevant metric is *isometric* to the original Ω.

The space $\tilde{\Omega}$ (*wave spectrum*) is the key element of the program of constructing a new functional model for symmetric semi-bounded operators [1]. The given results provide a step towards realization of this program.

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Neumann Problem in Polydomains

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In this presentation, we will discuss the Neumann problem for higher order model equations in the unit polydisc of C_2. We derive the integral representations of the functions defined in the unit polydisc of C_2 which may particularly be suitable for Neumann problems.

100 Years of Alma Mater

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A short survey of development of mathematical schools during 100 years of Ivane Javakhishvili Tbilisi State university.

Frobenius Lie Algebras and q-Hypergeometric Functions

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Classification of Frobenius Lie algebras is of interest because of their relationship with geometric approaches in quantum field theory.

A particularly tractable class of such algebras arises from the so called seaweed algebras, introduced by Dergachyov and Kirillov in 2000.

For seaweed algebras the Frobenius property can be expressed through a purely combinatorial problem – enumeration of meanders of special kind, which we call lieanders.

Enumeration general meanders is a long standing open problem, dating back at least to Poincaré. In 1988 Arnold gave new impetus to it in connection with the study of certain branched coverings. Some partial progress has been achieved. There is some interesting work by physicists (di Francesco with collaborators). In 1993 Lando and Zvonkin obtained functional equation for the generating function of irreducible meandric systems. In 2017 Zorich with collaborators have related meander statistics with quadratic differentials on Riemann surfaces.

Lieanders have their specifics which gives hope that their enumeration may be relatively simpler compared to the general case. We will describe some of our findinds about the generating functions of various lieandric systems. In particular, we will explain relationship between these generating functions and q-hypergeometric series.

A Problem about Partitions

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Let X be an arbitrary finite collection of sets (clusters), some of them may intersect and some of them may be disjoint. Denote by $|X|$ the total number of clusters in X. Given $n > 1$ the question is to count the total number of ways a cluster A in X can be written as a disjoint union of n other clusters in X. We will estimate this number in terms of $|X|$ and n.

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Approximate Matrix Wiener–Hopf Factorisation and Applications to Problems in Acoustics

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In this talk I will introduce some of the techniques which are employed in the study of Helmholtz equation with various boundary conditions.

I will introduce the Wiener–Hopf method which extends the separation of variables technique (in Cartesian coordinate) used to investigate PDEs. It provided analytic and systematic methodology for previously unapproachable problems. I will focus on constructive solutions to matrix Wiener-Hopf problems which come from acoustics. The first matrix Wiener–Hopf problems is motivated by studying the effect of a finite elastic trailing porous on noise production, joint work with Dr. Ayton. The approximate factorisation of this matrix with exponential phase factors is achieved using an iterative procedure which makes use of the scalar Wiener–Hopf problem arising for each junction. This is an extension to the pole removal method with explicit error and convergence criteria. We gained new important insight into noise reduction: the porous extension had the effect of changing the direction of emitted sound.

The second matrix problem arise from scattering of sound waves by an finite grating composed of rigid plates, joint work with Prof. Abrahams. The approximate factorisation of this matrix resulting from a periodic structure is performed using conformal mapping, rational approximation and the recent procedure by Mishuris and Rogosin.

Lastly, I will introduce some methods which relay on special function called Mathieu functions. They result in applying the change of variable techniques in elliptic coordinates to the Helmholtz equation and the boundary conditions. This allows to investigate the effect of various porosity in a plate. This is used to investigated the effect of a porous trailing edge on an airfoil motivated by the design of an owl wing.

Geometric Dynamics of a Harmonic Oscillator, Non-Admissible Mother Wavelets and Squeezed States

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We present a new method of geometric solution of a partial differential equation by a reduction through an integral transform to an equivalent first-order PDE. The new equation shall be restricted to a specific subspace with auxiliary conditions which are obtained from group a representation construction of the integral transform.

The method is applied to the fundamental case of the harmonic oscillator with the Hamiltonian $H = \frac{1}{2}(p^2/m + m\omega^2 q^2)$, where m is its mass and ω – the frequency. The coherent state transform is generated by the minimal nilpotent step three Lie group $\mathbb{A}$. Its Lie algebra has a basis $\{X_1, X_2, X_3, X_4\}$ with the following non-vanishing commutators

$$[X_1, X_2] = X_3, \quad [X_1, X_3] = X_4\,.$$

Note that $\{X_1, X_3, X_4\}$ span the Lie algebra of the Heisenberg group.

We demonstrated that the coherent state transform on the group $\mathbb{A}$ for an *arbitrary* minimal uncertainty state (aka squeezed state) used as a mother wavelet produces a geometric dynamic of the harmonic oscillator. In contrast, it is shown that the well-known Fock–Segal–Bargmann transform for the Heisenberg group requires the specific fiducial vector (with the squeeze parameter $E = m\omega$) to produce a geometric solution. The larger group $\mathbb{A}$ creates the image space with a bigger number of auxiliary conditions. These conditions give additional flexibility in reduction of the PDE's order, leading to a richer set of geometric solutions.

There are some natural bounds of a possible squeeze parameter, they are determined by the degree of singularity of the solution of the auxiliary condition in the form of the heat equation. Then, the radius of analytic continuation of the time parameter into the complex plane defines the limits for allowed squeeze. A technical aspect of the group $\mathbb{A}$ is that its representations are not square-integrable and a respective modification of a coherent state transform is required [1, 2].

The talk is based on joint work [3] with co-author Fadhel Almalki.

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Integral Transforms on Measure Metric Spaces and Applications

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The goal of our talk is to present our recent results dealing with the boundedness of integral operators in new function spaces defined on general structures. We plan to discuss mapping criteria of Hardy–Littlewood maximal functions and Calderón–Zygmund operators,including one-sided versions and integral transforms defined on product spaces.Application to BVP for analytic function will be given.

Three-Body Equations in Quantum Field Theory

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Faddeev's three-body equations represent a powerful tool in Quantum Mechanics.

Derivation of similar equations in Quantum Field Theory (QFT) encounters two problems: due to a possibility of particle creation and annihilation the number of them is not fixed, and particles can get ”dressed” the phenomenon that complicates analytic properties of particle propagation functions. It will be shown how to overcome these problems to derive the three-body equations in QFT and some applications will be discussed.

Representations of Free Algebras of Varieties and Hypervarieties

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The free Boolean algebra on n free generators is isomorphic to the Boolean algebra of Boolean functions of n variables. The free bounded distributive lattice on n free generators is isomorphic to the bounded lattice of monotone Boolean functions of n variables (R. Dedekind, 1897). A problem posed by B. I. Plotkin in 1970s has required finding the varieties (and hypervarieties) of algebras with analogous functional representations of free finitely generated algebras. In this talk we give a solution of this problem.

Differentiation of Integrals with respect to Translation Invariant Convex Density Bases

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For a translation invariant convex density basis B it is shown that its Busemann-Feller extension B_{BF} has close to B properties, namely, B_{BF} differentiates the same class of non-negative functions as B, moreover, the integral of an arbitrary non-negative function $f \in L(\mathbb{R}^n)$ at almost every point $x \in \mathbb{R}^n$ has one and the same type limits of indeterminacy with respect to the bases B and B_{BF}. This theorem provides a certain general extension principle of results obtained for Busemann–Feller bases to bases without the restriction of being Busemann–Feller. Some such type applications of the theorem are given.

Right-Angled Polytopes, Hyperbolic Manifolds and Torus Actions

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A combinatorial 3-dimensional polytope P can be realised in Lobachevsky 3-space with right dihedral angles if and only if it is simple, flag and does not have 4-belts of facets. This criterion was proved in the works of Pogorelov and Andreev of the 1960s. We refer to combinatorial 3-polytopes admitting a right-angled realisation in Lobachevsky 3-space as Pogorelov polytopes. The Pogorelov class contains all fullerenes, i.e. simple 3-polytopes with only 5-gonal and 6-gonal facets.

There are two families of smooth manifolds associated with Pogorelov polytopes. The first family consists of 3-dimensional small covers of Pogorelov polytopes P, also known as hyperbolic 3-manifolds of Loebell type. These are aspherical 3-manifolds whose fundamental groups are certain finite abelian extensions of hyperbolic right-angled reflection groups in the facets of P. The second family consists of 6-dimensional quasitoric manifolds over Pogorelov polytopes. These are simply connected 6-manifolds with a 3-dimensional torus action and orbit space P. Our main result is that both families are cohomologically rigid, i. e. two manifolds M and M' from either family are diffeomorphic if and only if their cohomology rings are isomorphic. We also prove that a cohomology ring isomorphism implies an equivalence of characteristic pairs; in particular, the corresponding polytopes P and P' are combinatorially equivalent. This leads to a positive solution of a problem of Vesnin (1991) on hyperbolic Loebell manifolds, and implies their full classification.

Our results are intertwined with classical subjects of geometry and topology such as combinatorics of 3-polytopes, the Four Colour Theorem, aspherical manifolds, a diffeomorphism classification of 6-manifolds and invariance of Pontryagin classes. The proofs use techniques of toric topology.

This is a joint work with V. Buchstaber, N. Erokhovets, M. Masuda and S. Park.

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Multipliers in Sobolev Spaces and Their Applications in the Theory of Differential Operators

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Denote by $H_p^s(\mathbb{R}^n)$, $p>1, s\in\mathbb{R}$, the Bessel potential spaces (for integer s they coincide with the Sobolev spaces $W_p^s(\mathbb{R}^n)$). We shall present the last results on the description of the spaces of multipliers acting from the space $H_p^s(\mathbb{R}^n)$ to another space $H_q^{-t}(\mathbb{R}^n)$. The main attention we will pay to the case when the smooth indices are of different signs, i.e. $s,t\geqslant 0$. Such a space of multipliers (we denote it by $M[H_p^s(\mathbb{R}^n)\to H_q^{-t}(\mathbb{R}^n)]$) consists of distributions $u\in\mathcal{D}'$ which obey the estimate

$$\|u\varphi\|_{H_p^{-t}}\leqslant C\|\varphi\|_{H_p^s}\quad\forall\ \varphi\in\mathcal{D},$$

where $\mathcal{D}$ is the space of the test functions and a constant C is independent of φ.

It turns out that always the following embedding with the norm estimate holds

$$M[H_p^s(\mathbb{R}^n) \to H_q^{-t}(\mathbb{R}^n)] \subset H_{q,\,unif}^{-t}(\mathbb{R}^n) \cap H_{p',\,unif}^{-s}(\mathbb{R}^n),$$

where $H^{\gamma}_{r,\,unif}(\mathbb{R}^n)$ is the so-called uniformly localized Bessel potential space.

In the case when $p \leqslant q$ and one of the following conditions

$$s \geqslant t \geqslant 0,\ s > n/p \quad \text{or} \quad t \geqslant s \geqslant 0,\ t > n/q' \quad (\text{where } 1/q + 1/q' = 1),$$

holds one has an explicit representation

$$M[H_p^s(\mathbb{R}^n) \to H_q^{-t}(\mathbb{R}^n)] = H_{q,\,unif}^{-t}(\mathbb{R}^n) \cap H_{p',\,unif}^{-s}(\mathbb{R}^n),$$

where p and p' are Holder conjugate numbers. Representations of this kind are impossible, provided that $s > n/p$ or $t > n/q'$. Then the spaces of multipliers should be described in terms of capacities. In the important case $s = t < n/\max(p, q')$ we can establish the two-sided embeddings with the norm estimates

$$H_{r_1,\,unif}^{-s}(\mathbb{R}^n) \subset M[H_p^s(\mathbb{R}^n) \to H_{q'}^{-s}(\mathbb{R}^n)] \subset H_{r_2,\,unif}^{-s}(\mathbb{R}^n),$$

where the numbers $r_1 > r_2 > 1$ can be written down explicitly.

The obtained results have important applications in the theory of differential operators with distribution coefficients. Some simple applications will be presented in the talk.

The talk is based on the joint papers with Alexey A. Belyaev. The work is supported by the Russian Scientific Fund, grant number No 17-11-01215.

The Riemann–Hilbert Problem for the Moisil–Teodoresku System

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The Moisil–Teodoresku system

$$M\left(\frac{\partial}{\partial x}\right)u(x) = 0, \quad M(\zeta) = \begin{pmatrix} 0 & \zeta_1 & \zeta_2 & \zeta_3 \\ \zeta_1 & 0 & -\zeta_3 & \zeta_2 \\ \zeta_2 & \zeta_3 & 0 & -\zeta_1 \\ \zeta_3 & -\zeta_2 & \zeta_1 & 0 \end{pmatrix}, \tag{1}$$

is considered in a bounded domain $D \subseteq \mathbb{R}^3$ with smooth boundary Γ. It is posed an analogue of the Riemann–Hilbert problem

$$Bu^+ = f \tag{2}$$

with (2×4)-matrixes

$$B = \begin{pmatrix} p_0 & p_1 & p_2 & p_3 \\ q_0 & q_1 & q_2 & q_3 \end{pmatrix},$$

whose rows are linear independent at each point of Γ.

Let us introduce the vector

$$l = p_0 q - q_0 p - [p, q],$$

where $p = (p_1, p_2, p_3)$, $q = (q_1, q_2, q_3)$. If Γ is homeomorphic to sphere the problem was investigated by V.I. Shevchenko[1]. He proved that after assumption

$$ln \neq 0, \tag{3}$$

where n is unit normal to Γ, the problem (1), (2) is Fredholmian and its index is equal to -1.

We give the analogues result for general domain D and we establish that under the assumption (3) the index of the problem is equal to $s - m - 1$. Here s is a number of connected component of Γ but m is an order of the first group cohomology $H^1(D)$ of the domain D. This result is based on the integral representation of a general solution of (1).

Note that the number m can be calculated explicitly. Let Γ_i, $1 \leq i \leq s$, be connected components of Γ and let m_i be genus of Γ_i. Then $m = m_1 + \ldots + m_s$.

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Compatible Topologies for Vector Spaces and Abelian Groups

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Dedicated to 130-th birthday anniversary of G. M. Fichtenholz

It can be said that the topic of my talk originates from articles [1] by Russian-Soviet mathematician Grigorii Mikhailovich Fichtenholz (June 5, 1888 – June 26, 1959) and [2] by American mathematician George Whitelaw Mackey (February 1, 1916–March 15, 2006) in which [1] is cited.

Given a real vector space X and a topology τ on it, let us write $(X,\tau)^*$ for the set of all τ-continuous linear functionals $f : X \to \mathbb{R}$. A topology η on X is said to be *compatible* with τ (or with the pair (X,Y), where $Y := (X,\tau)^*$) if $(X,\eta)^* = (X,\tau)^*$.

Similarly, given an Abelian group X and a topology τ on it, let us write $(X,\tau)^\wedge$ for the set of all τ-continuous group homomorphisms (characters) $\chi : X \to \mathbb{R}/\mathbb{Z}$. A topology η on X is said to be *compatible* with τ (or with the pair (X,Y), where $Y := (X,\tau)^\wedge$) if $(X,\eta)^\wedge = (X,\tau)^\wedge$.

We will survey some old and new results about compatible topologies in either cases. The talk is based mainly on [3]–[5].

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Braids, Lie Algebras and Homotopy Groups of Spheres

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We start with the standard definitions of braids as the system of curves in tree-dimensional space up to isotopy, and as a fundamental groups of configuration spaces. Then we present their classical properties and give some applications, in particular to Knot Theory.

Next we explain classical constructions based on braid groups, namely Lie algebras of pure and similar braids.

After that we introduce certain generalizations of braids as well as specific types of braids, in particular Brunnian braids. We describe the properties of these objects.

Connections between braids and homotopy groups of spheres will be given also.

The talk is based on author's survey articles [1, 2].

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Vibrodynamics as the Best Part of Applied Mathematics

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Vibrodynamics represents a high-impact and flourishing interdisciplinary research direction, unifying various phenomena and theories, which take place under the influence of time-oscillations and vibrations. Mathematically, Vibrodynamics deals with ODEs, PDEs, difference equations, integral equations, *etc.* with time-periodic (or oscillating) coefficients and/or oscillating right-hand sides. For the introducing this research direction to applied mathematicians, I start with its general idea, which can be classified as a two-timing method, or a multi-scale method, combined with an averaging method. The mathematical heart of the subject lies in the proper choosing of time-scales, which allows one to build up regular asymptotic solutions. I present the related ideas, which allow such a choice, using simple ODE examples. Then I give several winning examples, including the describing of self-propulsion of micro-robots and explanation of Langmuir circulations induced by water waves below the free surfaces of lakes, seas, and oceans. The latter case brought a theoretical breakthrough to the classical Craik-Leibovich equation and its generalizations to magneto-hydrodynamics, acoustics, and other areas of applied mathematics and physics. The talk is based on the papers quoted below.

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How Non-Positively Curved is the Mapping Class Group?

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The mapping class group Mod(S) of a surface S is the group of homeomorphisms $S \to S$ modulo the isotopy relation. It is also the (orbifold) fundamental group of the moduli space of Riemann surfaces. Particular examples include the braid group and the outer automorphism group of a surface group.

In this talk we shall discuss the study of the mapping class group from the point of view of geometric group theory. Geometric group theory is a flourishing and quickly-evolving area that has found many applications across various fields of mathematics from the solution of the virtual Haken conjecture in the study of 3-manifolds to the discovery of normal subgroups in the Cremona group.

A major theme in geometric group theory is the notion of non-positive curvature. Much can be learned about a group if it acts nicely by isometries on a non-positively curved space. There are two widely considered versions of non-positive curvature. The first is Gromov hyperbolicity which captures the large-scale geometry of the space and forgets the small scale. The second is more infinitesimal in flavour: a metric space is CAT(0) if all of its geodesic triangles are at least as thin as their comparison triangles in the Euclidean plane.

In the 90s Masur and Minsky proved that the mapping class group acts on an infinite-diameter, Gromov-hyperbolic metric space called the curve complex. Therefore some of the theory of Gromov-hyperbolic groups can be applied to study $\mathrm{Mod}(S)$. This has led to breakthroughs in the study of the geometry of the mapping class group and also its algebra e.g. for each countable group G there is some embedding $G \to Q$ for some quotient group Q of $\mathrm{Mod}(S)$. Despite much success, many notorious problems still remain open for $\mathrm{Mod}(S)$, which are much easier to answer for certain CAT(0) groups. For instance, does there exist a finite-index subgroup of $\mathrm{Mod}(S)$ with a surjective homomorphism to $\mathbb{Z}$? These problems would be more easily approached if $\mathrm{Mod}(S)$ had a suitable CAT(0) space to act on.

In this talk I will start by surveying some highlights of geometric group theory and the key properties of mapping class groups. I will explain why they are such interesting groups worth studying in their own right. I will define the curve complex and arc complex of S, and state their basic properties and applications to $\mathrm{Mod}(S)$. I will then prove that the arc complex does not admit a CAT(0) metric invariant under $\mathrm{Mod}(S)$ by using a theorem from combinatorics and some topology.

Abstracts of Participants' Talks

მონაწილეთა მოხსენებების თეზისები

Integral Modeling of the Filtration Process in Gas Wells

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The integral model of the non-stationary filtration process in gas wells is constructed and solutions of differential equations are presented. Methods are being developed to simplify the solution of the task. The obtained analytical expressions allow determining the parameters at the bottom hole and the formation by wellhead information, which is of great practical importance.

An integral model of the pressure build-up process is constructed and solutions of related differential equations are presented. An analytical expression is obtained to determine the dynamics of the pressure build-up process taking into account the dynamic connection between the formation and the well.

On Embedding Theorems between Variable Morrey Spaces

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In this abstract we introduce an embedding theorem between variable Morrey spaces.

We note that the variable Morrey spaces have recent history. The variable exponent Morrey spaces was introduced and studied in [1].

In particular, we obtained the embedding criterion between two different variable Lebesgue spaces (see [2]).

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Inverse System in the Category of Intuitionistic Fuzzy Soft Modules

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We introduce inverse system in the category of intutionistic fuzzy soft modules and prove that its limit exists in this category. Generally, limit of inverse system of exact sequences of intutionistic fuzzy soft modules is not exact. Then we define the notion $\varprojlim^{(1)}$ which is first derived functor of the inverse limit functor. Finally, using methods of homology algebra, we prove that the inverse system limit of exact sequence of intutionistic fuzzy soft modules is exact.

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Several New Inequalities about the Average

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Today, when science has achieved great progress in every aspect of math as a discipline, it is very difficult to find something new, even less important, such as not yet mentioned, despite this, math lovers try to find something new and by doing this, say their independent words in mathematics, which will not be repeated or replaced option. We think the subject of our research is just the latest and requires further survey to be more in-depth study of the issue.

Our task is to study the attitudes between the average values of positive, real numbers. In particular, as it is known that real, positive numbers can be determined by the following values: average harmonic, average geometric, average arithmetic, medium square, for some, some types of inequalities are true. These inequalities are very well known for those who love solving Olympic mathematical tasks. We think it is interesting that no information has been found in any of the mathematical books available at our own, nor on the Internet if the above mentioned four dimensions depend on each other, If we are going to pair them twice, specifically, if we compare the sum multiplication of the two and the sum multiplication of the rest. Of course, it is interesting to compare only the average square and the average harmonic "set" with the average arithmetic and average geometry set, because in other cases, the average square and its partner's advantages are obvious.

Several tasks discussed in the work are a kind of novelty, and we continue to work to get more interesting results, which will definitely introduce a wide audience.

On the Stochastic Property of the Continuous Transformations of Metric Compacts with n-adic Property

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The continues transformation $T : X \to X$ of the metric compact X induces a dynamic system $\{X, T\}$. The one of the stochastic property of the dynamic system is the existence of T-invariant the Borel ergodic measure on X entropy of which the is positive (see, [1]).

We proof that the continuous n-adic transformation of the metric compact has the analogous property.

Let n be an natural number. We say that the continuous transformation $T : X \to X$ has the n-adic property if there exist the the closed subsets $A_1, \dots, A_n$ from X such that

$$\bigcup_{k=1}^{n} A_k \subset \bigcap_{k=1}^{n} T(A_k) \ \text{ and } \ \bigcap_{k=1}^{n} A_k = \varnothing.$$

In addition, the cap of any $n-1$ elements from the set $A_1, \dots, A_n$ is nonempty.

Theorem. *If the continuous transformation $T : X \to X$ has the n-adic property, then on the metric compact X, there exists the Borel T-invariant ergodic measure with positive entropy.*

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Topology of Quadratic Endomorphisms of the Plane

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We discuss topological properties of quadratic endomorphisms of the plane. Let F be a quadratic endomorphism of the plane explicitly given by the components (F_1, F_2).

In particular, an algebraic criterion of properness of F is given in terms of coefficients of components F_1, F_2. Moreover, an algebraic formula for topological degree of map F using the signature formula of Khimshiashvili–Eisenbud–Levine. In addition a complete description of the possible structure of singularity set and bifurcation diagram of F is obtained.

The aforementioned results are used to obtain the criteria of surjectivity and stability of such an endomorphism. In special case, when F is the gradient of homogeneous polynomial of third degree, the structure of the local algebra at the origin is also determined.

The proofs are based on the normal forms of quadratic endomorphisms obtained in a recent paper *"Classification of critical sets and their images for quadratic maps of the plane"* (arXiv:1507.02732v1 [math.DS] 9 Jul 2015) by Chia–Hsing Nien, Bruce B. Peckham and Richard P. McGehee.

Keywords: quadratic map, endomorphism, singularity, critical set, topological degree of mapping.

The Mixed Problem for a System of Nonlinear Wave Equations with q-Laplasian Operators

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We study the initial boundary value problem

$$\begin{cases} u_{1tt} - \Delta_q u_1 + (-\Delta)^{\alpha} u_{1t} - f_1(u_1, u_2) = g_1(t,x), \\ u_{2tt} - \Delta_q u_2 + (-\Delta)^{\alpha} u_{2t} - f_2(u_1, u_2) = g_1(t,x), \end{cases}$$
$$u_k(0,x) = \varphi_k(x), \quad u_{kt}(0,x) = \psi_k(x), \quad x \in \Omega.$$

Here $t > 0$, $x \in \Omega$; $0 < \alpha_j \le 1$, $j = 1,2$; $f_1(u_1,u_2) = |u_1|^{\rho-1}|u_2|^{\rho+1}u_1$;

$$f_2(u_1,u_2) = |u_1|^{\rho+1}|u_2|^{\rho-1}u_2; \quad g_1(t,x), g_2(t,x) \in L_2([0,T]\times\Omega);$$
$$\Delta_q u = \sum_{i=1}^{\infty} \frac{\partial}{\partial x_i}\Big(\Big|\frac{\partial u}{\partial x_i}\Big|^{q-2}\frac{\partial u}{\partial x_i}\Big),$$

and Ω is a bounded domain in R^n, $n \ge 1$, with the smooth boundary $\partial\Omega$, $(-\Delta)^{\alpha}u = \sum\limits_{j=1}^{\infty} \lambda_j^{\alpha}(u,\varphi_j)\varphi_j$, where $0 < \lambda_1 < \lambda_2 \le \lambda_3 \le \cdots$, $\varphi_1, \varphi_2, \varphi_3, \ldots$ are the sequence of eigenvalues and eigenfunctions of $-\Delta$ in $H_0^1(\Omega)$, respectively.

Assume that q and ρ satisfy the conditions

$$0 < \rho < \frac{nq}{n-q} \ \text{ for } \ n > q,$$
$$0 < \rho < +\infty \ \text{ for } \ n \le q,$$
$$2 \le q < 2\rho + 1 \ \text{ or } \ q \ge \max\{2,\, 2\rho+1\}.$$

Under these conditions we investigate the existence and nonexistence of global solutions.

Solvability of a Boundary Value Problem for a Second Order Differential-Operator Equation with a Complex Parameter

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In separable Hilbert space H we consider the following boundary value problem for a second order elliptic differential equation:

$$L(\lambda, D)u := \lambda^2 u(x) - u''(x) + Au(x) = f(x), \quad x \in (0,1), \tag{1}$$

$$\begin{gathered} L_1(\lambda)u := u'(1) + (\beta_0 + \beta_1\lambda + \lambda^2)u(1) = f_1, \\ L_2 u := u(0) = f_2, \end{gathered} \tag{2}$$

where λ is a complex parameter; $D := \frac{d}{dx}$.

Theorem. *Let the following conditions be fulfilled:*

1. *A is a linear, closed, densely defined operator in H and $\|R(\lambda, A)\|_{B(H)} \le c(1+|\lambda|)^{-1}$ for $|\arg\lambda| \ge \pi - \varphi$, where $\varphi \in (0,\pi)$ is some number, $c > 0$ is some constant independent on λ.*
2. *β_0, β_1 are any complex numbers and $\beta_1 \neq 0$.*

Then the operator $\mathbb{L}(\lambda) : u \to \mathbb{L}(\lambda)u := (L(\lambda, D)u, L_1(\lambda)u, L_2u)$ for sufficiently large $|\lambda|$ from the angle $|\arg\lambda| \le \frac{\varphi}{2} < \frac{\pi}{2}$ is an isomorphisim from $W_p^2((0,1); H(A), H)$ to $L_p((0,1); H) \dotplus (H(A), H)_{\theta_1,p} \dotplus (H(A), H)_{\theta_2,p}$, where $\theta_1 = \frac{1}{2} + \frac{1}{2p}$, $\theta_2 = \frac{1}{2p}$, $p \in (1,\infty)$, and for these λ the following estimation is valid for solving the problems (1), (2)

$$|\lambda|^2 \|u\|_{L_p((0,1);H)} + \|u''\|_{L_p((0,1);H)} + \|Au\|_{L_p((0,1);H)} \le c\left[|\lambda|^2 \|f\|_{L_p((0,1);H)} + \sum_{k=1}^{2} \left(\|f_k\|_{(H(A);H)_{\theta_k,p}} + |\lambda|^{2(1-\theta_k)} \|f_k\|_H \right) \right].$$

Solvability of boundary value problems for second order differential-operator equations in the case when one and the same complex parameter is contained in the equation and in the boundary conditions, were studied in in different aspects in [1], [2].

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Optical Solutions in Higher Order Nonlinear Schrödinger Dynamical Equation

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In this research, we apply an analytical method on the modified Benjamin-Bona-Mahony which is one of the basic models in fluid mechanics and the coupled Klein-Gordon equations that is a relativistic version of the Schrodinger equation which considered as the basic model in the optical fiber. This method called extended simple equation method. We try to get exact and solitary wave solutions for both equations and we also try to investigate what is the difference between this method by making the comparison between the results that obtained by literature. We show how the method is very direct and powerful method and it ability to apply on different kinds of nonlinear evolution equations.

მათემატიკური ინდუქცია

ამირან ამბროლაძე

თბილისის თავისუფალი უნივერსიტეტის მათემატიკისა და კომპიუტერული მეცნიერების სკოლა, თბილისის თავისუფალი სკოლა, თბილისი, საქართველო

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მათემატიკური ინდუქცია არის ერთ-ერთი ყველაზე რთული თემა მათემატიკის სასკოლო პროგრამაში და არა მარტო სასკოლო პროგრამაში - უნივერსიტეტის სტუდენტებსაც ხშირად უჭირთ ამ პრინციპის სწორი გააზრება და გამოყენება.

ამ მოხსენებაში ვისაუბრებთ შემდეგ საკითხებზე:

1. ვიმსჯელებთ მათემატიკური ინდუქციის ვიზუალიზაციის ხერხებზე.
2. მოვიყვანთ არასწორი მსჯელობის რამდენიმე მაგალითს, სადაც არ არის ადვილი შეცდომის პოვნა.
3. მოვიყვანთ არასწორი მსჯელობის ერთ-ერთ ყველაზე გავრცელებულ მაგალითს, რასაც ხშირად ვპატიობთ სტუდენტებს (რადგანაც იშვიათად მივყავართ არასწორ შედეგამდე).
4. განვიხილავთ ძალიან უცნაურ ფაქტს, რაც გარკვეულად უნიკალურია დამტკიცების ამ მეთოდისთვის: ინდუქციის პრინციპით მეტის დამტკიცება შეიძლება უფრო ადვილი იყოს, ვიდრე ნაკლების (ანუ ზოგადს ვამტკიცებთ, მაგრამ ვერ ვამტკიცებთ კერძოს).

Column Sum Majorization

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Let $\mathbf{M}_{n,m}$ be the set of all $n \times m$ real or complex matrices. Let $c(A) := Ae$, where $e = (1, \dots, 1) \in \mathbb{R}^n$. For $A, B \in \mathbf{M}_{n,m}$, we say that A is column-sum majorized by B (written as $A \prec^{cs} B$) if $c(A) \prec c(B)$, i.e. there exists a doubly stochastic matrix D such that $c(A) = Dc(B)$. The structure of all linear operators $T : \mathbf{M}_{n,m} \to \mathbf{M}_{n,m}$ preserving or strongly preserving column-sum majorization will be characterized in this note. Also some other kinds of majorization are considered and their sum column majorization are investigated.

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On the Solvability of the Modification Cauchy Problem for Systems of Linear Impulsive Differential Equations with Singularities

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Let $I \subset R$ be an interval non-degenerate in the point, $t_0 \in I$, $I_{t_0} = I \setminus \{t_0\}$. Consider the linear system of impulsive differential equations with fixed points of impulses actions

$$\frac{dx}{dt} = P(t)\,x + q(t) \quad \text{for} \quad a.a. \quad t \in I_{t_0} \setminus \{\tau_l\}_{l=1}^{+\infty}, \tag{1}$$

$$x(\tau_l+) - x(\tau_l-) = G_l\,x(\tau_l) + g_l \ \ (l = 1, 2, \dots); \tag{2}$$

$$\lim_{t \to t_0+} \frac{x_i(t)}{|t - t_0|^{\mu_i}} = 0 \ \ (i = 1, \dots, n), \tag{3}$$

where $P \in L_{loc}(I_{t_0}, R^{n\times n})$, $q \in L_{loc}(I_{t_0}, R^n)$, i.e. P and q are, respectively, matrix- and vector-functions with integrable components on the every closed interval from I_{t_0}; $G_l \in R^{n\times n}$ $(l = 1, 2, \dots)$, $g_l \in R^n$ $(l = 1, 2, \dots)$, $\tau_i \neq \tau_j$ if $i \neq j$, $t_0 < \tau_{l+1} < \tau_l$ $(l = 1, 2, \dots)$ and $\lim_{l\to+\infty} \tau_l = t_0$; x_i is i-th component of the vector-function x for every $i \in \{1, \dots, n\}$, and $\mu_i \geq 0$ $(i = 1, \dots, n)$ are some numbers.

The singularity of the system (1), (2) is considered in the sense that the matrix P and vector q functions, in general, are not integrable at the point t_0.

We assume that $\det(I_n + G_l) \neq 0$ $(l = 1, 2, \dots)$, where I_n is the identity $n\times n$-matrix.

There are given the effective sufficient conditions for the existence of the unique solution of the problem (1), (2); (3). The solutions are finding in the set of the vector-functions whose restrictions on the every closed interval from the set I_{t_0} are absolutely continuous. In connection with these there is obtained the effective (un-improvable) condition guaranteeing absolutely continues of the restrictions of the solutions on the every closed interval from the interval I.

The analogous problem has been investigated in [1] for ordinary differential systems.

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On Asymptotics of the Function of Distribution of Spectrum for Higher Order Partial Operator-Differential Equation in Hilbert Spaces

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Let H be a separable Hilbert space. In Hilbert space $H_1 = L_2(H; R^n)$ we consider an operator

$$Lu = (-1)^m \sum_{k_1+k_2+\cdots+k_4=2m} A^{k_1k_2\cdots k_n}(x) \frac{\partial^{2m} u}{\partial x_1^{k_1} \partial x_2^{k_2} \cdots \partial x_n^{k_n}} + Q(x)\, u.$$

Here $x = (x_1, x_2, \ldots, x_n) \in R^n$, $A^{k_1k_2\cdots k_n}(x)$ are real valued functions bounded on all the space and satisfying the Lipshitz condition:

$$\left| A^{k_1k_2\cdots k_n}(x) - A^{k_1k_2\cdots k_n}(\xi) \right| \le k\, |x-\xi|^{\gamma} \quad \text{if} \quad |x-\xi| < 1, \quad 0 < \gamma < 1.$$

We suppose that the form of the leading terms is uniformly elliptic, i.e.

$$C_1 |\xi|^{2m} \le \sum_{k_1+k_2+\cdots+k_n=2m} A^{k_1k_2\cdots k_n}(x)\, S_1^{k_1} S_2^{k_2} \cdots S_n^{k_n} \le C_2 |\xi|^{2m},$$

where C_1, C_2 are positive constants.

Under some assumptions with respect to the operator function $Q(x)$ we show that the operator L has a discrete spectrum, and we find asymptotic formula for the function of distribution of eigenvalues of the operator L.

We note that, for a scalar operator of higher order given in all the space R^n, discreteness of spectrum and asymptotic distribution of eigenvalues were studied by A. G. Kostyucenko. Spectrum and asymptotic distributions of eigenvalues for elliptic operators given in bounded or unbounded domains were investigated by S. Clark, T. Karleman, G. I. Aslanov, Sh. G. Baimov.

On Two-Dimensional Models of Thermoelastic Shells in the Framework of Green–Lindsay Nonclassical Theory of Thermoelasticity

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The present paper is devoted to the investigation of Green-Lindsay nonclassical three-dimensional model of thermoelastic bodies, and construction and investigation of two-dimensional hierarchical models of shells in curvilinear coordinates in the framework of the three-dimensional model. We consider the nonclassical three-dimensional model, which was obtained by A. E. Green and K. A. Lindsay [1] to eliminate shortcomings of the classical thermoelasticity. Note that in Green-Lindsay model the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times.

Applying variation formulation and suitable a priori estimates the existence and uniqueness of solution of the linear three-dimensional initial-boundary value problem is proved for anisotropic inhomogeneous thermoelastic bodies. For general thermoelastic shells with variable thickness, which may vanish on a part of the lateral surface, two-dimensional hierarchical models in curvilinear coordinates are constructed by applying spectral approximation method, which is a generalization of the dimensional reduction method suggested by I. Vekua [2] in the theory of elasticity for plates with variable thickness. Note that the classical Kirchhoff-Love and Reissner-Mindlin models can be incorporated into the hierarchy obtained by I. Vekua so that it can be considered as an extension of the frequently used engineering plate models.

The existence and uniqueness of solutions of the obtained two-dimensional initial-boundary value problems is proved in suitable spaces of vector-valued distributions. The relationship between the hierarchy of dynamical two-dimensional models of thermoelastic shells obtained from Green-Lindsay model and the original three-dimensional initial-boundary value problem is investigated. The convergence of the sequence of vector-functions of three space variables constructed from the solutions of the reduced problems to the exact solution of the original three-dimensional initial-boundary value problem is proved in the corresponding Sobolev spaces pointwise with respect to the time variable and under additional conditions estimate of the rate of convergence is obtained. Note that the first approximations of the constructed hierarchies of two-dimensional initial-boundary value problems can be considered as independent nonclassical models for thermoelastic shells and can be used for mathematical modeling of engineering structures.

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About One Test for Homogeneity

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The new test for homogeneity for $p \geq 2$ independent samples based on Parzen's type estimators of distribution density is constructed. The limiting power of the constructed tests is found for Pitman's type "close" alternatives. Also is considered the comparison of constructed tests with Pearson's chi-square test for two samples. For this is found the limiting power of chi-square homogeneity test for above-mentioned alternatives. It is established the limiting power of constructed test is grater then the limiting power of chi-square homogeneity test.

On Some Goodness-of-Fit Tests Based on Wolverton–Wagner Type Estimates of Distribution Density

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Let $X_1, X_2, \dots, X_n$ be a sequence of independent, equally distributed random variables, having a distribution density $f(x)$. Based on sample $X_1, X_2, \dots, X_n$ it is required to check the hypothesis

$$H_0 : f(x) = f_0(x).$$

here we consider the hypothesis H_0 testing, based on the statistics

$$T_n = na_n^{-1} \int (f_n(x) - f_0(x))^2 r(x) dx,$$

where $f_n(x)$ is the recurrent Wolverton-Wagner kernel estimate of probability density defined by:

$$f_n(x) = n^{-1} \sum_{i=1}^{n} a_i K((a_i(x - X_i))),$$

where a_i is an increasing sequence of positive numbers tending to infinity, $K(x)$, $f_0(x)$ and $r(x)$ satisfy certain regularity conditions.

1. Question of consistency for the constructed criterion against any alternative $H_1 : f(x) = f_1(x)$, where $f_1(x)$ is such that $\int (f_n(x) - f_0(x))^2 r(x)\,dx > 0$ is studied.

2. The limiting behavior of the power is studied for sequence of close to hypothesis H_0 alternatives of type Pitmen and Rosenblatt [1] and it is shown that the tests based on T_n for above mentioned alternatives are more powerfull in limits than the tests based of Bickel–Rosenblatt [2].

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Non-Classical Problems for Second Order Quasi-Linear Equations with Rectilinear Characteristics

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One class of second order quasi-linear equations with rectilinear characteristics is considered [1]. For this special class of equations a general integral is constructed in terms of characteristics invariants. By using the method of characteristics, some variants of non-local problems are investigated. The conditions of existence of regular solutions are obtained.

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On Fejer–Steinhaus Theorem

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According to [1, Notes and remarks to Ch. VIII, p. 382] the following theorem was proved by L. Fejer [2] and H. Steinhaus [3].

Theorem 1 ([1, Ch. VIII, Theorem 1.13, p. 300])**.** *There exists a continuous function whose Fourier series converges pointwise, but not uniformly.*

Based on [4]we will discuss the questions whether the function from Theorem 1 can be taken odd or even.

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The (Co)shape and (Co)homological Properties of Continuous Maps

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The purpose of this paper is to investigate continuous maps from the standpoint of geometric topology and algebraic topology. Using a direct system approach and an inverse system approach of continuous maps, we study the (co)shape and (co)homological properties of continuous maps. Applications of the obtained results include:

I. Constructions of long exact sequences of continuous maps for the (co)homology pro-groups, (co)homology inj-groups, spectral Čech (co)homology groups, spectral singular (co)homology groups, Chogoshvili projective (co)homology groups groups.

II. Axiomatic characterizations of spectral and projective (co)homology groups without using the relative (co)homology groups.

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On (Co)homological Properties of Stone–Čech Compactifications of Completely Regular Spaces

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Using the set of functionally open finite covers of completely regular spaces in the paper are constructed Čech type functional homology functor $\check{H}^F_q(-,-;G) : \mathbf{Top^2_{cr}} \to \mathbf{Ab}$ and functional cohomology functor $\widehat{H}^q_F(-,-;G):\mathbf{Top^2_{cr}}\to\mathbf{Ab}$ from the category of pairs of completely regular spaces and their completely closed subspaces to the category of abelian groups, defined Bokstein–Nowak type functional coefficient of cyclicity $\eta^F_G:\mathbf{Top_{cr}}\to\mathbf{N}\cup\{-\mathbf{1},\infty\}$ from the class of completely regular spaces to the set of integers $t\geq -1$, proved the equalities $\check{H}^F_n(X,A;G)=\check{H}_n(\beta X,\beta A;G)$, $\widehat{H}^n_F(X,A;G)=\widehat{H}^n(\beta X,\beta A;G)$ and $\eta^F_G(X)=\eta_G(\beta X)$, where $\check{H}_n(\beta X,\beta A;G)$, $\widehat{H}^n(\beta X,\beta A;G)$ and $\eta_G(\beta X)$ are Čech homology group, Čech cohomology group and Bokstein–Nowak coefficient of cyclisity of Stone–Čech compactifications of pair $(X,A)\in ob(\mathbf{Top^2_{cr}})$ and space $X\in ob(\mathbf{Top_{cr}})$, respectively.

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Functions with the Thick Graphs and Measure Extension Problem

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The problem of investigating the measurability of sets and functions with respect to a concrete measure m on a base (ground) set E, to turn attention to the more general question of investigating the measurability of sets and functions with respect to a given class $\mathcal{M}$ of measures on E. We study the measurability properties of sets and real-valued functions with respect to various classes $\mathcal{M}$ of measures on the base set E.

We say that a function f is relatively measurable with respect to the class $\mathcal{M}$ if there exists at least one measure $\mu \in \mathcal{M}$ such that f is measurable with respect to μ.

Let (E_1, S_1, μ_1) and (E_2, S_2, μ_2) be measurable spaces equipped with sigma-finite measures. We Recall that a graph $\Gamma \subset E_1 \times E_2$ is $(\mu_1 \times \mu_2)$-thick in $E_1 \times E_2$ if for each $(\mu_1 \times \mu_2)$-measurable set $Z \subset E_1 \times E_2$ with $(\mu_1 \times \mu_2)(Z) > 0$, we have $\Gamma \cap Z \neq 0$.

Notice that, the thickness of graphs is pathological phenomenon for subsets of basic set. However, this feature plays an essential role in the problem of extensions of measures.

Theorem 1. *Let E_1 be a set equipped with a sigma-finite measure μ and let $f : E_1 \to E_2$ be a function satisfying the following condition: there exists a probability measure μ_2 on $ran(f)$ such that the graph of f is a $(\mu_1 \times \mu_2)$-thick of the product set $E_1 \times ran(f)$. Then there exists the measure μ' such that:*

1) *μ' is measure extending μ_1;*
2) *f is relatively measurable with respect to μ'.*

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Triangulation and the Graphs Associated with a Triangulation

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Certain triangulations of simple polygons in the plane $\mathbf{R}^2$ are presented and the graphs associated with these triangulations are considered. Analogous questions are studied for simple polyhedrons in the space $\mathbf{R^3}$. This topic is central in modern combinatorial geometry (see, for instance, [1]–[4]).

Let P be a simple polygon. The partition of the interior of P in triangles, by means of a set of non-crossing diagonals is called a triangulation of the polygon. Similarly, one can define a triangulation of a simple polyhedron in $\mathbf{R^3}$ into tetrahedrons, without adding new vertices.

Also, one can define some kinds of geometric graphs which are canonically associated with a triangulation $\mathbf{T} \in T(P)$, in particular, the flip graph and the dual graph.

We call the flip graph of a triangulation of P the graph, whose nodes are all the triangulations of P and whose edges are determined by elementary operations between the nodes.

We call the dual graph of a triangulation the graph whose vertices are some interior points of triangles of the triangulation (exactly one point in each triangle) and the edges connect those nodes which correspond to neighboring triangles (see [1], [3]).

In analogous way the concepts of flip graph and dual graph are introduced for triangulations of simple polyhedrons in $\mathbf{R^3}$.

We study some combinatorial properties of the dual graphs of triangulations in the space $\mathbf{R^3}$ and compare those properties with the ones of the dual graphs of triangulations in the plane $\mathbf{R^2}$.

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The Law of Large Numbers for Weakly Correlated Random Elements in Hilbert Spaces

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In this communication the law of large numbers for weakly dependent random elements with values in separable Hilbert spaces is presented and proved.

One Kind of Olympic Tasks in Mathematical School Curriculum and Methodical Peculiarities of Teaching Their Solutions

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Developing creative thinking of students has a strong effect on such tasks, which are not directly solved by famous algorithms. The process of solving such tasks requires cognitive thinking, which enhances the joy of finding the way of solving the task, which is an emotional factor and is a powerful tool for students' behavior. Its proper management has an utmost importance in all areas of human activity, in the process of forming a student as a perfect person and in the teaching process.

By the analysis of math teaching, it's enacted that the majority of students cannot solve the olympic tasks and the main reason is that the teaching program is almost never considered to teach students how to solve olympic tasks.

There is discussed some of the solution of olympic tasks related to division of numbers, which can be included in mathematics school courses, because they are selected by didactic principles and taking into consideration the age peculiarities of students, it serves to deepen and expand the study material, It is relevant to the level of intellectual development of students and has a developmental function.

In the process of solving such tasks, theorems related to division of whole and natural numbers are often used which should be delivered to students for introduction. Tasks with practical content are discussed, which may be used by teachers when passing relevant topics at the lesson for which there is no need for additional training time. The teacher can also use relatively difficult tasks in or out the classroom work also extra math lessons.

An experienced and innovative teacher can make up similar tasks and use them by his/her opinion in classroom or extracurricular work, thus enriching the area of tasks discussed in classroom.

Uniqueness Theorem of Exact Homology Theory on the Category Mor_C

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In the paper [1] Hu gave the new axiomatic system on the category of CW complexes and proved the uniqueness theorem for an exact homology theory. In this paper we formulate the Hu type axioms on the category Mor_{CW} of morphisms of the category CW and find the relation to the Hu's axioms. Using the methods developed in [2] and [3] we define an non-trivial extension of homology theory from the category Mor_{CW} to the category Mor_C and prove the uniqueness theorem.

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Solution an m-Point Nonlocal Boundary Value Problem for the Helmholtz Equations with Mathcad

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Let the domain $\overline{G}$ be a rectangle, $\overline{G}=[0,1]\times[0,1]$, Γ be the boundary of the domain G, $0<x_1<x_2<\cdots<x_m<1$, $\gamma_k=\{(x_k,y):\ 0\leq y\leq 1\}$, $k=1,\dots,m$, $\gamma=\{(1,y):\ 0\leq y\leq 1\}$, $f\in L_p(G)$, $p>2$, $0\leq q\in L_\infty(G)$. In the domain $\overline{G}$ we consider the following Bitsadze–Samarski boundary value problem for Helmholtz Equation:

$$\begin{gathered}
\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-q(x,y)u=f(x,y),\ \ (x,y)\in G,\\
u(x,y)=0,\ \ (x,y)\in\Gamma\setminus\gamma,\\
u(1,y)=\sum_{k=1}^{m}\sigma_k u(x_k,y),\ \ 0\leq y\leq 1,\ \ \sum_{k=1}^{m}\sigma_k<1,\ \ k=1,\dots,m.
\end{gathered}\tag{1}$$

For solving the problem (1), we consider the following iterative process

$$\begin{gathered}
\frac{\partial^2 u^{n+1}}{\partial x^2}+\frac{\partial^2 u^{n+1}}{\partial y^2}-q(x,y)u^{n+1}=f(x,y),\ \ (x,y)\in G,\\
u^{n+1}(x,y)=0,\ \ (x,y)\in\Gamma\setminus\gamma,\\
u^{n+1}(1,y)=\sum_{k=1}^{m}\sigma_k u^{n}(x_k,y),\ \ 0\leq y\leq 1,\\
\sum_{k=1}^{m}\sigma_k<1,\ \ k=1,\dots,m,\ \ n=0,1,2,\dots\,.
\end{gathered}\tag{2}$$

For each $n\in N$, problem (2) is a Dirichlet problem. For the numerical solution of the Dirichlet problem built-in functions was used Relax$\,(a,b,c,d,e,f,u,rjac)$ on Mathcad. In particular, for the Helmholtz equation coefficients are $a_{i,j}=b_{i,j}=c_{i,j}=d_{i,j}=1$, $e_{i,j}=-4-q_{i,j}$.

The iterative process in Mathcad was recorded by means of a software unit. The results of numerical solutions are presented graphically.

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ზოგად რგოლებზე განსაზღვრული მოდულების გეომეტრიული ასახვები

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ნაშრომში შესწავლილია ინვარიანტული ბაზისის მქონე რგოლებზე განსაზღვრული მოდულების გეომეტრიული ასახვები, კერძოდ კოლინეაციები, ჰარმონიული და პერსპექტიული ასახვები. ნაჩვენებია, რომ კოლინეაციები ინდუცირდება მოდულების ნახევარწრფივი ასახვებით, ჰარმონიული ასახვები კი ინდუცირდება წრფივი ასახვებით და რგოლების იზომორფიზმებით ან ანტიმორფიზმებით. შესწავლილია ჰარმონიული ოთხეულების ინვარიანტობა კოლინეარული ასახვების შემთხვევაში. პერსპექტიული ასახვები კი ინდუცირდება წრფივი ასახვებით დამატებული იგივური ავტომორფიზმით.

შენიშვნა: რგოლების ანტიიზომორფიზმი ეწოდება ისეთ ასახვას, რომელიც ინახავს კომუტატიურობის ოპერაციას ან ანტიკომუტატიურობის ოპერაციას. მოდულების ასახვას ეწოდება ანტიიზომორფიზმი, თუ იგი არის წრფივი ასახვა, ხოლო გამრავლების ოპერაციის მიმართ, კი კომუტაციური ან ანტიკომუტატიური.

Solution to the Two-dimensional Dynamic Problem of Thermodiffusion

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The paper deals with the investigation of a plane dynamic problem of the conjugate theory of thermodiffusion with mixed boundary conditions for multiple-connected domains. By the potential method, singular integral equations and Laplace transform, the theorems of existence and uniqueness of the solution are proved.

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Strongly Cofinitely $\oplus$-Supplemented Lattices

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In this work, strongly cofinitely $\oplus$-supplemented lattices are defined and some properties of these lattices are investigated. All lattices are complete modular lattices in this work. Let L be a cofinitely supplemented lattice. Then $1/r(L)$ is strongly cofinitely $\oplus$-supplemented.

Definition. Let L be a cofinitely supplemented lattice. If every supplement of any cofinite element of L is a direct summand of L, then L is called a strongly cofinitely $\oplus$-supplemented lattice.

Proposition 1. *Let L be a lattice with $(D1)$ property. Then L is strongly cofinitely $\oplus$-supplemented.*

Proposition 2. *Let L be a strongly cofinitely $\oplus$-supplemented lattice and a be a direct summand of L. Then $a/0$ is also strongly cofinitely $\oplus$-supplemented.*

Proposition 3. *Let L be a strongly cofinitely $\oplus$-supplemented lattice, $a \in L$ and $a = (a \wedge m) \oplus (a \wedge n)$ for every $m, n \in L$ with $m \oplus n = 1$. Then $1/a$ is strongly cofinitely $\oplus$-supplemented.*

Key words: lattices, small elements, supplemented lattices, cofinitely supplemented lattices.

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The Possibility of Devising a New Type of Logic Based on Both of Fuzzy Logic and Description Logic

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In this talk, we show by the very simple way the possibility of devising a new type of logic, based on Both of Fuzzy Logic and Description Logic.

In the previous, the scientists and researchers had found out the most suitable way for dealing with uncertainty and fuzziness of any kind of concepts by combining the Description Logic which is suitable, for managing structured knowledge and well-defined concepts, i.e. set of individuals with common properties. And the Fuzzy Logic which based on “degrees of truth” rather than the usual “true or false”.

Description Logic is limited in dealing with certain kinds of concepts which they do not have precisely defined criteria of membership, i.e. they are vague concepts like (height, weight, illness, happiness, etc.) Fuzzy logic can deal with such kind of vague concepts or even Uncertainty like (it will snow tomorrow, it will rain tomorrow, etc.)

The simple idea of the new logic is to generalize the dealing with all concepts by the same measure and vision, by using “degrees of truth” rather than using the usual and classical “true or false”, whatever the concepts are well-defined concepts, or they do not have precisely defined criteria of membership, i.e. they are vague or uncertain concepts and this new type of logic will allow the construction of the ontology, in a way that is closer to human thinking with its different probability of right and wrong, and thus will be a radical change in how to deal with computer applications that are used in different directions of life.

One Nonlinear Characteristic Problem for Nonlinear Oscillation

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In this work an attempt is made to state correctly one characteristic problem for a quasilinear equation, which arises in studying nonlinear oscillations. The conditions of the problem are set forth to various families. The problem makes it possible to simultaneously define regular solutions and its extension domains.

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Representation Formulas of General Solutions to the Static Equations of the Thermoelasticity Theory of Microstrech Materials with Microtemperature

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We consider the Two-dimensional differential equations of statics of the theory of thermoelasticity of microstrech materials with microtemperatures. The representation formula of a general solution of the homogeneous system of differential equations in the paper is expressed by means of three harmonic and four metaharmonic functions. These formulas are very convenient and useful in many particular problems for domains with concrete geometry. Here we demonstrate an application of these formulas to the Dirichlet and Neumann type boundary value problem for a circle. Solutions of the considered problems are obtained in the form of absolutely and uniformly convergent series.

Teaching Methods of Inverse Trigonometric Functions in Secondary School

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Exercise of Inverse trigonometric functions in the secondary school's current textbooks is given in the volume that is necessary for the solution of the simplest trigonometric equations and inequality. We believe that these issues require expansion in order to better understand the essence of the Inverse trigonometric function and the use of them to explore a more extensive circle of tasks. The report provides a methodical approach.

Finite Element Method for Lame Equation on Surface in Günter's Derivatives

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We investigate a mixed boundary value problem for Lame equation in a thin layer around a surface S with the boundary. We trace what happens in Γ-limit when the thickness of the layer tends to zero. The limit BVP for the Lame equation on the surface is described explicitly.

We prove that this problem possesses a unique solution in appropriate Bessel potential space. For this we apply the variational formulation and the calculus of Günter's tangential differential operators on a middle surface and layers, which allow global representation of basic differential operators and of corresponding boundary value problems in terms of the standard Euclidean coordinates of the ambient space $\mathbb{R}^n$.

We describe the discrete counterpart of the problem based on Finite Element Method. Employing Korn's inequalities we prove the existence and uniqueness of approximated solutions in suitable finite dimensional spaces and their convergence to the solution of the corresponding boundary value problem For Lame Equation On middle surface. We obtain this approximate solution in explicit form.

Teaching Perfect and Friendly Numbers at the First Level

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In the case of building the perfect number theory the first important steps were moved by Euclid, who gave us a "perfect number" formula in its "Initials" (Book IX). Leonard Euler conducted a serious study on friendly numbers in 1747–1750 And with its unique research discovered 60 new pairs of friendly numbers. At present, there are about 1100 pairs of friendly numbers.

Convexity of Certain Integral Operators Defined By Mittag–Leffler Functions

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In this paper, our aim is to study the convexity of certain integral operators defined by normalized Mittag–Leffler functions in the open unit disk.

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Intensification of Internal Gravity Waves in the Atmosphere – Ionosphere at Interaction with Nonuniform Shear Winds

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Intensification and further dynamics of internal gravity waves (IGW) in the ionosphere with non-uniform zonal wind (shear flow) is studied. It is revealed that the transient amplification of IGW disturbances due time does not flow exponentially, but in algebraic - power law manner. The frequency and wave-number of the generated IGW modes are functions of time. Thus in the ionosphere with the shear flow, a wide range of wave disturbances are produced by the linear effects, when the nonlinear and turbulent ones are absent. The effectiveness of the linear amplification mechanism of IGW at interaction with non-uniform zonal wind is analyzed. It is shown that at initial linear stage of evolution IGW effectively temporarily draws energy from the shear flow significantly increasing (by order of magnitude) own amplitude and energy.

An Extension of the Mixed Novikov–Kazamaki Condition

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Given a continuous local martingale M, the associated stochastic exponential $E(M) = \exp\{M - \frac{1}{2}\langle M\rangle\}$ is a local martingale, but not necessarily a true martingale. To know whether $E(M)$ is a true martingale is important for many applications, e.g., if Girsanov's theorem is applied to perform a change of measure. We give a several generalizations of Kazamaki's results and finally construct a counterexample which does not satisfy the mixed Novikov-Kazamaki condition, but satisfies our conditions.

Mathematical Model of Economic Cooperation Between the Two Opposing Sides

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The paper considers a nonlinear mathematical model of economic cooperation between two politically mutually opposing sides (possibly a country or a country and its subject) that takes into account economic or other type of cooperation between parts of the population of the sides aimed at convergence and peaceful resolution of the conflict. The model implies that the process of economic cooperation is free from political pressure, i.e. the governments of the sides and the third external side does not interfere in this process. A dynamic system has been obtained that describes the dynamics of parts of the population of the sides, focused on cooperation. The model also assumes that both sides have a zero demographic factor, i.e. during the process, the sum of supporters and opponents of cooperation is unchanged. In the case of constancy of the coefficients of the mathematical model, singular points of the nonlinear system of differential equations are found. The problem of stability of solutions is studied.

In the case of some dependence between the constant coefficients of the model, the first integral and the exact analytic solution are found. The exact solution obtained allows, within the limits of the given mathematical model and the dependence between its coefficients, to determine the conditions under which economic cooperation can peacefully resolve a political conflict (most of the populations of the sides want conflict resolution).

Nonlinear Mathematical Model of Process of Three-Level Assimilation

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Earlier we have offered nonlinear mathematical models of processes of bilateral and two-level assimilation [1]–[4].

In this work the new nonlinear mathematical model of process of three-level assimilation which is described by four-dimensional dynamic system is offered:

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1(t)u(t) + \beta_1(t)u(t)v(t) + \beta_2(t)u(t)w(t) + \beta_3(t)u(t)z(t), \\ \dfrac{dv(t)}{dt} = \alpha_2(t)v(t) - \beta_4(t)u(t)v(t) + \beta_5(t)v(t)w(t) + \beta_6(t)v(t)z(t), \\ \dfrac{dw(t)}{dt} = \alpha_3(t)w(t) - \beta_7(t)u(t)w(t) - \beta_8(t)v(t)w(t) + \beta_9(t)w(t)z(t), \\ \dfrac{dz(t)}{dt} = \alpha_4(t)z(t) - \beta_{10}(t)u(t)z(t) - \beta_{11}(t)v(t)z(t) - \beta_{12}(t)w(t)z(t), \end{cases} \tag{1}$$

$$u(0) = u_0, \quad v(0) = v_0, \quad w(0) = w_0, \quad z(0) = z_0, \tag{2}$$

$$\alpha_1(t) < 0, \quad \alpha_4(t) > 0, \quad \beta_i(t) > 0, \quad i = \overline{1,12}, \quad u, v, w, z \in C^1[0,T], \quad t \in [0,T].$$

In case of constancy of coefficients of model special points of dynamic system (1), (2) are found. Conditions on constants of coefficients of model at which special points are located in that part of four-dimensional space for which points all four coordinates are not negative are found.

At some ratios between constant coefficients of model the first integral (1),(2) which represents a three-dimensional surface in four-dimensional space is found.

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Mathematical Model of Competition Between Two Universities

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The paper deals with a nonlinear mathematical model of competition between two universities, which takes into account both competition for a limited contingent of enrollees and the attraction of students due to mobility (the transition of students from one university to another). Dynamic system describing quantitative dynamics (flows) as students of two universities and enrolles is received. In the case of constancy of the coefficients of the mathematical model, singular points of the nonlinear system of differential equations are found. Using the Routh–Hurwitz stability criterion, the question of the asymptotic stability of solutions is studied.

Predicting the Results of Political Elections with the Help of Mathematical and Computer Modeling

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At the moment it is actual to create a mathematical model, which would give an opportunity to define the dynamics of change in the number of supporters of different political subjects during the election period and a possible forecast of the election results.

The nonlinear mathematical models of two and three party elections were proposed in the works [1, 2], without transformation, i.e. without changing the number of competing parties in the period from elections to elections.

In [3] a mathematical and computer model of political elections is considered with subsequent forecasting of election results in case of an increase of the number of competing political parties from two to three between the elections.

This work considers the dynamics of the election process in the event of an increase or decrease of the number of participants in the period between elections. Numerous computer calculations were performed and the corresponding graphic illustrations were made, in which, depending on the choice of variable coefficients and initial data, various forecasts of election results were obtained.

The results of the numerical account can be used by both the ruling and opposition parties by selecting parameters and choosing the future strategy for achieving the desired goal.

The model makes it possible to create a database for different countries (with information on previous political elections in these countries) and to do the subsequent forecasting of the upcoming elections with a certain probability.

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Consistent Estimator of Tbilisi City Lepl Public Schools Internal Resources Financial Priorities of Addition Models of Regression

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Is carried out the sociological survey of Tbilisi city public schools (VIII–IX–X–XI classes) and pedagogues. Are identified factors that are presented the priorities of financial provision of internal resources. As a result of factorial analysis (IBM. SPSS, version 20) are determined the coefficients of those factors are constructed their consistent estimators.

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Provide Multilevel Access to Information Systems Using QR-Codes

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Any modern information system in its architecture implies the need to provide multilevel access to data. Existing mechanisms of customer interaction are oriented to users working with desktop computers and it is quite inconvenient to distribute mobile devices. The present work is presented by our protocol created by providing a security layer in the standard QR code and creation of comfortable conditions for the user. Theoretical and practical realization of this protocol is described on the basis of Application on Android Operating System, which can be used in components of smart buildings or smart cities.

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Numerical Modelling of Dust Aerosols Activity in Forming the Regional Climate of Georgia

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In the present study with the view of finding out the details of the dust aerosols influence on the Georgian climate change some numerical experiments were performed by WRF and RegCM models. Toward this purpose we have executed as short term

(WRF/Chem/dust) as well long term (RegCMv.4.7) calculations. Namely sets of 30 years simulations (1985-–2014) with and without dust effects has been executed by RegCM 4.7 model with 16.7 km resolution (over the Caucasus domain) and with 50 km resolution (encompassing most of the Sahara, the Middle East, the Great Caucasus with adjacent regions). Results of calculations have shown that dust aerosol is an inter-active player in the climate system of Georgia. Numerical calculations have shown that mineral dust aerosol influenced on temperature and precipitations (magnitudes) spatial and temporally inhomogeneous distribution on the territory of Georgia and obtained results generally agreed with MODIS satellite data.

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Calculation of Gas Non-Stationary Flow in Inclined and Branched Pipeline

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Natural gas distribution networks are complex systems with hundreds or thousands of kilometers of pipes, compression stations and many other devices for the natural gas transportation and distribution service. In the gas transmission pipelines to achieve the power consumption points with the required conditions is the main and the most difficult issue. For solving this problem properly determination of the gas pressure and flow rate distribution along the pipeline is necessary step. Searching of the gas flow pressure and flow rate distribution along the inclined and branched pipeline network is the more difficult issue. For this reason development of the mathematical models describing the non-stationary processes in the branched, inclined pipeline systems are actual. The purpose of this study is determination of gas pressure and flow rate special and temporally distribution along the inclined and branched pipeline. A simplified mathematical model (based on the hypothesis that the boundary conditions do not change quickly and the capacity of gas duct is relatively large) derived from the nonlinear system of one-dimensional partial differential equations governing the dynamics of gas non-stationary flow in the inclined, branched pipeline is obtained. In this case gas pressure special and temporally distribution along the branched pipeline is presented. Some results of numerical calculations of gas flow in the inclined branched pipelines are presented.

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Nonlocal Contact Problems for Some Non-Stationary Linear Partial Differential Equations with Variable Coefficients (The Method of Separation of Variables)

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Nonlocal boundary and initial-boundary problems represent very interesting generalizations of classical problems. At the same time, they quite often arise during the creation of mathematical models of real processes and the phenomena in physics, engineering, ecology, etc.

In the present report, the initial-boundary problems with nonlocal contact condition is investigated for non-stationary linear partial differential equations with variable coefficients. For the solution of these problems a method of separation of variables (also known as the Fourier method) is considered. Existence and uniqueness of regular solution is proved.

The Prevention of Expected Mistakes for the Evaluation of Pupils

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Evaluation of pupils is an inseparable part of an educational process. Right evaluation and purposefully designed exercises are important because a teacher plans every further step considering the results of evaluation, what to focus on, what activities to deploy to plan pupil-performance-oriented teaching.

Right evaluation of pupils should assist their development, the discovery of their abilities and hence, the adequate response from the teacher.

Sometimes, the teacher assigns problems to pupils to evaluate their competence; sometimes they give them tests with proper content by which pupils are expected to "fulfill" the goal of the teacher to evaluate him/her. Regretfully, as practice has shown, pupils can solve these types of problems with so-called ”correct answers” without clear idea about the knowledge of the issue the teacher wanted from him/her to understand with the means of these problems.

It is essential that huge attention and careful consideration should be practiced during evaluation and the process of designing math problems. The problems should be designed exactly in the way to serve the achievement of the goal.

This work contains some problems with their solutions, also, the prevention of expected mistakes is included.

სადოქტორო თემა - „ქართული ტექსტების ავტომატური ინტელექტუალური კლასიფიკაციის მეთოდები და ინსტრუმენტები“ - მიზნების, ამოცანებისა და მეთოდების ზოგადი მიმოხილვა

კონსტანტინე დემურჩევი, კონსტანტინე ფხაკაძე

ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი,
საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

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2017 წლიდან საქართველოს ტექნიკური უნივერსიტეტის „ინფორმატიკის“ სადოქტორო პროგრამის ფარგლებში ამოქმედდა სადოქტორო თემა „ქართული ტექსტების ინტელექტუალური კლასიფიკაციის მეთოდები და ინსტრუმენტები“ [1] (დოქტორანტი - კ. დემურჩევი, ხელმძღვანელი - სტუ ქართული ენის ტექნოლოგიზების ცენტრის დირექტორი პროფ. კ. ფხაკაძე), რომელიც ძირეულად ეყრდნობა სტუ ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [2] ორწლიანი ქვეპროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ ფარგლებში ფხაკაძის ქართული ენის ლოგიკურ გრამატიკაზე დაყრდნობით შემუშავებულ მოსაუბრე ქართულ ინტელექტუალურ ვებ-კორპუსს [3]. ამასთან, სადოქტორო კვლევის მიზანია ქართული ტექსტების ავტომატური ინტელექტუალური კლასიფიკაციის მეთოდებისა და ინსტრუმენტების შემუშავება. ეს გასაგებს ხდის ამ კვლევის მჭიდრო კავშირს გრძელვადიანი პროექტით „ქართული ენის ტექნოლოგიური ანბანი“ საქართველოს ტექნიკურ უნივერსიტეტში ქართული ენის დაცვის მიზნით 2012 წლიდან მიმდინარე კვლევებთან [2].

ამგვარად, ზემოაღნიშნულთა გათვალისწინებით, მოხსენებისას, მიმოხილული იქნება სადოქტორო კვლევის მიზნები, ამოცანები და ის ქართულისთვის ახალი მეთოდები, რითაც ჩვენ სადოქტორო კვლევით დასახული ამოცანების გადაწყვეტას ვგეგმავთ.

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The Radii of Parabolic Starlikeness and Uniformly Convexity of Bessel Functions Derivatives

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In this paper, we determine the radii of parabolic starlikeness and uniform convexity for three kinds of normalized Bessel function derivatives of the first kind. The key tools in the proof of our main results are the Mittag–Leffler expansion for nth derivative of Bessel function and properties of real zeros of it. The main results of the paper are natural extensions of some known results on classical Bessel functions of the first kind.

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The Existence of a Generalized Solution of an m-Point Nonlocal Boundary Value Problem for Quasi-linear Differential Equation

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Let G be the bounded domain on the complex plane E with the boundary Γ which is a closed simple Liapunov curve. We take two simple points A, B on Γ and assume that at these points there exists the tangent to Γ. It is obvious that that these points divide the boundary Γ into two curves. One of these parts denoted by γ is an open Liapunov curve with the parametric equation $z = z\,(s)$, $0 \le s \le \delta$. Let us choose simple points A_k, B_k, $k = 1, \dots, m$, on $\Gamma \backslash \gamma$ and assume that at these points the tangent to Γ exists. Besides, we draw in G the simple smooth curves γ_k, $k = 1, \dots, m$, which connect A_k and B_k. The curves γ_k are assumed to have the tangents at A_k and B_k which do not coincide with the tangent to the contour Γ at the same points. It is assumed that γ_k is the image of γ, diffeomorphic to $z_k = I\,(z)$ and with the parametric equation $z_k = z_k\,(s)$, $0 \le s \le \delta$, $k = 1, \dots, m$. Furthermore, it is assumed that $\gamma_i \cap \gamma_j = \varnothing$, $i \neq j$, $\gamma_i \cap \gamma = \varnothing$, $i, j = 1, \dots, m$, and the distance between every two lines $\gamma_1, \gamma_2, \dots, \gamma_m$ is larger than some positive number $\varepsilon = const > 0$.

Let us consider in $\bar{G}$ the following m-point nonlocal boundary value problem for quasi-linear differential equations of first order

$$\partial_{\bar{z}} = f\,(z, w, \bar{w})\,, \quad z \in G,$$

$$\operatorname{Re}\,[w\,(z)] = \varphi\,(z)\,, \quad z \in \Gamma \backslash \gamma, \quad \operatorname{Im}\,[w\,(z^*)] = c, \quad z^* \in \Gamma \backslash \gamma, \quad c = const,$$

$$\operatorname{Re}\,[w\,(z\,(s))] = \sum_{k=1}^{m} \sigma_k \operatorname{Re}\,[w\,(z_k\,(s))], \quad z\,(s) \in \gamma, \quad z_k\,(s) \in \gamma_k,$$

$$0 < \sigma_k = const, \quad k = 1, \cdots, m.$$

We prove a theorem on the existence and uniqueness of a generalized solution in the space $C_\alpha(\overline{G})$.

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Gravity and Maritime Navigation (experimental calculation of the gravitational constant with participation of students)

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Physics is the science of the simplest and, at the same time, the most general laws of nature, of matter, its structure and motion. The laws of physics are the basis of all natural science. The purpose of this science is to explain how the world works, to show what laws our universe is operated by. Allowing finding the answer to absolutely any question, physics does not cease to develop and improve.

According to the law of universal gravitation, all bodies are attracted to each other with a force directly proportional to the product of the masses of bodies and inversely proportional to the square of the distance between them $F = G\,\frac{Mm}{R^2}$. Thanks to this law, it is possible to answer many questions, including issues related to maritime navigation.

In this article, we draw attention to professional issues – the use of gravitational force in marine navigation. And also demonstrate on practice on the devices that we constructed with our own hands.

Keywords: Flow, star orientation, sea currents.

On the Optimal Stopping with Incomplete Data in Kalman–Bucy Scheme

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The Kalman–Bucy continuous model of partially observable stochastic processes is considered. The problem of optimal stopping of a stochastic process with incomplete data is reduced to the problem of optimal stopping with complete data. The convergence of payoffs is proved when $\varepsilon_1 \to 0, \varepsilon_2 \to 0$, where ε_1 and ε_2 are small perturbation parameters of the non observable processes respectively.

Laplace Equation in an Angular Domain

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Results on one dimensional Mellin pseudodifferential equations (ΨDOs) in the Bessel potential spaces will be presented. In contrast to the Fourier ΨDOs, BPOs and Mellin

ΨDOs do not commute and we derive an explicit formula for Mellin ΨDOs with meromorphic symbols. These results are applied to the lifting of the Mellin ΨDOs from the Bessel potential spaces to the Lebesgue spaces.

Consider the angle Ω_α of opening $0 < \alpha < 2\pi$ with the vertex at 0. Part of the boundary $\Gamma = \Omega_\alpha$ coincides with the semi axes $\mathbb{R}_+ := (0, \infty)$, while another we denote by $\mathbb{R}_\alpha$. Consider the mixed Dirichlet–Neumann boundary value problem for the Laplace equation in Ω_α

$$\begin{cases} \Delta u(t) = f(t), & t \in \Omega_\alpha, \\ u^+(\tau) = g(\tau), & \tau \in \mathbb{R}_\alpha, \\ -(\partial_2 u)^+(\tau) = h(\tau), & \tau \in \mathbb{R}_+. \end{cases} \tag{1}$$

Here $-\partial_2$ coincides on $\mathbb{R}_+$ with the normal derivative to the boundary.

Lax–Milgram Lemma applied to the BVP (1) gives that it has a unique solution in the classical setting $f \in \mathbb{L}_2(\Omega_\alpha)$, $g \in \mathbb{H}^{1/2}(\Gamma)$, $h \in \mathbb{H}^{-1/2}(\Gamma)$.

But in some problems, for example in approximation methods, it is important to know the solvability properties in the non-classical setting

$$f \in \widetilde{\mathbb{H}}^{s-2}_p(\Omega_\alpha), \ \ g \in \mathbb{H}^{s-1/p}_p(\Gamma), \ \ h \in \mathbb{H}^{s-1-1/p}_p(\Gamma), \ \ 1 < p < \infty, \ \ s > \frac{1}{p}\,. \tag{2}$$

Let those pairs of space parameters $(1/p, s) \in (0, 1) \times (1/p, \infty)$ for which the BVP (1), (2) is Fredholm call *regular pairs.*

Based on the above mentioned results for the Mellin ΨDOs we prove the following.

Theorem. *Let $1 < p < \infty$, $s \in \mathbb{R}$. The mixed boundary value problem* (1) *in the setting* (2) *is Fredholm if and only if the symbol is elliptic*

$$e^{4\pi i/p} \sin^2 \pi\Big(\frac{2}{p} - i\xi - s\Big) + \cos^2 \Big[(\pi - \alpha)\Big(\frac{2}{p} - i\xi - s\Big)\Big] \neq 0 \ \text{ for all } \ \xi \in \mathbb{R}. \tag{3}$$

If the symbol is elliptic, the strip $(0, 1) \times (1/p, \infty)$ of pairs of space parameters pairs $(1/p, s)$ decomposes into an infinite union of non-intersecting connected parts of regular pairs.

Let $\mathcal{R}_0$ be the regular connected part which contains the pair $(1/2, 1)$ (i.e. $p = 2$, $s = 1$). Then the BVP (1), (2) *is uniquely solvable for all pairs $(s, p) \in \mathcal{R}_0$.*

Similar results hold for the pure Dirichlet and pure Neumann BVPs for the Laplacian, although instead of condition (3) we have, respectively, the following

$$e^{4\pi i/p} \sin^2 \pi\Big(\frac{1}{p} - i\xi\Big) - \sin^2 \Big[(\pi - \alpha)\Big(\frac{1}{p} - 1 - i\xi\Big) - \pi\Big(s - \frac{1}{p}\Big)\Big] \neq 0 \ \text{ for all } \ \xi \in \mathbb{R},$$

$$e^{4\pi i/p} \sin^2 \pi\Big(\frac{1}{p} - i\xi\Big) - \sin^2 \Big[(\pi - \alpha)\Big(\frac{1}{p} - i\xi\Big) - \pi\Big(s - \frac{1}{p}\Big)\Big] \neq 0 \ \text{ for all } \ \xi \in \mathbb{R}.$$

The investigation was carried out in collaboration with V. Didenko (Brunei-Vietnam) and M. Tsaava (Georgia).

Solving Hedge Regular Language Equations

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In recent years, regular hedge languages [1] became very popular in programming languages, due to their expressive power. Languages supporting programming with regular hedge expressions are useful for Web-related applications. As examples, CDuce, PρLog, XDuce and XHaskell can be mentioned. Regular hedge expressions have been extensively used in search engines, rewriting, program verification, software engineering, lexical analysis, etc. Because of space limitation, we can not give an exhaustive overview of regular hedge expressions applications.

The theory of hedge languages generalizes the theory of word languages. Therefore, it is not surprising that regular hedge languages provide more expressive and powerful platform for semistructured data manipulation than regular word languages.

Solving regular word language equations with various restrictions have been intensively studied in the last decade. Solving regular word language equation systems without restrictions is hard and the class of smallest solutions of such systems corresponds to recursively-enumerable sets [2]. It should be noted that much less attention has been devoted to solving regular hedge language equations.

In this talk we propose a solving algorithm for one side ground regular hedge language equations. The solving algorithm is based on factorization of regular hedge languages, which generalizes factorization of regular word languages given in [3]. We show that, the algorithm computes maximal solutions and is sound and complete.

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საბჭოთა პერიოდამდე მათემატიკის ქართული სასკოლო სახელმძღვანელოების შესახებ

ომარ ძაგნიძე

ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის ანდრია რაზმაძის სახელობის მათემატიკის ინსტიტუტი, თბილისი, საქართველო

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„არითმეტიკის" პირველი ბეჭდური სახელმძრვანელო ქართულად გამოსცეს 1862 წელს მ. ზ. ყიფიანმა და ვ. თულაშვილმა. მათ წარმატებით დაძლიეს ტერმინოლოგიური სიძნელე და შექმნეს ტერმინები: გაყოფა, გასაყოფი, გამყოფი, ერთეული, ათეული, სამრავლი, მამრავლი, ნამრავლი, ნაშთი, შემოწმება, მარტივი რიცხვი, შეკრება, ასეული, ხარისხი, გამრავლება და სხვა. ამ მოვლენას დიდი მოწონებით შეხვდა საზოგადოება, თუმცა ცხრა წლის შემდეგ იყო არმოწონების შემცველი შეფასებაც. ამან აიძულა მ. ზ. ყიფიანი 1884 წელს გამოეცა 455 გვერდიანი „არითმეტიკა სოფლის სასწავლებელთათვის", სადაც მოცემულია ადვილად დასაძლევი 1257 ამოცანა-კითხვა. გეომეტრიის პირველი ორიგინალური 446 გვერდიანი ქართული სახელმძღვანელო „გეომეტრია" გამოსცა მ. ზ. ყიფიანმა 1888 წელს, რომლის ნაწილებია: პლანიმეტრია, სტერეომეტრია, უმდაბლესი გეოდეზია, ტრიგონომეტრიის ზოგიერთი კანონი და ლოგარითმული ცხრილების ხმარება.

ქართველი ახალგაზრდების მათემატიკური ცოდნის ამაღლების საქმეში მნიშვნელოვანი როლი შეასრულა ვ. რ. ყიფიანის მიერ 1893 წელს გამოცემულმა 666 გვერდიანმა სახელძღვანელომ „დაწყებითი ალგებრა", რომელიც შედგება რვა განყოფილებისგან და მოიცავს მთელი ალგებრის საკვანძო საკითხებს. საკითხთა სივრცელით და მასალის გადმოცემის მეთოდით გამოირჩევა ი. ავალიშვილის „არითმეტიკის სახელმძღვანელო, I" (1920 წ.). 1919 წელს ქუთაისში გამოიცა კ. მ. კანდელაკის 89 გვერდიანი „ალგებრულ ამოცანათა კრებული", ნაწ. II, წიგნი I. ასევე, 1918 წელს ქუთაისში გამოიცა ს. შარაშენიძის 102 გვერდიანი „სწორხაზოვანი ტრიგონომეტრია" საშუალო სასწავლებლის VII და VIII კლასებისთვის, სადაც გადმოცემულია ტრიგონომეტრიის მთელი კურსი.

1919 წელს დაარსდა „უმაღლესი დაწყებითი სკოლა", რომელსაც 1923 წელს ეწოდა „მთლიანი შრომის სკოლა". ამ სკოლების სანიმუშო პროგრამები (1920 წ.) შეიცავდა ბევრ ცვლილებას ყველა საგანში. იქ ნათქვამია: შეტანილია ცოტაოდენი ცვლილება არითმეტიკაში და რადიკალურადაა შეცვლილი გეომეტრიაო, რომლის სწავლებაში ნაცვლად ლოგიკური მსჯელობებისა შემოღებულია თვალსაჩინოებაო. ამის შესაბამისად, 1920 წელს ქუთაისში გამოიცა ივ. ერ. გაჩეჩილაძის 86 გვერდიანი „თვალსაჩინო გეომეტრია", ხოლო 1922-1923 წლებში თბილისში ნიკო შაფაქიძის „თვალსაჩინო-პრაქტიკული გეომეტრია, I-III". გეომეტრიის „თვალსაჩინო კურსი" მალე იქნა უარყოფილი.

On a Double Limit Connected with the Riemannian Method of Summation

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Along with any series convergent or not convergent

$$\sum_{k=0}^{\infty} a_k, \tag{1}$$

we can consider a series

$$\sum_{k=0}^{\infty} a_k \Big(\frac{\sin kh}{kh}\Big)^2,$$

depending on the variable h, in the assumption of its convergence for sufficiently small $h \neq 0$ and $\frac{\sin 0}{0} = 1$.

If under the above assumptions there exists a finite limit

$$\lim_{h\to 0} \sum_{k=0}^{\infty} a_k \Big(\frac{\sin kh}{kh}\Big)^2 = \sigma, \tag{2}$$

then the series (1) is called the Riemannian summation method, or R-summable to σ.

Obviously, equality (2) can be given the following form

$$\lim_{h\to 0} \lim_{n\to\infty} \sum_{k=0}^{n} a_k \Big(\frac{\sin kh}{kh}\Big)^2 = \sigma,$$

i.e., the form of a repeated limit

$$\lim_{h\to 0} \lim_{n\to\infty} A_n(h) = \sigma, \tag{3}$$

where it is assumed that

$$A_n(h) = \sum_{k=0}^{n} a_k \Big(\frac{\sin kh}{kh}\Big)^2.$$

Consequently, the fulfilment of equality (3) is equivalent to the R-summability of the series (1) to σ.

As for the existence of a finite double limit

$$\lim_{\substack{h\to 0\\ n\to\infty}} A_n(h) = s,$$

the theorem below is proved.

Theorem. *The convergence of the series* (1) *to s is the necessary and sufficient condition for the fulfilment of equality* (4)*.*

Difference Equations in Mathematical Modeling

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The general theory of difference equations is an important mathematical apparatus, since it has a wide range of applications for modeling various natural systems.

In the master's program of the Sukhumi University "Applied Mathematics" an important course is the obligatory course: "Difference equations in mathematical modeling", since for the analysis of discrete models in different fields of science, mathematical analysis of difference equations or systems of equations is required.

A course of lectures of this discipline has been created.

In this paper, sources of difference equations, types, their basic properties and basic methods of solution are considered; matrix, scalar and vector methods for solving systems of linear difference equations.

The study of the properties and methods of this mathematical apparatus is based on the construction of specific discrete mathematical models and their qualitative research.

Multiplication Groups of Topological Loops

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A set L with a binary operation $(x,y)\mapsto x\cdot y$ is called a loop if there exists an element $e\in L$ such that $x=e\cdot x=x\cdot e$ holds for all $x\in L$ and the equations $a\cdot y=b$ and

$x \cdot a = b$ have precisely one solution, which we denote by $y = a\backslash b$ and $x = b/a$. A loop L is proper if it is not a group.

The left and right translations $\lambda_a = y \mapsto a \cdot y : L \to L$ and $\rho_a : y \mapsto y \cdot a : L \to L$, $a \in L$, are permutations of L.

The permutation group $Mult(L) = \langle \lambda_a, \rho_a;\ a \in L \rangle$ is called the multiplication group of L. The stabilizer of the identity element $e \in L$ in $Mult(L)$ is called the inner mapping group $Inn(L)$ of L.

If L is a connected topological loop having a Lie group as the group of its left translations, then in general the multiplication group $Mult(L)$ of L is a differentiable transformation group of infinite dimension. The condition that the group $Mult(L)$ is a (finite dimensional) Lie group gives a strong restriction for the group $Mult(L)$ and also for the loop L: For every proper 1-dimensional topological loop L the multiplication group $Mult(L)$ has infinite dimension (cf. [2]). In [1] we proved that only the elementary filiform Lie groups $\mathcal{F}_n$, $n \geq 4$, are the multiplication groups $Mult(L)$ of 2-dimensional connected simply connected topological loops L.

We determine the structure of the Lie groups which are the multiplication groups of three-dimensional topological loops L. We use this result for the classification of Lie groups which occur as the group $Mult(L)$ of a three-dimensional loop L.

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On a Static Problem of Beam in the (0,0) Approximation

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The static problem of beam is considered and investigated in the (0,0) approximation of hierarchical models. There is considered beam whose length is L, width and thickness are given by the expressions:

$$2h_2 = h_2^0 \ \text{ and } \ 2h_3 = h_3^0 e^{-\frac{\kappa}{x_1}},$$

$$x_1 \in [0, L], \quad h_2^0 = h_3^0 = const > 0, \quad \kappa = const > 0, \quad L = const > 0.$$

We consider weighted boundary condition on the cusped end of the beam and Dirichlet boundary condition on the non-cusped end. The solution of the posed boundary value problem is given in an integral form.

Hilbert Functions of Morava $K(2)^*$-Theory Rings of Some 2-Groups

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This note presents some computer generated calculations of the Hilbert functions related to Morava $K(2)^*$-theory rings of classifying spaces BG, for some groups of order 32.

Reference

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The boundary problem for the Elliptic Equations in Generalized Weighted Morrey Spaces

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For the a weak solution Dirichlet boundary problem uniformly elliptic equations of higher order in generalized weighted Morrey spaces in a smooth bounded domain $\Omega \subset R^n$ a priori estimate is obtained. Weight function from in the Macenhoupt class A_p.

Numerical Model of Mesoscale Boundary Layer of the Atmosphere Taking into Account of Humidity Processes

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Using the non-stationary, 2–dimensional (in the vertical (x-z) plane) numerical model of the mesoscale boundary layer of the atmosphere, such cluster-like moisture processes as 3 layered clouds and fog are simulated, which eventually transform into 4 layered clouds.

The model takes into account a wider range of phase transitions of water, in particular, as the condensation of water vapor and freezing of water. We are also going to take into account water anomalies, in particular, sharply different values of dielectric permittivity of liquid water, water vapor and ice.

We continue to study foehn processes within the framework of this numerical model.

Schauffler Type Theorems for New Second Order Formulas

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In this talk the Belousov theorem on linearity of invertible algebras with the Schauffler $\forall\exists(\forall)$-identity of associativity is extended over the other $\forall\exists(\forall)$-identities. As a consequence we obtain the equivalency of the considered $\forall\exists(\forall)$-identities and non-trivial hyperidentities in systems of groups. For the considered second order formulas we prove the Schauffler type theorems too.

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On Formation of Massless Bose Particles *Hions* in the Quantum Vacuum. Problem of Dark Energy-Quintessence

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We have studied the possibility of formation of massless structure particles with spin 1 vector boson. Based on a stochastic differential equation of the Weyl–Langevin type, it is proved that as a result of multiscale random fluctuations of massless quantum vector fields, in the first phase of relaxation forms quasiparticles (later these particles called *hion*) in the form of randomly oscillating two-dimensional strings. It is shown that, in the limit of statistical equilibrium, the string is quantized and localized on a two-dimensional topological manifold. The wave state and geometric structure of the *hion* are studied in the case when the quasiparticle is free and when it interacts with a random environment. In the second phase of relaxation, the symmetry of the quantum state of the *hion* breaks down, which leads to spontaneous transitions of quasiparticle to other massless and mass states. The problem of entanglement of two *hions* with opposite projections of the spins $+1$ and -1 and the formation of a scalar zero-spin boson are studied in detail. The properties of the scalar field (dark energy-*quintessence*) are analyzed and it is shown that in fact it is a Bose–Einstein (BE) condensate. The problems of the decay of a scalar boson, as well as a number of features characterizing the stability of BE condensate, are investigated. The structure of the "*empty*" space-time is analyzed in the context of the new properties of the quantum vacuum, which allows us to assume the existence of a natural quantum computer with complex logic in the form of a dark energy-quintessence. The possibilities of space-time engineering are discussed.

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Maximum Inequalities and Their Applications to Orthogonal and Hadamard Matrices

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Using Hoeffding–Chernoff bound maximum inequalities for the signed vector summands and corresponding probabilistic estimations are established. By use of "transference technique" appropriate maximum inequalities are derived for the permutations. One application for Orthogonal and Hadamard matrices is suggested.

On Some Methods of Solving Textual Tasks in Secondary School

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Part of students of secondary school are difficult to learn mathematics. Special difficulties are encountered when solving textual tasks. The problem here is to understand correctly the condition of tasks. The report presents the methodological treatment of these issues.

Some Aspects of Mathematical Education of Computer Science Direction Students

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Modernity is characterized by the rapid development of computer sciences and technologies. The demand for corresponding specialists is increasing day by day. Demand for their high knowledge level also increases. The fundamental and conceptual shifts in this direction are importantly related to the mathematics' various applied aspects.

That's why, in the leading educational centers of the world, the role of mathematics in the preparation of students is increasingly reflected in the appropriate training programs. With the teaching of mathematical fundamental issues, it becomes more important to present a wide range of applications.

In our talk we focus on the growing role of discrete mathematics and, in particular, the theory of numbers in many different disciplines of computer science. First of all, we need to name modular arithmetic and its applications, their decisive role in the creation of new cryptographic systems. The creation of such public-key systems was based on Euler's theorem in the congruence theory and issues of linear congruencies. It is also important to get familiar with the solution of linear congruence systems and high order congruencies, properties of general multiplication functions and the continued fractions which allows us to solve many practical problems in computer sciences.

It should be noted that often learning boils down to acquainting only the final important theoretical results - the methods of receiving these results are ignored. The creative expert's knowledge of these methods often leads to a solution of new results. Obviously, some theoretical results can really be limited to the final results only, but such an approach can not be the only form of teaching.

Along with the above mentioned issues we will discuss some other topical problems of teaching and corresponding curricula.

Neural Network for Self-driving Car

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Neural Networks are effective tools for building driving management systems for self-driving cars. Self-driving cars have many sensors. The information from sensors are the input signals of the neural network. Neural Network makes decision about driving direction.

Presented system uses camera that is attached to the car. Camera captures road signes. Camera pictures are analized by the deep convolutional neural network. Neural Network recognizes the signs and makes decisions.

We use transfer learning on pretrained network in order to speed-up the learning process. We choose MobileNet [1] as our Neural Network. MobileNet was trained on ImageNet [2]. We retrained MobileNet on pictures captured by the car camera. We use Tensorflow [3] as the software library.

The recognition presizion is quite good. We think that our method, with combination with other sensors will be effective for self-driving cars.

References

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[2] ImageNet, `http://image-net.org`;

[3] TensorFlow, `http://tensorflow.org`.

Problems, Solvers and PageRank Method

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In recent years, intensive studies have been conducted to evaluate the effectiveness of various solvers and various methods for this purpose have been proposed in the literature. As well-known (see, e.g., [1]), most benchmarking tests utilize evaluation tables displaying the performance of each solver for each problem under a specific evaluation metric (e.g., CPU time, number of function evaluations, number of iterations). Different methods (based on suitable "statistical" quantities) are used to interpret data from these tables, including the mean, median, and quartiles, ranking, cumulative distribution function, etc. The advantages and disadvantages of each proposed method are often a source of disagreement; however, this only stimulates further investigation in the field.

The method discussed in this paper was proposed to introduce a new benchmark that directly accounts for the natural relationship between problems and solvers, which is determined by their evaluation tables. Namely, we introduces the benchmarking context concept as a triple $\langle S, P, J\rangle$, where S is a set of solvers, P is a set of problems, and J is an assessment function (a performance or evaluation metric). This concept is quite general and, furthermore, emphasizes that problem and solver benchmarking cannot be considered separately. Based on the data presented by the benchmarking context $\langle S, P, J\rangle$, a special procedure was defined allowing solvers and problems to be ranked. It should also be noted that the proposed procedure is a specific version (most probably the simplest) of the Google PageRank method (see, e.g., [2]). This study aimed to propose a PageRank procedure as an effective tool for benchmarking computational problems and their solvers.

In this study, as an illustrative example we conduct benchmarking analysis of Differential Evolution Algorithms (9 optimization algorithms) on a set of test problems (50 optimization problems for 25 well-known test functions) using the Random Sampling Equivalent–Expected Run Time (ERTRSE) measure as a performance metric [3]. The considered example demonstrate the viability and suitability of the proposed method for applications

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Renormalization of Mass in QCD

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We have shown in the most general way that the mass scale parameter may appear in QCD without violation of its exact $SU(3)$ color gauge invariance. For this purpose we investigate the color gauge structure of the Schwinger–Dyson (SD) equation of motion for the full gluon propagator [1]. The mass scale parameter, the mass gap, in what follows is dynamically generated by the self-interaction of the multiple massless gluon modes. The renormalization program for massive gluon fields is developed as well.

References

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Qausi-Conformal Estimates of Neumann–Laplace Eigenvalues

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We discuss quasi-conformal lower estimates of the first nontrivial Neumann Laplace eigenvalues for non convex domains including domains with fractal boundaries. Results are based on the geometric theory of composition operators.

About One Mathematical Model of Currency Arbitrage

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In today's Global Economy, multinational companies should make transactions in the currencies of the countries, where they operate. The majority of the firms on Georgian market have to convert different currencies while investing. Currency arbitrage, or simultaneous purchase and sale of the money on different markets, offer us better transaction from one currency to another. In this work, we have demonstrated how to formulate and solve arbitrage problem in case of Region's priority currency transactions. There is given simulation of real process of the transactions, we have demonstrated the mathematical model of currency arbitrage, relevant strategy of getting maximal revenue with the help of currency arbitrage is realized programmatically.

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Contractive Mapping Theorems in Generalized Soft Metric Spaces

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The purpose of this paper is to contribute for investigating on generalized soft metric space which is based on soft point of soft sets and give some of its properties. We define the concepts of sequential compact and totally bounded in generalized soft metric space and prove some important theorems on this space. Finally, we introduce contractive mappings on generalized soft metric spaces and prove a common fixed point theorem for a self-mapping on complete generalized soft metric spaces.

Keywords: Generalized soft metric space, soft contractive mapping, fixed point theorem.

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Neutrosophic Soft Seperation Axioms in Neutrosophic Soft Topological Spaces

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The neutrosophic soft topology based on operations of the neutrosophic soft union and intersection which are differently defined from the other studies. In this paper the neutrosophic soft null and absolute set will be re-defined differently from the study [1]. we also introduce some basic notions of neutrosophic soft topological spaces by using neutrosophic soft point concept. Later we give Ti- neutrosophic soft spaces and the relationships between them. Finally, we investigate some of its important properties.

Keywords: Neutrosophic soft set, neutrosophic soft separation axioms

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Hankel and Berezin Type Operators on Weighted Besov Spaces of Holomorphic Functions on Polydiscs

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Assuming that S is the space of functions of regular variation(see [2]) and $\omega = (\omega_1, \dots, \omega_n)$, $\omega_j \in S$, by $B_p(\omega)$ we denote the class of all holomorphic functions defined on the polydisk U^n such that

$$\|f\|^p_{B_p(\omega)} = \int\limits_{U^n} |Df(z)|^p \prod_{j=1}^{n} \frac{\omega_j(1-|z_j|)dm_{2n}(z)}{(1-|z_j|^2)^{2-p}} < +\infty,$$

where $dm_{2n}(z)$ is the $2n$-dimensional Lebesgue measure on U^n and D stands for a special fractional derivative of f defined here. As in the one-dimensional case, $B_p(\omega)$ is a Banach space with respect to the norm $\|\cdot\|_{B_p(\omega)}$. For properties of holomorphic Besov spaces see [1].

In this paper we consider also the generalized Berezin type operators on $B_p(\omega)$ (and on $L_p(\omega)$) and prove some theorems about the boundedness of these operators. Let us define the little Hankel operators as follows: denote by $\overline{B}_p(\omega)$ the space of conjugate holomorphic functions on $B_p(\omega)$. For the integrable function f on U^n we define the generalized little Hankel operator with symbol $h \in L^\infty(U^n)$ by

$$h^\alpha_g(f)(z) = \overline{P}_\alpha(fg)(z) = \int\limits_{U^n} \frac{(1-|\zeta|^2)^\alpha}{(1-\zeta\overline{z})^{\alpha+2}} f(\zeta)g(\zeta)dm_{2n}(\zeta),$$

$$\alpha = (\alpha_1, \dots, \alpha_n), \ \ \alpha_j > -1, \ \ 1 \le j \le n.$$

For $n = 1$, $\alpha = 0$ this includes the definition of the classical little Hankel operator, see [3]. we consider the boundedness of little Hankel operator on $B_p(\omega)$. For the case $0 < p < 1$ and for the case $p = 1$ we have the following results.

Theorem 1. *Let $0 < p < 1$, $f \in B_p(\omega)$ (or $f \in \overline{B}^p(\omega)$), $g \in L^\infty(U^n)$. Then $h^\alpha_g(f) \in \overline{B}_p(\omega)$ if and only if $\alpha_j > \alpha_{\omega_j}/p - 2$, $1 \le j \le n$.*

Theorem 2. *Let $f \in B_1(\omega)$, $g \in L^\infty(U^n)$. Then $h^\alpha_g(f) \in \overline{B}_1(\omega)$ if and only if $\alpha_j > \alpha_{\omega_j} - 2$, $1 \le j \le n$.*

The case $p > 1$ is different from the cases of $0 > p < 1$ and from the case of $p = 1$. Here we have the following

Theorem 3. *Let $1 < p < +\infty$, $f \in B_p(\omega)$ (or $f \in \overline{B}_p(\omega)$), $g \in L^\infty(U^n)$. Then if $\alpha_j > \alpha_{\omega_j}$, $1 \le j \le n$, then $h_g^\alpha(f) \in \overline{B}_p(\omega)$.*

For the integrable function f on U^n and for $g \in L^\infty(U^n)$ we define the Berezin-type operator in the following way

$$B_g^\alpha f(z) = \frac{(\alpha+1)}{\pi^n}(1-|z|^2)^{\alpha+2} \int\limits_{U^n} \frac{(1-|\zeta|^2)^\alpha}{|1-z\overline{\zeta}|^{4+2\alpha}} f(\zeta) g(\zeta)\, dm_{2n}(\zeta).$$

In the case $\alpha = 0$, $g \equiv 1$ the operator B_g^α will be called the Berezin transform. We have the following results:

1. for the case of $0 < p < 1$ we have

Theorem 4. *Let $0 < p < 1$, $f \in B_p(\omega)$ (or $f \in \overline{B}_p(\omega)$), $g \in L^\infty(U^n)$ and let $\alpha_j > \alpha_{\omega_j}/p - 2$, $1 \le j \le n$. Then $B_g^\alpha(f) \in L^p(\omega)$.*

2. the case $1 < p < +\infty$ gives the next theorem

Theorem 5. *Let $1 < p < +\infty$, $f \in B_p(\omega)$ (or $f \in \overline{B}_p(\omega)$), $g \in L^\infty(U^n)$ and let $\alpha_j > (\alpha_{\omega_j}/p - 2$, $1 \le j \le n$. Then $B_g^\alpha(f) \in L_p(\omega)$.*

3. we consider now the case of $p = 1$.

Theorem 6.*Let $f \in B_1(\omega)$ (or $f \in \overline{B}_1(\omega)$), $g \in L^\infty(U^n)$. Then $B_g^\alpha(f) \in L_1(\omega)$ if and only if $\alpha_j > \alpha_{\omega_j}$, $1 \le j \le n$.*

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Issues of Building a Microclimate Management Model in the Building

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The microclimate is a climate artificially constructed in a residential and workplace, the purpose of which is to protect the human body from adverse environmental impacts. Because the microclimate is artificially created in living and working areas, it is possible to control and manage it.

The modern level of automation and tool building makes it easier to solve the problem of creating a micro-climate in public and residential buildings.

In some private cases, the model of a micro-clock management system can be brought down before the equilibrium equals the mathematical description of the air exchange process taking into consideration various external climatic impacts.

The Algorithm for Functioning the Digital Meter Device

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Different measuring control devices and devices are significantly different from functions and components and components, but they can still be represented as their structural block schemes.

This is due to the fact that any control-control device can be identified by its main determinant blocks and nodes. This in turn allows us to describe a formalized (mathematical) measuring-control device, its model construction and its functional algorithm.

Hybrid Encryption Model of Symmetric and Asymmetric Cryptography with AES and ElGamal Encryption Algorithms

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Computer networks and internet application are growing fast, so data security is the challenging issue of today that touches many areas. To prevent unauthorized access to the user data or database, any transmitting process should be securely encrypted. Different cryptography techniques and algorithms are used to provide the needed security to the applications. Cryptography methods provides authentication, integrity, availability, confidentiality, identification, security and privacy of user data. Data security and authenticity are used in our daily life such as in banking, smart card, business discussion and insurance. There are two types of cryptography algorithms such as symmetric key cryptography and asymmetric key cryptography.

This paper provides a comparison between two symmetric, asymmetric algorithms and new hybrid cryptography algorithm model. The factors are achieving an effectiveness and security. Currently many encryption algorithms are available to secure the data but some algorithms consume lot of computing resources such as memory and CPU time. Comparative analysis was done on encryption algorithms such as AES and ElGamal. Is designed new hybrid model using combination of two cryptography algorithms AES and ElGamal.

The objective of this research is to evaluate the performance of AES, ElGamal cryptography algorithms and AES&ElGamal hybrid cryptography algorithm. The performance of the implemented encryption algorithms is evaluated by means of encryption and decryption time and memory usage. To make comparison experiments, for those algorithms is created program. The programming language Java is used in implementing the encryption algorithms. As the result shows, provided hybrid algorithm model is comparatively better than ElGamal in terms of encryption / decryption time and better than AES in terms of its security.

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On the Mannheim Rational Bezier Curve Pairs in 3-Space

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In literature it is well known that curve pairs $\{\alpha, \beta\}$ are called Mannheim curve pairs if the unit principal normal vector of the curve α is the same as the unit binormal vector of the curve β.

In this paper we study Mannheim rational Bezier curve pairs. Let two rational Bezier curves of degree n with control points $\{b_i\}_{i=0,1,\ldots,n}$ and the weights $\{w_i\}_{i=0,1,\ldots,n}$, and $\{c_i\}_{i=0,1,\ldots,n}$ and the weights $\{m_i\}_{i=0,1,\ldots,n}$ be given in the space of E^3. We investigated the conditions of being Mannheim curve pairs of these rational Bezier curves. So we stated these conditions as control points $\{b_i\}_{i=0,1,\ldots,n}$ and $\{c_i\}_{i=0,1,\ldots,n}$ and weights $\{w_i\}_{i=0,1,\ldots,n}$, $\{m_i\}_{i=0,1,\ldots,n}$ of given rational Bezier curves.

Keywords: Mannheim curve, Bezier curve, control points.

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On the Mannheim Bezier Curve Pairs in 3-Space

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Asymptotic Behaviour of Solutions of One Fourth Order Hyperbolic Equation with Memory Operator

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Let $\Omega \subset R^N$ $(N \geq 1)$ be a bounded, connected set with a smooth boundary Γ. We consider the following problem:

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial t}\left[u + \mathrm{F}(u)\right] + \Delta^2 u + |u|^p u = h \ \text{ in } \ Q = \Omega \times (0, T), \tag{1}$$

$$u = 0, \quad \Delta u = 0, \ \ (x,t) \in \Gamma \times [0,T], \tag{2}$$

$$\left[u + \mathrm{F}(u)\right]\Big|_{t=0} = u^{(0)} + w^{(0)}, \quad \frac{\partial u}{\partial t}\Big|_{t=0} = u^{(1)} \ \text{ in } \ \Omega, \tag{3}$$

where $p > 0$ and F is a memory operator (at any instant t, $\mathrm{F}(u)$ may depend not only on $u(t)$, but also on the previous evolution of u), which acts from $\mathrm{M}(\Omega; C^0([0,T]))$ to $\mathrm{M}(\Omega; C^0([0,T]))$. Here $\mathrm{M}(\Omega; C^0([0,T]))$ is a space of strongly measurable functions $\Omega \to C^0([0,T])$. We assume that the operator F is applied at each point $x \in \Omega$ independently: the output $[\mathrm{F}(u(x,\cdot))](t)$ depends on $u(x,\cdot)|_{[0,t]}$, but not on $u(y,\cdot)|_{[0,t]}$ for any $y \neq x$ (see [1]).

The similar problem for second order hyperbolic equation with memory operator was studied in [2] and asymptotic results for solutions of the corresponding problem were obtained.

In this work we prove the existence and uniqueness of solutions, the existence of an absorbing set, the asymptotic compactness of a semigroup, generated by problem (1)–(3) in $E = H_0^2(\Omega) \times L^2(\Omega) \times L^2(\Omega)$ and then the basic theorem about existence of a minimal global attractor for this problem under conditions, set in [2].

We prove this theorem by time discretization method.

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Simultaneous Approximation in the Variable Exponent Smirnov Classes

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In this talk we discuss the simultaneous approximation problems for the polynomials having maximal convergence property on the continuums of the complex plane.

Let K be a bounded continuum with simple connected complement $G:=\mathbb{C}\backslash K$, in the complex plane $\mathbb{C}$. We denote by φ the conformal mapping of G onto domain $D^{-}:=\{w:|w|>1\}$ and by Γ_R, $R>1$, the Rth level line of K defined by $\Gamma_R:=\{z:|\varphi(z)|=R\}$ and let $G_R:=int\,\Gamma_R$.

The class of Lebesgue measurable functions $p(\cdot):\Gamma_R\to[0,\infty)$, satisfying the conditions

$$1<p_{-}:=\operatorname*{es\,sin\,f}_{z\in\Gamma}p(\cdot)\le\operatorname*{ess\,sup}_{z\in\Gamma}p(\cdot)=:=p^{+}<\infty$$

$$|p(z_1)-p(z_2)|\le c(p)\ln 1/|z_1-z_2|\,,\ \forall z_1,z_2\in\Gamma_R,\ |z_1-z_2|<diam\Gamma_R$$

we denote by $\mathcal{P}(\Gamma_R)$.

For a given $p(\cdot)\in\mathcal{P}(\Gamma_R)$ we define the variable exponent Lebesgue spaces $L^{p(\cdot)}(\Gamma_R)$ of functions f with the norm

$$\|f\|_{L^{p(\cdot)}(\Gamma_R)}:=\inf\left\{\lambda\ge 0:\int\limits_{\Gamma_R}|f(z)/\lambda|^{p(\cdot)}|dz|\le 1\right\}<\infty.$$

Let $E^1(G_R)$ be the classical Smirnov class of analytic functions in G_R. We define the variable exponent Smirnov class as $E^{p(\cdot)}(G_R):=\left\{f\in E^1(G_R):f\in L^{p(\cdot)}(\Gamma_R)\right\}$.

It is well known that if $f\in E^1(G_R)$, then for every $n\in\mathbb{N}$ the following Faber series representation $f^{(n)}(z)=\sum\limits_{k=0}^{\infty}a_k(f)F_k^{(n)}(z)$, $z\in K$, holds, where F_k, $k=0,1,2,\dots$, are the Faber polynomials of degree k for continuum K.

We estimate the error $\sup\limits_{z\in K}\left|f^{(n)}(z)-\sum\limits_{k=0}^{n}a_k(f)F_k^{(n)}(z)\right|$ with dependence of the best polynomial approximation number $E_n(f,G_R)_{p(\cdot)}:=\inf\{\|f-p_n\|:p_n\in\Pi_n\}$ and R where Π_n is the class of the algebraic polynomials of degree not exceeding n.

Approximation in the Variable Exponent Lebesgue Spaces

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In this talk we discuss the approximation problems in the variable exponent Lebesgue spaces defined on the interval $[0, 2\pi]$ and also in the variable exponent Smirnov classes of analytic functions defined on the simple connected domains of the complex plane. Under some restrictive conditions on the variable exponent the direct and inverse theorems of approximation theory and also theirs improvements are proved. Here the speed of approximation in term of the modulus of smoothness, constructed via Steklov mean operator of given function is estimated. In particular case, the constructive approximation problems in the generalized Lipschitz subclasses are considered and the appropriate theorems are formulated.

The Basic Transmission Problem of the Thermoelastostatics for Anisotropic Composite Structures

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In the paper we consider the basic transmission problem for piecewise homogeneous three-dimensional space consisting of two adgasent anisotropic elastic components with differential material constants, when one of them is a bounded region and the second one is its complement to the whole space. The basic rigid transmission conditions are given on the interface.

Recurrence Relations for the Moments of the Order Statistics from the Generalized Beta Distributions

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The use of recurrence relations for the moments of order statistics is quite well-known in statistical literature (see, for example, Arnold et al., 1992 and Malik et al., 1988). Balakrishnan et al. (1988) have reviewed many recurrence relations and identities for the moments of order statistics arising from several specific continuous distributions such as normal, Cauchy, logistic, gamma and exponential. For improved forms of some of these results, see Jabbari Khamnei and Makouyi (2018), Thomas and Samuel (2008), Samuel and Thomas (2000) and Thomas and Samuel (1996). In this paper, the main focus is to study the recurrence relations for the single and product moments of the order statistics of a random sample of size n arising from the generalized beta distributions.

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WRF Model's Installation, Parameterization and Some Results of Numerical Calculations

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We have elaborated and configured Whether Research Forecast – Advanced Researcher Weather (WRF-ARW) model for Caucasus region considering geographical-landscape character, topography height, land use, soil type and temperature in deep layers, vegetation monthly distribution, albedo and others. Porting of WRF-ARW application to the grid was a good opportunity for running model on larger number of CPUs and storing large amount of data on the grid storage elements on the platform Linux-x86. The real time outputs of global model – GFS (Global Forecast System), as lateral boundary and initial conditions for regional domain and recalculating its results adjusted for local physical-geographical parameters and some meso and micro atmosphere, biological and chemical processes was used. In searching of optimal execution time for time saving different model directory structures and storage schema was used. Simulations were performed using a set of 2 domains with horizontal grid-point resolutions of 18 and 6 km. The coarser domain is a grid of 94×102 points which covers the South Caucasus region, while the nested inner domain has a grid size of 70×70 points mainly territory of Georgia. Both use the default 31 vertical levels.

Due to the importance of precipitation forecast in South Caucasus area, the main attention was paid to assess the model sensitivity to several configurations of convective and microphysical parameterizations. The options defined as tested schemes include cumulus and microphysics parameterizations, which have been combined in 3 combinations of different physical schemes for the coarser domain and 2 configurations for the inner one.

Two case studies which are generally characterize model simulation behavior for western and eastern type synoptic processes are presented.

About New Properties of the Well-Known Integrals

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We have studied the properties of the following functional:

$$J\Big[f_\nu\Big(\ln\frac{1-t}{t}\Big)\Big](\tau)=\int\limits_0^1 dt\, t^{z-1}(1-t)^{-z-1}f_\nu\Big(\ln\frac{1-t}{t}\Big). \tag{1}$$

The parameter z is a complex number and $\nu=1,2$.

If $z=\varepsilon+i\tau$, one can show that:

$$J\left[f_1\left(\xi\right)\right]\left(\tau\right)=L\left[f_1\left(\xi\right)\right]\left(\tau\right), \quad \xi=\ln\frac{1-t}{t},$$

where $L\left[f_1\left(\xi\right)\right]\left(\tau\right)$ denotes a Laplace transform and $f_1\left(\xi\right)=\theta\left(\xi\right)f\left(\xi\right)$. The function $f\left(\xi\right)$ satisfies the condition $f\left(\xi\right): R\in C$ (see [1]), and $\theta\left(\xi\right)$ is the Heaviside Unit function.

For $z=i\tau$, one obtains:

$$J\left[f_2\left(\xi\right)\right]\left(\tau\right)=F\left[f_2\left(\xi\right)\right]\left(\tau\right), \quad \xi=\ln\frac{1-t}{t}. \tag{2}$$

In the previous expression $F\left[f_2\left(\xi\right)\right]\left(\tau\right)$ denotes the Fourier transform and $f_2\left(\xi\right)=f\left(\xi\right)$.

In addition, when $z=i\tau$, one gets the following relation:

$$J\left[f_2(-\ln u)\right]\left(\tau\right)=M\left[f_2(-\ln u)\right]\left(\tau\right), \quad u=\frac{t}{1-t}. \tag{3}$$

We have denoted the Mellin transform as follows: $M\left[f_2(-\ln u)\right]\left(\tau\right)$, where $f_2(-\ln u)=f\left(-\ln u\right)$, $u^{i\tau-1}f\left(-\ln u\right): R\in C$, and $f\left(\xi\right)$ is bounded for arbitrary real ξ (see [2, 3]).

Let note, that from the equalities (2) and (3) one obtains the relation:

$$F\left[f(\xi)\right]\left(\tau\right)=M\left[f(-\ln u)\right]\left(\tau\right).$$

Thus, the Laplace, Fourier and Mellin transforms one can derive from the expression (1).

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The New Representations of the Fourier Transform

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In our previous studies, we have shown that (see, e, g., [1] and [2, p. 139])

$$\mathcal{F}[1(x)](\tau)=\int\limits_0^1 dt\, t^{i\tau-1}(1-t)^{-i\tau-1}=2\pi\delta(\tau),$$

where $F[1(x)](\tau)$ denotes the Fourier transform of the function $f(t)=1$.

Some of the similar results that we got for the generalized functions will be suggested below.

For instance, one can show that the relations hold (see, e, g., [2, p. 141]):

$$F[\theta(x)](\tau)=\int\limits_1^\infty dt\, t^{-i\tau-1}(t-1)^{i\tau-1}=2\pi\delta_+(\tau), \tag{1}$$

$$F[\theta(-x)](\tau)=\int\limits_1^\infty dt\, t^{i\tau-1}(t-1)^{-i\tau-1}=2\pi\delta_-(\tau), \tag{2}$$

where $\mathcal{F}[\theta(x)](\tau)$ and $\mathcal{F}[\theta(-x)](\tau)$ are the Fourier transforms of the Heaviside functions $\theta(x)$ and $\theta(-x)$ accordingly. The singular generalized Heisenberg functions $2\pi\delta_+(\tau)$ and $2\pi\delta_-(\tau)$ are defined by the Sokhotsky formulas as follows (see, e, g., [2, p. 161]):

$$2\pi\delta_+(\tau)=\frac{i}{\tau+i0}\,,\quad 2\pi\delta_-(\tau)=-\frac{i}{\tau-i0}\,.$$

By virtue of the inverse transform of the formula (1), one can calculate the singular integral of the following kind:

$$i\int\limits_{-\infty}^{+\infty}\frac{d\tau}{\tau}\,t^{i\tau}(1-t)^{-i\tau}=\begin{cases}+\pi, & \dfrac{1}{2}\le t\le 1\\ -\pi, & 0\le t<\dfrac{1}{2}\,.\end{cases}$$

The new representations of the Fourier transform that we found are very useful to calculate singular integrals of the new type.

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Chipot's Method of Solution of Elliptic Kirchhoff Type Equation

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Kirchhoff type equation is called the following boundary value problem

$$-\varphi\bigg(\int\limits_{\Omega}|\nabla w|^2\,dx\bigg)\Delta w=f(x),\ \ x\in\Omega, \tag{1}$$

$$w(x)=0,\ \ x\in\partial\Omega, \tag{2}$$

where Ω is an open subset of $\mathbb{R}^n$, $n\ge 1$, and $\partial\Omega$ is its boundary. The function $f(x)$ is twice continuously differentiable function on Ω and function $\varphi(z)$, $0\le z<\infty$, satisfies the condition

$$\varphi(z)\ge\alpha>0. \tag{3}$$

In [3] the problem (1), (2) is studied when $n = 1$. For the solution is used Chipot's approach (see [1], [2]) and accuracy of the method is discussed. Here also are given numerical examples. In [4] the problem (1), (2) is studied when $n = 2$. For the solution is used Chipot's approach and accuracy of the method is discussed.

In this paper, we also use Chipot's approach to solve the problem (1), (2). In order to estimate the error of the method we consider the numerical example for $n = 3$. For this purpose we use Matlab.

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Time-cost Trade-off Method in Project Management

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In many countries around the globe various construction, research or other kinds of projects are being conducted. The main factors in such pieces of work are time and cost, which are necessary components for accomplishing them. The aim of the manager is to complete project in due date, with minimal costs possible. It's possible to represent construction, research and some other kinds of projects with networks flows. Therefore, network theory provides us with opportunity of modeling project and finding the efficient solution - minimization of costs and time. Following article discusses the opportunity of using network flows in project management. One of the ways to do so is using time-cost trade-off method. This method leads us to reducing the time needed to complete different actions in project (and time for the whole project, consequently) by consuming minimum amount of money. This ability is discussed in an example. On this example is built the model, respective network flow and it's solution.

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Boundary Value Two-dimensional Problems of Stationary Oscillation of the Thermoelasticity of Microstrech Materials with Microtemperature

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In the paper we consider the stationary oscillation case of the theory of linear thermoelasticity with microstrech materials with microtemperatures and microdilations. The representation formula of a general solution of the homogeneous system of differential equations in the paper is expressed by means of seven metaharmonic functions. These formulas are very convenient and useful in many particular problems for domains with concrete geometry. Here we demonstrate an application of these formulas to the Dirichlet and Neumann type boundary value problem for a circle. We construct an explicit solutions in the form of absolutely and uniformly convergent series.

Certain Properties of Matrices in the Schemes of Finite Differences with Variable Accuracy

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Finite difference schemes with constant coefficients are constructed for the Dirichlet problem with the Poisson equation, which order of approximation and convergence is depend on the number of knots (lines). There are considered properties of matrices included into these schemes. In the case of discretization of one variable with the corresponding transformations they are reduced to the equivalent canonical form, in which each equation contains just one unknown function. The solution of the received system is given in explicit form. In the case of discretization of two variables a linear system of mn equations with so many variables (n is a number of inner knots according to x variable and m is a number of inner knots according to y variable) is received. It is not difficult to solve such system of equations.

The Radii of Parabolic Starlike of Some Special Functions

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In this paper, we determine the radii of β-parabolic starlike of order α for six kinds of normalized Lommel and Struve functions of the first kind. In the cases considered the normalized Lommel and Struve functions are β-parabolic starlike functions of order α on the determined disks.

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On the Cotorsion Hull of Corner's Group

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A. L. S. Corner considered an abelian separable primary group which endomorphisms rings additive group is presented as a direct sum of an additive group of separable closed ring and additive group of a ring small endomorphisms. It is shown that the cotorsion hull of Corner's group is not fully transitive and it is constructed the meet-semilattice the lattice of dual ideals of which is isomorphic to the lattice of fully invariant subgroups of a cotorsion hull Corner's group.

Discrete Models of Information Warfare

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The discrete analogue of the continuous mathematical model of attraction of adepts of Samarskiy-Mikhaylov is provided. The computer experiment of this mathematical model is made. The also discrete mathematical model of information warfare with limitations on technological capabilities [1].

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Hydrodynamic Model of Formation of Karst Voids

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The physical representation of the dynamic picture of the karst development pursues various objectives, among which, in particular, is to assess the characteristic time scale of their formation. Obviously, this problem is quite complicated because of the many-sidedness of the process of karsting, proceeding with both: general characteristics and local features. In particular, karst voids can have a variety of forms, some of which have some regularity due to similarity with a certain geometric figure. For example, for karst, a funnel-shaped form with a base on the earth's surface is quite common. The effectiveness of the leaching factor is directly dependent on the geological quality of the medium and the duration of the action of the water. It seems that to confirm the uniformity of the mechanism, the action of which leads to the elution of the solid rock, one can turn to the approximation of the hydrodynamic boundary layer arising when flowing over a solid surface. The rate of washing out of solid rock from the karstic cavity depends on the flow of water, which can vary depending on the flow regime. However, we can talk about some average characteristic, if we assume, for example, that the water movement is laminar. it should be noted that the value of the rate of karst leaching used for numerical evaluation is very approximate. it nevertheless seems that with the help of the model we have used, it is possible to obtain more accurate quantitative estimates.

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A Focal Line in the Improper Hyperplane

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A two-dimensional surface is considered in the four-dimensional extended affine space. A moving frame is attached to the surface mentioned above, according to a certain rule. When we choose a conjugated net on the normal of Voss for the net under the certain condition. So the line described by one of the points represents a focal line that may be either an oval line or a real pair of straight lines.

On the Non-Smooth Solitonic Solutions of the Non-linear Schrödinger Equation

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The nonlinear Scrödinger equation describes wide range of physical phenomena [1–5]. In the Euclidian space R^3 we consider the cubic nonlinear Scrödinger equation (cNLS)

$$\Delta r + \lambda_0 r^3 - A_0 r = 0, \tag{1}$$

where r is unknown function, λ_0 and A_0 are some parameters, $A_0 > 0$. r is the modulus of the wave function Ψ

$$\Psi = re^{iA_0 t + iA_1},$$

where t is a time, $i^2 = -1$ and A_1 is a definite number.

By using previous results of the author [5] we have obtained the approximate non-smooth solitonic solutions of the equation (1)

$$r = R\sin\{\exp[-\alpha|x| - \beta|y| - \gamma|z| - D]\} + \sum_{k=1}^{m} R_k \exp[-\alpha_k|x| - \beta_k|y| - \gamma_k|z| - D_k], \tag{2}$$

where α, β, γ, α_k, β_k, γ_k, R_k, D_k, R, D, $k = 1, 2, \dots, m$, are some parameters satisfying the conditions

$$\alpha^2 + \beta^2 + \gamma^2 = \alpha_k^2 + \beta_k^2 + \gamma_k^2 = A_0;\ \alpha,\ \beta,\ \gamma,\ \alpha_k,\ \beta_k,\ \gamma_k > 0,$$

$$\lambda_0 R^2 = 4A_0/3; \quad R,\ A_0 > 0; \quad D_k \geq 3D; \quad D > 0,$$

m is an arbitrary natural number and the constant D is chosen for the desired accuracy in such a way, that e^{-5D} is negligible.

The function given by the formula (2) vanishes at infinity exponentially and is the approximate solution of the equation (1) with the accuracy $A_0 \exp(-5D)/2$.

The graph of (2) is constructed for the different parameters.

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Composing a Syllabus on Finance Mathematics for Different Faculties

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In the talk I shall give some possibilities of composing the syllabuses on finance mathematics for different faculties

On Cofinitely e-Supplemented Modules

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In this work, some properties of cofinitely e-supplemented modules are investigated. All rings are associative with identity and all modules are unital left modules in this work.

Proposition 1. *Let M be a finitely generated R-module. Then M is e-supplemented if and only if M is cofinitely e-supplemented.*

Lemma. *Let M be an R-module, U be a cofinite essential submodule of M and $M_1 \leq M$. If M_1 is cofinitely e-supplemented and $U + M_1$ has a supplement in M, then U has a supplement in M.*

Corollary. *Let U be a cofinite essential submodule of M and $M_i \leq M$ for $i = 1, 2, \ldots, n$. If M_i is cofinitely e-supplemented for $i = 1, 2, \ldots, n$ and $U + M_1 + M_2 + \cdots + M_n$ has a supplement in M, then U has a supplement in M.*

Proposition 2. *Let M be a cofinitely e-supplemented module. Then every M-generated R-module is cofinitely e-supplemented.*

Keywords: cofinite submodules, essential submodules, small submodules, supplemented modules.

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ინფორმატიკის ფაკულტეტზე მათემატიკის სწავლების ზოგიერთი თავისებურების შესახებ

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დღესდღეობით რაიმე დარგის განვითარება კომპიუტერის გარეშე წარმოუდგენელია. კომპიუტერული ტექნოლოგიები მთელ მსოფლიოში სულ უფრო სწრაფად ვითარდება და, შესაბამისად, იზრდება მოთხოვნა ინფორმატიკის სხვადასხვა მიმართულების სპეციალისტებზე. სწორედ ამის გამო, მაღალკვალიფიციური სპეციალისტების აღზრდა ქვეყნებისთვის მნიშვნელოვანი საკითხია. უნივერსიტეტების წინაშე კი დგას განათლების ამაღლების საკითხი სწორედ ამ მიმართულებებით. თანამედროვე განათლება მკაფიოდ ჩამოყალიბებულ მიზნებს და პროფესიაზე მორგებულ სწავლებას მოითხოვს.

მეორეს მხრივ, ინფორმატიკა დისციპლინათშორისი ბუნებით ხასიათდება. განსაკუთრებით მნიშვნელოვანია მისი კავშირი მათემატიკასთან, რაც ძირითადად, მოვლენათა აბსტრაგირებასა და სიმბოლურ ასახვაში გამოიხატება. ამის და კიდევ მრავალი მიზეზის გამო მათემატიკის გარკვეული საგნების ცოდნის გარეშე ინფორმატიკოსის აღზრდა წარმოუდგენელია.

მათემატიკა ინფორმატიკის სპეციალობისათვის ერთ-ერთი მთავარი საგანია, რომელზეც შემდეგ სპეციალობის უამრავი საგანია დაშენებული. ამის გამო, მათემატიკური საგნების გავლა სტუდენტებს სწავლების პირველივე საფეხურზე უწევთ და ხშირად მათემატიკის სწავლების საჭიროება მათთვის გაუგებარია, რთულია და მოსაწყენია. სწორედ ამიტომ, შემოთავაზებული იქნება ზოგიერთი მოსაზრება ინფორმატიკის ფაკულტეტებზე მათემატიკური საგნების სწავლების შესახებ. მაგალითად, როგორიცაა გამოყენებითი ხასიათის მათემატიკური ამოცანების ამოხსნა კომპიუტერის გამოყენებით. მათემატიკური ამოცანების დაპროგრამება გაცილებით გაზრდის მოტივაციას. კომპიუტერული პროგრამების გამოყენებით ისეთი ფუნდამენტური მათემატიკური საკითხების სწავლება, როგორიცაა ფუნქცია, მრავალი ცვლადის ფუნქცია, ზღვარი, წარმოებული და სხვა, გაცილებით ამარტივებს საკითხის ათვისებას და ხდის მას უფრო საინტერესოს.

გარდა ამისა, მნიშვნელოვანია სტუდენტმა დაინახოს მათემატიკის კავშირი დაპროგრამებასთან. ამასთან დაკავშირებით, საჭიროა ისეთი მაგალითების განხილვა მათემატიკაში, რომელიც რაიმე პროცესს აღწერს სხვადასხვა სფეროში, ანუ მარტივი მათემატიკური მოდელების შექმნა და მათი კომპიუტერული რეალიზაცია. სტუდენტებისთვის ნათლად წარმოჩენა, თუ ამ მოდელებში რა როლს თამაშობს ფუნქცია და როგორ გამოიყურება ის დაპროგრამებაში. დაპროგრამების კუთხით ისეთი მათემატიკური ცნებების განხილვა, როგორიცაა რთული ფუნქცია, რეკურსია, მიმდევრობები, ვექტორები, მატრიცები, ოპტიმიზაციის საკითხები მრავალგანზომილებიან სივრცეებში და სხვა. რა პრობლემები შეიძლება გამოიწვიოს პროგრამირებაში ფუნქციის თვისებების არცოდნამ, როგორიცაა მაგალითად,

ფუნქციის უწყვეტობა, მიმდევრობის კრებადობა და მრავალი მსგავსი საკითხის განხილვა ხელს შეუწყობს არა მარტო ფუნდამენტური საკითხების შესწავლას, არამედ გაზრდის სტუდენტის მოტივაციას.

The Constructed by Chain Depend Observations of Kernel Comparative Precision of the Density by L_1 Metrics

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Due the selection of chain depend elements is constructed the Rosenblatt–Parzen type kernel density estimation. In the certain conditions are determined the precision of estimation with L_1 metrics. Is considered the example for specific kernel function case.

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Charge Distribution and Currents in Nuclei

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RGEs for currents describing interaction of a two nucleon system with external probes are derived. The nonrelativistic pionless EFT is studied on the basis of these equations. To this end a non-trivial fixed point solution for the interaction current is identified. The linear equation for the perturbations near this fixed point current is derived and solved.

The Radii of Starlikeness of Some Integral Operators

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The object of the present paper is to study of radius of starlikeness two certain integral operators as follows

$$F(z) := \int_0^z \prod_{i=1}^n (f_i'(t))^{\gamma_i}\,dt$$

where $\gamma_i \in \mathbb{C}$, f_i $(1 \le i \le n)$ belong to the certain subclass of analytic functions.

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Exponential Snooding Techniques in Exchange Rate Forecasting

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This paper aims to investigate the behavior of daily exchange rate of the US Dollar/Georgian Lari and US Dollar/ERO. We make use of daily data to evaluate the parameters of each model and produce volatility estimates. Exchange rates forecasting challenging task in finance and for this task we will use statistical smoothing techniques and the forecasting ability of these methods are subsequently assessed using the symmetric loss functions which are the Mean Absolute Error(MAE) and Root Mean Square Error (RMSE).

Matrix Wiener–Hopf Problems Related to Propagation of Cracks in Elastic Structures

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In many problems of mechanics, especially fracture mechanics, the presence of mixed boundary conditions allows to apply integral transforms which lead to Wiener-Hopf problem [1]. This applies to both static problems within a continuous model [2] and dynamic problems, especially when it comes to steady-state regime. In addition, this technique is also effective in the case of discrete problems [3,4], which concern both lattice structures composed of masses and connecting springs [5] and structures made of masses and beams [6]. Wiener-Hopf technique allows us to determine the basic properties of the solution and to identify important physical applications relating to the nature of crack propagation or phase transitions [7]. On the other hand, in the case when there is a significant process zone in the vicinity of the crack tip, the application of this method is much more complicated. Sometimes it can be done by reducing to a matrix Wiener-Hopf problem [8]. In the present paper some of these problems are considered. Some numerical examples are presented and discussed.

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On Some Controlled Multi-channel Queueing Models

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Recent years, queueing theory has acquired new practical importance as a primary tool for studying, designing and optimizing real-world systems with interacting components for which queueing models (systems and networks) provide a simple but extremely useful representation. The role of the networks is constantly increasing in epidemiology, genetics, economics, in the study of cellular communication networks, computer viruses, computer support. Queueing models are successfully used at all levels of organization of such network structures.

The main model we consider is a queueing network consisting of r service nodes. Each node is a queueing system and it consists of an infinite number of servers. Therefore, if a customer arrives at such a system, then it begins processing immediately. Input flow arriving at the network is controlled by a Markov process. We define a service process in the network as an r-dimensional stochastic process $Q(t) = (Q_1(t), \dots, Q_r(t))'$, $t \geq 0$, where $Q_i(t)$, $i = 1, 2, \dots, r$, is the number of customers at the i-th node at instant t.

We study such a network in two cases. Firstly, we consider one-dimensional case, where the network has the only service node. It is assumed that the instants of customers' arrivals to the system are the same as jump instants of a homogeneous continuous-time Markov chain with a finite set of states. A customer arrived to the system immediately begins to be served anywhere on a free server. The service time is distributed exponentially. In this case generating function of the stationary distribution for the process $Q(t)$ is obtained. The form of the generating function is a matrix version of the Takacs formula.

Further, the network with $r > 1$ service nodes is studied. A common input flow of customers arrives at servicing nodes. This flow is controlled by a Markov chain $\eta(t)$ according to the following algorithm. As before, the instants of customers arrivals are the same as jump instants t_n, $n = 1, 2, \dots,$ of the chain $\eta(t)$. If the chain $\eta(t)$ jumps into state i at the instant t_n, the customer numbered n arrives for service into the ith node. Note, that the number of states for controlling Markov chain $\eta(t)$ coincides with the number of network nodes. At the node the customer occupies a free server for the time distributed exponentially with parameter μ_i. After service in the ith node the customer is transferred to the jth node with probability p_{ij}, $j = 1, 2, \dots, r$, or leaves the network with probability $p_{ir+1} = 1 - \sum\limits_{j=1}^{r} p_{ij}$. For a multivariate service process the condition of a stationary regime existence and a correlation matrix are found.

Finally, the stochastic network with controlled input flow is considered in heavy traffic. It is proved, that under certain heavy traffic conditions on the network parameters, the service process converges in the uniform topology to a Gaussian process. Correlation characteristics of the limit process are written via the network parameters.

Methodology of Teaching Geometric Construction Tasks in Secondary School

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It should be noted that geometric construction tasks are very narrow in secondary school. The process of solving geometric construction tasks is accompanied by a logical reasoning that is necessary for mathematical education. Therefore, it must take a proper place in mathematics. The report presents the methodological treatment of these issues.

On the Absolute Continuity for Random Measures under Nonlinear Transformations

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Random measures and their nonlinear transformations are considered. The conditions of absolute continuity for this measures are obtained in case of nonlinear and random transformation of a space. There is given explicit formula for Radon–Nikodym derivative. The notion of measurable functional is used and the logarithmic derivative technique of measures is developed.

Analysis of the Consumer's Choice under Risk Condition Using Utility Function

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Utility function, In case of choice under risk conditions, can be represented as a function of not only consumption level, but of probabilities as well. Denote c_1, c_2, and c_3 to be consumption for different conditions, and π_1, π_2, and π_3 to be probability of appearing of these three different conditions. If those probabilities independence condition is satisfied the utility function should obtain the following form:

$$U(c_1, c_2, c_3) = \pi_1 u(c_1) + \pi_2 u(c_2) + \pi_3 u(c_3).$$

This is the function that we call expected utility function. The substitutions marginal norm of the first and the second goods has the following form:

$$MRS_{12} = \frac{\Delta U(c_1, c_2, c_3)/\Delta c_1}{\Delta U(c_1, c_2, c_3)/\Delta c_2} = \frac{\pi_1 \Delta u(c_1)/\Delta c_1}{\pi_2 \Delta u(c_2)/\Delta c_2}\,.$$

MRS depends only on the first and second goods amount that we have and is independent of the third goods amount.

The Consumer's optimal choice of insurance will be determined under condition, that MRS of consumption of two expected values is equal to the ratio of the prices of corresponding consumptions:

$$MRS = -\frac{\pi \Delta u(c_2)/\Delta c_2}{(1-\pi)\Delta u(c_1)/\Delta c_1} = -\frac{\gamma}{1-\gamma}\,. \tag{1}$$

The expected value of insurance is exactly equal to the insurance price when insurance company does not get profit or loss the. Thus, $P = \gamma K - \pi K = 0$. This means that $\gamma = \pi$.

If we plug this value in Equation (1) and divide by π we will obtain condition that should satisfy optimal amount of insurance:

$$\frac{\Delta u(c_1)}{\Delta c_1} = \frac{\Delta u(c_2)}{\Delta c_2}\,. \tag{2}$$

Optimal amount of investment is defined with the condition that derivative with x of expected utility should be equal to zero. This will be global maximum because the second derivation of utility is automatically negative due to it's curvature.

If equation $EU'(x) = \pi u'(w + xr_g)r_g + (1 - \pi)u'(w + xr_b)r_b$ is equal to zero we will obtain:

$$EU'(x) = \pi u'(w + xr_g)r_g + (1 - \pi)u'(w + xr_b)r_b = 0.$$

This equation defines the optimal choice of x for a given consumer.

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Necessary and Sufficient Conditions for Weighted Boundedness of Integral Transforms Defined on Product Spaces in Generalized Grand Lebesgue Spaces

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Our talk deals with one-weighted boundedness criteria for the integral transforms generated by the strong maximal functions, multiple conjugate functions and Hilbert transforms in grand Lebesgue spaces with respect to measurable functions. We characterize both the weak and strong type weighted inequalities. Both cases of weighted spaces differing by position of the weight function in the norms are explored.

The Expansion Formula for Sturm–Liouville Equations with Spectral Parameter Nonlinearly Contained in the Boundary Condition

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Differential equations with nonlinear dependence on the spectral parameter arise in various problems of mathematics as well as in applications [1, 6]. Some aspects for boundary value problems in various formulations have been considered in [3, 4].

In this work, operator theoretic formulation is given for the boundary value problem, resolvent operator is constructed and expansion formula was obtained by using Titchmarsh's [5] method.

The singular Sturm–Liouville problem with spectral parameter in the boundary condition arise from applied problems such as the study of heat equation by [1, 2]. Spectral analysis involving linear dendence on the eigenvalue in the boundary condition was studied in [2].

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On Linear Stochastic Differential Equations in a Banach Space

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Linear stochastic differential equation in an arbitrary separable Banach space is considered.The corresponding linear stochastic differential equation for generalized random processes is constructed and is produced its solution as a generalized process Ito.It is found the conditions under which the received generalized random process is the Ito process in a Banach space; in such a way it is received the solution of the considered linear stochastic differential equation.

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შალვა მალიძე, კონსტანტინე ფხაკაძე

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On Kunneth's Correlation and it's Applications

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Let K be an abelian category and has enough injective objects, $T : K \to A$ be an any left exact covariant additive functor to an abelian category A and $T^{(i)}$ be right derived functors, $i \geqslant 1$. If $T^{(i)} = 0$ for $i \geqslant 1$ and $T^{(i)}C_n = 0$ for all $n \in Z$, then there is an exact sequence

$$0 \to T^{(1)}H_{n+1}(C_*) \to H_n(TC_*) \to TH_n(C_*) \to 0,$$

where C_* is a chain complex in the category K, $H_n(C_*)$ is the homology of the chain complex C_*, TC_* is a chain complex in the category A, $H_n(TC_*)$ is the homology of the chain complex TC_*. This exact sequence is the well-known Kunneth's correlation.

In the present work the conditions are found under which the infinite exact sequence

$$\begin{aligned} \cdots \to T^{(2i+1)}H_{n+i+1} \to \cdots \to T^{(3)}H_{n+2} \to T^{(1)}H_{n+1} \to H_n(TC_*) \to \\ \to TH_n(C_*) \to T^{(2)}H_{n+1} \to T^{(4)}H_{n+2} \to \cdots \to T^{(2i)}H_{n+i} \to \dots \end{aligned}$$

holds.

The formula makes it possible to generalize Milnor's formula for the cohomologies of an arbitrary complex, to relate the Kolmogorov and Sklyrenko homology to the Alexandrov–Čech homology, to generalize result of W. Massy for a locally compact Housdorff space and the direct system $\{U\}$ of an open subsets U of X such that $\overline{U}$ is a compact subset of X.

Teaching Mathematics with Generalization

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Methodology of teaching, in particular, the teaching of mathematics is a teacher's purposeful action to develop fitting competence in pupils and is only effective when it meets students' abilities and experiences, triggering their interest. Educational literature divides teaching methodologies in two groups: methodologies that promote learning of a specific material and methodologies that develop general/transferable skills. However, the division is nominal, since most of the methodologies help pupils develop material specific as well as general competencies. In order to use the teaching methods effectively, the student must have the appropriate learning strategies. Among the learning strategies one of the central partsis held by cognitive strategies that enable the learner to gain knowledge, process, analyze, critically evaluate, store and use it on demand in different situations. Cognitive strategies are studied by generalization, which is particularly effective during mathematics. The author discusses the positive sides of the generalization strategy with the generalization of trigonometric functions. In particular, the functions introduced using the axiomatic method are reminiscent of trigonometric sin and cos functions, but differ from them, for example, by determining the range and period values.By determining the features of the new functions and comparing them with those of the known functions, students gain abilities of researching, analyzing the retrieved information and critical evaluation.

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Investigation of Loss Stability of the Ring under the Action of Nonuniform External Pressure

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The purpose of this paper is a numerical analysis of the critical time for the loss of stability of a multilayer linear viscoelastic ring composed of various materials and under the action of an unevenly distributed external pressure of a given intensity. The search for reserves and the saving of material with a simultaneous increase in the bearing capacity of the structure is an actual and important problem of mechanics. When solving such a class of problems, it is necessary to take into account the geometric nonlinearity.

The material object of the study is a ring of radius R and thickness $2h$, and the study is carried out in the polar coordinate system (z, φ) . We denote by v and w, respectively, the displacement in the tangential direction and the deflection. The basis of the theory of compressed multilayer rings proposed here is as follows:

a) in the process of deformation, the nonlinearity is taken into account both for the deflection and for the tangential displacement (total nonlinearity);

b) neglecting the tangential displacement, we restrict ourselves to nonlinearity only of the deflection (partial nonlinearity);

c) when $v \approx 0$ is satisfied the inequality $w/R \ll 1$(simple non-linearity).

The considered ring is compressed by an unevenly distributed radial load, which varies in magnitude and direction according to law

$$q = q_0(1 + \mu \sin^2 \varphi),$$

here the parameter $\mu > 0$ u characterizes the non-hydrostatic nature of the compressible pressure, and q_0 is the control parameter of the loading.

Obtaining effective analytical solutions to the task is very difficult, and sometimes impossible. This is due to the need to integrate nonlinear boundary value problems with discontinuous coefficients. Therefore, to overcome the mathematical difficulties that arise, the solution of the problem is carried out by means of a variational method of mixed type in combination with the Rayleigh-Ritz method. The influence of the number of layers in the packet and the parameter of the unevenness of the external pressure on the critical buckling time are numerically revealed. Comparison of numerical results is performed in cases of total nonlinearity, partial nonlinearity and simple nonlinearity.

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Lim Colim Versus Colim Lim

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The use of $\mathbf{lim}^1$ (and in extreme cases also $\mathbf{lim}^2, \mathbf{lim}^3, \dots$) provides a reasonable description of any limiting behaviour in homology and cohomology for infinite polyhedra and, on the other hand, for compact spaces. In contrast, homology and cohomology (even ordinary) of non-triangulable non-compact spaces have been poorly understood until recently, due to the lack of any clues on how direct limits (**colim**) interact with inverse limits (**lim**). I will talk about a few first steps in this direction (some of them).

Story A. Here is a model situation in which **lim** and **colim** do not commute, but their "commutator" can be computed in terms of $\mathbf{lim}^1$ and a new functor $\mathbf{lim}^1_{\mathrm{fg}}$. There are two well-known approximations of the Steenrod–Sitnikov homology of a Polish space X: "Čech homology" $qH_n(x)$ and "Čech homology with compact supports" $pH_n(X)$. The homomorphism $pH_n(X) \to qH_n(X)$, which is a special case of the natural map **colim lim** $\to$ **lim colim**, need not be either injective (P. S. Alexandrov, 1947) or surjective (E. F. Mishchenko, 1953), but it is still unknown whether it is surjective for locally compact X. It turns out that for locally compact X, the dual map in cohomology $pH^n(X) \to qH^n(X)$ is surjective and we are able to compute its kernel. The original map $pH_n(X) \to qH_n(X)$ is surjective and its kernel is computed if X is a "compactohedron", i.e. contains a compactum whose complement is a polyhedron.

Story B. What happens if we permute **colim** with **lim** (or rather homotopy limit) in the definitions of Steenrod–Sitnikov homology and Čech cohomology? This very natural question has a well-known but very unnatural answer: the resulting "strong homology" and "strong cohomology" cannot be computed in ZFC already for simplest non-compact non-triangulable spaces (Mardešić–Prasolov, 1988). The reason being, already $\mathbf{lim}^1$ cannot be computed in ZFC for certain very simple inverse systems with uncountable indexing sets. We explain how to "correct" the functors $\mathbf{lim}^p$ for uncountable indexing sets so that the whole issue disappears. Namely, for a Polish space X the "corrected" strong (co)homology, as expressed in terms of $\mathbf{lim}^p$ of qth (co)homology of open neighborhoods of X in the Hilbert cube (resp. of compact subsets of X) via a Bousfield–Kan type spectral sequence, turns out to be nothing but the usual Steenrod–Sitnikov homology (resp. Čech cohomology). The correction of $\mathbf{lim}^p$ takes into account the topology of the indexing sets.

Story C. "Fine shape" of Polish spaces is a common correction of strong shape and compactly generated strong shape (which differ from each other essentially by permuting a **lim** with a **colim**), obtained by taking into account the topology on the indexing sets. For compacta, fine shape coincides with strong shape, and in general, its definition can be said to reconcile Borsuk's and Fox's approaches to shape. Both Steenrod–Sitnikov homology and Čech cohomology are proved to be invariant under fine shape, which cannot be said of any of the previously known shape theories of non-compact spaces. In fact, for a (co)homology theory, fine shape invariance is a strong form of homotopy invariance which implies the map excision axiom.

A Categorical Approach to Tilting Theory

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Tilting modules play a prominent role in representation theory of finite dimensional algebras. Tilting theory is mainly described as torsion theory in module categories. We show that it can also be accessed through the (co)monad associated to the tilting module and that some constructions related with "tilting" can be described at this level of generality. As an application we define and characterize tilting objects in the non additive case

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დადგენილია ფურიეს ჯერადი ტრიგონომეტრიული მწკრივების განზოგადებული აბსოლუტური კრებადობის საკმარისი პირობები ფუნქციის δ-ცვლილების მოდულის ტერმინებში. დამტკიცებული თეორემიდან მიიღება ცნობილი თეორემები ფურიეს ჯერადი მწკრივების აბსოლუტური კრებადობის შესახებ.

The Gibbs Phenomenon for Some Orthonormal Systems

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The Gibbs Phenomenon discovered by Henry Wilbraham in 1848 and rediscovered by Josiah Willard Gibbs in 1899, is the peculiar manner in which the Fourier series of some function behaves at a jump discontinuity. The n-th partial sum of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. The overshoot does not die out as n increases, but approaches a finite limit. We studied the Gibbs phenomenon for general Franklin systems and for Stromberg systems.

The general Franklin system corresponding to a given dense sequence of points $T = (t_n, n \geq 0)$ in $[0, 1]$ is a sequence of orthonormal piecewise linear functions with knots from T, that is, the n-th function from the system has knots $t_0, \ldots, t_n$.

Stromberg system is m-order spline system on $\mathbb{R}$, particularly, it is a modified classical Franklin system in the case $m = 0$. It was defined by Jan-Olov Stromberg in 1983 (see [1]). Stromberg system is obtained using Stromberg's wavelet.

The Gibbs Phenomenon has been studied for Fourier series with respect to several famous systems (see [2]–[7]). We proved that the Gibbs phenomenon for both of general Franklin systems and Stromberg systems occurs almost everywhere. In particular, for

Stromberg systems in the case of $m = 0$ the Gibbs phenomenon occurs everywhere in $\mathbb{R}$ and the Gibbs function is constant almost everywhere.

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The Existence of Unchangeable Sets for Non-linear Dynamic Systems (Neural Network Approach)

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While modeling public systems, it is comfortable to use a neural network approach. That is why the issues appearing in such systems are important in neural network theory too.

Occasional processes in public systems are often described by differential equations, and their evolution takes place in discrete time. The components in them are connected to each other in non-linear form and there may be a whole group of random sets that take the values from the set given in advance. This creates mindful behavior of the system.

The paper shows the existence of unchangeable set for such systems, in which the discrete dynamic system trajectory after some point of time, regardless of whether the initial value of the movement belonged to it or not, enters and stays in it.

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Necessary Conditions for Optimal Control of the Stationary Process in Conditions of Heat Exchange

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Let $u \in U$ limited and poor closed sen in U. Let rewrite the heat exchange boundary problem in operator form

$$\overline{A}(u, \overline{y}) = f, \quad f \in X^*, \quad \text{where} \quad \overline{A}(u, \overline{y}) = A(\overline{y}) - \overline{F}(u, \overline{y} + v(u)).$$

Objective functions, which minimizes with help of the operator u optimal control has the form

$$J(u) = \int\limits_{\Omega} |\overline{\Theta}(x,y) + \widehat{\Theta}(x,y) - \Theta^*(x,y)|^2 \, dx \, dy.$$

Objective functions which minimizes with help of the operator u optimal control has the form

$$L(u, \overline{y}(u)) = \int\limits_{\Omega} |\overline{\Theta}(x,y) + \widehat{\Theta}(x,y) - \Theta^*(x,y)|^2 \, dx \, dy.$$

We assume that the problem is regular (otherwise it will be regulated) and the optimum conditions of vibration are ascertained. To do so, the operator L must meet certain conditions of smoothness, namely:

1) Reflection $\overline{A}: U \times X \longrightarrow X^{*}$, V in some areas has partial derivatives.

2) Functional (function) $\overline{A}: U \times X \longrightarrow R$, V in some areas has partial derivatives with Gateaux.

1) conditions is not fulfilled $\overline{A}(u, \overline{y}) = A(\overline{y}) - \overline{F}(u, \overline{y}) + \nu(u)$ because of operator $\overline{F}$. So we build $\overline{F}_n$ operator build a family that meets the condition

$$\lim_{n\to\infty} \overline{F}_n(u, \overline{y}) = \overline{F}(u, \overline{y}) \ \ \forall\, \overline{y} \in X, \ \ u \in U.$$

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The Optimal Conditions for Optimal Control in the Conditions of Heat Exchange for Dynamic Process

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Consider the problem

$$L(u, \overline{y}(u)) = J(u) \longrightarrow \inf_{u \in U}, \tag{1}$$

where

$$J(u) = \int\limits_{\Omega} |\overline{\Theta}(t,x,y) + \widehat{\Theta}(t,x,y) - \Theta^{*}(x,y)|^2 \, dx\, dy$$

for heat exchange boundary problem whose operator form is

$$\overline{y}' + \overline{A}(u, \overline{y}) = f, \ \ f \in X^{*}, \ \ \overline{y}(0,x,y) = y_0(x,y), \ \ y_0(x,y) \in L_2(\Omega),$$

where $\overline{A}(u, \overline{y}) = A(\overline{y}) - \overline{F}(u, \overline{y}) + \nu(u)$.

Theorem 1. *$\overline{A}: U \times X \to X^{*}$ operator, which corresponds to the nonlinear boundary problem, where U is the limited poor closed subsystem on $\overline{U} = L_2(\Gamma_1 \times S)$ and represents nonlinear, limited, coercive operator with equally semi-limited variation and satisfy to the property (M).*

Theorem 2. *Problem* (1) *with the restriction* $\overline{y} \in K(u, \overline{y})$*, where* K *is convex and closed set on* $X = [L_2(S; \overset{\circ}{W}{}_2^1(\Omega) \cap L_p(S; L_p(\Omega)))]$*,* $p \geq 3$*, has a solution if and only if when it is regular.*

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Techniques of Debugging and Adjusting the LAN

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The brief description of programming capability for debugging the LAN software is given. Operating in the menu mode, the user can set up a connection with the nodes hooked up to the computer, load the programs prepared for the further debugging, look through, copy and modify the files with the net software. The program could be easily transferred to any computer of CP/M operation system. Operating in the menu mode, the user can specify a node architecture, the number of ports to be serviced, and give parameters for the initial adjusting of the ports for specific equipment. Using screen menu mode of operation, the user can copy, rename, restore, find and change phrases, to print files.

Parabolic Fractional Integral Operators with Rough Kernels in Parabolic Local Generalized Morrey Spaces

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Let P be a real $n \times n$ matrix, whose all the eighenvalues have positive real part, $A_t = t^P$, $t > 0$, $\gamma = trP$ is the homogeneous dimension on R^n and Ω is an A_t-homogeneous of degree zero function, integrable to a power $s > 1$ on the unit sphere generated by the corresponding parabolic metric. We study the parabolic fractional integral operator $I^P_{\Omega,\alpha}$, $0 < \alpha < \gamma$, with rough kernels in the parabolic local generalized Morrey space $LM^{\{x_0\}}_{p,\varphi,P}(R^n)$. We find conditions on the pair (φ_1, φ_2) for the boundedness $I^P_{\Omega,\alpha}$ from the space $LM^{\{x_0\}}_{p,\varphi_1,P}(R^n)$ to another one $LM^{\{x_0\}}_{p,\varphi_2,P}(R^n)$, $1 < p < q < \infty$, $\frac{1}{p} - \frac{1}{q} = \frac{\alpha}{\gamma}$, and from the space $LM^{\{x_0\}}_{p,\varphi_1,P}(R^n)$ to the weak space $WLM^{\{x_0\}}_{p,\varphi_2,P}(R^n)$, $1 \le q < \infty$, $1 - \frac{1}{q} = \frac{\alpha}{\gamma}$.

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Effect of Electrokinetic Processes on the Propagation of Non-Linear Waves in Gas Saturated Liquid

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It is known that there are many applications of a liquid with gas bubbles in nature, industry and medicine. Non-linear wave processes in a gas-liquid mixture were studied for the first time in works [1–3]. The Burgers, the Korteweg-de Vries and the Burgers -Korteweg-de-Vries equations were obtained in [1–5] for the description of long weakly non-linear waves. Non-linear waves in a liquid with gas bubbles in the three-dimensional case were considered in [6]. Linear waves in a gas-liquid mixture under the van Wijngaarden's theory were studied in [7, 8]. In [9] propagation of linear waves in a liquid containing gas bubbles at finite volume fraction was considered. We investigate non-linear waves in a gas saturated liquid taking into consideration influence of internal electrokinetic process. To the best of our knowledge the influence of potential difference parameter on non-linear waves propagation simultaneously was not considered previously. The aim of our work is to study non-linear waves in a liquid with gas bubbles taking into account electric potential differences for non-linear waves. The nonlinear waves described by the KdV-Burgers nonlinear equation as follows:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial z} - \eta\frac{\partial^2 U}{\partial z^2} + \beta\frac{\partial^3 U}{\partial z^3} = 0, \tag{1}$$

where

$$\eta = \frac{(\frac{4\mu}{R_0} + \frac{2}{3}\,\sigma E R_0)R_0}{6\alpha_1\alpha_2\rho_f}, \quad \beta = \frac{R_0 C_e}{6\alpha_1\alpha_2}.$$

We have investigated numerically the nonlinear wave process described by equation (1) as well. We defined that the more potential difference increase, the less radius of bubbles decrease [10]. Accordingly this result, we have demonstrated that when potential difference increases, the amplitude of waves attenuate gradually.

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Acoustic Scattering by Inhomogeneous Anisotropic Obstacle with Lipschitz Boundary

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We consider the time-harmonic acoustic wave scattering by a bounded anisotropic inhomogeneity embedded in an unbounded anisotropic homogeneous medium assuming that the boundary of the obstacle and the interface are Lipschitz surfaces. The material parameters may have discontinuities across the interface between the inhomogeneous interior and homogeneous exterior regions. The corresponding mathematical model is formulated as a boundary-transmission problem for a second order elliptic partial differential equation of Helmholtz type with peace wise Lipschitz-continuous variable coefficients. The problem is studied by the so-called nonlocal approach which reduces the problem to a variational equation containing sesquilinear forms over a bounded region occupied by the inhomogeneous obstacle and over the interfacial surfaces. This is done with the help of the theory of layer potentials on Lipschitz surfaces. The coercivity properties of the corresponding sesquilinear forms are analyzed and unique solvability of the variational equation is established, which in turn implies unique solvability of the acoustic scattering problem in appropriate Sobolev–Slobodetskii and Bessel potential spaces.

GE-Supplemented Modules

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Let M be an R-module. If every essential submodule of M has a g-supplement in M, then M is called a GE-supplemented module. In this work, some properties of these modules are investigated.

Lemma 1. *Let M be an R-module, U be an essential submodule of M and $M_1 \leq M$. If M_1 is GE-supplemented and $U + M_1$ has a g-supplement in M, then U has a g-supplement in M.*

Lemma 2. *Let $M = M_1 + M_2$. If M_1 and M_2 are GE-supplemented, then M is also GE-supplemented.*

Corollary 1. *The finite sum of GE-supplemented modules is GE-supplemented.*

Lemma 3. *Every factor module a GE-supplemented module is GE-supplemented.*

Corollary 2. *Every homomorphic image of a GE-supplemented module is GE-supplemented.*

Key words: essential submodules, small submodules, supplemented modules, G-supplemented modules.

2010 Mathematics Subject Classification: 16D10, 16D80.

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Cofinitely $\oplus - G$-Supplemented Modules

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Let M be an R-module. If every cofinite submodule of M has a g-supplement that is a direct summand of M, then M is called a cofinitely $\oplus - G$-supplemented module. In this work, some properties of these modules are investigated.

Proposition 1. *Any direct sum of cofinitely $\oplus - G$-supplemented modules is cofinitely $\oplus - G$-supplemented.*

Proposition 2. *Let M be a cofinitely $\oplus - G$-supplemented module. If every g-supplement of any cofinite submodule in M is a direct summand of M, then every direct summand of M is cofinitely $\oplus - G$-supplemented.*

Lemma 1. *Let M be a distributive and cofinitely $\oplus - G$-supplemented R-module. Then every factor module of M is cofinitely $\oplus - G$-supplemented.*

Corollary. *Let M be a distributive and cofinitely $\oplus - G$-supplemented R-module. Then every homomorphic image of M is cofinitely $\oplus - G$-supplemented.*

Key words: essential submodules, small submodules, cofinite submodules, G-supplemented modules.

2010 Mathematics Subject Classification: 16D10, 16D80.

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On a Problem of Minimization

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We plan to discuss a solution of a general problem of minimization, which implies in particular that the following inequality is true: let $n \geq 2$ be a natural number and $x_k \geq 1$, $k = 1, \ldots, n$ be real numbers; then

$$\prod_{k=1}^{n} x_k \geq \sum_{k=1}^{n} x_k - (n-1)\,.$$

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Mathematical Modeling of Dynamics of the Disk-Shaped Flying Device

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The disk-shaped flying devices were developed in many countries and they bring in mind the UFO forms. Chance Vought designed the first such device in the USA in the 1911. In 1939, in Nazi Germany Henrich Focke got a patent for the plane of a disk-shaped form with turbine powered vertical take-off capability. Development of such machines are still under work, although the information on such activities is classified. In his work, which is based on the theory of hypercomplex numbers (quaternions), the mathematical model of kinematics and dynamics of the disk-shaped flying device are developed based on Dr. G. Kvaratskhelia projects. This device flies and maneuvers. Dr. G. Kvaratskhelia also developed a new design of a disk-shaped military shell, flight mechanics of which is again, described by quaternions.

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Mathematical Modeling of Mud Flow

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For mud flows it is characteristic, macroscopic composite structure and obviously expressed not stationarity. The mud flow on the one hand consists of the stone and detrital weight and soil, on the other hand of water. Its education is, as a rule, caused by erosive processes in slopes of the bed of the mountain rivers that is in return caused by destruction of a green cover. Well-known destroying results of Mud Fow's. Practically, for all mountainous areas emergence of mud flow is characteristic though, on the basis of ant torrential actions perhaps considerably to reduce a loss for engineering constructions and agricultural grounds. In this regard, mathematical modeling of a mud flow and identification of the defining parameters is obviously important; definition of the expected loads of engineering constructions and optimally ant torrential actions to what this work is devoted. In work, the mud stream is represented as mix of two liquids, on the one hand it is a mud-stone soil component which we represent as the baroviscosy circle of Geniyev-Gogoladze and on the other hand is an incompressible liquid of Naiver-Stokes. Two-component mix is averaged by T. G. Voynich-Syanozhentsky's method and becomes isolated the equation of nonlinear diffusion for a water component and the law of a fractal filtration of water through soil under D. Dzhanelidze's law.

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Solution of Cauchy Problem of Non-Linear Mathematical Model of Rheymatoid Arthritis

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Our model of rheumatoid arthritis is a system of non-linear ordinary differential equations [1] and describes immunopathogenic dynamics in patients with rheumatoid arthritis. We improved this model by providing the treatment components that allows evaluating the effect of the drug and choosing a treatment scheme. It is considered the solution of Cauchy problem determined by the system.

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On One Problem of Plane Theory of Elasticity with Partially Unknown Boundary for Plate Weakened with a Hole

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The problem of plane theory of elasticity for plate with partially unknown boundaries weakened with hole is investigated. The uniformly distributed normal strees is applied to the hole boundary.The tangential stresses and the normal displacements are zero along the entire boundary of the body. The shape of the contour of the required hole and the stressed state of the given body are determined, provided that the tangential normal stress arising at contour of required hole would take the constant value. Equistable hole is found by means of complex analysis. The considered problem with partially unknown boundaries is reduced to the known boundary value problems of the theory of analytic functions by means of the developed method. The solutions are presented in quadratures. Equistable contour is constructed.

The Stability Problem of Differential Equations in the Sense of Ulam

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In this study we consider the stability problem of a general class of differential equations in the sense of Hyers–Ulam and Hyers–Ulam–Rassias with the aid of a fixed point technique. We extend and improve the literature by dropping some assumptions of some well known and commonly cited results in this topic. Some illustrative examples are also given to visualize the improvement.

A Study of the Fundamentals of Neutrosophic Soft Sets Theory

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Some operations on neutrosophic soft sets defined in the studies [1, 2]. In the present paper, we re-defined this operations differently from other studies. We have constructed the neutrosophic soft topological spaces differently from the study [1] in the direction of these re-defined operations. Finally, some basic notions and theorems on neutrosophic soft topological spaces are investigated and interesting examples are given.

Keywords: Neutrosophic soft set, neutrosophic soft interior, neutrosophic soft closure.

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Construction and Numerical Realization of Algorithms for approximate solution of Some Nonlinear Integro-Differential Equations

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In the first part of the work the problems for both dynamic beams are studied. The complex nonlinear problem of Timoshenko type for dynamic beam, which was solved using by us algorithm [1]. The algorithm we constructed gives an approximation for both spatial and temporal variables. An algorithm for the resulting system of discrete equations is constructed, considering the nonlinear, namely, cubic structure of the model. In order to simplify the iterative process in this part of the algorithm, we used Cardano's formulas, which allowed us to optimize the algorithm in certain sense, and positively affected the number of iterations.

In the second part of the thesis the problem of approximate solution of the nonlinear integro-differential equation for a static beam of Kirchhoff type is studied [2]. We used an approach, which reduces the problem to a nonlinear integral equation, using Green's functions, and for its solution we use the Picard's iterative method. The condition of convergence of considered method is established and the accuracy is estimated. The theoretical results related to the convergence of approximate solutions are confirmed by the numerical experiments.

The author express hearing thanks to Prof. J. Peradze for his active help in problem statement and solving.

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The Splitting of a System of Timoshenko Equations for a Plate

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We consider a boundary value problem for the following system of Timoshenko static equations [1]

$$\frac{\partial^2 u_i}{\partial x_i^2}+\frac{1-\mu}{2}\,\frac{\partial^2 u_i}{\partial x_j^2}+\frac{1+\mu}{2}\,\frac{\partial^2 u_j}{\partial x_1\partial x_2}+\frac{\partial w}{\partial x_i}\,\frac{\partial^2 w}{\partial x_i^2}+\frac{1+\mu}{2}\,\frac{\partial w}{\partial x_j}\,\frac{\partial^2 w}{\partial x_1\partial x_2}$$
$$+\frac{1-\mu}{2}\,\frac{\partial w}{\partial x_i}\,\frac{\partial^2 w}{\partial x_j^2}+\frac{1-\mu^2}{Eh}\,p_i=0,\quad i,j=1,2,\quad j\neq i,$$

$$k^2\,\frac{1-\mu}{2}\Big(\Delta w+\frac{\partial\psi_1}{\partial x_1}+\frac{\partial\psi_2}{\partial x_2}\Big)+\sum_{i=1}^{2}\frac{\partial}{\partial x_i}\bigg\{\frac{\partial w}{\partial x_i}\bigg[\frac{\partial u_i}{\partial x_i}+\mu\,\frac{\partial u_j}{\partial x_j}+\frac{1}{2}\Big(\frac{\partial w}{\partial x_i}\Big)^2\bigg]$$
$$+\frac{1-\mu}{2}\,\frac{\partial w}{\partial x_j}\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}+\frac{\partial w}{\partial x_1}\,\frac{\partial w}{\partial x_2}\Big)\bigg\}+\frac{1-\mu^2}{Eh}\,q=0,\quad j=1,2,\quad j\neq i,$$

$$\frac{\partial^2\psi_i}{\partial x_i^2}+\frac{1-\mu}{2}\,\frac{\partial^2\psi_i}{\partial x_j^2}+\frac{1+\mu}{2}\,\frac{\partial^2\psi_j}{\partial x_1\partial x_2}-6k^2\,\frac{1-\mu}{h^2}\Big(\frac{\partial w}{\partial x_i}+\psi_i\Big)=0,\quad i,j=1,2,\quad j\neq i,$$
$$a_i<x_i<b_i,\quad i=1,2,\quad 0<\mu<1,\quad E,h,k>0.$$

By analogy with [2], this system yields a nonlinear integro-differential equation of Kirchhoff type for the function $w(x_1,x_2)$, while for each pair of functions $u_1(x_1,x_2)$, $u_2(x_1,x_2)$ and $\psi_1(x_1,x_2)$, $\psi_2(x_1,x_2)$ we write a system of linear differential equations. The first of these systems is the Lame system of plane elasticity.

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Some New Results Concerning Strong Convergence of Partial Sums and Fejer Means with Respect to Vilenkin Systems

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This lecture is devoted to review some new strong convergence theorems for partial sums and Fejer means with respect to the Vilenkin systems.

In particular, we show that there exists an absolute constant c, such that

$$\sup_{n\in\mathbb{N}} \frac{1}{n\log n}\sum_{k=1}^{n} \|S_k f\|_{H_1} \le c\|f\|_{H_1} \ \text{ for all } \ f\in H_1, \tag{1}$$

and

$$\sup_{n\in\mathbb{N}} \frac{1}{n\log n}\sum_{k=1}^{n} \|\sigma_k f\|_{H_{1/2}}^{1/2} \le c\|f\|_{H_{1/2}}^{1/2} \ \text{ for all } \ f\in H_{1/2}. \tag{2}$$

Sharpness of inequalities (1) and (2) will be also obtained.

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The Density Nonparametric Estimates of a Dependent Observations Some for Class

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On the probabilistic space (Ω, F, P) a two-component stationary (in the narrow sense) sequence $\{\xi_i, X_i\}_{i\geq 1}$ is given, where $\{\xi_i\}_{i\geq 1}$ $(\xi_i : \Omega \to \Xi)$ is a controlling sequence, and the members of the sequence $\{X_i\}_{i\geq 1}$, $(X_i : \Omega \to R)$ are observations on some random variable. The cases of conditional independence and chainwise dependence of these observations are considered.Using observations $\{X_i\}_{i\geq 1}$ kernel observations of Rosenblatt–Parzen type of an unknown density of the variable X are constructed. The upper bounds of the mathematical expectations are established for the integral of the standard deviation of the obtained estimates from $f(x)$.

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On the Statistical Estimation of the Probability Distribution Density

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An asymptotic behavior of the integral mean square deviation for projective estimation of the probability distribution density with the use of the integrals according to a Feinmann measure examined for the certain class of the functionals is considered in the work.

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Improved Understanding of Aqueous Solubility Modeling through Topological Data Analysis

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Topological data analysis is a family of recent mathematical techniques seeking to understand the 'shape' of data, and has been used to understand the structure of the descriptor space produced from a standard chemical informatics software from the point of view of solubility. We have used the mapper algorithm, a TDA method that creates low-dimensional representations of data, to create a network visualization of the solubility space. While descriptors with clear chemical implications are prominent features in this space, reflecting their importance to the chemical properties, an unexpected and interesting correlation between chlorine content and rings and their implication for solubility prediction is revealed.

A parallel representation of the chemical space was generated using persistent homology applied to molecular graphs. Links between this chemical space and the descriptor space were shown to be in agreement with chemical heuristics.

The use of persistent homology on molecular graphs, extended by the use of norms on the associated persistence landscapes allow the conversion of discrete shape descriptors to continuous ones, and a perspective of the application of these descriptors to QSPR problems is presented.

აფხაზური ხმოვანმართვიანი მკითხველი სისტემის საკომპიუტერო და საინტერნეტო ვერსიებისათვის - შედეგები და პერსპექტივები

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სტუ ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [1] ქვეპროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული

თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ ფარგლებში შემუშავდა ქართული ჭკვიანი კორპუსი [2] და ქართული ხმოვანმართვიანი მკითხველი სისტემის საცდელ-სამომხმარებლო ვერსიები. ამასთან, დღეს უკვე ქართული ჭკვიანი კორპუსი გაფართოვდა და მოიცავს აფხაზურ, ყაბარდოულ, ჩეჩნურ, ლეკურ და მეგრულ კორპუსებს. ამდენად, ჩვენი კვლევის ერთ-ერთი მიზანია ამ იბერიულ-კავკასიური ენების კორპუსების აღჭურვა ხმოვანმართვიანი მკითხველით, რის პირველ ეტაპადაც ჩვენ განვიხილავთ აფხაზური ხმოვანმართვიანი მკითხველი სისტემების აგებას. აქვე, ხაზს ვუსვამთ: 1. ჩვენი აფხაზური და მეგრული კორპუსები საცდელი სახით უკვე აღჭურვილია აფხაზური და მეგრული საინტერნეტო მკითხველებით [3]; 2. უკვე აგებული გვაქვს აფხაზური ხმოვანმართვიანი ვორდის მკითხველის საცდელი საკომპიუტერო სისტემა [3].

ამგვარად, მოხსენებისას მიმოვიხილავთ უკვე აგებულ იბერიულ-კავკასიური ენების კორპუსებს. ამასთან, ყურადღებას გავამახვილებთ აფხაზურ ხმოვანმართვიან საკომპიუ-ტერო მკითხველსა და აფხაზურ და მეგრულ საინტერნეტო მკითხველებზე და იმ პერსპექ-ტივებზე, რაც დღეს ჩვენ აფხაზური ხმოვანმართვიანი საინტერნეტო მკითხველის აგების მიმართულებით გვაქვს [3].

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ქართული უნივერსალური ჭკვიანი კორპუსი როგორც ნაბიჯი ერთიანი ქართული საინტერნეტო ქსელისკენ - შედეგები და პერსპექტივები

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ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

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მოხსენებისას მიმოვიხილავთ ჩვენი კვლევის ერთ-ერთ მიზანზე, რაც ქართული უნივერსალური ჭკვიანი კორპუსის ისეთ ქართულ მრავალენოვან საინტერნეტო ქსელად ფორმირებას გულისხმობს, რომელიც ქართულთან ერთად თავისუფლად გამოყენებადი იქნება აფხაზური და ყველა სხვა კორპუსში არსებული იბერიულ-კავკასიური ენებითაც, რაც, ცხადია, ქართულთან ერთად ამ ენებსაც საშუალებას მისცემს ციფრულ ეპოქაში მყარად დაიმკვიდრონ ადგილი კულტურის მაწარმოებელ და მატარებელ ენებს შორის.

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A Sequent-Type Calculus for Three-Valued Circumscription

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Circumscription is an important formalism for nonmonotonic reasoning introduced by John McCarthy [3, 4] based on the idea of minimal-model reasoning. Roughly speaking, for checking whether a formula is a logical consequence of a given theory, T, instead of considering all models of T, only models satisfying a certain minimality condition are taken into account. In this work, we consider *three-valued circumscription* [5], a generalisation of the basic circumscription approach based on three-valued logic, and introduce a sequent-type calculus for it. Our calculus generalises a similar one for standard circumscription as introduced by Bonatti and Olivetti [2], who relied their system on a sequent-type calculus for valid formulas and a so-called *complementary sequent-type calculus* for invalid formulas. In our approach, in order to accommodate the underlying three-valued logic, L, we use an approach based on *many-sided sequents* [1]. Intuitively, a many-sided sequent is a triple of form $\Gamma_1 \mid \Gamma_2 \mid \Gamma_3$, where each Γ_i $(i = 1, 2, 3)$ is a finite set of formulas, corresponding to one of the three truth values of L. Overall, our calculus, then, comprises a many-sided sequent calculus for formulas valid in L, a complementary calculus for formulas invalid in L, and specific nonmonotonic inference rules similar to the one for standard circumscription by Bonatti and Olivetti [2].

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Mathematical Modeling of the Operation of the Consumer Gas Consumer Safety System

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The report describes issues of building mathematical model of the security of gas consumption.

There is presented the gas leak monitoring system in the high-rise buildings. The results of calculations have shown that Device and monitoring system with its main technical parameters are in compliance with the existing foreign analogs.

Giorgi Nikoladze's Merits in the Development of Computer Technologies

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George Nikoladze was a prominent Georgian scientist, primarily a mathematician, a specialist in the field of geometry, an engineer-metallurgist, the author of many works on mathematics, Doctor of Sorbonne University. He was one of the initiators of Georgian mountaineering. He was among the founders of Georgian scientific and technical terminology.

The paper deals with one of the interesting aspects of G. Nikoladze's activity.

Brilliant knowledge of mathematics and eternal aspiration for technical novelties led him to the idea of designing a computing machine based on electric devices. The model was made. G. Nikoladze obtained a patent for his device, but failed to realize the idea because of lack of finances

The Martingale Approach in the Problem of the Stochastic Integral Representation of Functionals

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It is known that any functional of Brownian motion with finite second moment can be expressed as the sum of a constant and an Ito stochastic integral. A corollary of this result is that any martingale (on a closed interval) that is measurable with respect to the increasing family of σ-fields generated by a Brownian motion is equal to a constant plus a stochastic integral. The general result only asserts the existence of the representation and does not help to find it explicitly. Sufficiently well-behaved Frechet-differentiable functionals have an explicit representation as a stochastic integral in which the integrand has the form of conditional expectations of the differential.

The first proof of the martingale representation theorem was implicitly provided by Ito (1951) himself. Many years later, Dellacherie (1974) gave a simple new proof of Ito's theorem using Hilbert space techniques. One of the pioneer work on explicit descriptions of the integrand is certainly the one by Clark. In general, the finding of explicit expression for integrand $\varphi(t,\omega)$ of stochastic integral is very difficult problem. According to Ocone (1984) $\varphi(t,\omega)=E[D_t^BF|\Im_t^B]$ (so called Clark–Ocone formula), where D_t^B is the so called Malliavin stochastic derivative. A different method for finding the process $\varphi(t,\omega)$ was proposed by Shiryaev, Yor and Graversen (2003, 2006), which was based on the Ito (generalized) formula and the Levy theorem for the Levy martingale $M_t=E[F|\Im_t^B]$ connected with the considered functional F. Later on, using the Clark–Ocone formula, Renaud and Remillard (2006) have established explicit martingale representations for path-dependent Brownian functionals.

We study the problem of stochastic integral representation of stochastically nonsmooth functional. In [2], we also considered the stochastically nonsmooth path-dependent Brownian functional. It turned out that the requirement of smoothness of the functional can be weakened (see [1]). In particular, we generalized the Clark-Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method for finding of the integrand.

Here, by conditional averaging of the stochastic functional, we pass to the deterministic function of two variables, study the properties of its smoothness, and on the basis of the martingale approach we derive the stochastic integral representation formula with an explicit form of the integrand.

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Inverse Trigonometric Functions

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Function and its inverse is one of the most important topics in the high school math curriculum. Some of the important applications of the topic are following: to determine the range of a rational function, we find the domain of its inverse; to learn exponential and logarithmic functions; to investigate equivalency of irrational equations and inequalities, we use the properties of the inverse of a power function.

Graphical representation of inverse functions is tightly connected with line symmetry. Indeed, to build the graph of the inverse, one needs to reflect the graph of the given function over the angle-bisector of the I and III quadrants (line $y = x$). Therefore, understanding line symmetry is a crucial pre-requisite to learning inverse functions.

When it comes to inverse trigonometric functions, their role in the curriculum is limited to writing down the solutions of trigonometric equations using specific values of inverses. We believe, that function and its inverse should be taught simultaneously to ensure that students have a deep understanding of the topic. This, of course, includes trigonometric functions as well. It turns out, that "Geogebra" is an extremely effective tool to teach and learn trigonometric functions and their inverses.

On the Products of Algebraic K-Functors of a Crossed Hopf Algebras

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Let k be a commutative ring with identity, H be a Hopf algebra over k and let A be a commutative H-module algebra. If $\sigma \in Reg_{+}^{2}(H, A)$ is a 2-cocycle of Sweedler's cohomology [1] of H with coefficients in A then a crossed product $A\#_{\sigma}H$ (see [1]) of the Hopf algebra H and the commutative H-module algebra A is defined as an A-module $A \otimes_k H$ with multiplication

$$(a\#_{\sigma}g)(b\#_{\sigma}h) = \sum_{(g),(h)} a(g_{(1)}b)\sigma(g_2 \otimes h_{(1)})\#_{\sigma}g_{(3)}h_{(2)}.$$

We construct some product for algebraic K-functors of the crossed product of the commutative ring and the cocommutative Hopf algebra and investigate its some properties. This product generalises corresponding results for (crossed) group rings and for restricted (crossed) enveloping algebras of (super) Lie p-algebras.

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Usage of TSR Logic Methods in Natural Event Problems

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Regional climate formation above the territory of complex terrains is conditioned dominance due to of joint action of large-scale synoptic and local atmospheric processes which is basically stipulated by complex topographic structure of the terrain. The territory of

Georgia is a good example for that. Indeed, about 85% of the total land area of Georgia is mountain ranges with compound topographic sections which play an important role for spatial-temporal distribution of meteorological fields. As known, the global weather prediction models can well characterize the large scale atmospheric systems, but not enough the mesoscale processes which are associated with regional complex terrain and land cover. With the purpose of modelling these smaller scale atmospheric phenomena and its characterizing features, it is necessary to take into consideration the main features of the local complex terrain, its heterogeneous land surfaces and at the same time influence of large scale atmosphere processes on the local scale processes.

The Weather Research and Forecasting (WRF) model is a mesoscale numerical weather prediction system, designed for forecasting needs. WRF consists of several solvers and it is quite flexible to be extended for different needs. One of such example is the Polar WRF. One of our goals is to combine WRF model with the solver developed by us, to get better prediction of temperature, wind velocity, showers and hails for different set of physical options in the regions characterized with the complex topography.

Rule-Based Techniques in Access Control

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Access control is a security technique that specifies which users can access particular resources in a computing environment. Formal description of access control is extremely important, since it should be defined, unambiguously, how rules regulate what action can be performed by an entity on the resource, how to guarantee that each request gets an authorization decision, how to ensure consistency, etc. It is also important that such a formal description is at the same time declaratively clear and executable, to avoid an additional layer between specification and implementation.

Over the years, numerous access control models have been developed to address various aspects of computer security. In this talk, we describe traditional models: discretionary access control (DAC), mandatory access control (MAC) and role-based access control (RBAC). Despite successful practical applications of these traditional models, they have certain disadvantages, which was the reason why new approaches emerged. We will focus on one modern approach, attribute-based access control, which has been proposed in order to overcome limitations of traditional models.

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Numerical Computation of Rayleigh-Benard Problem for Dilatant Fluids

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In this study, we consider flow properties of Dilatant fluids motion generated by thermal gradients in an enclosed cavity region. As far as Rayleigh-Benard convection is concerned, share rate equals to the vorticity function. As a result, since the share rate is zero on the boundary, vorticity function must be zero as well. Pseudo time derivative is used to solve the continuity, momentum and energy equations with these conditions. Therefore, the governing equations of fluid of vorticity-stream function and temperature formulations are solved numerically using finite difference method. The stream function, vorticity and temperature results are obtained for the steady, two-dimensional, incompressible Diatant flow. These results are presented both in tables and figures. The stream function, vorticity and energy equations are solved separately with the numerical solution method used in this study. Each equation with pseudo time parameter on very fine grid mesh is solved step by step with a pair of tridiagonal system. The advantage of this process is that it gives the solution of the flow problems effectively and accurately.

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Initial-Boundary Value Problems Related to Integrable Nonlinear Equations

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We consider several important initial-boundary value problems related to integrable nonlinear equations. We start with the well-known *compatibility condition* (zero curvature equation). Then we solve *second harmonic generation* equation using the evolution of the Weyl function. Finally, since initial-boundary value problems for integrable nonlinear equations are mostly overdetermined, we discuss some cases where initial condition is determined by the boundary condition. There is also a simple connection between Weyl functions and reflection coefficients. Thus, one can discuss reflection coefficients instead of Weyl functions. The talk is based on the papers [1–4].

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Sun's Direct Radiation Impact on Glaciers Melting Index for Some Glaciers of Georgia

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The glaciers of Georgia have undergone significant changes against the background of global warming. Most of them have disappeared, and some have suffered degradation. The glacier area has decreased during the retreat, but at the same time the total number of glaciers has increased. Generally the glaciers play a major role in formation the water balance of the region and their reduction or disappearance poses significant damage to the natural ecosystems. The paper deals with major meteorological factors operating on glaciers and the melting of direct solar radiation on the basis of the melting energy model of the Enguri basin glacier.

On Regularity Results for Localization Operators

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In this work we introduce localization operators by means of time frequency analysis. Regularity results for localization operators with symbols and windows living in various function spaces (such as Lebesgue spaces, modulation spaces or mixed Lorentz type modulation spaces) are discussed.

Some key references are given below.

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One Method of Teaching the Bellman's Optimality Principle

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When teaching mathematic modeling to students of economics and not only them, the greatest importance is attributed to the solutions of the examples from the practice by using mathematic methods and mathematic modeling elements. Among those practical problems, which are solved by using mathematic modeling, the problems that are solved by dynamic programming method are exceptionally interesting. Our goal is to discuss the teaching of solving a specific problem with dynamic programming method. It's important to show students the principle and action mechanism of dynamic programming while solving the problem.

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Improved Algorithm of Customers Segmentation

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Finding, developing and retaining a customer is becoming a priority for companies. If a company has more than a thousand customers, it is too difficult to take into account the needs of each of them. It is advisable to analyze the needs of several client groups, rather than each of them. It is advisable to analyze the needs of several client groups rather than each of them, to effectively solve the problem of segmentation the customer's market based on the clients consumer needs and preferences in order to further target marketing tasks [1].

According to the formally defined client market segmentation problem [2], the distances between customers are represented as points of the m-dimensional space R^m, and for the analysis Fuzzy C-Means and Gustavson–Kessel algorithms [3, 4], the Euclidean distance, as the geometric distance in m-dimensional space, as well as the Mahalanobis distance [3, 5] for its correction are used. It makes possible to form clusters of a spherical shape with an arbitrary orientation of the axes.The algorithms of Fuzzy C-Means and Gustavson–Kessel are based on using a fuzzy neural network. Both algorithms train the network in order to minimize the target function $J(M, U, C)$ according to the self-organization algorithm, which assumes the clustering of the customer market.

The output of the algorithm is a list of clients and the probability of their belonging to a particular market segment.

The proposed mathematical model and an improved algorithm of client market segmentation will allow to divide the customer's market into groups, taking into account the algorithm type (fast or accurate) selected by user, client attributes for the segmentation, fuzzyness parameter w, stopping criterion δ and a given number of segments g on which it is necessary to divide the market of customers. In the situations where the value of the client attribute vector is on the boundary between clusters, the proposed information technology gives a more accurate result of customer segmentation. It was proved based on the conducted tests, which used the solution of the client market segmentation, by using the developed information technology, which is based on the improved algorithm and Deductor Academic [6].

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Feasibility Study of Using Associative Rules During the Software Development

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Various data mining technologies can be used to extract useful information that will be used during software development (SWD), and help to discover patterns between software applications.

The purpose of this study is to justify the feasibility of using the technology to search for associative rules (AR) in the software development process.

Associative rules can be used to accomplish these goals during software development:

1. To detect defects in the software, by searching for AR among all extracted versions of the program code. This approach will identify vulnerabilities in the architecture of the software in the early stages and reduce the material costs of their correction in the future [1].

2. To determine the necessary resources for the software development process and effectively manage them. It is proposed to do this, by using the theory of fuzzy sets and search for AR. This will effectively plan the actions of each participant during the software development process [2].

3. To identify the developer, who will be assigned to correct the defect. For this purpose, it is proposed to use the search technologies of AR and such characteristics of

the defect as: importance, priority, short description and developer. This approach will automate the process of assigning a developer to fix a certain defect [3].

Therefore, the use of technology of searching for associative rules is feasible in the software development. Therefore, it is proposed to use a modified Frequent Pattern Growth method of searching AR to determine the time required to perform a particular task by a particular developer. The modification is that the data before the search for associative rules is classified, which will allow to obtain more informative associative rules.

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On the Number Of Representations of Integers by the Quadratic Forms of Eight Variables

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We form the spherical polynomials of second order with respect to some quadratic form $Q(X)$, we form also the basis of the space of corresponding generalized theta-series and obtained the formulae for the number of representations of positive integers by quadratic form of eight variables.

Let

$$Q(X) = x_1^2 + 2x_2^2 + 2x_3^2 + 4x_4^2 + x_1x_2 + x_1x_4 + x_2x_3 + x_2x_4 + 2x_3x_4$$

be a quadratic form of type $(-2, 13, 1)$ (see [3]) and

$$F = Q(x_1, x_2, x_3, x_4) + Q(x_5, x_6, x_7, x_8)$$

be the quadratic form of eight variables. For this quadratic form we have proved the following

Theorem. *The number of representations of positive integers n by quadratic form F is given by*

$$r(n,F) = \frac{24}{17}\,\sigma_3^*(n) + \frac{91}{34}\Big(\sum_{Q(x)=n} x_1^2 - \frac{4}{13}n\Big) - \frac{247}{34}\Big(\sum_{Q(x)=n} x_1x_2 - \frac{4}{13}n\Big) - \frac{26}{17}\Big(\sum_{Q(x)=n} x_1x_3\Big),$$

where

$$\sigma_3^*(n) = \begin{cases} \sigma_3(n), & \text{if } (13,n)=1, \\ \sigma_3(n) + 169\sigma_3\left(\dfrac{n}{13}\right), & \text{if } 13|n. \end{cases}$$

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მათემატიკური მეთოდებით აგებული ქართულ-ფრანგულ-ინგლისური შინაარსულად მთარგმნელი სისტემის პირველი საცდელი ვერსია

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სტუ ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი" [1] ქვეპროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ" [2] ფარგლებში ფხაკაძის ქართული ენის ლოგიკური გრამატიკით შემუშავებულ მათემატიკურ მეთოდებზე დაყრდნობით [3] აიგო ქართული-ინგლისურ-გერმანული მთარგმნელის საცდელ-სამომხმარებლო ვერსია [2], რომელიც ეყრდნობოდა 2005 წელს აგებულ საცდელ ქართულ-გერმანულ მთარგმნელს [4]. ამგვარად, სამაგისტრო კვლევით დაიგეგმა ქარ- თულ-ფრანგულ-ინგლისური მთარგმნელის საცდელ-სამომხმარებლო ვერსიის აგება, რითაც ეს კვლევა ნაწილი გახდა პროექტით „ქართული ენის ტექნოლოგიური ანბანი" ქართული ენის სრული ტექნოლოგიური უზრუნველყოფის და, შესაბამისად, ციფრული კვდომის საფრთხისგან დაცვის მიზნით სტუ-ში 2012 წლიდან მიმდინარე კვლევებისა.

მოხსენებისას მოკლედ მიმოვიხილავთ სამაგისტრო თემის „მათემატიკური მეთოდებით აგებული ქართულ-ფრანგულ-ინგლისური შინაარსულად მთარგმნელი სისტემის პირველი საცდელი ვერსია" [5] მიზნებს, ამოცანებს, მეთოდებსა და უკვე მიღწეულ შედეგებს.

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On the Eigenvalue Problem of Functionally Graded Cylindrical Shells with Mixed Boundary Conditions in an Elastic Medium

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The cylindrical shells are widely used as structural elements in many engineering applications, aerospace, nuclear reactors, petrochemicals, marine industry, civil engineering and mechanical engineering. The vibration analyses of cylindrical shells have been investigated for a long time. Recently, a new class of composite materials known as functional graded materials (FGMs) has become attractive due to the increased requirements for structural characteristics in the industry, especially at extremely high temperatures and high speeds. FGMs are designed to achieve functional characteristics with variable characteristics in one or more directions. The concept of FGMs was first expressed in 1984 by a group of Japanese scientists [1]. FGM cylindrical shells interact with the elastic medium in some special applications. Most floors can be represented by a mathematical model based on Pasternak, and sandy soils and liquids can be represented by the Winkler model. Vibration analysis of the FGM shells on the elastic foundations is done by some scientists [2, 3]. There are limited studies on the vibration of unconstrained FCM shells under mixed boundary conditions [4]. The purpose of this work is to examine the free vibration of FGM cylindrical shells under mixed boundary conditions resting on the Winkler elastic foundation. Donnell shell theory is taken into account in the derivation of the governing equations for FGM cylindrical shells. The closed form solution for circular frequency on the Winkler elastic foundation is obtained. Using the obtained formula, numerical analyzes are performed and the influences of various profiles of FGM and Winkler elastic foundation on the circular frequency of FGM cylindrical shells with mixed boundary conditions are analyzed.

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On Some Paradoxes in the Theory of Elastic Contact with Sliding

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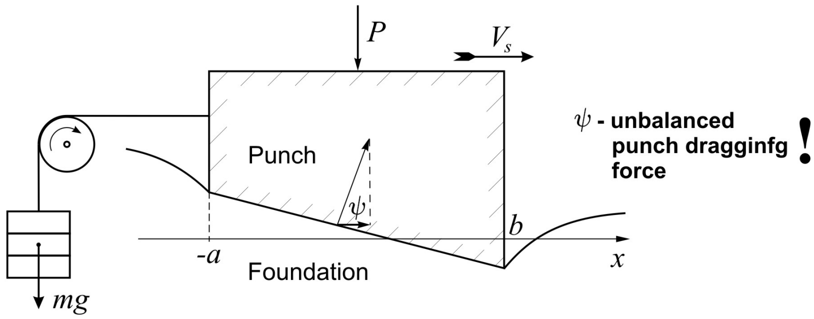
Two problems on sliding contact of a punch with elastic foundation are considered, notably: case of punch with corners (fixed contact region) without friction and case of smooth punch with friction.

The formal statement of the 1-st problem leads to paradox of "perpetuum mobile" (figure). To solve the paradox a concept of corner tangential forces $\psi_{\pm}$ is introduced. Such kind of forces are defined by limiting passage from the smooth punch to the corner one using known equations of contact problem for elastic half-plane [1]:



$$\psi_\pm = \pm \frac{C_\pm^2}{4\pi\theta(a+b)}; \quad C_\pm = P \pm \begin{Bmatrix} a \\ b \end{Bmatrix} A_0 + A_1,$$

$$A_k = -\theta \int_{-a}^{b} \frac{s^k g'(s)\, ds}{\sqrt{(a+x)(b-x)}}, \quad \theta = \frac{E}{2(1-\nu^2)}.$$

Friction sliding of the smooth punch (2-nd case) is taking place under action of the following tangential forces: external drag force T, Coulomb friction force F and deformation friction force ψ. The powers of these forces are related by energy conservation law $M_T = M_F + M_\psi$. The last equality means that work of the external force T is only partially expended on covering Coulomb friction losses. In connection with it a paradoxical issue arises: what the remainder M_ψ of the power M_T is expended on, whereas there is no dissipation of energy in the elastic foundation?

For elimination of the contradiction it is proposed to take into account variation of the sliding velocity over contact region: $V(x) = V_s(1 + u'(x))$ [2] in calculation of Coulomb friction losses.

The work was supported by the Russian Foundation for Basic Research (17-01-00352).

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On the Regularity of Positive Ternary Quadratic Forms

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Let F be a positive ternary quadratic form, which belongs to some genus G of quadratic forms.

B. W. Jones in [1] has proved that a form F of genus G is regular if and only if it represents all the integers represented by the every form of G.

Therefore, whenever F is the only reduced form of a genus, it is regular. Conversely, if the form F is regular, it may not belong to one class genera. There are just small amount of such forms and it is difficult to find them.

Based on the theory of a modular forms, general approach for representation of numbers by positive quadratic forms was developed by G. A. Lomadze and then by his disciples.

Using J. S. Hsia's [2] consideration and based on the properties of modular forms simple formulas for the representation of integers by two ternary quadratic forms belonging two class generas are obtained and all arithmetical progressions, whose members and only they are not represented by these quadratic forms are founded. Therefore, these two quadratic forms are regular.

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Two-Parametric Analysis of an elastic Half-Space Coated by a Soft/Stiff Thin Layer

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Static equilibrium of an elastic half-space with a thin soft/stiff coating, subject to a vertical load, is considered. The two small parameters arise from relative small thickness of the coating, as well as from the contrast in stiffnesses of the layer and the substrate. A version of the method of direct asymptotic integration, see e.g. [1], is then developed. First, a solution of a toy plane-strain problem for a harmonic load is obtained, in order to establish the asymptotic scaling for general setup for both limits of soft and stiff coating layer. For a sufficiently soft coating the Winkler-Fuss hypothesis is justified at leading order, and higher-order corrections to the formulation, such as the Pasternak model, are also studied [2]. In case of a rather stiff coating the traditional thin plate model is validated. Alternative approximations for less pronounced contrast are also addressed.

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The Direct and Reverse Relationships of Generalized Zener Body, when the Constitutive Relationship Contains Conformable Fractional Derivatives

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Be Considered generalized Zener body, when the constitutive relationship contains conformable fractional Derivatives. The constitutive relationship for generalized Zener body has been written down by means of the kinematic and dynamic relations. The direct and reverse relationships have been obtained for generalized Zener body as well as the expressions of the creep and relaxation functions. The case is considered when in model the spring is replaced with an element of fractional calculus.

On the Decomposition into a Direct Sum of Locally Linear Compact Topological Abelian Group

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Locally linear compact abelian group G is a weakly compact group, which has such a linear compact open subgroup H, that corresponding quotient group G/H is a discrete. For such group G with a subgroup H of the given property, a decomposition into a direct product with groups of rank 1 is studied:

Theorem. *For the decomposition into a direct product with groups of rank* 1 *of a locally linear compact abelian group* G *it is necessary and sufficient:*

1. *A decomposition into a direct product with groups of rank* 1 *of subgroup* H*;*
2. *A decomposition into a direct product with groups of rank* 1 *of quotient groups* $G|p^{\infty}H$ *with a subgroup* $f(H)$ *of the given property.*

On the Solution of Some Non-Classical Problems of Statics of the Theory of Elastic Mixture in Infinite Plane with a Circular Hole

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In the work for homogeneous equation of statics of the linear theory of elastic mixture in an infinite domain with a circular hole are considered two boundary value problems. In the case of problem I on the boundary of domain there are prescribed projections of the partial displacements vectors on the normal and rotation, and in the case of problem II on the boundary of domain there are prescribed projections of the partial displacements vectors on the tangent and divergence. The problems are uniquely solvable and the solutions are represented in quadratures.

The Task of Chemical Synthesis with the Modelling of Differential Equations

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Differential equations take an important place in math appendixes of different fields of science. They are quite effective and common way to solve the tasks by using natural science and technique. Many real processes are described easily and completely with differential tasks. Therefore the interest toward creating differential equations is quite natural.

Our aim is to show different tasks of natural science and technique and support the modern methods to study the creation of differential equations. These tasks are derived in the process of scientific research or production. Chemical kinetics equation for example: molecular reaction, stationary diffusion and many others are brought on the solution of differential equation or differential equation system.

We decided to study the chemical synthesis in its theoretical model of differential equation. Therefore, we deal with the task of chemical synthesis with the modelling of differential equations.

Convergence and Summability of the One- and Two-Dimensional Vilenkin–Fourier Series in the Martingale Hardy Spaces

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The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves. Such waves have already been used frequently in the theory of signal transmission, multiplexing, filtering, image enhancement, codic theory, digital signal processing and pattern recognition. The development of the theory of Vilenkin–Fourier series has been strongly influenced by the classical theory of trigonometric series. Because of this it is inevitable to compare results of Vilenkin series to those on trigonometric series. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, which studies orthonormal systems from the point of view of the structure of a topological group.

This lecture is devoted to review theory of martingale Hardy spaces. We present central theorem about atomic decomposition of these spaces and show how this result can be used to derive necessary and sufficient conditions for the modulus of continuity such that partial sums with respect to one- and two-dimensional Vilenkin–Fourier series converge in norm. Moreover, we also present some strong convergence theorems of partial sums of the one- and two-dimensional Vilenkin systems.

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Boundary Value Properties of Canonical Blaschke Product in a Unit Circle

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The necessary and sufficient conditions for the canonical Blaschke product to have angular limits are obtained.

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Robust Stochastic Control of the Exchange Rate

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We consider the problem of a Central Bank that wants the exchange rate to be as close as possible to a given target, and in order to do that uses the interest rate level and interventions in the unspecified foreign exchange market model. We represent this as a robust stochastic control problem, and provide for the first time a solution to that kind of problem. We give examples of solutions that allow us to perform an interesting economic analysis of the optimal strategy of the Central Bank. For the fully specified model such type problems were solved in [1], [2].

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About Convergence of Stochastic Integral

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Let us consider a sequence of continuous processes with bounded variation X_t^n, which converges to a continuous semimartingale X. Let $\int\limits_0^t f(s, X_s^n)\, dX_s^n$ be a sequence of Lebesgue–Stieltjes integrals where $f = f(t,x)$ $(t \in [0,T], x \in \mathfrak{R})$ is a function of two variables. It is interesting, when this sequence of integrals has a limit, can this limit be presented as a stochastic integral and in which sense we should consider this integral.

This question was answered first by Wong and Zakai in 1965, for the case when $X = W$ is Brownian motion. It turned out, that given limit approaches not to the Ito's Stochastic integral, but to the integral in Stratanovich sense. Further it has been also proved for continuous semimartingales.

We generalize this result for nonatisipating functionals using the generalized Ito's formula derived by R. Chitashvili (1982).

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Convolutions and Fourier Analysis Generated by Riesz Bases

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In this talk we will discuss notions of convolutions generated by biorthogonal systems of elements of a Hilbert space. We develop the associated biorthogonal Fourier analysis and the theory of distributions, discuss properties of convolutions and give a number of examples.

The talk is based on the joint work with Professor Michael Ruzhansky.

Boundary Value Problem for the Bi-Laplace–Beltrami Equation

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The purpose of the present paper is to investigate the boundary value problems for the bi-Laplace-Beltrami equation $\Delta^2_{\mathcal{C}}\varphi = f$ on a smooth hypersurface $\mathcal{C}$ with the boundary $\Gamma = \partial\mathcal{C}$. The unique solvability of the BVP is proved, based upon the Green formulae and Lax–Milgram Lemma.

We also prove the invertibility of the perturbed operator in the Bessel potential spaces $\Delta^2_{\mathcal{C}} + \mathcal{H}\, I \ : \ \mathbb{H}^{s+2}_p(\mathcal{S}) \to \mathbb{H}^{s-2}_p(\mathcal{S})$ for a smooth closed hypersurface $\mathcal{S}$ without boundary for arbitrary $1 < p < \infty$ and $-\infty < s < \infty$, provided $\mathcal{H}$ is smooth function, has non-negative real part $\operatorname{Re}\mathcal{H}(t) \geqslant 0$ for all $t \in \mathcal{S}$ and non-trivial support $\operatorname{mes}\operatorname{supp}\operatorname{Re}\mathcal{H} \neq 0$.

On Relative Topological Finiteness

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In [1] the notion of topologically finite space was given: a topological space is called topologically finite provided it is homeomorphic to none of its proper subspaces (otherwise the space is called topologically infinite). Later, in [2], the concept of relative topological finiteness was introduced: a topological space is called topologically finite relative to some subclass of its proper subspaces if the given space is homeomorphic to none of its subspaces belonging to the mentioned subclass (otherwise the space is called topologically infinite relative to the given subclass). In [2] a Hausdorff compact topologically finite space is constructed which is topologically infinite relative to the class of all its proper G_δ subspaces. In the present work we give an example of a topologically finite (Hausdorff) hereditarily normal (not perfectly normal) space which is topologically infinite relative to the class of all its proper F_σ subspaces.

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On Fiber Strong Shape Equivalences

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Using the notion of fiber double mapping cylinder are given the characterizations of fiber strong shape morphisms. Here are found necessary and sufficient conditions under which a map over fixed space B_0 is a fiber strong shape equivalence.

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აფინური ასახვები და კოორდინატიზაცია (აქსიომატიკა) ზოგად რგოლებზე განსაზღვრული მოდელებისათვის

მზია ცხომელიძე

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შესწავლილია აფინური გრადაქმნების გეომეტრიული ინტერპრეტაცია კლასიკური ალგებრის კუთხით, დაგროვილია საკმაო მასალა ძირითადად მთავარ იდეალთა რგოლებზე, კერძოდ, ინვარიანტული ბაზისის მქონე მოდულების კოორდინატიზაცია, კერძოდ, აქსიომატიკური აღწერა. აქსიომათა სისტემა არის დამოუკიდებელი, თავსებადი აქსიომები გეომეტრიულ ინტერპრეტაციების ხასიათს ატარებს და ისინი დამოუკიდებელნი არიან. აქსიომატიკაში გვხვდება დეზარგისა და პაპის თეორემა, რომელიც განაპირობებს ზოგადი რგოლების კომუტატიურობას. არაკომუტატიური რგოლების შემთხვევაში კი დეზარგის

და პაპის აქსიომებს დამოუკიდებელი ხასიათი აქვთ, ხოლო კომუტატიური რგოლების შემთხვევაში ისინი იდენტურები არიან. ამ საკითხების პარალელურად განვიხილავთ გარდაქმნას, ინვერსიას (ასახვას). ამ გარდაქმნების კოორდინატიზაციის, აფინური გარდაქმნების მოდელზე დაყრდნობით, ინვერსიის გამოყენებით, დამტკიცებულია დეზარგის თეორემა და აგებულია წერტილთა ოთხეული და სხვა მრავალი. თეორემის შინაარსი და მისგან გამომდინარე შედეგები მნიშვნელოვანი ნაბიჯია ელემენტარული 29 გეომეტრიული ამოცანიდან პროექციული გეომეტრიის უფრო რთულ ამოცანებზე გადასვლისას.

Some Issues of Conducting Fluid Unsteady Flows in Constant Cross Section Pipes in Transverse Magnetic Field

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In this article is considered the unsteady flow of viscous incompressible electrically conducting fluid in an infinitely long pipe placed in an external uniform magnetic field perpendicular to the pipe axis. It is considered that the motion is created by applied at the initial time in constant longitudinal pressure fall. The exact general solution of problem is obtained.

In this section is given a formulation of problem and are stated the general considerations, related with its solution for an arbitrary profile of transverse cross-section pipe. The next three sections of work (§§ 2-4) are devoted to the detailed study of flow in rectangular pipes. Finally in the last §5 is considered special case of motion in a circular pipe.

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On the Summability of Fourier Series of Abstract Almost Periodic Functions

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Let W be a set of almost periodic (resp., weak almost periodic) functions, defined on a locally compact Abelian group G, with values in a Banach space X. There are considered some summability methods of Fourier series for functions $f \in W$ with respect to the norm (resp., weak norm) of the space X. These methods are obtained by varying the coefficients of Fourier series $\sum_k a_{\chi_k}(f)\chi_k(g)$, where χ_k are characters of the group G, $a_{\chi_k}(f)$ are the Fourier coefficients of f, belonging to X and defined by the formula $a_{\chi_k} = M_g\{f(g)\chi_k(g)\}$. $M_g : G \to X$ is the average (resp., weak average) value on G of the function in brackets. There are studied the cases which are connected with the consideration of accumulation points of the set $\{\chi_k(g)\}$. The varying of the Fourier coefficients is realized with the help of some multiplier function φ_T which is defined on the dual group $\widehat{G}$, is equal to 1 in some symmetric with respect to the unity compact set $T \subset \widehat{G}$ and the Fourier transform $\widehat{\varphi_T}$ of which is integrable on G. In the case when X is the space of complex numbers, similar problems were considered in [1].

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On Grand Lebesgue Spaces on Sets of Infinite Measure

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The theory of grand spaces is intensively developed during last two decades. Such spaces $L^{p)}(\Omega)$, $1<p<\infty$, on bounded sets $\Omega\subset\mathbb{R}^n$ were introduced by T. Iwaniec and C. Sbordone [1] in connection with application to differential equations.

Last years, operators of harmonic analysis were widely studied in such spaces. Some of these results are presented in the book [2]. In all the above mentioned studies only sets Ω of finite measure were allowed, based on the embedding $L^p\subset L^{p-\varepsilon}$.

In the papers [3] and [4] there was suggested an approach to define grand spaces $L_a^{p)}(\Omega)$ on sets $\Omega\subseteq\mathbb{R}^n$ of not necessarily finite measure. In the general form given in [4], this approach is based on introducing the small power a^ε of a weight a into the norm of grand space. We call this function a, which determines the grand space $L_a^{p)}(\Omega)$, the *grandizer* of this space.

The significance of the approach based on the use of the, is first of all in the fact that it allows to consider variants operators of harmonic analysis on $\mathbb{R}^n$. For the Riesz potential operator I^α this approach was realized in [5], where in particular it was shown that the known inversion of I^α by hypersingular operators is possible also in grand spaces under an appropriate conditions for the grandizer.

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Neutrosophic Soft Modules

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In this paper we study the concept of neutrosophic set. We have introduced this concept in soft sets and defined neutrosophic soft set. Some definitions and operations have been introduced on neutrosophic soft set. The main purpose of this paper is to introduce a basic version of neutrosophic soft module theory, which extends the notion of module by including some algebraic structures in soft sets. Finally, we investigate some of neutrosophic soft module basic properties.

The Number of Representations Function for Binary Forms Belonging to Multi-class Genera

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We show how the representations number of some positive integers by binary forms belonging to multi-class genera can be computed by linking those forms to other forms whose genus consists of a single class. The formulas for the number of representations of a positive integer by a binary form which belongs to one-class genus are known. The relationship between forms belonging to multi-class genera with forms of a single class is obtained by elementary means.

Approximation of Certain Linear Growth Functionals

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We consider certain functionals $\int_\Omega g(x,Du)$ defined for $u \in BV(\Omega)$ where the integrand $g(x,p) : \Omega \times \mathbb{R}^N \to \mathbb{R}$ is measurable in x for each p, convex in p for a.e. x, is of the form $g(x,p) = g(x,|p|)$, and satisfies a linear growth condition. If $\int_\Omega g(x,Du)$ is lower semicontinuous in $L^1(\Omega)$, we construct a family of functionals $\int_\Omega \varphi(x,Du)$ that Γ-converge to $\int_\Omega g(x,Du)$ in $L^1(\Omega)$, whose minimizers converge to minimizers of $\int_\Omega g(x,Du)$, and are of the form

$$\int_\Omega \varphi(x,Du) := \sup_{\phi \in \mathcal{V}} \bigg\{ -\int_\Omega u \operatorname{div} \phi + \varphi^*(x,\phi(x))\,dx \bigg\}.$$

Importantly, continuity in the x variable is not assumed.

Compactness in Soft Bigeneralized Topological Spaces

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The soft bigeneralized topological spaces have been firstly introduced in [1] by Ozturk et. al. and some basic notions and theorems have been presented. In this paper we will present some researches and investigations by defining the concept of compactness in soft bigeneralized topological spaces.

Keywords: Soft bigeneralized topological spaces, soft bigeneralized continuity, soft bigeneralized compactness

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About Subject and Learning of Logical-Analytical Thinking

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Teaching of analytical thinking and conclusions is crucial for pupils and students. It has given stimulus to the documentation created and translated for the logical, analytical and critical thinking of the Ministry of Education [1]–[3]. By D. Zarnadze was created the “General Skills Logic” Guide [4]. With the participation of D. Zarnadze, D. Ugulava, M. Kublashvili was created the Standard of subject of “Logical-Analytical Thinking” in the X, XI, XII Classes, Study/Teaching Methodology and Syllabus for teaching in the general education system that was financed by the Patriarch International Foundation. According this standards was written textbook “logical-analytical thinking grounds” [5] in which it represents as an inter-disciplinary subject. Is being studied not only the right thinking rules but also their Understanding-Learning Practical Exercises and Learning Analytical Thinking.

The unanimous thought of these issues showed the shortcomings and errors that were actually legalized in the guiding texts on which we were mentioned in the works [6]–[7]. These mistakes have been map into legal laws and national exam tests.

Based on the beginning research of classical logic operations for three statements were made their grammatical, set interpretations [8] and the construction of the appropriate circuits in the digital electronics [9]–[10]. These studies have led to the study of similar issues in English, German and other languages.

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Linear Consistent Criteria for Testing Hypotheses

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Let (X, B) is separable Hilbert space with σ-algebra of Borel sets in X and $\{\mu_h,\ h \in H\}$ is the family of measures on B. Let a_h average value: $(a_h, z) = \int\limits_X (z, x)\mu_h(dx)$, $z \in X$

and B_h correlation operator

$$(B_h z, u) = \int\limits_X (z,x)(u,x)\mu_h(dx), \quad \forall\, z, u \in X,$$

where (z,u) denotes scalar product in X. Let us suppose $B_y = B$ is independent of h. We assume that the hypothesis parameters are the average values:

$$(h,z) = \int\limits_X (z,x)\mu_h(dx), \quad H \subset X.$$

In linear theory is assumed that H is a linear manifold.

Definition. We will say that the statistical structure $\{X, B, \mu_h, h \in H\}$ admits a weakly (strongly) sequential consistent linear criteria if there exists the sequence of continuous mappings $g_n : X \longrightarrow X$ such that

$$\lim_{n\longrightarrow\infty} \int\limits_X (z, g_n(x) - h)^2 \mu_h(dx) = 0 \ \text{ (respectively, } \lim_{n\longrightarrow\infty} \int\limits_X ||g_n(x) - h||^2 \mu_h(dx) = 0).$$

Theorem 1. *Let $X_n \subset X_{n+1}$ be some increasing sequence of finite dimensional subspaces of Hilbert space X, Q_n is projector on X_n and operator $Q_n^{'}$ satisfies the relation $Q_n^{'}(u) = u$, if $u \in X_n - Q_n X$; $Q_{\nu_n}^{'}$ $\forall \nu_n \in X_n$; $Q_n^{'} B = B Q_n^{'}$ on X. If X is a complete separable space, then the condition*

$$\lim_{n\to\infty} (BQ_n^{'} Q_n Z, Q_n^{'} Q_n Z) = (Bz, z)$$

is necessary and sufficient for existence unbiased consistent criteria coordinated with sequence of subspaces X_n.

Theorem 2. *Let $X_n \subset X_{n+1}$ be some increasing sequence of finite dimensional subspaces of Hilbert space X and*

$$\lim_{n\to\infty} \inf_{X_n} [(Ba, a) + \sup_{\|h\|\leqslant 1} (a - Q_n z, h)^2] = 0,$$

then there exists unbiased consistent criteria.

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Linear Consistent Criteria for Gaussian Statistical Structure

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Let (X,B) is separable Hilbert space with σ-algebra of Borel sets in X and $\{\mu_h,\ h\in H\}$ is the family of Gaussian probability measures with correlation operator B and means $h\in H\subset X$.

Definition 1. We will say that the statistical structure $\{X,\mathrm{B},\mu_h,\ h\in H\}$ admits a weakly sequential consistent linear criteria for testing hypotheses if there exist a sequence of continuous linear mappings $g_n: X\to X$ such that

$$\lim_{n\to\infty}\int\limits_X (z,g_n(x)-h)^2\mu_h\,(dx)=0\ \ \forall\, z\in X,\ \ h\in H.$$

Definition 2. We will say that the statistical structure $\{X,\mathrm{B},\mu_h,\ h\in H\}$ admits a strongly sequential consistent linear criteria for testing hypotheses if there exist a sequence of continuous linear mappings $g_n: X\to X$ such that

$$\lim_{n\to\infty}\int\limits_X \|g_n(x)-h\|^2\mu_h\,(dx)=0\ \ \forall\, h\in H.$$

Definition 3. Weakly measurable linear mapping $g: X\to X$ admits a weakly consistent linear criteria for statistical structure $\{X,\mathrm{B},\mu_h,\ h\in H\}$ if $\mu_h\{x:\ g(x)=h\}=1\ \forall\, h\in H$.

Definition 4. Strongly measurable linear mapping $g: X\to X$ admits a strongly consistent linear criteria for statistical structure $\{X,\mathrm{B},\mu_h,\ h\in H\}$ if $\mu_h\{x:\ g(x)=h\}=1$ $\forall\, h\in H$.

Theorem. The Gaussian statistical structure $\{X,\mathrm{B},\mu_h,\ h\in H\}$ admits a consistent linear criteria if and only if there exists the sequence of positive symmetric operators A_n such that the following relations are fulfilled

(1) $\lim\limits_{n\to\infty}(A_nh,h)=0\ \forall\, h\in H$;

(2) $\lim\limits_{n\to\infty}\mu_0\{x:\ |(A_nx,x)-(x,x)|>\varepsilon\}=0\ \forall\,\varepsilon>0$;

(3) $\sum\limits_k (B^{1/2}A_nB^{1/2}e_k,e_k)$ uniform converges with respect n and for all complete orthonormal system $\{e_k\}$ of operator B.

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Strain Control of Infinite Elastic Body with Circular Opening and Radial Cracks by Means of Boundary Condition Variation

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A two-dimensional boundary value problem of elastic equilibrium of a plane-deformed infinite body with a circular opening is studied. A part of the opening is fixed and from some points of the unfixed part of the cylindrical boundary there come radial finite cracks. The problem is to find conditions for the fixed parts of the opening so that the damage caused by the crack, i.e. stresses on its surface, should be minimal. We should note that the crack ends inside the body are curved. The curve radii vary similar to boundary conditions. The solution of the given problem can be immediately applied to the construction of different kinds of structures, in particular, to underground structures. The problem is solved by the boundary element method [1–3].

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Definition of Deflected Mode of Cylindrical Body under the Influence of Temperature, Volumetric and Surface Forces

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As it is known, determination of tensed-deformated condition of log of cylindrical shape belongs to the group of spatial objectives of mechanics of deformative bodies. The spatial objectives like this, on the basis of the classical elasticity, as a rule is accompanied to solve the flat objectives with the help of several hypothetic assumptions (among them the principle of Saint-Venant), with what, it is evident, their exact solving is not reached. In the work the task of deflected isotropic cylindrical circular-section body, mounted on either end motionlessly is studied under the influence of temperature, volumetric and surface forces. Without the usage of hypothetical assumptions (among them Saint-Venant principle) the exact solution of the task is given. The coordinate system without dimension is used in cylindrical coordinates. With the help of corresponding transformation of set objectives, basic equations of theory of elasticity are portrayed. The internal strain and the relocate components are found, which satisfy the corresponding initial and boundary conditions of the task, also the equilibrium equations and physical equations.

Some Issues about the Graphical Expression of the Direct Proportional Attitudes

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Study of the proportionate attitude between the values is very important in the mathematical school course. The student should be able to describe the quality and quantity of the given dependence on the effect of the change in the size of one degree. You should be able to draw examples from everyday life associated with constant and unmatched quantities. The practice proved to be that the students can well articulate the relationship between the values, but unfortunately they are not able to graphically represent them, or vice versa, with a graphic representation of the proportional attitude between the size. In the paper we will pay attention to the graphical depiction of the direct proportional attachment between the values.

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