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თეზისების კრებული
BOOK OF ABSTRACTS

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ბათუმი, 23 – 28 აგვისტო

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Contents

DRAFT

DRAFT

DRAFT

DRAFT

DRAFT

DRAFT

DRAFT

DRAFT

DRAFT

Abstracts of Plenary Talks

DRAFT

Tarski Problems, Algebraic Geometry Over Groups, and Fraisse Limits

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Algebraic geometry over free groups plays a prominent part in solutions to Tarski problems on elementary theories of free groups. To see the relation it suffices to notice that finitely generated groups universally equivalent to a free non-abelian group F are precisely the coordinate groups of irreducible algebraic sets over F, which are happen to be precisely the limits of the group F in Gromov–Hausdorff topology (which are also known as limit groups). Lyndon's exponential group $F^{\mathbb{Z}[t]}$ (which is a free group in the variety of groups with exponents in $\mathbb{Z}[t]$) is the universal group in the class of limit groups, i.e., it contains all limits groups as subgroups, and every finitely generated subgroup of $F^{\mathbb{Z}[t]}$ is a limit group. Similar results hold if one replaces the free group F by an arbitrary non-abelian torsion-free hyperbolic group G.

Recently Kharlampovich, Miasnikov, and Sklinos showed that the Lyndon's group $F^{\mathbb{Z}[t]}$ is the Fraisse limit in the category of iterated free extensions of centralizers of F [1].

We prove that for arbitrary torsion-free non-abelian CSA group G, in particular, for arbitrary torsion-free hyperbolic group G, the Lyndon's $\mathbb{Z}[t]$-completion $G^{\mathbb{Z}[t]}$ of G is the Fraisse limit in the category of iterated centralizer extensions of G.

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DRAFT

Trilinear Embedding Theorem for Elliptic Partial Differential Operators in Divergence Form with Complex Coefficients

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We introduce the notion of p-ellipticity of a complex matrix function and discuss basic examples where it plays a major role, as well as the techniques that led to the introduction of that notion. We focus on a recent result featuring p-ellipticity, namely, the so-called trilinear embedding theorem for complex elliptic operators and its corollaries.

The talk is based on collaboration with Andrea Carbonaro (U. Genova) and Vjekoslav Kovač and Kristina Škreb (U. Zagreb).

DRAFT

Nonlinear Composition Operators in Generalized Morrey Spaces

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Let Ω be an open subset of $\mathbb{R}^n$. Let f be a Borel measurable function from $\mathbb{R}$ to $\mathbb{R}$. We prove necessary and sufficient conditions on f in order that the composite function $T_f[g] = f \circ g$ belongs to a generalized Morrey space $\mathcal{M}_p^w(\Omega)$ whenever g belongs to $\mathcal{M}_p^w(\Omega)$. Then we prove necessary conditions and sufficient conditions on f in order that the composition operator $T_f[\,\cdot\,]$ be continuous, uniformly continuous, Hölder continuous and Lipschitz continuous in $\mathcal{M}_p^w(\Omega)$ or in its 'vanishing' generalized Morrey subspace $\mathcal{M}_p^{w,0}(\Omega)$. For the uniform, Hölder and Lipschitz continuity we have also conditions that are both necessary and sufficient.

The talk is based on joint work with Alexey Karapetyants.

DRAFT

Composition Operators on Sobolev Spaces (with Applications to Spectral Theory and Nonlinear Elasticity)

Vladimir Gol'dshtein, Valery Pchelintsev, Alexander Ukhlov

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In the present talk we give a review of the theory of composition operators on the Lebesgue and Sobolev spaces with applications to spectral theory of elliptic operators and we shall discuss a new point of view to nonlinear elasticity problems. This talk will be focused on lower estimates for Laplace–Neumann eigenvalues, spectral stability for divergence type elliptic operators and some applications of a new concept of composition duality to nonlinear elasticity problems.

Joint with Valery Pchelintsev and Alexander Ukhlov.

DRAFT

Classical Multiplier Theorems and Recent Improvements

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A multiplier operator alters the frequency of input functions/signals via multiplication by a fixed function called a multiplier. Multiplier theorems provide sufficient conditions for multiplier operators to preserve integrability. The classical multiplier theorems of Marcinkiewicz and of Hrmander on Euclidean spaces will be reviewed and comparisons between different versions of these results (and examples) will be given. The main focus of the talk is to discuss recent optimal improvements of these theorems in terms of membership of the multipliers in appropriate scales of Sobolev classes.

DRAFT

Integration over Brownian Motion Sample Paths; the Green Formula

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We introduce a deterministic integral (not a random variable) for continuous functions $f(t,x)$ with respect to continuous functions $g(t)$ which is similar to the one in [1], and prove that this integral exists for almost all by Wiener measure (W-a.a.) Brownian motion sample paths (BMSP) g and for any β-Hölder functions f, $\beta > 0$. This theorem completes the result of L. C. Young [4] ($\beta > 1/2$) for Hölder functions in the Riemann–Stieltjes integration and provides the integral which is the limit of linear Riemann integrals over approximation of sample paths.

The first integral for functions $f \in L_2([a,b])$ with respect to BM was introduced by Paley, Wiener, Zygmund [3] which was constructed relying on the Hilbert space structure of $L_2([a,b])$. For general stochastic processes the stochastic integral with respect to BM was introduced by Itô [2] as a limit of mean-square convergent sequence in a probability space (Ω, Σ, P). This stochastic integral and Itô's formula (1951) play a fundamental role in the stochastic calculus and many applications in finance and physics. Meanwhile, the stochastic processes are also observed as their realizations or sample paths but the Itô integral is defined non-constructively as a limit in $L_2(\Omega \times R_+)$ which does not allow one to evaluate the integral over sample paths.

To compare with the later one, the new deterministic integral defines an integration over realizations of sample paths of stochastic processes and defined as the limit of linear Riemann integrals. For the new integral we prove the continuity of it by the upper limit and the Green formula that reduces its calculation to the standard Riemann integral. It is important to emphasize that the introduced deterministic integration works as well for W-a.a. BMSPs. The constructive definition of the new integral as a limit of the linear Riemann integrals and formulas for its calculations are useful in the evaluation of the integral over realization of the diffusion processes based on their quantifiable information. A simple application of these results to the stochastic differential equations is given.

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DRAFT

Floating Mats and Sloping Beaches: Spectral Asymptotics of the Steklov Problem on Polygons

Stas Krymski, Michael Levitin, Leonid Parnovski, Iosif Polterovich, David Sher

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I will discuss asymptotic behaviour of the eigenvalues of the Steklov problem (aka Dirichlet-to-Neumann operator) on curvilinear polygons. The answer is completely unexpected and depends on the arithmetic properties of the angles of the polygon.

The talk is based on joint work with M. Levitin, I. Polterovich, D. Sher, and S. Krymski.

DRAFT

Stability of the Inverses and Fredholm Properties of Interpolated Operators

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We will discuss the stability of isomorphisms and Fredholm properties of operators on Banach spaces generated by abstract interpolation methods. We prove that interpolated isomorphisms satisfy uniqueness of inverses on interpolation scales of Banach spaces.

DRAFT

The talk is based on the joint work with Irina Asekritova and Natan Kruglyak.

Harish–Chandra Pairs and Group Superschemes

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In my talk I am going to discuss the recent results in the theory of (not necessary affine) group superschemes, obtained in collaboration with Prof. Akira Masuoka (Tsukuba University, Japan).

First, we proved that the category of locally algebraic group superschemes is equivalent to the category of Harish–Chandra pairs. A Harish–Chandra pair is a couple (G, V), where G is a group scheme (for example, an elliptic curve or more generally, an abelian variety) and V is a finite dimensional G-module equipped with a certain G-equivariant bilinear symmetric map $V \times V \to Lie(G)$.

Second, using the above fundamental equivalence we proved a superversion of Barsotti–Chevalley theorem. The latter states that any group superscheme G has an ascending series of normal group super-subschemes $H_1 \leq H_2 \leq G$ such that H_1 is an affine group superscheme, H_2/H_1 is an abelian group variety and G/H_2 is an affine group superscheme as well.

Third, we proved that for any algebraic group superscheme G and its group super-subscheme H the sheaf quotient G/H is a superscheme of finite type. This theorem is a cornerstone result to develop a standard cohomology theory of linear bundles on G/H.

DRAFT

Existence, Uniqueness and Regularity of Space-Periodic Solutions to Stokes and Navier–Stokes Equations

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He uniqueness and existence of solution to stationary anisotropic (linear) Stokes system in a compressible framework on n-dimensional torus are analysed first in a range of periodic Sobolev (Bessel-potential) spaces. By employing the Leray–Schauder fixed point theorem, the linear results are used to show the existence of solution to the stationary anisotropic (non-linear) Navier–Stokes incompressible system on torus in a periodic Sobolev space. Then the solution regularity for stationary anisotropic Navier–Stokes system on torus is analysed. Some counterparts of these results for the corresponding non-stationary problems are presented as well.

DRAFT

Behavior at Infinity for Null-Solutions of Higher-Order Elliptic Systems

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The main aim in this talk is to elucidate the structure of null-solutions of higher-order elliptic systems in exterior domains, which exhibit power growth at infinity. We show that the source of this type of growth is a polynomial function, itself a null-solution of the elliptic system, in the sense that the difference between the given function and this polynomial has the same magnitude as the fundamental solution of the elliptic system. Such asymptotic expansions are important in the study of boundary value problems in exterior domains.

This is joint work with Irina Mitrea.

DRAFT

On the Solvability of the Neumann Problem for the Laplacian

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The Laplacian may be written in infinitely many ways as a generic second-order operator, using various coefficient matrices. In turn, corresponding to each such writing, there corresponds a specific Neumann problem. These boundary value problems exhibit distinct solvability properties, and in this talk we will the elaborate on the following basic issue: characterize the classes of domains and coefficient matrices for which the corresponding Neumann problem is well posed.

DRAFT

Effective Properties of Random Composites

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Various analytic formulas for random composites were deduced by means of self-consistent methods (Maxwell's approach, effective medium approximation, differential scheme, Mori–Tanaka method and so forth) for dilute composites when the concentration of inclusion is sufficiently small. Extensions of Maxwell's approach from single- to n-inclusions problems called cluster methods. Many years it was thought that cluster methods can be developed by taking into account interactions between pairs of spheres, triplets of spheres, and so on. However, as it will be demonstrated in the talk the field around a finite cluster of inclusions without clusters interactions can yield a formula for the effective conductivity only for dilute clusters. Therefore, all the known analytical formulae for random composites based on the self-consistent methods including cluster methods yield the same Clausius–Mossotti formula and analogous formulas for elastic composites. We establish analytical approximate formulae for random composites valid for high concentrations following the books [1] and [2]. The main result is based on the constructive solution of the Riemann–Hilbert and $\mathbb{R}$-linear problems for a multiply connected domain by a method of functional equations.

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DRAFT

Dirac Operators on $\mathbb{R}^n$ with Singular Potentials

Vladimir Rabinovich

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We will consider the Dirac operators with singular potentials

$$\mathbb{D}_{\mathbf{A},\Phi,Q_{\sin}} = \mathbb{D}_{\mathbf{A},\Phi} + Q_{\sin}, \tag{1}$$

where

$$\mathbb{D}_{\mathbf{A},\Phi} = \sum_{j=1}^{n} \alpha_j(-i\partial_{x_j} + A_j) + \alpha_{n+1}m + \Phi(x)I_N \tag{2}$$

is a Dirac operator on $\mathbb{R}^n$ with variable magnetic and electrostatic potentials $A = (A_1, \dots, A_n) \in L^\infty(\mathbb{R}^n, \mathbb{C}^n)$ and $\Phi \in L^\infty(\mathbb{R}^n)$, $\{\alpha_j\}_{j=1}^{n+1}$ are $N \times N$ Dirac matrices, that is $\alpha_j\alpha_k + \alpha_k\alpha_j = 2\delta_{jk}I_N$, I_N is the unit $N \times N$ matrix, $N = 2^{[(n+1)/2]}$. In formula (1), $Q_{sin} = \Gamma\delta_\Sigma$ is a singular delta-potential with support on a uniformly regular C^2-hypersurface $\Sigma \subset \mathbb{R}^n$ being the common boundary of the open sets $\Omega_\pm$. Let $H^1(\Omega^\pm, \mathbb{C}^N)$ be the Sobolev space of N-dimensional vector-valued distributions $\boldsymbol{u}$ on $\Omega_\pm$, and $H^1(\mathbb{R}^n \setminus \Sigma, \mathbb{C}^n) = H^1(\Omega_+, \mathbb{C}^n) \oplus H^1(\Omega_-, \mathbb{C}^n)$. We associate with the formal Dirac operator $\mathbb{D}_{\mathbf{A},\Phi,Q_{\sin}}$ an unbounded in $L^2(\mathbb{R}^n, \mathbb{C}^n)$ operator $\mathcal{D}_{\mathbf{A},\Phi}$ defined by the Dirac operator $\mathbb{D}_{\mathbf{A},\Phi}$ with domain

$$\operatorname{dom} \mathcal{D}_{\mathbf{A},\Phi} = \Big\{\boldsymbol{u} \in H^1(\mathbb{R}^n \setminus \Sigma, \mathbb{C}^n) : \ \ a_+(s)\boldsymbol{u}_+(s) + a_-(s)\boldsymbol{u}_-(s) = 0, \ \ s \in \Sigma\Big\},$$

where $\boldsymbol{u}_\pm(s) = \gamma_\Sigma(P^\pm\boldsymbol{u})(s)$, $P^\pm : H^1(\mathbb{R}^n, \mathbb{C}^n) \to H^1(\Omega_\pm, \mathbb{C}^n)$ are the operators of restrictions, and $\gamma_\Sigma : H^1(\Omega_\pm, \mathbb{C}^4) \to H^1(\Sigma, \mathbb{C}^4)$ is the trace operator, $a_\pm(s)$ are $N \times N$ matrices defined as

$$a_+(s) = \frac{1}{2}\,\Gamma(s) - i\alpha \cdot \boldsymbol{\nu}(s), \quad a_-(s) = \frac{1}{2}\,\Gamma(s) + i\alpha \cdot \boldsymbol{\nu}(s), \ \ s \in \Sigma,$$

where $\alpha \cdot \boldsymbol{\nu} = \sum\limits_{j=1}^{n} \alpha_j\boldsymbol{\nu}_j$ and $\boldsymbol{\nu} = (\nu_1, \dots, \nu_n)$ is the unit normal vector to Σ.

The main aims of the talk are:

1) conditions of self-adjointness of the operators $\mathcal{D}_{\mathbf{A},\Phi}$;
2) the description of their essential spectra for bounded and unbounded interaction hypersurfaces $\Sigma \subset \mathbb{R}^n$;
3) connections $\mathcal{D}_{\mathbf{A},\Phi}$ with the operators of boundary value problems for Dirac operators (2) in domains $\Omega_\pm$.

DRAFT

Old and New Results on the Boundedness of the Riesz Potential Operator in the Borderline Case

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We define the grand Lebesgue space corresponding to the case $p = \infty$ and similar grand spaces for Morrey and Morrey type spaces, also for $p = \infty$, on open sets in $\mathbb{R}^n$. We show that such spaces are useful in the study of mapping properties of the Riesz potential operator in the borderline cases $\alpha p = n$ for Lebesgue spaces and $\alpha p = n - \lambda$ for Morrey and Morrey type spaces, providing the target space "more narrow" than BMO. While for Lebesgue spaces there are known results on the description of the target space in terms better than BMO, the results obtained for Morrey and Morrey type spaces are entirely new. We also show that the obtained results are sharp in a certain sense.

DRAFT

This talk is based on joint work with S. Samko and S. Umarkhadzhiev.

Sharp Estimates for Conditionally Centred Moments and for Compact Operators on L^p Spaces

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Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space, ξ be a random variable on $(\Omega, \mathcal{F}, \mathbf{P})$, $\mathcal{G}$ be a sub-σ-algebra of $\mathcal{F}$, and let $\mathbf{E}^{\mathcal{G}} = \mathbf{E}(\cdot \mid \mathcal{G})$ be the corresponding conditional expectation operator. We obtain sharp estimates for the moments of $\xi - \mathbf{E}^{\mathcal{G}}\xi$ in terms of the moments of ξ. This allows us to find the optimal constant in the bounded compact approximation property of $L^p([0,1])$, $1 < p < \infty$.

DRAFT

Variational Measures and Descriptive Characterization of Generalized Integrals

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We consider integrals which play an important role in various fields of harmonic analysis and in some application.

The known descriptive characterization of the indefinite Lebesgue integral in terms of absolutely continuous functions is equivalent to the following statement: a function f is L-integrable on $[a,b]$ if and only if there exists a function F of bounded variation on $[a,b]$ which generates an absolutely continuous Lebesgue–Stieltjes measure and $F'(x)=f(x)$ a.e.; the function $F(x)-F(a)$ being the indefinite L-integral of f.

In case of non-absolute generalizations of the Lebesgue integrals (of Denjoy–Perron- or Henstock-type) indefinite integrals fail to be of bounded variation and so cannot generate a finite Stieltjes measure. In this case a descriptive characterization can be obtained in terms of some generalized σ-finite outer measure (so-called variational measure) generated by the indefinite integral.

Variational measures can be defined with respect to various derivate bases. In the simplest case of interval basis on an interval $[a,b]$ the definition is as follows. Let a *tagged interval* be a pair $([c,d],x)$ with $x\in[c,d]$ being a *tag*, $[c,d]\subset[a,b]$, and let δ be a strictly positive function on $[a,b]$ called a *gauge*. We say that a finite collection of tagged intervals $\{(I_i,x_i)\}$ is a *δ-fine partition $\{(I_i,x_i)\}$ on $[a,b]$ tagged in $E\subset[a,b]$* if $I_i\subset[a,b]\cap[x-\delta(x),x+\delta(x)]$, I_i are pairwise non-overlapping and $x_i\in E$ for all i.

For a function F on $[a,b]$ and a set $E\in[a,b]$ the variational measure of E generated by F is defined by $\mathrm{V}_F(E):=\inf\limits_{\delta}\big\{\sup\sum\Delta F(I_i)\big\}$ where sup is taken over all δ-fine partition $\{(I_i,x_i)\}$ on $[a,b]$ tagged in E and inf is taken over all gauges defined on E.

It can be checked that V_F is a metric outer measure on all subset of $[a,b]$.

A descriptive characterization of the Denjoy–Perron integral is given by the following statement (see [1]): A function f is DP-integrable on $[a,b]$ if and only if there exists a function F on $[a,b]$ which generates an absolutely continuous variational measure with $F'(x)=f(x)$ a.e.; the function $F(x)-F(a)$ being the indefinite DP-integral of f.

Most of integrals serve to recover a primitive from its derivative which in turn depends on derivate basis and a space where it is defined. We discuss in this talk constructions and descriptive characterizations of the above type of integrals defined on various spaces, in particular on compact groups and related to various derivatives, in particular the derivative defined for functions in L^r-space.

DRAFT

Acknowledgment

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Kippenhahn Curves and Numerical Ranges of Some Structured Matrices

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The numerical range $W(A)$ (a.k.a. the field of values, or the Hausdorff set) of an n-by-n matrix A is defined as the image of the unit sphere of $\mathbb{C}^n$ under the mapping $f_A\colon x \mapsto x^*Ax$. It is a compact subset of $\mathbb{C}$, which is also convex due to the celebrated *Toeplitz–Hausdorff theorem*. Moreover, as was first observed in [7], $W(A)$ is the convex hull of a certain algebraic curve $C(A)$ of class n, thus nowadays called the *Kippenhahn curve* of A. This provides an insight into the *Elliptical range theorem*: for $n = 2$ the numerical range is an elliptical disk (degenerating into the line segment connecting the eigenvalues of A when A is normal).

As n increases, there is more variety in possible shapes of $W(A)$. Surprisingly though, for some classes of matrices $W(A)$ stays elliptical (or ends up being the convex hull of a small, compared to n, number of ellipses). The state of the matter, as of the beginning of the century, has been described in [3]. It became clear later that the phenomenon at hand is caused by $C(A)$ consisting of several components, the "exposed" of them being ellipses.

In this talk, we describe several such classes. It is based on [1], [2], [4]–[6] and some work in progress.

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DRAFT

Space-Time Methods for Partial Differential Equations

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For the numerical solution of time-dependent partial differential equations we apply space-time discretization methods which are based on a variational formulation in the space-time domain. This approach allows an adaptive resolution of the solution in space and time simultaneously, and parallelization in space and time for an efficient iterative solution.

We first discuss a standard space-time variational formulation in Bochner spaces, with applications to the solution of distributed optimal control and inverse problems, subject to the heat equation. More recent work is on time-varying computational domains in order to do a shape optimization of electrical machines.

An alternative approach is a space-time variational formulation in anisotropic Sobolev spaces, where we use a modified Hilbert transformation to end up with a stable scheme in the space-time domain. This approach also allows to consider the acoustic wave equation, where we present first results for an unconditionally stable space-time finite element method, and new coercivity estimates for related space-time boundary element methods.

The talk is based on joint work with U. Langer (Linz), F. Tröltzsch (Berlin), H. Yang (Korneuburg), P. Gangl, (Graz), M. Gobrial (Graz), M. Zank (Wien), C. Urzua-Torres (Delft), and R. Löscher (Darmstadt).

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DRAFT

New Estimates for the Maximal Functions and Applications

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We discuss sharp pointwise inequalities for maximal operators, in particular, an extension of DeVore's inequality for the moduli of smoothness and a logarithmic variant of Bennett–DeVore–Sharpley's inequality for rearrangements.

Joint work with Oscar Domíngues.

DRAFT

Changes in the National Curriculum in the Field of Mathematics

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The report addresses changes implemented within the national curriculum.

Teaching the appropriate level of mathematical literacy poses a challenge to all countries. The present-day life puts different demands on the mathematical knowledge and skills of the graduating student of our secondary schools. Simultaneously, qualitatively new teaching tools have appeared, which are: computer programs, simulations, online teaching systems, electronic resources, etc.

In my report I present a vision of how we intend to deal with the challenges of quality teaching of mathematics in schools providing general education with the use of modern teaching methods and tools.

The report will consider the following issues:

1. Overview of the current situation, problems;
2. Vision for resolving existing problems;
3. Changes in the third generation national curriculum and the challenges of this plan implementation process;
4. How to bridge a gap between secondary and high schools.

DRAFT

Coupling and Decoupling of Free Flow and Flow in Porous Media and Related Interfacial Conditions

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Many natural and engineering problems involve the interaction of free fluid flow with fluid flow in porous media. Well-known examples include flows in karst aquifers, fluid filtration processes, proton exchange membrane fuel cell, hyporheic zone among many others.

In this talk, we address the following physically important questions mathematically:

1. How do free flows interact with flows in the porous media?
2. Can the interface conditions be derived in a systematic manner?
3. How are the various boundary conditions related to each other?
4. Are there physically important regimes where the two subsystems decouple?

DRAFT

Dispersive Estimates for the Schrödinger Equation

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The importance of the dispersive estimates for Schrödinger equations in spectral theory and in nonlinear analysis will be discussed. Furthermore, the literature on the $L^p - L^{p'}$ estimates will be reviewed, starting with the early results in the 1990th, and with an emphasis in the results in one dimension. New results will be presented, in $L^p - L^{p'}$ estimates for matrix Schrödinger equations in the half-line, with general selfadjoint boundary condition, and in matrix Schrödinger equations in the full-line with point interactions. In both cases we consider integrable matrix potentials that have a finite first moment.

The talk is based on joint work with Tuncay Aktosun and Iván Naumkin.

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DRAFT

Limiting Weak-type Behaviors for Singular Integral Operators and Maximal Function

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The best constant problem for some important operators' norm inequalities play important roles in Harmonic analysis and other fields. It is known that the best constant problem is closely connected with the limiting weak-type behaviors of these operators. In this talk, we will recall some known results about limiting weak-type behavior for several operators, and introduce our recent works on singular integral operators with rough kernels and [illegible] maximal functions.

DRAFT

Abstracts of Sectional Talks

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DRAFT

Design Model for Creation of Intelligent Learning Systems Based on Bayesian Networks

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Recently, there has been a rapid growth in networked Intelligent Learning Systems (ILS) to support learning processes to help students navigate responsively in interactive learning materials.

Web-based educational systems facilitate distance learning and offer easy access to any area of knowledge and learning process at any time, for learners from different walks of life, with different needs, preferences and characteristics. Differences in student characteristics play an important role in the design of an Internet-based learning environment. Hence, successful online learning requires multimedia techniques, adaptive techniques, and reasoning skills in addition to a user-friendly interface. Students often enjoy the ease of use and communication in a virtual learning environment, as well as customized learning paths. Thus, the challenge is to design web-based learning systems that dynamically adapt to each individual user in order to deliver knowledge efficiently. Consequently, web-based learning systems must be dynamically adaptable to the individual student, and they must be able to monitor student activities and provide personalization based on specific needs, preferences and knowledge. Moreover, these systems should offer students more freedom to navigate online course content and control their learning pace and learning flow. With these features, educational institutions, whether traditional or online, have rapidly begun to implement adaptive learning environments, virtual learning environments, and e-learning management systems to increase the number of online course students.

A technology that supports adaptive navigation, which helps learners to acquire knowledge faster and improve learning outcomes, is one of the common technologies used in educational web systems. Many ILS use technology such as the Intelligent Learning Systems platform, which supports learners' navigation in cyberspace by adapting to the goals and knowledge of the individual user. Ability of ILS to provide adaptability is based on student modeling technology. Student modeling, which we talked about above, is a process that is responsible for representing the goals and needs of students, analyzing student performance, and defining prior and acquired knowledge. However, the student modeling process is not black and white, but it often deals with uncertainty, and it is impossible to say for sure whether the student has assimilated the concept or not. Thus, the challenge is to build an effective student model, which is a key component of the ILS for overcoming uncertainty.

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DRAFT

Controlling the Transmission Dynamics of Schistosomiasis: Mathematical Model Approach

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Schistosomiasis is one of the neglected tropical disease affecting communities in flood prone environment or where fishing activities take place. This has been a draw back to the health and economic life of the citizens in these areas. This study is to evaluate the impact of public health education and snail control activities on the spread of schistosomiasis.The model is developed with attention given to the snail and human populations which are the hosts of the miracidia and cercariae respectively. The existence and stability of disease free-equilibrium and endemic states are established. The disease-free equilibrium state is showed to be locally and globally asymptotically stable whenever the basic reproduction number is less than unity. The stability of endemic equilibrium state of the model is also analysed using centre manifold theory and Lyapunov function for its local and global stability when basic reproduction number is more than unity. The numerical simulations of the model are carried to evaluate the impact of these control strategies, public health education and snail control on schistosomiasis transmission. It was observed that public health education and snail control activities play an important role in mitigating the spread of the disease.

DRAFT

Weighted Estimates and Compactness of a Class of Integral Operators with Logarithmic Singularity

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Let $I=(0,\infty)$ and let v, u be almost everywhere positive and locally integrable functions on the interval I. Let $1<p,q<\infty$ and $p'=\frac{p}{p-1}$. Let us denote by $L_{p,v}\equiv L_p(v,I)$ the set of measurable functions f on I for which

$$\|f\|_{p,w}=\bigg(\int\limits_0^\infty |f(x)|^p w(x)\,dx\bigg)^{\frac1p}<\infty.$$

Let W be a strictly increasing and locally absolutely continuous function on the interval I. Let $\frac{dW(x)}{dx}=w(x)$, for almost all $x\in I$.

Consider the operator

$$T_{\alpha,\beta}f(x)=\int\limits_0^x \frac{(ln\frac{W(x)}{W(x)-W(s)})^\beta u(s)f(s)w(s)\,ds}{(W(x)-W(s))^{1-\alpha}}\,,\ \ x\in I, \tag{1}$$

where $\alpha>0$, $\beta\geq 0$.

The boundedness of the operator (1) from $L_{p,w}$ to $L_{q,v}$ when $\beta=0$ is obtained in the paper [1] for $\alpha>1/p$, $1<p\leq q<\infty$ and $0<q<p<\infty$.

Further, we assume that W is non-negative on I and $\lim\limits_{x\to 0^+}W(x)=0$. The following theorem holds.

Theorem. *Let $0<\alpha<1$, $\frac1\alpha<p\leq q<\infty$ and $\beta\geq 0$. Let the function u be non-increasing on I. Then the operator $T_{\alpha,\beta}$, defined by formula (1), is bounded from $L_{p,w}$ to $L_{q,v}$ if and only if*

$$A_{\alpha,\beta}=\sup_{z>0}A_{\alpha,\beta}(z)<\infty,$$

$$A_{\alpha,\beta}(z)=\bigg(\int\limits_z^\infty v(x)W^{q(\alpha-\beta-1)}(x)\,dx\bigg)^{\frac1q}\bigg(\int\limits_0^z W^{\beta p'}(s)u^{p'}(s)w(s)\,ds\bigg)^{\frac{1}{p'}},$$

and operator $T_{\alpha,\beta}$ is compact from $L_{p,w}$ to $L_{q,v}$ if and only if $A_{\alpha,\beta}<\infty$ and

$$\lim_{z\to0^+}A_{\alpha,\beta}(z)=\lim_{z\to\infty}A_{\alpha,\beta}(z)=0.$$

Moreover, $\|T_{\alpha,\beta}\|\approx A_{\alpha,\beta}$, where $\|T_{\alpha,\beta}\|$ is the norm of the operator (1) from $L_{p,w}$ to $L_{q,v}$.

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DRAFT

Convergent Affine Bounding Functions for Polynomials: Bernstein Least Squares

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In this work, we consider the computation of bounding functions for multivariate polynomials over simplices. Bounding the range of functions is an important issue in many areas of applied mathematics and computing like global optimization, computer aided geometric design, stability analysis and robust control. The expansion of a (multivariate) polynomial function F is given into the so-called simplicial Bernstein basis over a simplex (triangles), [2]. The Bernstein expansion is used due to the tightness of the enclosure and its rate of convergence properties to the exact range, [1]. The expansion of Bernstein can be extended to rational polynomial functions. The Bernstein lower bound is also used to certify whether a given function is positive over a simplex. Our main goal is providing convergent affine lower bound for polynomials over the whole domain. We show the linear and quadratic convergent with respect to raising the degree and subdivision of a simplex.

The bound can be used to certify the positivity of polynomials over subsimplices. Certifying the positivity of functions can be improved by subdividing the given simplex at a specific point. Specifically, if the domain is shrunk or subdivided to smaller subdomains, then the Bernstein bound shrinks too. On the other hand of applications, the key to finding a Lyapunov function for a polynomial linear system is to find positivity certificates, where the subdivision method satisfies local positivity certificates over subsimplices.

However, computing Bernstein coefficients on the face values of a simplex is the fastest way for computing the minimum bound. In this work, we hold this property onto the simplicial case together with proving further important properties of the Bernstein basis over a simplex. First, we provide a representation for the Bernstein coefficients and the enclosure bound of polynomials over a simplex. Subsequently, we provide an convergent affine lower bound for polynomials over subsimplices. Finally, convergent properties and applications on stability of control systems are addressed.

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DRAFT

On Singular Extensions of Continuous Functionals From $C([0,1])$ to Variable Lebesgue Spaces

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Valadier and Hensgen proved independently that the restriction of functional

$$\phi(x)=\int\limits_0^1 x(t)\,dt,\ \ \in L^{\infty}([0,1])$$

on the space of continuous functions $C([0,1])$ admits a singular extension back to the whole space $L^{\infty}([0,1])$. Some general results in this direction for the Banach lattices were obtained by Abramovich and Wickstead. In present note we investigate analogous problem for variable exponent Lebesgue spaces, namely we prove that if the space of continuous functions $C([0,1])$ is closed subspace in $L^{p(\cdot)}([0,1])$, then every bounded linear functional on $C([0,1])$ is the restriction of a singular linear functional on $L^{p(\cdot)}([0,1])$.

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Ramsey Numbers

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In modern mathematics, it is difficult to find open-ended problems, such problems that are still unsolved and at the same time, whose main idea and problem description can be formulated in the language of elementary mathematics so that the problem is not difficult for lower grade students. For a mathematician, all of the above are additional challenges and motivators to tackle such problems, although they are very difficult to solve successfully and with great work, justification is probably necessary.

The paper focuses on one of these types of problems, presents the already known results on the topic, and the content of the still-unsolved problems related to the topic, in particular, discusses Ramsey-type problems and the so-called Ramsey numbers.

Before discussing Ramsey numbers and their problems, the paper implies that each of the two people either knows or does not know each other. Also, the encounter is two-way (if A knows B so B knows A) The Ramsay number $R(f,e)$ is the smallest natural number that has the following property: For any $R(f,e)$ person, there are f people, each of whom knows each other, or e person, none of whom knows each other. Here, we can notice that $R(f,e) = R(e,f)$.

The paper also shows that the number $R(f,e)$ exists for any f and e numbers, but specifically, what the number $R(f,e)$ is equal to, it is very difficult to figure out. For today, the exact value of none of the numbers $R(f,e)$ is known, except that $f \le e \le 5$.

The paper discusses some basic problems related to Ramsey numbers.

DRAFT

BVP for the First Order Elliptic Systems in the Plane

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In this presentation the Riemann–Hilbert type boundary value problem in some classes of solutions for general first order elliptic systems in plane domains bounded by smooth curves will be discussed. In some cases, Noethericity conditions of the problem are given.

Acknowledgement

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DRAFT

The Consistent Estimators of Beta ("B") Statistical Structure

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The density of Beta ("B") low is determined by the equality

$$f(x,\alpha,\beta)=x^{\alpha-1}(1-x)^{\beta-1}/B(\alpha,\beta),$$

where

$$B(\alpha,\beta)=\int\limits_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx,\ \ \alpha>0,\ \ \beta>0.$$

The probability measure μ corresponding to the Beta ("B") density: $\mu(A)=\int\limits_A f(x,\alpha,\beta)\,dx$ is called "B" measure.

Definition 1. A statistical structure $\{E,S,\mu_i,i\in I\}$ is called Beta ("B") Statistical structure if μ_i ($\forall\, i\in I$) are "B" measure.

For each $i\in I$ we denote by $\overline{\mu}_i$ the completion of the measure μ_i, and by $dom\,(\overline{\mu}_i)$ – the σ-algebra of all $\overline{\mu}_i$-measurable subsets of E. We denote

$$S_1=\bigcap_{i\in I} dom\,(\overline{\mu}_i).$$

Definition 2. The "B" statistical structure $\{E,S_1,\overline{\mu}_i,i\in I\}$ is called strongly separable if there exists a family of S_1-measurable sets $\{Z_i,i\in I\}$ such that the relations are fulfilled:

(1) $\mu_i(Z_i)=1\ \ \forall\, i\in I$;

(2) $Z_{i_1}\cap Z_{i_2}=\varnothing\ \ \forall\, i_1\neq i_2,\ \ i_1,i_2\in I$;

(3) $\bigcup\limits_{i\in I} Z_i=E$.

Definition 3. We will say that the "B" statistical structure $\{E,S_1,\overline{\mu}_i,i\in I\}$ admits a consistent estimators of parameters $i\in I$ if there exists at least one measurable mapping $\delta:(E,S_1)\to(I,B(I))$ such that $\overline{\mu}_i(\{x:\ \delta(x)=i\})=1\ \forall\, i\in I$.

Theorem. *In order that the Beta* ("B") *orthogonal statistical structure* $\{E,S_1,\overline{\mu}_i,i\in I\}$, $card\, I=c$, *admitted a consistent estimators of parameters* $i\in I$ *it is necessary and sufficient that this statistical structure be strongly separable.*

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On the Uniformization of Some Function Couples of Three Variables

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In the connection with D. Siersma's approach to the investigation of non-complete intersection singularities it is proved that all polynomials algebraically dependent on equations of simple curve singularities are polynomials of the corresponding left-hand sides from the list of M. Giusti.

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Lorenz-Like System as a Set of Coupled Driven and Damped Nonlinear Oscillators

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We develop a mathematical formalism within which a Lorentz-like system is represented as a set of second-order ordinary differential equations which describe three nonlinearly coupled oscillators with damping and self-excitation. The advantage of this formalism is not in reducing the mathematical difficulties of solving nonlinear dynamics of such systems, but rather to gain deeper insight it affords into their formal structure. It explicitly shows the intrinsic damping as well as the modulations of the natural frequencies of each oscillator resulting from their nonlinear coupling. This approach enables us to construct an effective Lagrangian and generalized Hamiltonian (energy) for the system. The "grounded" and transparent formalism is the basis for developing a prototypical, minimal phenomenological model which accounts for the negative and positive feedback loops that underlie the cyclic processes and rhythmicity in living organisms as typical representatives of open systems with a dissipative nature and gen[illegible].

DRAFT

Apollonius Problem on the Number of Normals Passing Through a Point of a Conic

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We study a variant of Apollonius problem on determining the number of normals to an ellipse passing through a given point. The origin of the problem is Book V of Apollonius' Conics. We mention also the case of hyperbolas and parabolas. We study the following problem: Let A be a general point on the plane of the ellipse with equation $\frac{x^2}{a^2}+\frac{a^2}{b^2}=1$, where $a>b>0$. How many points B, different from A, do exist on this ellipse, such that the line AB is perpendicular to the tangent line of this ellipse at the point B? We denote this number by $n(A)$. We do not exclude the case where the point A is on the given ellipse. In fact the main aim of the present paper is to study this case in more detail.

Firstly, we suppose that the point A is not on the ellipse. In this variant the problem (Apollonius Problem) was considered in [4, §13]; [3, p. 71 (Problem 13), pp. 257–258 (solution)]; [1, Section 17.7.4]. For historical background of this problem see [5, Chapter VII] and [2, Chapter 12]. We will present here a new solution using elementary calculus. After this we find intersection points of the given ellipse and its astroida. The obtained points surprisingly have elegant coordinates. These points completely solve the problem for the points of the given ellipse.

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DRAFT

Poisson Brackets and Poisson Prime Ideals in Polynomial Algebras

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The concept of Poisson algebra is one of the most important concepts in mathe- matics that makes a link between commutative and noncommutative algebra. In this talk, I will give the definition of the Poisson algebra, talk about some properties of polynomial Poisson algebras, Poisson prime ideals, Poisson spectra, simple Poisson algebras and examples.

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On Temporal Heyting Algebras

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Modalized Heyting calculus mHC introduced by Leo Esakia in his work [1], is the augmentation of the Intuitionistic logic Int by a modal operator $\Box$. This modalized Heyting calculus is a weakening of the proof-intuitionistic logic KM of Kuznetsov and Muravitsky by discarding Löb axiom. There is exact embedding of the mHC calculus into the modal system $K4.Grz$.

Temporal Heyting calculus tHC is a temporal enrichement of mHC. This calculus was introduced by Leo Esakia. The temporal Heyting calculus tHC is defined on the basis of mHC with additional axioms for the "adjoint" modality $\Diamond$, namely:

(t1) $p \to \Box\Diamond p$;

(t2) $\Diamond\Box\, p \to p$;

(t3) $\Diamond(p \vee q) \to \Diamond p \to \Diamond q$;

(t4) $\Diamond\bot \to \bot$

and an additional rule:

$$p \to q \Longrightarrow \Diamond p \to \Diamond q.$$

Algebraic models of mHC are fHA-algebras (frontal Heyting algebras). In their work [2] Jose Luis Castiglioni, Marta Sagastume, Hernan Javier San Martin have extended Heyting duality to the category **fHA**.

We investigate variety of temporal Heyting algebras **tHA**, which represent algebraic models of temporal Heyting calculus tHC and have the following results:

1. We develop a theory of temporal Heyting algebras.
2. We generalize Heyting duality to the category **tHA**.
3. Characterization of simple tHA-algebras and subdirectly irreducible tHA-algebras is given.

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Towards a Georgian Controlled Language

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Controlled natural languages (CNLs) are engineered languages that are based on natural language, but have their vocabulary, syntax, and/or semantics restricted [1]. The motivation is to have a language that, on one hand, looks as natural as possible and, on the other hand, is simple and unambiguous.

Application areas of CNL are quite broad. They serve improving communication and mutual understanding between humans (especially for people with specific reading and understanding restrictions), facilitate manual or automatic translation, are suitable for reasoning, writing technical documents and legal texts, interchanging business rules among organizations, for efficient communication in emergency situations and crisis management, etc. Examples include technical documentations of Boeing and IBM, special English CNLs for communications between ships and harbors, etc.

While English dominates the landscape of CNLs ([1] analyzes 100 English CNLs), controlled (sub)languages have been developed for many other languages (e.g., Bulgarian, French, Spanish, Chinese, Russian, German, Greek, Spanish, and Japanese). However, Georgian is not among them. In general, from the computational point of view, despite some recent and ongoing attempts, Georgian is still pretty under-resourced.

In this talk we discuss challenges on constructing Georgian controlled language, which aims at establishing clear and unambiguous communication between different parties in emergency situations.

Acknowledgement

This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant # FR-19-18557.

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DRAFT

Anti-Unification: Recent Advances and Applications in Natural Language Processing

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Anti-unification [7] is a well-known method to compute generalizations in logic. Given two objects, the goal of anti-unification is to reflect commonalities between these objects in the computed generalizations, and highlight differences between them. This technique has many interesting applications in the areas such as, e.g., inductive logic programming, inductive and analogical reasoning, software code close detection, program analysis, proof generalization, automated program repair, etc.

In natural language processing, anti-unification has been used in tasks such as semantic parsing, grammar and grounded language learning, semantic classification of sentences based on their parse trees (used in chatbot development), detecting semantic textual similarity, modeling metaphors, etc., see, e.g., [1] for an overview. The major technique in these applications is still the original Plotkin–Reynolds algorithm with some adaptations.

Our group at RISC, together with collaborators, contributed in the area of anti-unification by designing, studying, and implementing new powerful algorithms in various unranked, higher-order, equational, and fuzzy theories, see, e.g., [2]–[6]. In this talk, we overview some of them in the context of applicability in natural language processing.

Acknowledgement

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Model of Coronavirus Spread in the Light of Vaccination

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Understanding the principle and speed of virus spread is critically important to take the right decisions in relation to the safe behavior of society.

All predictions of coronavirus spread are based on opinions, expectations, anticipations, and hopes that the virus "soon reach a plateau", "the peak will be passed soon", "we are at the beginning of the fourth wave", etc. The situation is rapidly changing and requires adoption of responsible decisions both from government authorities and ordinary citizens. The lives, health, and financial wellbeing of people depend on prompt adoption and correctness of such decisions.

Many prediction models were discussed specifically for coronavirus, from which we emphasize [1], where a quite accurate model is implemented by means of math apparatus. In this paper, we consider a model, where the impact of vaccination on the spread of coronavirus and its mortality is additionally taken into account.

The virus propagation model, which we consider, is based on the combination of parameters, whose values are unique for each country depending on population density, population behavioral patterns, time of virus introduction, and government actions. Additionally, we introduce two parameters: NV – number of vaccinated people and NE – vaccine efficiency. Because the number of infected persons is in inverse proportion to the number of vaccinated, therefore, let's multiply NT(ti) by inverse number of the product of vaccinated people's number (NV) and vaccine efficiency (NE), obtaining the vaccination impact formula:

$$\mathrm{NTVAC}\,(ti) = \mathrm{NT}\,(ti) \cdot \frac{1}{(\mathrm{NE} * \mathrm{NV})}$$

and for calculation of vaccination impact on the number of deceased persons by ti time the following formula:

$$\mathrm{NLVAC}\,(ti) = \mathrm{NL}\,(ti) \cdot \frac{1}{(\mathrm{NE} * \mathrm{NV})}\,.$$

We build a hybrid model from various prediction models using parallel data [2], including for such actual issues as a coronavirus spread model.

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DRAFT

Numerical Methods in the Question on Lattice Points in Shifted Balls

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In Number Theory one studies the question on the number of lattice points enclosed into various domains of multidimensional spaces. Let's denote by $T(N)$ the number of lattice points in the sphere $x^2 + y^2 + z^2 = N$ and put

$$R = R(r) = T(r^2) - \frac{4}{3}\pi r^3.$$

The question on estimation of this variance is known as Sphere Problem. There is a conjecture which states that $R \ll r^{1+\varepsilon}$.

Besides, in series of works (see [1]) the problem was studied from other points of view. Authors of those works investigated the fluctuations in the number $N_\alpha(r)$ of lattice points reminder term inside a sphere of radius r centered at a point $\alpha \in [0,1)^3$ different from the origin.

We apply computer calculations for some r and points (x, y, z) to analyze the behavior of the relative deviation of the number of lattice points from the volume. The results' analyze show that the center of the sphere with minimal deviation is random. There is a point for which a relative deviation for the sphere centered at this point is very large, which indicates its propensity to growth, as a radius stands large. This fact agrees, at the origin, with the following result of the work [1]:

$$\lim_{T\to\infty} (T\log T)^{-1} \int\limits_1^T \Big|R\Big(\sqrt{u}\Big)\Big|\, du = \frac{32}{7}\,\frac{\zeta(2)}{\zeta(3)}\,.$$

The program written in Python used for some values of ball's radiuses. We have computed the relative deviation d of the number of lattice points in shifted balls. There were considered shiftings of the balls with radiuses 60, 99, 200, 300. As the time of calculations grows proportional to the volume of the ball, we suffice with shifting in four steps in the case of radiuses 60 and 99. For the radius 200 we consider shifting in three steps, and in the case of the radius 300 consider two steps. For got results of calculations we preserved 4 decimal digits after of the point, when the relative deviation is calculated. Since the value of math.pi is taken with an error no more than 10^{-11}, then the nominator of the fraction d is known with an error no more than 10^{-3}. Since $r \le 300$, then among these 4 decimal digits at least one after the point is a right digit.This is sufficient for establishing approximate shifting for which the relative deviation d is small or large. We fixing the first coordinate x, computed a minimal absolute relative deviation and indicated the centers where the minimum is occurs.

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DRAFT

Derivations Period 2 of Semiprime Rings

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Ring theory is a show-piece of mathematical unification, bringing together several branches of the subject and creating a powerful machine for the study of problems of considerable historical and mathematical importance. Rings derivations are not the kind of subject that undergoes tremendous revolutions. Moreover, this has been studied by many algebraists in the last years, especially the relationships between derivations and the structure of rings.The study of derivation was initiated during the 1950s and 1960s. Precisely, derivations of rings got a tremendous development in 1957. A map $d: R \to R$ is called a derivation if $d(x+y) = d(x) + d(y)$ and $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$.

In other words, d is called a derivation if the Leibniz's rule satisfy.

A mapping $d: R \to R$ is called period 2 on R if $d^2(x) = x$ for all $x \in R$. Recall that R is semiprime if $aRa = 0$ implies $a = 0$ and R is prime if $aRb = 0$ implies $a = 0$ or $b = 0$. The main purpose of this paper is to introduce and study the definition of derivations period 2 via semiprime ring and prime ring as following:

An additive mapping $d: R \to R$ is called derivation period 2 if d is a derivation and has the property period 2 gather together. Accurately, we prove the commutativity with other cases of a ring that satisfied certain conditions.

Note on the Multifractal Formalism of Covering Number on the Galton–Watson Tree

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We consider, for t in the boundary of Galton–Watson tree $(\partial \mathsf{T})$, the covering number $\mathsf{N}_n(t)$ by cylinder of generation n. For a suitable set I and a sequence $(s_{n,\gamma})$, we establish almost surely, and uniformly on γ, the Hausdorff and packing dimensions of the set

$$\big\{t \in \partial \mathsf{T}: \ \ \mathsf{N}_n(t) - nb \sim s_{n,\gamma}\big\} \ \text{ for } \ b \in I.$$

Keywords: Random covering, Hausdorff dimension, indexed martingale, Galton–Watson tree.

DRAFT

On the One Nonparametric Estimator of the Bernoulli Regression Function

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The estimate for the Bernoulli regression function is constructed using the Bernstein polynomial. The question of its consistency and asymptotic normality is studied. Testing hypothesis is constructed on the form of the Bernoulli regression function. Also, the test is constructed for the hypothesis on the equality of two Bernoulli regression functions. The question of consistency of the constructed tests is studied.

DRAFT

Inverse Problems and Their Discrete Analogs for the Second Order Quasi-Linear Equations of Mixed Type

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We describe the process of construction of general integrals for some quasi-linear non-strictly hyperbolic equations of second order. General integrals are obtained by means of characteristic differential relations. We use these General integrals, as a powerful instrument for investigating and solving of Cauchy problem posed for various quasi-linear partial differential equations with an admissible parabolic degeneration. The given functions describing the initial conditions are defined on a closed interval. We also study a variant of the inverse problem and prove that the considered problem has a solution under certain conditions. We illustrate the Cauchy and inverse problems in some concrete examples. In these cases, the definition domains of the solution are investigated. In some specific cases the families of the characteristic curves have either common envelopes or singular points. Also, a corresponding numerical scheme is constructed for the problem and the convergence of the scheme is proved.

DRAFT

Polynomial Generators of $MSU_*[1/2]$ Related to Classifying Maps of Certain Formal Group Laws

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We provide a set of polynomial generators of $MSU_*[1/2]$ defined by the formal group law in spherical cobordism. One aspect is to obtain the genera on $MSU_*[1/2]$ with values in polynomial ring as the restrictions of the classifying map of the Abel formal group law and the Buchstaber formal group law. The latter is associated with the Krichever–Hoehn complex elliptic genus.

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DRAFT

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On Menshov–Rademacher Inequality

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In [5] the following version of Menshov–Rademacher inequality is contained: let $\varphi_n \in H = L_2[0,1]$, $n = 1, 2, \dots$ be an orthonormal sequence and $(a_n)_{n\in\mathbb{N}} \in l_2$ be a sequence of numbers; then the estimate

$$\int\limits_0^1 \Big(\sup_{n\geq 1}\Big|\sum_{k=1}^{n} a_k\varphi_k(t)\Big|\Big)^2\,dt \leq K\sum_{n=1}^{\infty}|a_n|^2(1+\log_2 n)^2$$

holds with some universal constant K.

The further study of the constant was made in [1, 2, 4].

We plan to discuss these results and their some consequences.

The talk is based mainly on [3].

Ackowledgement

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DRAFT

On Axiomatic Characterization of Alexander–Spanier Normal Homology Theory of General Topological Spaces

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On the category $\mathcal{K}^2_{CM}$ of pairs of compact metric spaces the exact homology theory was defined by N. Steenrod. First axiomatic characterization of the Steenrod homology theory on the category $\mathcal{K}^2_{CM}$ was done by J. Milnor. There are exact homology theories defined by other authors (A. N. Kolmogoroff, G. Chogoshvili, K. A. Sitnikov, A. Borel and J. C. Moore, H. N. Inasaridze, D. A. Edwards and H. M. Hastings, W. S. Massey, E. G. Sklyarenko). Later, on the category $\mathcal{K}^2_{C}$ of pairs of compact Hausdorff spaces the axiomatic characterization of an exact homology theory was obtained by N. Berikashvili, L. Mdzinarishvili and Kh. Inasaridze, L. Mdzinarishvili, Kh. Inasaridze. For paracompact spaces S. Saneblidze generalized the result obtained by N. Berikashvili.

In the paper [1] we have generalized the result for general topological spaces. We constructed an exact, the so-called Alexander–Spanier normal homology theory on the category $\mathcal{K}^2_{Top}$, which is isomorphic to the Steenrod homology theory on the subcategory of compact pairs $\mathcal{K}^2_{C}$. Moreover, we gave an axiomatic characterization of the constructed homology theory [1].

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DRAFT

Solving the Linear Difference Equation with Periodic Coefficient via the Analytic Formula of Generalized Fibonacci Sequences

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The aim of this talk is to establish explicit solutions of the homogeneous linear difference equations with periodic coefficients. For this purpose, we get around the problem by converting each equation of this class to an equivalent linear difference equation with constant coefficients. The approach used reposes on the analytic formula of the Generalized Fibonacci Sequences.

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DRAFT

On Some Models of Set Theory

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According to well-known set-theoretical results, there are models of **ZFC** in which all projective subsets of **R** are well-behaved from the point of view of descriptive set theory, in particular, they all are Lebesgue measurable. For instance, the Axiom of Projective Determinacy (**PD**) implies prominent regularity properties of the projective sets: Lebesgue measurability, the Baire property, the perfect subset property and the Ramsey property.

This implies that definable absolutely nonmeasurable functions on **R** can only exist in certain models of **ZFC** without substantial large cardinals. Assuming that there exists a well-ordering of **R** whose graph is Δ^1_2-subset of the plane, there exists a Vitali set in **R** which is a Δ^1_2-subset of **R**. Consequently, under this assumption, there are projective sets which are Lebesgue non-measurable and do not have the Baire property. Recall that this assumption holds true in Gödel's Constructible Universe **L**. (cf. [2], [4]).

In [1] we have shown, that there exists a model of ZF, such that there is no well-ordering of the reals but there is a Hamel basis.

A. Miller has shown that in **L**, there is Π^1_1-Hamel basis. It is an old result by S. Feferman that the existence of Vitali sets doesn't imply that there is a well-ordering of the reals (cf. [2], [4]). Still in **ZFC**, there is a Mazurkiewicz set which is simultaneously a Hamel basis, cf. [3]. In joint work with R. Schindler we formulate a sufficient criterion for a model of **ZF** to have a Mazurkiewicz set.

Theorem 1 (**ZF**)**.** *Assume that there is some sequence $(A_i, r_i :\ i < \lambda)$ such that for all $i \le j < \lambda$,*

(a) $A_i \subset A_j$;

(b) $\mathbb{R} = \bigcup\limits_{k<\lambda} A_k$;

(c) r_i *is a real which is not in* $\operatorname{comp}(A_i)$;

(d) $\operatorname{comp}\left(A_i \cup \{r_i\}\right) \subset A_{i+1}$.

There is then a Mazurkiewicz set.

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DRAFT

Some Class of Polyhedrons in R^3 which are Not Primitive

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Decomposing a geometric object into simpler parts is one of the most fundamental problems in computational geometry. In the Euclidean plane $\mathbf{R^2}$ a triangulation of polygon is called decomposing into triangles without adding new vertices and every simple polygon can be triangulated. It is known that it is possible to divide any polygon with $n-3$ diagonals into $n-2$ triangles. The generalization of this process to higher dimensions is also called a triangulation. For example, polyhedron in $\mathbf{R}^3$ can be decomposing with tetrahedrons. In higher dimensional spaces polyhedrons we can decompose with simplices. By the investigation of the triangulations of convex polyhedrons, I have crossed (connected) and considered a certain class of convex polyhedrons (primarily, in the space $\mathbf{R}^3$), which are called primitive polyhedrons.

Notion 1. Let Q be a convex three-dimensional polyhedron in the space $\mathbf{R}^3$.

We recall that a convex polyhedron Q' is a primitive extension of Q if there exist a triangular facet D of Q and a tetrahedron T in $\mathbf{R}^3$ such that D is also a faced of T and

(*) $T \cap Q = D$;

(**) The set of all vertices of Q' coincides with the union of the sets of all vertices of Q and T.

Notion 2. Let $\{Q_1, Q_2, \dots, Q_k\}$ be a finite sequence of convex tree-dimensional polyhedra in $\mathbf{R}^3$.

We say that this sequence is primitive if Q_1 is a tetrahedron and, for each natural index $i \in [1, k-1]$, the polyhedron Q_{i+1} is a primitive extension of Q_i.

Notion 3. A convex tree-dimensional polyhedron $Q \subset \mathbf{R}^3$ is called primitive if $Q = Q_k$ for some primitive sequence $\{Q_1, Q_2, \dots, Q_k\}$ of convex polyhedra in $\mathbf{R}^3$.

It is shown that in the three-dimensional Euclidean space some prism is not primitive polyhedron.

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DRAFT

The Law of Large Numbers for Weakly Correlated Random Elements in The Spaces l_p, $1 \le p < \infty$

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The Law of Large Numbers (*LLN*) is a popular area of research and has been intensively studied by many authors for the sequences of independent (or uncorrelated) random variables. One of the first results of the *LLN* for the dependent random variables was obtained in 1928 by A. Khinchin [1]:

Theorem (A. Khinchin, 1928)**.** *Let $\xi_1, \xi_2, \dots, \xi_n, \dots$ be a sequence of random variables such that for any positive integer n, ξ_n has a mathematical expectation μ_n and variance σ_n^2. Furthermore, let g be a nonnegative function, defined on the set of nonnegative integers such that for the coefficient of correlation ϱ_{mn} of random variables ξ_m and ξ_n the following inequalities hold*

$$|\varrho_{mn}| \le g(|m-n|), \quad m,n = 1,2,\dots .$$

If

$$\lim_{n\to\infty} \frac{1}{n^2} \Big(\sum_{i=0}^{n-1} g(i)\Big)\Big(\sum_{i=1}^{n} \sigma_i^2\Big) = 0,$$

then the sequence of random variables $\xi_1, \xi_2, \dots, \xi_n, \dots$ satisfies the LLN.

The purpose of this presentation is to show an analogue of this theorem for random elements with values in the Banach spaces l_p, $1 \le p < \infty$.

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DRAFT

Boundedness and Compactness Criteria of a Certain Class of Matrix Operators with Variable Limits of Summation

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Let $1 < p, q < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$, $\{\omega_i\}_{i=1}^{\infty}$, $\{u_i\}_{i=1}^{\infty}$ be non-negative, $\{v_i\}_{i=1}^{\infty}$ positive sequences of real numbers, which will be referred to as weights. Let $l_{p,v}$ be the space of sequences $f = \{f_i\}_{i=1}^{\infty}$, for which the following norm is finite:

$$\|f\|_{p,v} := \Big(\sum_{i=1}^{\infty} |f_i v_i|^p\Big)^{\frac{1}{p}}.$$

We consider the matrix operators of the following form

$$(Af)_n = \sum_{k=\alpha(n)}^{\beta(n)} a_{n,k} f_k, \quad n \geq 1, \tag{1}$$

where $(a_{n,k})$ is a non-negative matrix of operator A, which satisfy the following Oinarov's discrete general condition: there exists $d \geq 1$, a sequence of positive numbers $\{\omega_i\}_{i=1}^{\infty}$ and a non-negative matrix $(b_{i,j})$, such that the inequalities

$$\frac{1}{d}\,(b_{n,k}\omega_m + a_{k,m}) \leq a_{n,m} \leq d(b_{n,k}\omega_m + a_{k,m})$$

holds for all $1 \leq k \leq n$, $\alpha(n) \leq m \leq \beta(k)$, where $\alpha(n)$, $\beta(n)$ are sequences of the natural numbers such that:

$$\begin{aligned} &\text{(i)} \quad \alpha(n) \ \text{ and } \ \beta(n) \ \text{ are strictly increasing sequences;} \\ &\text{(ii)} \quad \alpha(1) = \beta(1) = 1 \ \text{ and } \ \alpha(n) < \beta(n), \ \text{ for } \ n \geq 2. \end{aligned} \tag{2}$$

Note that from (2) it follows $n \leq \alpha(n) < \beta(n)$ for $n \geq 2$.

In this work we obtained necessary and sufficient conditions for the boundedness and compactness of the matrix operator (1) from $l_{p,v}$ into $l_{q,u}$ when $1 < p \leq q < \infty$.

DRAFT

Striving to Transcend the Fuzzy Ontology

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The importance of the need for ontology languages is increasing with time and nowadays the biggest ontologies are used in medicine since automatized medical tools are based exactly on such background knowledge bases.

An example of the biggest medical ontology is the Systematized Nomenclature of Medicine – Clinical Terms (SNOMED CT) which is a standardized, multilingual vocabulary of clinical terminology that is used by physicians and other health care providers for the electronic exchange of clinical health information.

The importance of ontology lies not only in the medical sector but also extends to many other fields as well like biology, bioinformatics, e-commerce, geography, etc.,

Although the ontology languages are standardized by W3C, there are still many problems remaining with these ontologies, where information is vague and imprecise which is so-called, fuzzy ontologies, Fuzzy ontologies are obtained by integrating fuzzy logic with ontologies.

As a result of this increasing importance of ontology, there have been continuous attempts by scientists and researchers to solve the problem of the so-called, fuzzy ontologies, and this, in turn, led them to what is called Unranked Fuzzy Logic.

The final step that has been achieved in Unranked Fuzzy Logic is that its tableaux calculus was sound and complete, but the Unranked Fuzzy Logic has not been implemented yet.

In this research paper, we will introduce the answers to many questions about the implementation of such kind of logic.

DRAFT

About the Issues of Ambulatory Monitoring Devices

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Technological improvements have become increasingly intertwined with daily living and consumers are beginning to control their health using consumer-grade software and hardware.

As ambulatory monitoring devices that can be worn on the body as an accessory or implanted into garments are known as smart wearables like smartwatches, wristbands, and other gadgets which can provide health information, particularly in measuring heart rates.

The heartbeat is measured by the number of contractions of the heart as beats per minute, the lower the number, the better the heart performs. Activity, fitness level, mental state, and medicine can all impact heart rates.

Therefore Heart rate inconstancy can be a gigantic threat for people, particularly for those who are analyzed with any kind of heart disease or recuperated from a previous cardiovascular condition. In order to avoid this threat, continuous monitoring of heart disorder is preferred.

Thus a lot of Ambulatory monitoring devices are allowing a new paradigm in health care for their continuous monitoring and they are becoming increasingly popular. However, the problem with these devices is that they are limited only to monitoring the individual's heart rate and since the individual may lack medical knowledge, an abnormal heartbeat could result in a life-threatening situation.

In this paper, we will be covering ambulatory monitoring devices and their limitations as well as how to take advantage of their benefits while avoiding their drawbacks, and the importance of an individual's heart rate data to the medical authority.

DRAFT

About Unranked Fuzzy Unification

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Solving equations between logic terms is a fundamental problem with many important applications in mathematics, computer science, and artificial intelligence. It is needed to perform an inference step in reasoning systems and logic programming, to match a pattern to an expression in rule-based and functional programming, to extract information from a document, to infer types in programming languages, to compute critical pairs while completing a rewrite system, to resolve ellipsis in natural language processing, etc. Unification and matching are well-known techniques used in these tasks.

Unification (as well as matching) is a quite well-studied topic for the case when the equality between function symbols is precisely defined. This is the standard setting. There is quite some number of unification algorithms whose complexities range from exponential [4] to linear [3]. Besides, many extensions and generalizations have been proposed. Those relevant to our interests are unification with sequence variables and flexible-arity (unranked) function and predicate symbols [2].

Unranked fuzzy logic [1] is an extension of first-order Łukasiewicz product logic (ŁΠ∀) with sequence variables and unranked function and predicate symbols. In this talk we consider unification problem for this logic. Unranked fuzzy unification is divided into two parts: unranked unification and solving sets of linear inequalities. Unranked unification, and thus unranked fuzzy unification, is non-terminating in general, but there are some well-known terminating classes. We discuss challenges of unranked fuzzy unification and identify corresponding terminating fragments.

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Acknowledgement

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The Tensor Completions in Varieties Hall's W-Power Groups

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In the present paper some problems of the theory of the varieties of Hall's W-power groups and tensor completions of Hall's W-power groups in a variety are considered.

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DRAFT

Program for Enrollment of Entrants in Higher Education on the National Exams, Problems and Ways to Solve Them

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The program of admission of applicants to universities at passing state exams has been partially studied. It has been proven that the recruitment of applicants to higher education institutions and the distribution of grants occurs with significant errors. A way to solve this problem is also given.

DRAFT

Solving the Linear Ordering Problem Using the Faceted Cuts $(NP = P)$

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In this paper, a survey of the results is given, giving a polynomial algorithm for the NP hard of linear ordering problem. We consider the linear ordering problem as an integer linear programming problem. Solving the linear programming problem and obtaining a non-integer solution, we find all the necessary cutting facets using a polynomial algorithm and with the obtained facets we again solve the linear programming problem. This approach to solving the problem continues until now until we get an integer solution. Every time we can find all the necessary facets using a polynomial algorithm. Therefore we obtain a polynomial algorithm for solving the linear ordering problem.

DRAFT

Compactness of Commutators for the Calderon–Zygmund Type Singular Operator on Global Morrey-Type Spaces

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In this paper we give sufficient conditions of compactness of Commutators for singular operators $[b,T]$ from Global Morrey-type space $\mathrm{GM}^{w_1}_{p\theta_1}$ to $\mathrm{GM}^{w_2}_{p\theta_2}$. Let $1 \le p, \theta \le \infty$, w be a measurable non-negative function on $(0,\infty)$. The Global Morrey-type space $\mathrm{GM}^{w}_{p\theta} \equiv \mathrm{GM}^{w}_{p\theta}(\mathbb{R}^n)$ is defined as the set of all functions $f \in L^{loc}_p(\mathbb{R}^n)$ with finite quasi-norm

$$\|f\|_{\mathrm{GM}^{w}_{p\theta}} \equiv \sup_{x\in\mathbb{R}^n} \big\| w(r)\|f\|_{L_p(B(x,r))}\big\|_{L_\theta(0,\infty)},$$

where $B(t,r)$ the ball with center at the point t and of radius r [2].

In this paper we consider the Calderon–Zygmund type singular operator

$$Tf(x) = \int\limits_{\mathbb{R}^n} K(x,y) f(y)\, dy,$$

where K(x,y) is standart singular kernel.

For a function $b \in L_{loc}(\mathbb{R}^n)$ by M_b denote multiplier operator $M_b f = bf$, where f-measurable function. Then the commutator between T and b is defined by $[b,T] = M_b T - T M_b$. By $\mathrm{VMO}(\mathbb{R}^n)$ we denote the BMO-closure $C^\infty_0(\mathbb{R}^n)$, where $C^\infty_0(\mathbb{R}^n)$ the set of all functions from $C^\infty(\mathbb{R}^n)$ with compact support.

Theorem. *Let $1 < p < \infty$, $1 < \theta_1 \le \theta_2 < \infty$, $b \in \mathrm{VMO}(\mathbb{R}^n)$, the functions w_1, w_2 satisfy the conditions*

$$\Big\| w_2(r) \Big(\frac{r}{t+r}\Big)^{\frac{n}{p}} \Big\|_{L_{\theta_2}(0,\infty)} \le \|w_1(r)\|_{L_{\theta_1}(t,\infty)}.$$

Then the commutator $[b,T]$ is a compact operator from $\mathrm{GM}^{w_1}_{p,\theta_1}$ to $\mathrm{GM}^{w_2}_{p,\theta_2}$.

The compactness of Commutators for singular integrals on Morrey spaces was considered in [3]. The condition of compactness of set in Global Morrey-type spaces was obtained in [1].

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In Regard to Multidimensional Density Distribution of Projection Type of Nonparametric Statistical Assessment

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The study of empirical analogies of density distribution is based on the main statistical challenge, which is represented by density terminology of theoretical distribution. Nonparametric statistical assessment task of an unknown density distribution is among them, which is one of the basic challenges of mathematical statistics.

Recently attention of many authors was drawn towards Lebega's variations in force properties and size, towards the search of probable sizes of density of empirical analogies.

The founder of Georgian probability school, professor G.Mania, together with the member correspondent of Georgian science academy, E. Nadaraia, studied and based on independant observation versatilely generalized E.Glivenko and N. Smirnov's "histogramic" and Rozemblat–Parzen's "hearty" type assessments. Together with this, E. Nadaraya discussed and generalized "projection" type assessments structured by M. Chencovi in which spectral disintegration of "heart" is used on the basis of orthonormal function. He got new results and also determined density deviation of framed assessment with L_2 metric precision.

In the presented study an unknown multidimensional density distribution of Chencov's type nonparametric statistical assessment of root-mean-square integral deviation of asymptotic behaviour is discussed. For the deviation of μ size square integral function in L_2 space Laplace transformation is offered.

Random processes and Gauss's random field's general methods, which enable the generalization of the existing results are chosen as the main methods of the study.

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DRAFT

Asymptotic Analysis and Regularity Results of Dynamical Boundary-Transmission Problems of the Generalized Thermo-Electro-Magneto-Elasticity for Composed Structures

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The talk is devoted to the investigation of mixed boundary-transmission problems of dynamics for composed elastic structures consisting of two contacting anisotropic bodies which occupy two three-dimensional adjacent regions with a common contacting interface. The structure contains an interfacial crack. The contacting elastic bodies are subject to different mathematical models. In particular, we consider Green-Lindsay's model of generalized thermo-electro-magneto-elasticity in one component, while in the other one, we considered Green–Lindsay's model of generalized thermo-elasticity. The interaction of the thermo-mechanical and electro-magnetic fields in the composed homogeneous piecewise elastic structure is described by the fully coupled systems of partial differential equations of dynamics and the appropriate boundary-transmission conditions. This problem by means of Laplace transform is reduced to the corresponding problem of pseudo-oscillations.

Using the potential method and the theory of pseudodifferential equations on manifolds with a boundary, we prove the uniqueness and existence of the solutions of the pseudo-oscillation problem in suitable function spaces, investigate the regularity of solutions and singularities of the corresponding thermo-mechanical and electro-magnetic fields near the interfacial crack edges, derive the explicit expressions for the stress singularity exponents, and show that they depend essentially on the material parameters. A special class of composed elastic structures is considered, where the so-called oscillating stress singularities do not occur.

Finally, by the inverse Laplace transform the solutions of the original dynamical problems are constructed with the help of the solutions of the corresponding elliptic problems of pseudo-oscillations.

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DRAFT

Shell Equations in Terms of Günter's Derivatives, Derived by the Γ-Convergence

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A mixed boundary value problem for the L'ame equation in a thin layer $\Omega^h : \mathcal{C} \times [-h, h]$ around a surface $\mathcal{C}$ with the Lipshitz boundary is investigated. We show that when thickness h of the layer tends to zero $h \to 0$, the corresponding energy functional, scaled properly, converges in the Γ-limit sense to some functional defined on mid-surface $\mathcal{C}$ of the layer, which corresponds to two-dimensional boundary value problem for associated Euler–Lagrange equation in terms of Gúnter's derivatives. The obtained equations together with boundary conditions can be considered as a boundary value problem defined on a shell model. This BVP on $\mathcal{C}$, considered as the Γ-limit of the initial BVP, is written in terms of Günter's tangential derivatives on $\mathcal{C}$ and represents a new form of the shell equation. It is shown that the Neumann boundary condition from the initial BVP on the upper and lower surfaces transforms into a right-hand side term of the basic equation of the limit BVP.

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DRAFT

Generalization of Schwartz's Theorem

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The equality in the weak sense of second order mixed derivatives is proved for function having isolated singularities. This is a generalization of Schwartz's theorem (in some sources it is called Mixed Partials theorem or Clairaut's theorem or Young's theorem):

A number of physical problems, in particular those related to point (idealized) objects, have to be described by generalized functions (such as the Dirac function and related to it). Thus, it seems interesting to investigate whether the conditions of Schwartz's theorem can be expanded. We have proved the

Theorem. *Let a function $F(x) = F(x_1, \dots, x_N)$ and its second order derivatives be defined and continuous at all points of a closed area $\overline{D}$ of the Euclidian space $\mathbb{E}^N$ ($\overline{D} \subset \mathbb{E}^N$, $N = 2, 3, \dots$) except one interior point P,*

$$P = x^{(P)} = (x_1^{(P)}, x_2^{(P)}, \dots, x_N^{(P)}) \in D, \ \ P \notin \partial D,$$

where the function F has an isolated singularity (for example, a pole of order k, $k \in \mathbb{N}$). Then the second order derivatives $\partial_\alpha \partial_\beta F(x)$ ($\alpha, \beta = 1, \dots, N$) are commutative (in the weak sense) at all points of $\overline{D}$:

$$\partial_\alpha \partial_\beta F(x) - \partial_\beta \partial_\alpha F(x) \doteq 0, \ \ x = (x_1, x_2, \dots, x_N) \in \overline{D}, \ \ \alpha, \beta = 1, \dots, N.$$

Here the symbol $\doteq$ is used for equalities in the weak sense. That is, one has

$$\int\limits_{D_1} \big[\partial_\alpha \partial_\beta F(x) - \partial_\beta \partial_\alpha F(x)\big] f(x)\, dx_1 \cdots dx_N = 0, \ \ D_1 \subset \overline{D},$$

where $f(x)$ is any continuous function in $\overline{D}$.

On an Algorithm for Integer Optimization

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1. Formulation of problem

Let $n > 1$ be a natural number, $\mathbb{Z}_+$ be the set of non-negative integers, $L \in \mathbb{Z}_+$, $L > 0$, $\mathbf{k}_n = (k_1, \dots, k_n) \in \mathbb{Z}_+^n$ and $\mathbf{s}_n = (s_1, \dots, s_n) \in \mathbb{Z}_+^n$.

Optimization problem. *Find*

$$b(L, n;\, \mathbf{k}_n,\, \mathbf{s}_n) = \max \prod_{i=1}^{n} (x_i + s_i)$$

when x_i, $i = 1, \dots, n$ are non-negative integers such that

$$\sum_{i=1}^{n} x_i = L \ \text{ and } \ x_i \geq k_i, \ \ i = 1, \dots, n.$$

2. Algorithm for solution

We plan to discuss the following steps, which provide an algorithm for solution of the problem.

Write

$$I_n := \{1, \dots, n\}, \quad q := \frac{L + \sum\limits_{i=1}^{n} s_i}{n} \ \text{ and } \ r := L + \sum_{i=1}^{n} s_i - n[q]\,.$$

Assume further that

$$d := L - \Big(\sum_{i=1}^{n} k_i + n \Big) \geq 0$$

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and denote

$$\alpha = \min_{1 \leq i \leq n} (k_i + s_i), \quad \beta = \max_{1 \leq i \leq n} (k_i + s_i),$$
$$\mathcal{I}_1 = \big\{ i \in I_n : \ s_i + k_i \leq q \big\}, \quad \mathcal{I}_2 = \big\{ i \in I_n : \ s_i + k_i > q \big\},$$
$$m := \operatorname{card}(\mathcal{I}_1), \quad \mathcal{I}_1 := \{i_1, \dots, i_m\}.$$

The following statements are valid.

(a) $q \geq 1 + \frac{d}{n} + \alpha > \alpha$, in particular, $m \geq 1$ and

$$\Big(\prod_{i=1}^{n-1} (1 + k_i + s_i) \Big) \cdot (1 + k_n + d + s_n) \leq b(L, n; \mathbf{k}_n, \mathbf{s}_n) \leq \big(1 + [q]\big)^r [q]^{n-r}.$$

(b) If $m = n$, i.e. if $q \geq \beta$, then we have the following *equality*:

$$b(L, n; \mathbf{k}_n, \mathbf{s}_n) = \big(1 + [q]\big)^r [q]^{n-r}.$$

(c) If $m < n$, i.e. if $q < \beta$, then

$$q = [q] \Longrightarrow b(L, n; \mathbf{k}_n, \mathbf{s}_n) < q^n$$

and

$$b(L, n; \mathbf{k}_n; \mathbf{s}_n) = b(L_m, m; , \mathbf{k}_m; \mathbf{s}_m) \prod_{i \in \mathcal{I}_2} (k_i + s_i),$$

where

$$L_m := L - \sum_{i \in \mathcal{I}_2} k_i, \quad \mathbf{k}_m := (k_{i_1}, \dots, k_{i_m}), \;\; \mathbf{s}_m := (s_{i_1}, \dots, s_{i_m}).$$

(d) If $m = 1$, then

$$b(L, n; \mathbf{k}_n; \mathbf{s}_n) = (L_1 + s_{i_1}) \prod_{i \in I_n \setminus \{i_1\}} (k_i + s_i),$$

where

$$L_1 := L - \sum_{i \in I_n \setminus \{i_1\}} k_i.$$

The talk is based on [1].

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In the European Union with the Georgian and Abkhazian Languages, i.e. Short Overview of the Results Achieved for Aims of Defense Georgian and Abkhazian Languages from Dangers of Digital Extinction

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At the presentation on the base of European language policy and European expert studies directed on the protection of languages, we explain clearly the meaning of the phrase "In The European Union With The Georgian and Abkhazian Languages" and its direct connection with the aims of defense of the Georgian state languages – Georgian and Abkhazian from danger of digital extinction [1], [2]. Also, during the presentation we will shortly overview the social, theoretical and practical results achieved for aims of defense Georgian and Abkhazian languages from dangers of digital extinction within projects funded by the Shota Rustaveli Georgian National Science Foundation, which were performed under the direction of K. Pkhakadze. They are: MG-ISE-19-1315 (Tbilisi First International Summer School "Logic, Language, Artificial Intelligence"), PHDF-18-1228 ("In the European Union with Georgian and Abkhazian Languages, i.e. the Doctoral Thesis – Elaboration of the New Developing Tools and Methods of the Georgian Smart Corpus and Improvement of Already Existing Ones" (Docoral Student – Sh. Malidze), AR/122/4-105/14 ("One More Step Towards Georgian Talking Self-Developing Intellectual Corpus"), DO/308/4-105/14 (In the European Union with the Georgian Language, i.e., the Doctoral Thesis – Georgian Grammar Checker (Analyzer), (Doctoral Student – M. Chikvinidze)), DO/305/4-105/14 ("In the European Union with the Georgian Language, i.e., the Doctoral Thesis – Georgian Speech Synthesis and Recognition" (Doctoral Student – G. Chichua)) and FR/362/4-105/12 ("Foundations of Logical Grammar of Georgian Language and Its Application in Information Technology").

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DRAFT

Asymptotical Solution of the Mixed Problem for a System of Equations in Partial Derivatives Describing the Process of Propagation of Explosive Shock Waves in Nonhomogeneous Gravitational Gas Bodies

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Mathematical modeling of explosive processes in gravitational gas bodies is one of the actual problems in astrophysics [1]–[6].

The work considers a nonautomodel problem about the central explosion of nonhomogeneous gas body (star) bordering vacuum, which is in equilibrium in its own gravitational field. Asymptotic method of thin impact layer is used to solve the mixed problem for a system of equations in partial derivatives describing the process of propagation of explosive shock waves in nonhomogeneous gravitational gas bodies. The solution of the problem in the vicinity behind the shock wave (discontinuity surface of the first kind) is sought in the form of a singular asymptotic decomposition by a small parameter. Analytically, the main approximation for the law of motion and the thermodynamic characteristics of the medium was accurately found. The Cauchy's problem for zero approximation of the law of motion of the shock wave is solved exactly, in the form of elliptic integrals of the first and second kind. Corresponding asymptotics of solutions are calculated.

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DRAFT

Nonlinear Mathematical Models Describing the Initial Stage of Infection Spread

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It is of great interest to predict the extent of the epidemic by creating appropriate mathematical models [1]. Mathematical and computer models of the possible spread of infectious disease, in particular malaria, are discussed in [2]. Since the beginning of 2020, the incurable infectious disease COVID-19 has spread around the world, which has already killed many people and changed the world (economy, development rate, etc.).

Naturally, at this stage, several vaccines have already been developed that have different probabilities (percentages) of acquiring immunity to a given disease, but not complete immunity (100%). In many scientific centers, attempts are being made to create mathematical models for predicting the extent of the spread of this infection.

The work discusses new mathematical models that describe the early stages of the spread of certain infectious diseases, including COVID-19. In the first model, two groups of people are considered: those without healthy immunity; asymptomatic infected. In the second model, three groups of people are considered: those without healthy immunity; asymptomatic infected; identified infected.

In the first model, the infectivity variable coefficient is taken as a linear incremental function of two unknown function variables. The first integral is obtained. Cauchy's problem is solved analytically exactly. In the second model, in the case of constant infection coefficients, the first two integrals of a three-dimensional dynamic system are found and the problem is reduced to the Cauchy's problem for one unknown function.

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DRAFT

The Spectral Picture and Joint Spectral Radius of the Generalized Spherical Aluthge Transform

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For an arbitrary commuting d-tuple $\boldsymbol{T}$ of Hilbert space operators, we fully determine the spectral picture of the generalized spherical Aluthge transform $\Delta_t(\boldsymbol{T})$ and we prove that the spectral radius of $\boldsymbol{T}$ can be calculated from the norms of the iterates of $\Delta_t(\boldsymbol{T})$.

We first determine the spectral picture of $\Delta_t(\boldsymbol{T})$ in terms of the spectral picture of $\boldsymbol{T}$; in particular, we prove that, for any $0 \leq t \leq 1$, $\Delta_t(\boldsymbol{T})$ and $\boldsymbol{T}$ have the same Taylor spectrum, the same Taylor essential spectrum, the same Fredholm index, and the same Harte spectrum. We then study the joint spectral radius $r(\boldsymbol{T})$, and prove that

$$r(\boldsymbol{T}) = \lim_n \|\Delta_t^{(n)}(\boldsymbol{T})\|_2 \;\; (0 < t < 1),$$

where $\Delta_t^{(n)}$ denotes the n-th iterate of Δ_t. For $d = t = 1$, we give an example where the above formula fails.

The talk is based on recent research with Chafiq Benhida, Sang Hoon Lee and Jasang Yoon.

The Factorized Difference Schemes for the Numerical Solution of a Quasi-linear System of Hyperbolic Type Equations

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In this paper, factorized difference schemes for a two-dimensional system of hyperbolic type equations with mixed derivatives are considered.

When constructing a difference scheme, the method of regularization of difference schemes, proposed by A. A. Samarsky, is used. The convergence of the scheme is proved. The factorized difference schemes for a second-order general hyperbolic system are used to solve numerically the system of equations of elasticity theory, in the case of two spatial variables. The constructed algorithm can be effectively used for parallel computing systems.

DRAFT

Smartphone Sensor-Based Fall Detection Using Machine Learning Algorithms

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Human Activity Recognition and particularly detection of abnormal activities such as falls have become a point of interest to many researchers worldwide since falls are considered to be one of the leading causes of injury and death, especially in the elderly population. The prompt intervention of caregivers in critical situations can significantly improve the autonomy and well-being of individuals living alone and those who require remote monitoring. This paper presents a study of accelerometer and gyroscope data retrieved from smartphone embedded sensors, using iOS-based devices. In the project framework there was developed a mobile application for data collection with the following fall type and fall-like activities: Falling Right, Falling Left, Falling Forward, Falling Backward, Sitting Fast, and Jumping. The collected dataset has passed the preprocessing phase and afterward was classified using different Machine Learning algorithms, namely, by Decision Trees, Random Forest, Logistic Regression, k-Nearest Neighbour, XGBoost, LightGBM, and Pytorch Neural Network. Unlike other similar studies, during the experimental setting, volunteers were asked to have smartphones freely in their pockets without tightening and fixing them on the body. This natural way of keeping a mobile device is quite challenging in terms of noisiness however it is more comfortable to wearers and causes fewer constraints. The obtained results are promising that encourages us to continue working with the aim to reach sufficient accuracy along with building a real-time application for potential users.

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DRAFT

The Role of Complex Assignments in the Development of Students Critical Thinking in Mathematics

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The National Curricula aims at developing critical thinking skills in students. A student should critically analyze facts and ideas; ask and answer questions; discuss with arguments, make assumptions, identify problems, look for alternative ways, and make decisions. With this respect, the role of complex assignments in math teaching is important already on the primary level.

This paper represents the content, structure and major features of complex assignments. It is with emphasis that working on a real life related long term assignment, a complex assignment, helps students to get challenged, activated and motivated and exchange knowledge. All these are essential for the advancement of critical thinking.

Final Result of the Doctoral Thesis "Methods and Tools for Intellectual Classification of Georgian Texts"

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Within the framework of the doctoral program "Informatics" of the Georgian Technical University doctoral thesis *"Methods and Tools for Intellectual Classification of Georgian Texts"* (Ph.D. student – K. Demurchev, Supervisor – Director of the Scientific-Educational Center for Georgian Language Technology of the Georgian technical University K. Pkhakadze) was launched in 2017. The doctoral thesis is mainly based on the Georgian Intellectual Web-Corpus developed based on Pkhakadze's logical grammar of the Georgian language within the framework of two-year AR/122/4-105/14 project *"One More Step Towards Georgian Talking Self-Developing Intellectual Corpus"* (project supervisor K. Pkhakadze, project was funded by Shota Rustaveli Georgian National Scientific Foundation) [2], which, in turn, is a sub-project of the long-term project *"Technological Alphabet of Georgian Language"* [1] of the Center for the Georgian Language Technology of the Georgian Technical University. At the same time, the doctoral thesis is aimed to develop methods and tools for automatical and intellectual classification of Georgian texts, that makes this research as an important part of the long-term project *"Technological Alphabet of Georgian Language"*, which was launched in 2012 and aimed to complete technology support of the Georgian language [3]. Thus, in view of all above mentioned, at the presentation, we will review the first experimental version of the automatic intellectual Georgian text classification, which is a final result of the doctoral thesis *"Methods and Tools for Intellectual Classification of Georgian Texts"* and, also, we will review that methods and tools, which were used in construction of it.

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DRAFT

Extremal Problems for Bicentric Polygons

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There will be considered some extremal problems for families of bicentric polygons with fixed inscribed and circumscribed circles. Each such family is completely determined up to a congruence by three positive numbers (R, r, d), where R is the radius of circumscribed circle, r is the radius of inscribed circle and d is the distance between their centers. Extremal problems for such families of bicentric polygons will be studied for three objective functions: perimeter P, sum of diagonals D, and will be also described Coulomb potential E of unit charges placed at the vertices.

Namely, we describe the shapes of extremal polygons and calculate the maximal and minimal value as explicit functions of variables (R, r, d). As follows from the Fuss relations, for a fixed R all these extremal values become functions of single variable d. We describe several properties of these functions and present drawings of their graphs. Some further geometric properties of bicentric polygons will also be given.

DRAFT

On the Free S_1^ω-Algebras

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MV-algebras are the algebraic counterpart of the infinite valued Łukasiewicz sentential calculus, as Boolean algebras are with respect to the classical propositional logic. As it is well known, MV-algebras form a category which is equivalent to the category of abelian lattice ordered groups (ℓ-groups, for short) with strong unit.

It is known that any subvariety of MV-algebras is generated by finitely many algebras, and explicit axiomatizations have been obtained [1], [2], [4]. Notice that the free algebras over the subvarieties of MV-algebras have been described functionally in [5] using McNaughton functions. Finitely generated free MV_n-algebras (that correspond to n-valued Lukasiewicz logic) was described algebraically in [1], [2].

In this paper we give an algebraic description of finitely many generated free MV-algebras in the variety $\mathcal{V}(S_1^\omega)$ generated by the algebra S_1^ω (denoted as C) that was introduced by Chang and later by Komori who also introduced S_n^ω-algebras for $n \geq 2$. Moreover, we give ordered spectral spaces of the free algebras. Non-exact algebraic description of free m-generated S_1^ω-algebra (or $MV(C)$-algebra) have been given in [3] where the free algebras are represented by subdirect product of infinite family of chains. In present talk we represent free algebras by means of subdirect product of the finite family of chains according to Panti's result in [5]. Notice, that although 1- and 2-generated free S_1^ω-algebras have been represented by the authors before, here we give more clear and simple visualization of spectral space.

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Involutive Symmetric Goedel Spaces, their Algebraic Duals and Logic

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It is introduced a new algebra $(A, \otimes, \oplus, *, \rightharpoonup, 0, 1)$ called L_PG-algebra if $(A, \otimes, \oplus, *, 0, 1)$ is L_P-algebra (i.e. an algebra from the variety generated by perfect MV-algebras [1]) and $(A, \rightharpoonup, 0, 1)$ is a Gödel algebra (i.e. Heyting algebra satisfying the identity $(x \rightharpoonup y) \vee (y \rightharpoonup x) = 1$).

The lattice of congruences of an L_PG-algebra $(A, \otimes, \oplus, *, \rightharpoonup, 0, 1)$ is isomorphic to the lattice of Skolem filters (i.e. special type of MV-filters) of the MV-algebra $(A, \otimes, \oplus, *, 0, 1)$. The variety $\mathbf{L_PG}$ of L_PG-algebras is generated by the algebras $(C, \otimes, \oplus, *, \rightharpoonup, 0, 1)$ where $(C, \otimes, \oplus, *, 0, 1)$ is Chang MV-algebra. Any L_PG-algebra is bi-Heyting algebra. The set of theorems of the logic L_PG is recursively enumerable. Moreover, the description of finitely generated free L_PG-algebras and their dual objects – MV-spaces is given.

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DRAFT

Random Elastic Composites with Circular Inclusions

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Consider 2D multi-phase random composites with different circular inclusions. A finite number n of inclusions on the infinite plane forms a cluster. The corresponding boundary value problem for Muskhelishvili's potentials is reduced to a system of functional equations. Solution to the functional equations can be obtained by a method of successive approximations or by the Taylor expansion of the unknown analytic functions. Next, the local stress-strain fields are calculated and the averaged elastic constants are obtained in symbolic form. An extension of Maxwell's approach and other various self-consisting cluster methods from single- to n- inclusions problems is developed. An uncertainty when the number of elements n in a cluster tends to infinity is analyzed by means of the conditionally convergent series. Application of the Eisenstein summation yields new analytical approximate formulas for the effective constants for macroscopically isotropic random 2D composites.

DRAFT

Convolution Equations on the Abelian Group $\mathcal{A}(-1,1)$ and Their Applications

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The interval $\mathcal{J} := [-1,1]$ turnes into the Abelian group $\mathcal{A}(\mathcal{J})$ if endowed with the group operation $x +_{\mathcal{J}} y := (x+y)(1+xy)^{-1}$, $x, y \in \mathcal{J}$. The invariant Haar measure is $d_{\mathcal{J}} x := (1-x^2)^{-1} dx$ and the Fourier transformation is

$$(\mathcal{F}_{\mathcal{J}} v)(\xi) := \int\limits_{-1}^{1} \Big(\frac{1-y}{1+y}\Big)^{i\xi} \frac{v(y)\,dy}{1-y^2} = \int\limits_{-1}^{1} \Big(\frac{1-y}{1+y}\Big)^{i\xi} v(y)\, d_{\mathcal{J}} y, \quad \xi \in \mathbb{R}. \tag{1}$$

These tools allow to solve exactly convolution equations on this group

$$c_0 \varphi(x) + (k *_{\mathcal{J}} \varphi)(x) := c_0 u(x) + \int\limits_{-1}^{1} k\Big(\frac{x-y}{1-xy}\Big) \frac{v(y)\,dy}{1-y^2} = h(x), \quad x \in \mathcal{J}. \tag{2}$$

To the class of equations (2) belongs

$$\sum_{k=0}^{n} \bigg[a_k(t) \mathfrak{D}^k u(t) - b_k(t) \int\limits_{-1}^{1} \Big(\frac{1-\tau^2}{1-t^2}\Big)^{d_k} \frac{\mathfrak{D}^k u(\tau) d\tau}{\tau - t} \bigg] = f(t), \quad t \in \mathcal{J}, \tag{3}$$

where $d_k \in \mathbb{C}$ are complex numbers, coefficients are sufficiently smooth. Here $\mathfrak{D} u(x) := (1 - x^2)\, \frac{d}{dx} u(x)$ is the natural derivative of functions on the group $\mathcal{A}(-1,1)$.

It turned out that to the class of convolution equations (2) belong the following cebrated equations with important applications-Prandtl equation

$$\boldsymbol{P} u(x) = \frac{c_0 u(x)}{1-x^2} + \frac{c_1}{\pi i} \int\limits_{-1}^{1} \frac{u'(y)\,dy}{y-x} = f(x), \quad x \in \mathcal{J} \tag{4}$$

Singular Tricomi equation

$$\boldsymbol{T} v(x) = c_0 v(x) + \frac{c_1}{\pi i} \int\limits_{-1}^{1} \frac{v(y)\,dy}{y-x} + \frac{c_2}{\pi i} \int\limits_{-1}^{1} \frac{v(y)\,dy}{1-xy} = g(x), \quad x \in \mathcal{J} \tag{5}$$

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and also Lavrentjev–Bitsadze equation. Moreover, Laplace–Beltrami equation on the unit sphere in $\mathbb{S}^2 \subset \mathbb{R}^3$ is also a $\mathcal{J}$-convolution operator with a parameter.

These equations have ample of applications in Mechanics and Mathematical physics and were investigated by many authors. Equations (4) and (5) were investigated by V. E. Petrov in [1], [2] in the general Banach spaceless setting, while in [3] equation (1) was investigated in the Bessel potential space setting $\widetilde{\mathbb{H}}^s(\mathcal{J})$, $-1 \leqslant s \leqslant 1$. These equations were considered as the image of classical Fourier convolutions on $\mathbb{R}$ under the diffeomorphism $x = -\tanh t : \mathbb{R} \to \mathcal{J}$ mapping the real axes $\mathbb{R}$ to the interval $\mathcal{J}$.

We solve equations (2)–(5) in the full scale of Bessel potential spaces $\mathbb{H}^s_p(\mathcal{J}, d_{\mathcal{J}}x)$ for $-\infty < s < \infty$, $1 < p < \infty$. For an integer $s = m = 1, 2, \dots$, $\mathbb{H}^m_p(\mathcal{J}, d_{\mathcal{J}}x)$ coincides with the Sobolev space $\mathbb{W}^m_p(\mathcal{J}, d_{\mathcal{J}}x)$ of functions which have weighted p-integrable derivatives $\mathfrak{D}^k u \in \mathbb{L}_p(\mathcal{J}, d_{\mathcal{J}}x)$ for $k = 1, 2, \dots, m$.

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DRAFT

On Alexander–Spanier Normal Cohomology Groups of Locally Finitely Valued Cochains

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For a topological space X and an abelian group G let $C^n(X;G)$ be the group of all functions $\varphi : X^{n+1} \to G$. Following V. Baladze a function φ is called *locally zero* if there exists a normal covering $\mathcal{U}$ such that for every $U \in \mathcal{U}$ and for each points $x_i \in U$, $i = 0, 1, \dots, n$ we have that $\varphi(x_0, x_1, \dots, x_n) = 0$, i.e., φ vanishes on $\mathcal{U}^{n+1}$.

Let $C_0^n(X;G)$ be the group of all locally zero functions. The group of all locally finitely valued functions [2] we denote by $C_L^n(X;G)$.

There exist cohomology groups $H_N^n(X;G)$ and $H_{LN}^n(X;G)$ of cochain complexes

$$C^*(X;G) = (C^n(X;G)/C_0^n(X;G), \delta)$$

and

$$C_L^*(X;G) = \left(C_L^n(X;G)/C_L^n(X;G) \cap C_0^n(X;G), \delta\right),$$

where δ is the coboundary homomorphism [1, 2].

Each continuous map $f : X \to Y$ induces a cochain map $f^\sharp$ and a homomorphism f^* of the defined cochain complexes and cohomology groups.

For a subset, resp., a closed subset, $A \subset X$ and the inclusion map $i : A \to X$ consider the homomorphisms

$$i^\sharp : C^*(X;G) \to C^*(A;G), \quad i^\sharp : C_L^*(X;G) \to C_L^*(A;G)$$

and denote by $C^*(X, A;G)$, resp. by $C_L^*(X, A;G)$ their kernels.

There exist cohomology groups

$$H_N^*(X, A;G) \ \text{ and } \ H_{LN}^*(X, A;G),$$

which are called respectively the Alexander-Spanier normal cohomology Groups and *the Alexander–Spanier normal Cohomology Groups of locally finitely valued cochains* .

The first group was defined by V. Baladze; he and his pupils established some properties of the group.

Our main aim is to prove that for the group $H_{LN}^*(X, A;G)$ Steenrod–Eilenberg's type axioms and tautness properties are satisfied.

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DRAFT

Development of the Math Skills and Competences in the Engineering-Technological Curriculums

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The present research discusses the international experience in developing Math skills and competences in the Engineering-Technological curriculums. The paper deals with the study of the Georgian higher educational institutions in the regard of the mentioned as well as involvement of stakeholders in elaboration and implementation of curriculums.

In conclusion, the recommendations, based on the comparative analysis of the Georgian and International practice, are presented.

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Schwarz Gradients and Differentiability for Functions of Two Variables

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In the present paper, the Schwarz derivative is generalized for functions of two variables. For this we introduce the Schwarz concepts of differentiability and differential, an ordinary gradient, an angular gradient, a strong gradient, a generalized angular gradient and a generalized strong gradient.

It is proved that the Schwartz differentiability is equivalent to the existence of a generalized angular Schwarz gradient whose components are the coefficients of a Schwartz differential. Also, a necessary and sufficient condition and a sufficient condition of alternative form are established.

We prove that the existence of a generalized angular partial Schwarz derivative implies a generalized angular smoothness with respect to this variable.

It is established that the Schwarz differentiability of a function at some point implies the smoothness of this function at the same point. An obvious outcome of this fact is that the set of smooth functions contains both a set of ordinary differentiable functions and a set of Schwarz-differentiable functions.

DRAFT

Two-Dimensional Unsteady Flow of a Viscous Incompressible Fluid in a Porous Channel

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Two-dimensional unsteady flow of a viscous incompressible fluid through a porous channel is considered. This motion gets excited from the periodical time change of a pressure drop and a percolation velocity.

The approximate method presented below gives the possibility to find a solution in a practically convenient form, as well as take into account the final conductivity of the channel walls [1]–[3].

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DRAFT

Modeling of Wave Structures in the Upper Ionospheric Plasma in Spherical Coordinates

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Physical mechanism of generation of the new modes of ultra-low-frequency (ULF) electromagnetic planetary waves in F-region of the spherical ionosphere due to the latitudinal inhomogeneity of the geomagnetic field is suggested. The frequency spectra, phase velocity, and wavelength of these perturbations are determined. It is established, that these perturbations are self-localized as nonlinear solitary vortex structures in the ionosphere and moving westward or eastward along the parallels with velocities much greater than the phase velocities of the linear waves. The properties of the wave structures under investigation are similar to those of low-frequency perturbations observed experimentally in the ionosphere at middle latitudes.

DRAFT

Asymptotic Behavior and Numerical Solution of Initial-Boundary Value Problem for One Nonlinear System

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The following initial-boundary value problem is considered:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\Big(V^{\alpha}\,\frac{\partial U}{\partial x}\Big),$$

$$\frac{\partial V}{\partial t} = -aV^{\beta} + bV^{\gamma}\Big(\frac{\partial U}{\partial x}\Big)^2 + cV^{\gamma-\alpha}\,\frac{\partial U}{\partial x}\,,$$

$$U(0,t)=0, \quad V^{\alpha}\,\frac{\partial U}{\partial x}\bigg|_{x=1} = \psi,$$

$$U(x,0)=U_0(x), \quad V(x,0)=V_0(x),$$

where a, b, c, α, β, γ, ψ are constants and U_0, V_0 are given functions on $[0;1]$. Such type problems are studied in many works (see, for example, [1], [2]). Large time behavior of solution is studied. The possibility of a Hopf bifurcation is established. Approximate solutions are also constructed.

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DRAFT

From Plato's Linguophilosophical Views to Challenges of the Digital Age

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The report is based on a Bachelor's thesis "Plato's Linguophilosophical Views" (Student – N. Gegenava, Thesis Advisor – Prof. N. Tomashvili) [1], [2], [6], [7], as well as on the research that was done after the completion of the thesis to further understand the depths and widths of the challenges of the digital age, the most important of which is constructing computers almost fully knowledgeable in different languages [3]–[5]. Thus, the presentation will be discussing Plato's views of language, its characteristics and their importance in the development of philosophical and lingual researches. In addition, the presentation, taking into account Plato's ideas about language, will underline the dire need to have right understanding of the nature of language to finding most excellent solution to the momentous problem of constructing computers that possess knowledge of language. This shines a light upon the cruciality of intensive research on of philosophy of language in general and, accordingly, on philosophy of Georgian language, as we stand on the verge of entering the digital age.

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Further Improvement of the Numerical Model of the Mesoscale Boundary Layer of the Atmosphere

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A numerical model of the full cycle of cloud and fog genesis in the mesoboundary layer of atmosphere has been created. The critical values of relative humidity at which the formation of humidity processes takes place have been determined.

A numerical model of the distribution of aerosol from an instantaneous point source into the mesoboundary layer of the atmosphere has been created. The time intervals at which the deposition of aerosol on the earth's surface begins and ends have been determined.

The formation of smog is simulated based on the synthesis and "overlay" of the two above models.

DRAFT

Boundary Value Problems of Thermoelastic Diffuzion Theory with Microtemperatures and Micriconcentrations

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The talk deals with the linear theory of thermoelastic diffusion for elastic isotropic and homogeneous materials with microtempeatures and microconcetrations. For the system of the corresponding differential equations of pseudo-oscillations the fundamental matrix is constructed explicitly in terms of elementary functions. With the help of Green's identities the general integral representation formula of solutions is derived by means of generalized layer and Newtonian potentials. The basic Dirichlet and Neumann type boundary value problems are formulated in appropriate function spaces and the uniqueness theorems are proved. The existence theorems for classical solutions are established by using the potential method.

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DRAFT

On Some Aspects of the Current Education Reform of Secondary School

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The new school model in the secondary education system introduces radical innovations. Among the multifaceted organizational and qualitative innovations, the assessment system of teachers and students was clearly identified, which, in particular, included the current assessment system.

The abolition of all levels of centralized exams in secondary school has completely changed the existing system of student assessment. It is currently planned to significantly improve the quality of formation and development of students' creative and research skills by working on complex assignments, and this activity and its assessment will become the main focus of students' assessment. Such a strategy, without the involvement of experienced and highly qualified teachers, can even give us negative results. Not excluded a replication of already prepared assignments among the students, to avoid work on their creation and so it will limit the students' activity only to this.

It is essential that teachers discover new and current topics for complex assignments. In this regard, many interesting topics of research and discussion must be selected.

The report will illustrate the construction of a number of complex assignments on the topics of polyhedra. We will cover the topics with interesting materials: from the history of mathematics (including issues of the identification and research of Platonic bodies, mystical representations related to them); analytic geometry, polyhedra's symmetry issues; issues of graph theory and, in general, topology; matrix algebra; combinatorics; crystallography; physics; chemistry (namely, the structure of some molecules); architecture; famous issues of art.

The connections of the named issues will be considered in the talk and also underlined the importance of understanding the relationships of the research of current events in the world and the importance of a unified representation of various events with multilateral aspects in the student's perception.

Working on creating of interesting practical complex assignments will help to develop students' synthetic thinking, will make the subject more interesting and multifaceted and increase the motivation to study the assigned tasks.

DRAFT

On the One Mnemonic Scheme for Studying a Two-Dimensional Random Process

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A complex redundant system with two types of operation is considered. The formalization of the process under consideration leads us to the study of a two-dimensional random process.

It is well known that the study of a one-dimensional random process is simplified by the introduction of a mnemonic scheme for the state graph of systems. According to the researchers, the difficulties associated with the study of such processes grow enormously with an increase in dimension.

We have proposed a mnemonic scheme for a two-dimensional random process. The usefulness of using this scheme for modeling and researching a complex redundant system is shown.

Some explicit formulas are obtained for the constructed concrete model. The results of numerical experiments are presented.

DRAFT

On the Convergence of the Perturbation Algorithm for the Semidiscrete Scheme for the Evolution Equation with Variable Operator in the Banach Space

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We study the semi-discrete schemes for the following evolutionary problem in the Banach space X:

$$\frac{du(t)}{dt} + A(t)u(t) = f(t), \quad t \in \,]0,T], \quad u(0) = u_0, \tag{1}$$

were $A(t)$ is a closed linear operator with the domain $D(A)$, which is everywhere dense in X ($D(A)$ does not depend on t); $f(t)$ is a continuously differentiable function with values in X; u_0 is a given vector from X; $u(t)$ is the sought function.

On the interval $[0,T]$, we define the grid $t_k = k\tau$, $k = 0,1,\dots,n$, with the step $\tau = T/n$. Using the difference formula of second order approximation for the approximation of the first derivative equation (1) can be represented at the point $t = t_{k+1}$ as:

$$\frac{\Delta u(t_k)}{\tau} + \frac{\tau}{2}\frac{\Delta^2 u(t_{k-1})}{\tau^2} + A(t_{k+1})u\,(t_{k+1}) = f(t_{k+1}) + \tau^2 R_{k+1}(\tau,u), \quad R_k(\tau,u) \in H, \tag{2}$$

where $\Delta u(t_k) = u(t_{k+1}) - u(t_k)$, $\tau^2 R_k(\tau,u)$ is the approximation error of the first derivative at the point $t = t_k$. Using the perturbation algorithm on the basis of representation (2) we obtain the following system of equations:

$$\frac{\Delta u_k^{(i)}}{\tau} + A(t_{k+1})u_{k+1}^{(i)} = f(t_{k+1}) + \frac{1}{2}\,\frac{\Delta^2 u_{k-1}^{(i-1)}}{\tau^2}\,, \quad k = i+1,\dots,n, \quad i = 0,1, \quad u_k^{(-1)} = 0.$$

Let the vector $v_k = u_k^{(0)} + \tau u_k^{(1)}$ ($k = 2,\dots,n$) be an approximate value of the exact solution of problem (1) for $t = t_k$, $v_k \approx u(t_k)$.

The following theorem is valid.

Theorem. *Let $A(t)$ is a closed linear operator with the domain $D(A)$, which is everywhere dense in X and for any z with $Re(z) \geq 0$ the operator $(zI + A)$ has a bounded inverse, and $\|(zI+A)^{-1}\| \leq c(1+|z|)^{-1}$, $c = const > 0$. Let solution $u(t)$ sufficiently smooth function. Then, if*

$$D(A^m(t)) = D(A^m(0)), \quad (m = 2,3),$$
$$\big\|(A^m(t') - A^m(t''))A^{-m}(s)\big\| \leq c|t' - t''| \ \ \forall\, t',t'',s \in [0,T], \quad m = 1,2$$

and

$$\big\|(A(t_{k+1}) - 2A(t_k) + A(t_{k-1}))A^{-1}(t_{k+1})\big\| \leq c\tau^2, \quad k = 1,\dots,n-1.$$

Then, it holds that

$$\|u(t_k) - v_k\| = O(\tau^2), \quad k = 2,\dots,n.$$

Mathematical Modeling of Dynamics of Suspension

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In work features of mathematical modeling of dynamics of suspensions are considered and the corresponding mathematical model is under construction. The particle of suspension is considered as disperse mix which mass represents the sum of masses firm components and a liquid disperse component of the corresponding volume. On the basis of the law of constancy of weight, expression for mix density is based. The rheological equation of mix differs from Navier–Stokes's rheology in viscosity coefficient. The coefficient of viscosity of suspension, depends on concentration of a firm component. On the basis of Cauchy's equations and the equation of continuity the corresponding system of the nonlinear equations of suspension is under construction. The received system becomes isolated by the diffusion equation. On the basis of the constructed model, the problem of sedimentation firm component is solved.

The Electrogravitational Instability of an Oscillating Streaming Fluid Cylinder

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The electrogravitational instability of an oscillating streaming fluid cylinder surrounded by a selfgravitating tenuous medium pervaded by transverse varying electric field is discussed under the action of selfgravitating, capillary and electro dynamic forces. This has been done for all modes of perturbation. A second order integro-differential equation of Mathieu type has been drived. Several published works are obtained as limiting cases from the present general one. The model is stable due to the stabilizing effect of the transverse electric field for all modes of perturbation. The capillary force has a strong destabilizing influence on the selfgravitating instability of the model. The streaming has a strong destabilizing effect in all modes of perturbation.

DRAFT

On an Integral Equation of Fredholm Type for the Class of Bounded Functions on Real Axes

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In the present work one considers the question on solution of integral equations of Fredholm type

$$\phi(x) = f(x) + \lambda \lim_{T\to\infty} \frac{1}{T} \int\limits_0^T K(x,\xi)\phi(\xi)\,d\xi \tag{1}$$

in the class of bounded functions given on real axes, where $f(x)$ is bounded and continuous in $\mathbb{R}$. We suppose that the function $K(x,\xi)$ (kernel) is bounded and continuous function in $\mathbb{R}\times\mathbb{R}$. For simplicity we consider the case of symmetric kernel.

If we take the case of ordinary Fredholm equation of second type, the question on solvability is the question demanding special investigation in some classes of functions (for example in the class of Bohr almost periodic functions). This is natural, because of that fact that here we consider the class of functions on all real axes. Consideration of the equations of the type (1) is free of some difficulties. The main result of the present work is as follows.

Theorem. *Let the function $K(x,\xi)$ be continuous and bounded function in $\mathbb{R}\times\mathbb{R}$. Let the ordinary equation*

$$\phi(x) = f(x) + \frac{\lambda}{T} \int\limits_0^T K(x,\xi)\phi(\xi)\,d\xi$$

have solutions for some unbounded sequence of numbers $0 < T_1 < T_2 < \cdots$. Then the equation (1) has solutions.

On the base of this result we construct the complete theory of the Fredholm type equation (1). It is possible introduce the notions of Fredholm determinant and show that this is an integral function. Defining the eigenvalues of the kernel we give the criterion of solubility non-homogenious equation. We give an explicit solution of the equation (1).

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DRAFT

Trace Inequalities for Fractional Integrals in Central Morrey Spaces

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Let (X, d, μ) be a quasi-metric measure space with doubling measure μ. Suppose that ν is another measure on X. Necessary condition and sufficient condition on a measure ν guaranteeing the trace inequality

$$\|K_\alpha f\|_{\mathcal{L}_r^q(X,\nu)} \le C\|f\|_{\mathcal{L}_s^p(X,\mu)}$$

are established, where $\mathcal{L}_r^q(X,\nu)$ and $\mathcal{L}_s^p(X,\mu)$ are central Morrey spaces defined with respect to measures ν and μ respectively, and K_α is the fractional integral operator defined on (X, d, μ). The results are new even for fractional integral operators and Morrey spaces defined on $\mathbb{R}^n$. In particular, we have necessary condition and sufficient condition for a measure ν defined on $\mathbb{R}^n$ governing the trace inequality

$$\|I_\gamma f\|_{\mathcal{L}_r^q(\mathbb{R}^n,\nu)} \le C\|f\|_{\mathcal{L}_s^p(\mathbb{R}^n)},$$

where I_γ is the Riesz potential operator on $\mathbb{R}^n$, $0 < \gamma < n$.

The problem is studied for multilinear fractional integrals as well.

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DRAFT

On Automorphisms and Fixing Number of co-Normal Product of Graphs

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An automorphism of a graph describes its structural symmetry and the concept of fixing number of a graph is used for breaking its symmetries (except the trivial one). In this paper, we evaluate automorphisms of the co-normal product graph $G_1 * G_2$ of two simple graphs G_1 and G_2 and give sharp bounds on the order of its automorphism group. We study the fixing number of $G_1 * G_2$ and prove sharp bounds on it. Moreover, we compute the fixing number of the co-normal product graph of some families.

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DRAFT

Using the Function of Two Variables to Solve Elementary Mathematical Problems

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In order to solve mathematical problems, a connection need to be established between the given values. This connection can be described by a logical reasoning or we can make an equation corresponding to their relationship. Describing this relationship through a formula is more convenient than using logical reasoning.

The goal is to describe certain types of proportional relationships as function of two variables, where independent and dependent variables will be identified from the beginning. It is not necessary to pre-determine whether the relationship is directly proportional or inversely proportional.

Writing functions with two variables makes easier to solve the problem. It should be highlighted that while making a function, the so-called proportionality coefficient comes in, which physically connects the argument with the function.

I use this approach successfully in solving elementary mathematical problems in the school course – it makes easier for students to solve difficult logical problems.

DRAFT

On Investigation and Approximate Solution of Nonlinear Model Describing Electromagnetic Field Diffusion Process

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The nonlinear Maxwell system is considered, which describes the propagation of magnetic field in the medium and the temperature change at the expense of Joule heating and heat conductivity. If there is not heat conductivity this system may be rewritten in the integro-differential form [1].

Some aspects of the investigation and numerical solution of this partial differential system and the above-mentioned integro-differential analogs are studied in many works (see, for example, [1]–[3] and references therein).

Our aim was to investigate and numerical solution of the one-dimensional version of the Maxwell system and its integro-differential analogs. Especially, the semi-discrete and finite difference scheme for initial-boundary value problems for some kind of nonlinearities are constructed and investigated. The asymptotic behavior of solutions is also studied. The corresponding numerical experiments are done.

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DRAFT

Entropy Research of Enhanced Hybrid Model Based on Classical Cryptosystem

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In the modern world, privacy is one of the key issues addressed by information security.Security of transferred digital data is becoming vital. Various attackers are trying to have unauthorized access to private data. There are many cryptographic systems available today to ensure the security of digital data, but each has its advantages and disadvantages. A big amount of data is transferred over the internet and it must be encrypted with some methods. Nowadays, there are asymmetric cryptographic algorithms that have a very high rate of information security, but they have also one big disadvantage, they consume a lot of system resources, also need a lot of encryption time.

This article presents a novel encryption algorithm that is based on classical encryption methods and Unicode information technology standards. The main principle is to use of substitution encryption method. To make experiments on this novel cryptosystem was developed a web application through modern web programming technologies. This software product is easy to implement and fast for usage. The cryptosystem was tested by encrypting and decrypting the plaintext using the proposed software.

Experimental research showed the following advantages of the proposed cryptosystem: transferred information security rate is very high and good encryption speed. The security of the proposed cryptosystem is based on diffusion, randomness, and substitution. The security rate of the algorithm was also researched based on an entropic study. The encrypted text is very random and the attacker cannot even detect the original language of plaintext. The empirical results revealed that the proposed hybrid model is secure and can be used in real-time applications.

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DRAFT

Testing Hypotheses Concerning Equal Parameters of Normal Distribution

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The problem of testing complex hypotheses with respect to the equal parameters of normal distribution using the constrained Bayesian method is discussed. Hypotheses are tested using maximum likelihood and Stein's methods. The optimality of decision rule is shown by the criteria: the mixed direction false discovery rate, the false discovery rate, the Type I and Type II errors, under the conditions of providing the desired level of constraint. The algorithms for the implementation of the created methods and the programs for their realization are given. Simulation results show the correctness of the theoretical results and their superiority over the classical Bayes method.

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Weighted Differential Inequality and Oscillatory and Spectral Properties of a Class of Fourth Order Differential Operators

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Let $I = (0, \infty)$ and $1 < p \leq q < \infty$. Let u be a continuous nonnegative function. Suppose that v and r are positive functions sufficiently times continuously differentiable on I such that $v^{-1} = \frac{1}{v} \in L^{loc}_{p'}(I)$ and $r^{-1} = \frac{1}{r} \in L^{loc}_1(I)$, where $p' = \frac{p}{p-1}$. Consider the following inequality

$$\bigg(\int\limits_0^\infty |u(t)f(t)|^q \, dt \bigg)^{\frac{1}{q}} \leq C \bigg(\int\limits_0^\infty |v(t)D_r^2 f(t)|^p \, dt \bigg)^{\frac{1}{p}}, \quad f \in C_0^\infty(I), \tag{1}$$

where

$$D_r^2 f(t) = \frac{d}{dt}\, r(t)\, \frac{df(t)}{dt}\, .$$

We first establish criteria for the fulfillment of inequality (1). Then we apply the obtained results to find the oscillatory properties of the differential equation

$$D_r^2(v(t)D_r^2 y(t)) - u(t)y(t) = 0, \quad t \in I,$$

and the spectral properties of the operator generated by the differential expression

$$ly(t) = \frac{1}{u}\, D_r^2 \big(v(t) D_r^2 y(t) \big)$$

in the Hilbert space $L_2(u, I)$.

DRAFT

Some Remarks on Equidecomposability of Sets

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The present report is devoted to various definitions of equidecomposability of sets. The connection between finitely equidecomposable and countable equidecomposable sets will be shown.

In particular:

(a) if X and Y are finitely equidecomposable, then they are also countable equidecomposable;

(b) in $\mathbf{R}^n$ there exist two sets X and Y, with $\lambda_n(X) > 0$ and $\lambda_n(Y) = 0$, which are not countable equidecomposable under than of all affine translations of $\mathbf{R}^n$;

(c) in $\mathbf{R}^n$ there exist two sets X and Y such that $card\,(X) = card\,(Y) = c$ and X is not countably equidecomposable with Y.

Acknowledgement

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DRAFT

On Endomorphisms of a Separable p-Group

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We consider a separable abelian p-group G whose basic subgroup is unbounded and G is not direct sum of cyclic p-groups or torsion complete group. It is shown that endomorphism of group G is of finite order or is the multiplication by p-adic number.

Models of Spreading False Information and SARS-CoV-2 Virus

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Computer and mathematical models of spread of false information [2] and SARS-CoV-2 virus [3] are proposed. Mathematical models are constructed using a system of nonlinear ordinary differential equations. The models cover the stages of denial of false information and vaccination. Variable coefficients are also considered in the models [1]. Computer simulation is performed in MatLab environment.

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Various Definitions of Almost Invariant Sets and their Applications

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Almost invariant sets have important applications in various directions of mathematics. In particular, in the theory of invariant and quasi-invariant measures, infinite combinatorial analysis, etc.

In my presentation I will be concerned with so-called almost invariant sets in infinite (mostly, in uncountable) sets. There are various possibilities to define such sets (see, [2]). We will consider below only three variants of their definitions, their equivalence and some applications of those sets in the theory of invariant and quasi-invariant measures (see, [1]).

Acknowledgement

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DRAFT

The Parameter Estimation in Stochastic Volatility Models

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We consider a stochastic differential equation

$$dX_t = \theta a(t, X_t)\, dt + \sigma(t, X_t, Y_t)\, dW_t, \tag{1}$$

with a general diffusion coefficient $\sigma(t, X_t, Y_t)$, where Y is some additional adapted stochastic process, and θ is an unknown drift parameter. Such equations arise as the models of some markets in mathematical finance. We investigate the existence and uniqueness weak and strong solutions of this equation under diverse conditions on the coefficients a, σ and the process Y.

A first particular interest is given to the case where $\sigma(t, X_t, \cdot\,) := \beta(t, X_t)$ is independent of Y. In this case, we investigate the problem of estimation of the unknown drift parameter in (1), with the coefficients supplying standard existence uniqueness demands. We also consider a particular situation when the ratio of drift and diffusion coefficients is non-random, and establish the strong consistency of the estimator with different ratios, from many classes of non-random standard functions.

A second intersect is given to the particular case where the diffusion coefficient is the product of two factors:

$$\sigma(t, X_t, Y_t) = \sigma_1(t, X_t)\sigma_2(t, Y_t).$$

In this case we also estimate the drift parameter θ and we establish the strong consistency of the maximum likelihood estimator for this unknown parameter.

In the general case, we study the drift parameter estimation and we take some examples of the main equation and of the process Y to supply the strong consistency.

We illustrate our results with [illegible] simulations.

DRAFT

Keywords. maximum likelihood estimator, parameter estimation, strong consistency, weak and strong solution.

On the Approximate Solutions of the Schrödinger Equation

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The problem of the approximation of solutions of stationary linear as well as nonlinear Schrödinger equations is considered.

The linear Schrödinger equation is studied in the areas of polygonal configuration. By means of the conformal mapping method 2D linear Schrödinger equation is replaced by the approximate elliptic equation. The initial area is mapped on the rectangle and the exact solution of the elliptic equation is obtained.

The results has applications to the quantum mechanics and biophysics [1], [2], [4], [5], [7].

Also, the multi-dimensional cubic non-linear Schrödinger equation is considered in the infinite area.The equation is replaced by the approximate non-linear elliptic equation.The exact solutions of this equation vanishing at infinity are obtained. These solutions represent solitary waves [2], [6]. Their profiles are plotted by means of "Maple".

The problem of approximation of non-stationary Schrödinger equation by means of finite-difference schemes and integral equations has been discussed in [3], [8].

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DRAFT

Torricelli Point as a Maximizer

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Classically, the Torricelli point of a triangle is defined as the unique point for which the sum of distances to the vertices of triangle is minimal. In other words, the Torricelli point is the unique minimizer of the function defined as the sum of distances to the vertices of triangle. The aim of this talk is to show that the Torricelli point is also the unique maximizer of another natural function on the interior of triangle.

Let us consider a triangle $\triangle$ with vertices A, B, C such that each of its angles is less than $2\pi/3$. As is well known such a triangle possesses a uniquely defined Torricelli point T inside $\triangle$ characterized by the property that the angles ATB, BTC, CTA are all equal to $2\pi/3$. Introduce now a function f of point P inside $\triangle$ defined as

$$f(P) = \sin p \cdot \sin q \cdot \sin r,$$

where p, q, r are the angles APB, BPC, CPA respectively. It is easy to see that f is a smooth positive function on the interior of $\triangle$ which tends to zero as point P approaches the boundary of $\triangle$.

Our first main result is that the function f has a unique critical point inside $\triangle$ which coincides with the Torricelli point of triangle $\triangle$ and is a point of non-degenerate maximum of f. In other words, T is the unique maximizer of f. The second main result states that the level sets of function f are smooth convex curves which yield a foliation of the interior of $\triangle$. We will also explain relation of these results to a certain problem of electrostatics.

DRAFT

More on Relations Between Crossed Modules of Different Algebras

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Groups, associative algebras and Lie algebras are related by the well-known adjoint functors: the group algebra functor is left adjoint to the unit group functor, as well as the universal enveloping algebra functor is left adjoint to the Liezation functor. These classical facts have been recently extended to the respective categories of crossed modules in [2] and [3].

In the non-commutative framework, when Lie algebras are replaced by Leibniz algebras, the analogous objects to associative algebras are dialgebras, introduced and studied by J.-L. Loday [4]. There is an adjunction between the categories of Leibniz algebras and dialgebras, which is analogous and related to the one between the categories of Lie and associative algebras.

We present the construction of adjoint functors between the categories of crossed modules of dialgebras and Leibniz algebras. Moreover, we extend the well-known relations between the categories of Lie, Leibniz, associative algebras and dialgebras to the respective categories of crossed modules.

The results of this research are presented in the papers [1]–[3].

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DRAFT

Influence of Chiral Asymmetry on Phase Structure of the Two-Color Quark Matter

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It is well known that quantum chromodynamics (QCD) is the theory of hot and dense strongly interacting matter.

And the great interest are properties and phase diagram of dense baryon (quark) matter, which may be realized in heavy-ion collision experiments or inside compact stars. However, at the corresponding values of temperature and baryon density, the QCD interaction constant is quite large. Therefore, using the usual perturbation method, it is impossible to obtain an adequate picture of the phenomena of dense matter. In this case different effective low-energy QCD-like models, among which are the well-known Nambu–Jona-Lasinio (NJL) type models [2], etc, can be used to describe the corresponding parts of the QCD phase diagram.

Dense baryonic matter, which can exist in neutron stars or be even observed in heavy-ion collision experiments, is characterized by not only μ_B chemical potential. In the real case there appears an additional isospin chemical potential μ_I in the system. Moreover, since nuclear matter is usually under the influence of extremely strong external magnetic fields, chiral asymmetry of the medium can also be observed. In the most general case this phenomenon is described by two chemical potentials, chiral μ_5 and chiral isospin μ_{I5}. Thus, the phase structure of real dense quark (baryonic) matter should be described in terms of QCD with several chemical potentials.

In the present work, an attempt is performed using NJL-model to find out how the phenomenon of condensation of diquark pairs, which in the real 3-color case corresponds to color superconductivity (CSC) phenomenon, can affect the duality between the CSB and charged PC phases. Moreover, it is also under the influence of strong magnetic field, leading to chiral asymmetry of quark medium. In our recent paper [1] the phase structure of the 2-color and 2-flavor massless NJL model was investigated in the mean-field approximation at three nonzero chemical potentials, $\mu_B \neq 0$, $\mu_I \neq 0$ and $\mu_{I5} \neq 0$. In the ground state of the $N_c = 2$ system under consideration there can be a condensation of σ particles, and in this case the CSB phase is realized. Finally, the condensation of baryonic colorless diquarks leads to the phase of quark matter with spontaneous breaking of baryonic $U(1)_B$ symmetry, and we call it the baryonic superfluid (BSF) phase. It is shown in the paper [1] that one more, diquark, channel of quark interactions does not spoil at all the dual symmetry between CSB and charged PC phenomena at large μ_B. Moreover, in this case there appear two additional dual symmetries of the (μ_B, μ_I, μ_{I5})-phase portrait of the model: between the BSF and CSB phases, as well as between the BSF and charged PC phases.

DRAFT

In heavy-ion collisions due to large temperatures and non-trivial gluon configurations chiral imbalance μ_5 may appear. Therefore, it would be interesting to clarify the situation with the dual symmetries of the phase diagram of this system in the most general case.

The main results of the work are as follows. It turns out that the full $(\mu_B, \mu_I, \mu_{I5}, \mu_5)$-phase diagram of the model is interconnected by the dualities and possesses a very high symmetry. Chiral μ_5 is the only chemical potential that keeps this symmetry intact but is not involved in it itself, it only deforms the whole phase diagram.

The phase diagram itself is studied numerically. One could see two interesting features of chiral μ_5 imbalance, its chameleon nature and its property of being universal catalyzer. It is shown that it can catalyze every phenomena in the system. Chameleon nature signifies that chiral μ_5 chemical potential can take a role of any chemical potential and have the same influence on the phase structure, being universal catalyzer is one of manifestations of chameleon properties, chiral chemical potential can take a role of isospin one and in equal degree catalyze charged PC, or it can mimic baryon one and catalyze diquark condensation.

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DRAFT

The Consistent Criteria of Hypotheses Testing for Gamma ("G") Statistical Structure

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The density of Gamma ("G") low is determined by the equality

$$f(x,\alpha,\theta)=x^{\alpha-1}e^{-x/\theta}\theta^{-\alpha}G^{-1}(\alpha),\ \ x\geq 0,$$

where $G(\alpha)=\int\limits_0^\infty t^{\alpha-1}e^{-t}\,dt$, $\alpha>0$, $\theta>0$.

The probability measure μ corresponding to the Gamma ("G") density: $\mu(A)=\int\limits_A f(x,\alpha,\theta)\,dx$ is called "G" measure.

Definition 1. A statistical structure $\{E,S,\mu_h,h\in H\}$ is called Gamma ("G") Statistical structure if each μ_h $(h\in H)$ is "G" measure.

For each $h\in H$ we denote by $\overline{\mu}_h$ the completion of the measure μ_h, and by $dom(\overline{\mu}_h)$ – the σ-algebra of all $\overline{\mu}_h$-measurable subsets of E. We denote $S_1=\bigcap\limits_{h\in H} dom\,(\overline{\mu}_h)$. Let H be the set oh hypotheses and let $B(H)$ be σ-algebra of subsets of H which contains all finite subsets of H.

Definition 2. We will say that the statistical structure $\{E,S_1,\overline{\mu}_h,h\in H\}$ admits a consistent criterion of hypotheses testing if there exists at least one measurable mapping $\delta:(E,S_1)\to(H,B(H))$ such that $\overline{\mu}_h(\{x:\ \delta(x)=h\})=1\ \forall\,h\in H$.

Definition 3. A statistical structure $\{E,S_1,\overline{\mu}_h,h\in H\}$ is called strongly separable if there exists a family of S_1-measurable sets $\{Z_h,h\in H\}$ such that the relations are fulfilled:

(1) $\mu_h(Z_h)=1\ \ \forall\,h\in H$;

(2) $Z_{h_1}\cap Z_{h_2}=\varnothing\ \ \forall\,h_1\neq h_2,\ \ h_1,h_2\in H$;

(3) $\bigcup\limits_{h\in H} Z_h=E$.

Theorem. *In order that the Gamma ("G") orthogonal statistical structure* $\{E,S_1,\overline{\mu}_h,h\in H\}$, $card\,H=c$, *admitted a consistent criteria of hypotheses testing it is necessary and sufficient that this statistical structure be strongly separable.*

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DRAFT

The Method of Almost Surjective Homomorphisms and the Relative Measurability of Functions

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Some interconnections between the relative measurability of real-valued functions and certain classes of mappings and group homomorphisms with thick graphs will be given for solving certain versions of the general measure extension problem in its algebraic setting.

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DRAFT

The Use of the Differentiation Elements in a Primary-Level Mathematics Lesson

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The third generation of the National Curriculum primarily aims at implementing a student-oriented educational process, in which a teacher should plan the educational process where all the decisions regarding the selection of methods, strategies and teaching resources; classroom management; creation of an inclusive classroom environment; evaluation system and etc., are based on the needs of a student. With this respect, using the differentiation approach throughout the educational process is essential for a teacher.

This paper offers differentiation activity patterns for lower, medium and higher mathematical proficiency students in math classes. The principles of differentiation are distinctly represented. Despite different levels of assignments, its important that National Curriculum requirements are universal for every student. These students walk along different trails to get to the same goal.

DRAFT

On Numerical Solving of the Dirichlet Generalized Harmonic Problem for Regular n-Sided Pyramidal Domains by the Probabilistic Method

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The probabilistic method (PM) is proposed for numerical solving of the Dirichlet generalized harmonic problem for regular n-sided pyramidal domains. The term "generalized" indicates that a boundary function has a finite number of first kind discontinuity curves. In the considered case edges of the pyramid represent the mentioned curves. Application of the PM consists of following main stages:

(a) computer modelling of the Wiener process;

(b) construction of an algorithm for finding the intersection point of the trajectory of the simulated Wiener process and the surface of the pyramid;

(c) checking of a scheme and corresponding calculating program needed for numerical implementation and reliability of obtained results;

(d) finding of the probabilistic solution of generalized problems at any fixed points of the considered pyramid. The algorithm does not require approximation of a boundary function,which is main of its important properties.

For illustration of the effectiveness and simplicity of the suggested method several numerical examples are considered. Numerical results are presented.

DRAFT

Generalized Multilinear Sobolev Inequality in Grand Product Lebesgue Spaces Defined on Non-Homogeneous Spaces

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Necessary and sufficient condition governing the boundedness of the multilinear fractional integral operator $T_{\gamma,\mu}$ with measure μ from the grand product space $\prod\limits_{j=1}^{m} \mathcal{L}^{p_j),\theta}(X,\mu)$ to grand Lebesgue space $L^{q),\theta q/p}(X,\mu)$ is established, where $\frac{1}{p} = \sum\limits_{j=1}^{m} \frac{1}{p_m}$, $p < q$ and $\theta > 0$. As a corollary we have a complete characterization of the multilinear Sobolev inequality in these spaces.

For such a characterization in the case of classical Lebesgue spaces we refer to [2] in the multilinear setting, and [1], [3] in the linear case.

The talk is based on the joint work with A. Meskhi.

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DRAFT

On the Low Dimensional Cohomologies of Biparabolic Subalgebras

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It is proved, that the dimension of the center of a biparabolic subalgebra P of the prime Lie algebra $so\,(N)$, i.e. the dimension of the zero regular cohomologies $H^0(P,P)$ is the number of partial nontrivial equal sums of the partition of $n=[N/2]$

$$n=n_1+\cdots+n_r=m_1+\cdots+m_s, \tag{1}$$

which corresponds to the biparabolic subalgebra P. Let us remark that similar result for the center of a biparabolic subalgebra of the prime Lie algebra $sl\,(n)$ was obtained by G. Rakviashvili and E. Kuljanishvili earlier.

Suppose we have the partition, similar to (1), for a biparabolic subalgebra Q of the prime Lie algebra $sl\,(n)$. It is proved, that the dimension of the first regular cohomologies $H^1(Q,Q)$, i.e. the dimension of the outer derivations of Q is equal to

$$\dim Q=(r+s-d-2)d,$$

where d is the dimension of the center of Q.

DRAFT

Shape Differentiability of Semilinear Equilibrium-Constrained Optimization

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A class of semilinear optimization problems linked to variational inequalities is studied with respect to its shape differentiability. One typical example stemming from quasi-brittle fracture describes an elastic body with a Barenblatt cohesive crack under the inequality condition of non-penetration at the crack faces. The other conceptual model is described by a generalized Stokes–Brinkman–Forchheimer's equation under divergence-free and mixed boundary conditions. Based on the Lagrange multiplier approach and using suitable regularization, an analytical formula for the shape derivative is derived from the Delfour–Zolesio theorem. The explicit expression contains both primal and adjoint states and is useful for finding descent direction of a gradient algorithm to identify an optimal shape, e.g., from boundary measurement data.

Acknowledgement

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On Convergence of Variation of Normed Sum of Chain Dependent Sequence

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On the probabilistic space (Ω, F, P) there is discussed a two-component narrow-sense stationary sequence $\{\xi_n, Y_n\}_{n\geq 1}$. The control sequence $\{\xi_n\}_{n\geq 1}$ is finite, stationary, ergodic Markov chain with one class of ergodic (is possible to be included the cyclic sub-classes). $\{Y_n\}_{n\geq 1}$ is a chain- dependent sequence [1].

In the certain conditions is shown the convergence of

$$S_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (Y_i - EY_1)$$

by variation of sum distribution to normal distribution

$$Var(P_{S_n}, \Phi_{R_0+T_\mu}) \underset{n\to\infty}{\longrightarrow} 0.$$

R_0 and T_μ covariation matrixes are defined by Y_1 value and chain characteristics. The rate of this convergence is determined to be

$$Var(P_{S_n}, \Phi_{R_0+T_\mu}) \leq c\, \frac{(\ln n)^{k/2}}{\sqrt{n}}\,.$$

The order of estimation is similar to

$$U_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \big(E(Y_i|\xi_i) - EY_1\big)$$

sum to Φ_{T_μ} distribution of convergence rate by Levi–Prokhorov metric [2]

$$L_\Pi(P_{U_n}, \Phi_{T_\mu}) \leq O\Big(\frac{\ln n}{\sqrt{n}}\Big).$$

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DRAFT

Stability of Spectral Characteristics of Boundary Value Problems for 2×2 Dirac Type Systems

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Boundary value problems associated in $L^2([0,1];\mathbb{C}^2)$ with the following 2×2 Dirac type equation

$$L_U(Q)y = -iB^{-1}y' + Q(x)y = \lambda y, \quad B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \ \ b_1 < 0 < b_2, \ \ y = col\,(y_1, y_2), \tag{1}$$

with a potential matrix $Q \in L^p([0,1];\mathbb{C}^{2\times 2})$, $p \geq 1$, and subject to the regular boundary conditions $Uy := \{U_1, U_2\}y = 0$ has been investigated in numerous papers. If $b_2 = -b_1 = 1$ this equation is equivalent to one dimensional Dirac equation.

In this talk we present recent results concerning the stability property under the perturbation $Q \to \widetilde{Q}$ of different spectral characteristics of the corresponding operator $L_U(Q)$. Our approach to the spectral stability relies on the existence of the triangular transformation operators for system (1) with $Q \in L^1$, which was established by us in [1]. The starting point of our investigation is the local Lipshitz property of the mapping $Q \to K_Q^{\pm}$, where $K_Q^{\pm}$ are the kernels of transformation operators for system (1):

$$\|K_Q^{\pm} - K_{\widetilde{Q}}^{\pm}\|_{X_{\infty,p}^2} + \|K_Q^{\pm} - K_{\widetilde{Q}}^{\pm}\|_{X_{1,p}^2} \leq C \cdot \|Q - \widetilde{Q}\|_p, \quad \|Q\|_p, \|\widetilde{Q}\|_p \leq r, \ \ p \in [1,\infty],$$

It is new even for $\widetilde{Q} = 0$. Here $X_{\infty,p}^2$, $X_{1,p}^2$ are the special Banach spaces naturally arising in such problems. We also obtain similar estimates for Fourier transforms of $K_Q^{\pm}$.

Assuming boundary conditions to be strictly regular, let $\Lambda_Q = \{\lambda_{Q,n}\}_{n\in\mathbb{Z}}$ be the spectrum of $L_U(Q)$. It happens that the mapping $Q \to \Lambda_Q - \Lambda_0$ sends $L^p([0,1];\mathbb{C}^{2\times 2})$ into the weighted space $\ell^p(\{(1+|n|)^{p-2}\})$ as well as into $\ell^{p'}$, $p' = p/(p-1)$. One of our main results is the Lipshitz property of this mapping on compact sets in $L^p([0,1];\mathbb{C}^{2\times 2})$, $p \in [1,2]$. The proof of the inclusion into the weighted space $\ell^p(\{(1+|n|)^{p-2}\})$ involves as an important ingredient inequality that generalizes classical Hardy–Littlewood inequality for Fourier coefficients. Similar result is proved for the eigenfunctions of $L_U(Q)$ using the deep Carleson–Hunt theorem for "maximal" Fourier transform. Certain modifications of these spectral stability results are also proved for balls in $L^p([0,1];\mathbb{C}^{2\times 2})$, $p \in (1,2]$.

The talk is based on joint preprint [2] with Mark M. Malamud.

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DRAFT

On Angular Limits of Cauchy Type Integral

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1. Definitions

Let $\Gamma : \zeta = \zeta(t)$, $t \in [a,b]$, be a rectifiable Jordan curve on the complex plane E_2 (which is identified with the set of finite complex numbers). The expression

$$\mathcal{K}_\Gamma(F,z) = \int\limits_\Gamma \frac{F(\zeta)}{\zeta - z}\, d\zeta, \quad z \notin \Gamma, \quad F \in L(\Gamma),$$

where $L(\Gamma)$ is a space of summable functions on Γ, is called a Cauchy type integral.

For each $z \in E_2$ and real value α we denote by $\mu_\Gamma(z,\alpha)$ the number of points where the half-line $\zeta = z + \rho e^{i\alpha}$, $\rho > 0$, $\alpha \in [0, 2\pi]$, meets the curve Γ and define (see [4]):

$$R_\Gamma(z) = \int\limits_0^{2\pi} \mu_\Gamma(z,\alpha)\, d\alpha.$$

2. Result

Theorem 2.1. *The angular boundary values of a Cauchy type integral $\mathcal{K}_\Gamma(F,z)$ exists for every $F \in L(\Gamma)$ almost everywhere on the curve Γ if and only if for Γ the following condition is satisfied:*

(K) *for almost every points ζ of the curve Γ we have*

$$R_\Gamma(\zeta) < +\infty.$$

See [5] for several proofs of this theorem and related results.

In [1] A.-P. Calderon proved that that a singular operator $\mathcal{K}_\Gamma(F,\zeta)$ is bounded in the space $L_2(\Gamma)$ if the curve Γ satisfies the Lipschitz condition. In [2] it was formulated without proving a theorem asserting that there exist angular boundary values of a Cauchy type integral $\mathcal{K}_\Gamma(F,z)$ almost everywhere on rectifiable Jordan curves for all summable functions on the curve Γ and that the operator $\mathcal{K}_\Gamma(F,\zeta)$, $\zeta \in \Gamma$, is bounded in spaces $L_p(\Gamma)$, $p > 1$, when Γ is a smooth curve. In [3] G. David proved that for the boundedness (continuity) of a singular operator $\mathcal{K}_\Gamma(F,\zeta)$ in spaces $L_p(\Gamma)$, $p > 1$, it is necessary and sufficient that the curve Γ satisfy the condition

$$\sup_{r>0} r^{-1} \operatorname{mes}\{\eta;\ \eta \in \Gamma,\ \ |\eta - \zeta| < r\} < C \tag{1}$$

for all $\zeta \in \Gamma$, where the constant C depends on the curve Γ.

In [4] it was given an example of a rectifiable Jordan curve for which $R_\Gamma(\zeta) = +\infty$ almost everywhere on the curve. In [5] J. Kral proved that the set of smooth curves for which $R_\Gamma(\zeta) < +\infty$ at one point at least is the set of the first category in the class of smooth curves.

Note that Lipschitz curves and the more smooth curves satisfy the condition (1).

DRAFT

It is not difficult to construct examples of curves for which smoothness is violated only at one point $\zeta \in \Gamma$ (at which $R_\Gamma(\zeta) = +\infty$) and on this curve find a function $F \in C(\Gamma)$, such that $\mathcal{K}_\Gamma(F,\zeta) \notin L(\Gamma)$.

Note that we have also

$$\operatorname{mes}\big\{\eta;\ \eta \in \Gamma,\ |\eta - \zeta| < r\big\} < C R_\Gamma(\zeta) r.$$

Summarizing the above argumentation and taking into account Theorem 2.1 and [4], [5], it turns out that the proofs of the works [1]–[3] have some gaps. As for smooth curves and Lipschitz curves and, moreover, for curves satisfying the condition (1), a singular integral may not exist at all for every $F \in C(\Gamma)$ and therefore it is meaningless to consider the boundedness of the operator $\mathcal{K}_\Gamma(F,\zeta)$ in spaces $L_p(\Gamma)$, $p > 1$.

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DRAFT

Evaluation of Profit from Illegal Copies of Intellectual Property

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Consider simple model of illegal distribution firm and denote number of copies with x. Suppose there is an ability of arrest with probability $\pi(x)$, in which case an owner of the firm must pay a fine F. Probability $\pi(x)$ is an increasing function of x. The firm's expected profit can be represented as follows [1]: $[1-\pi(x)]px-\pi(x)F$. The first term of the equation is the expected profit, and the second term is the expected cost. The manufacturer wants to maximize his expected profit, which means that he will equal the marginal (expected) profit to the marginal (expected) cost. Assume that there is no barrier for entry into production, so firms continue to enter production until expected profits are reduced to zero; it follows that:

$$[1-\pi(x)]px-\pi(x)F=0. \tag{1}$$

It is strange, but as it turns out, the level of production does not depend on the size of the fine F. Consider the problem of such maximization: $\max\limits_x [1-\pi(x)]px-\pi(x)F=0$ has the following condition of the first order:

$$[1-\pi(x)]p-px\pi'(x)F-\pi'(x)F=0.$$

After transformation of this equation we get:

$$\frac{p}{px+F}=\frac{\pi'(x)}{1-\pi(x)}\,. \tag{2}$$

The zero-profit condition (1) can be written as follows:

$$\frac{p}{px+F}=\frac{\pi(x)}{x}\,. \tag{3}$$

The combination of equations (2) and (3) gives us:

$$x=\frac{\pi(x)[1-\pi(x)]}{\pi'(x)}\,.$$

This shows, that equilibrium level of production does not depend on the size of the fine. These two conditions – profit maximization and zero profit from free entry – can be used to determine the optimal level x^* of the average distributor's "production". The form of $\pi(x)$ function finally is determined by firm operating level. Because the firm's operating level is x^*, from equation (3) we can define p^*:

$$p^*=\frac{\pi(x^*)}{[1-\pi(x^*)]}\,F.$$

This means that although x^* does not depend on the size of F, the price will still depend on F. In fact, it is directly proportional to F. The more fines the payer has to pay if its illegal action is exposed, the price of intellectual property will be higher.

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DRAFT

On the Extension of Calderón–Zygmund Inequality

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The goal of our talk is to present the results concerning the behavior of the solution of the Poisson equation

$$\Delta u(x) = f(x) \;\text{ a.e. }\; x \in \Omega,$$

where Ω is a bounded Lipschitz domain in R^n, $n \geq 3$ and f belongs to the weighted grand Lorentz space. The latter one is defined as the set of all measurable functions f and weight w defined on Ω, for which the norm

$$\|f\|_{L_w^{p),s,\theta}} = \sup_{0<\varepsilon<p-1} \varepsilon^{\frac{\theta}{p-1}} \|f\|_{L_w^{p-\varepsilon,s}}$$

is finite, where $1 < p < \infty$, $1 \leq s < \infty$.

DRAFT

On Gaussian Random Variables in Stochastic Scheduling Problems

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In stochastic scheduling we offer to use Gaussian and *symmetrically truncated* independent Gaussian random variables.

We plan to discuss their role in finding the mean optimal and almost surely optimal *makespan* in one-machine case.

Acknowledgement

The authors were supported by the Shota Rustaveli National Science Foundation Grant # DI-18-1429: *"Application of Probabilistic Methods in Discrete Optimization and Scheduling Problems"*.

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DRAFT

Weighted Extrapolation and Boundedness in Generalized Grand Morrey Spaces, and Applications to Partial Differential Equations

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Weighted extrapolation and one-weight inequalities for operators of Harmonic Analysis under the Muckenhoupt condition on weights are derived in generalized grand Morrey spaces $\|f\|_{L_w^{p),r,\varphi(\cdot)}(X)}$. The operators and spaces under consideration are defined on quasi-metric measure spaces. One-weight norm estimates for commutators of singular integrals defined on domains in $\mathbb{R}^n$ are applied to study regularity properties of solutions of second order partial differential equations with discontinuous coefficients in generalized grand Morrey spaces under the Muckenhoupt condition on weights. For these results in the classical weighted grand Morrey spaces we refer to [1].

The investigation was carried out jointly with V. Kokilashvili and M. A. Ragusa.

Acknowledgment

The author was supported by the Shota Rustaveli National Science Foundation Grant (Project # DI-18-118).

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DRAFT

On Teaching the Composition of Functions

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The paper deals with problems of teaching the composition of functions. The main attention is paid to general properties of composition of functions, establishing the range of definition and set of values, finding the inverse function of composition of functions, graphic methods of solving some equations with iteration of functions. We think that the examples given are interesting and important for studying this subject and their solution shows many considerable properties of composition of functions.

DRAFT

Brief Preliminary Overview of the Aims and Methods of the PhD Thesis "Formalism and Applications of Georgian Language Processing by Machine Learning Methods"

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Within framework of doctoral program "Informatics" of Georgian Technical University doctoral thesis *"Formalism and Applications of Georgian Language Processing by Machine Learning Methods"* (Doctoral Student – B. Mikaberidze, Supervisor – K. Pkhakadze, Director of the Scientific-Educational Center for Georgian Language Technology of the Georgian technical University) was launched in 2020. The doctoral thesis is mainly based on the Pkhakadze's Logical Grammar of the Georgian Language [1], which were developed within framework of FR/362/4-105/12 project *"Foundations of Logical Grammar of Georgian Language and Its Application in Information Technology"* (Supervisor K. Pkhakadze, project were funded by Shota Rustaveli Georgian National Scientific Foundation) and, also, it is mainly based on the Georgian Intellectual Web-Corpus, which, in turn, were developed within the framework of AR/122/4-105/14 project *"One More Step Towards Georgian Talking Self-Developing Intellectual Corpus"* (Supervisor K. Pkhakadze, project were funded by Shota Rustaveli Georgian National Scientific Foundation) [3]. In addition, the doctoral thesis is aimed to develop the formalism of processing Georgian language by machine learning methods and highlight the advantages of the applied aspects of this formalism, that makes this study closely related to the long-term project *"Technological Alphabet of the Georgian Language"* [2], which was launched in 2012 and which is aimed to the complete technology support of the Georgian language. Thus, at the presentation, we will briefly review the basic visions, aims, and methods of the doctoral thesis.

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DRAFT

Neumann Type Boundary-Transmission Problem for Composed Elastic Structures

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We investigate a boundary-transmission problem for a two-layered elastic anisotropic structure when in different adjacent components of the composed body different refined models of elasticity theory are considered. In particular, we analyse the case when we have the generalized thermo-electro-magneto elasticity model (GTEME model) in one region of the composed body and the generalized thermo-elasticity model (GTE model) in another adjacent region. Both models are associated with Green–Lindsay's model [1], [2].

This type of mechanical problem mathematically is described by systems of partial differential equations with appropriate boundary-transmission conditions. In the GTEME model part we have six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function), while in the GTE model part we have four dimensional unknown physical field (three components of the displacement vector and temperature distribution function).

The diversity in dimensions of the interacting physical fields complicates mathematical formulation and analysis of the corresponding boundary-transmission problems.

On the interface surface of the two-layered structure we consider the transmission conditions for the mechanical and thermal characteristics together with the Dirichlet type boundary conditions for the electric and magnetic potentials, while on the exterior boundary of the layered body the Neumann type conditions are prescribed.

Using the potential method and the theory of pseudodifferential equations we investigate existence and uniqueness of solutions to the boundary-transmission problem in appropriate Sobolev–Slobodetski function spaces.

Acknowledgement

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DRAFT

On Lyapunov Dimension of Nonlinear Dynamic Systems

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Suppose we have generalized Henon map of 6th degree:

$$\begin{aligned} &x \longmapsto 1 - a_1x^2 - a_2x^4 - a_3x^6 + by,\\ &y \longmapsto bx, \end{aligned} \tag{1}$$

where $a_1, a_2, a_3 > 0$, $b \in (0, 1)$.

It is proved that under certain restrictions Lyapunov dimension of dynamical system (1), over the bounded invariant set K involving stationary points is equal to

$$dim_L K = 1 + \frac{1}{1 - \ln b^2/\ln a_1(x_-)}\,, \tag{2}$$

where $a_1(x_-)$ corresponds to singular value of the map F at stationary point x_-.

Let us remark, that formula (2) for the generalized Henon map of degree 4, under certain restrictions was obtained by G. Rakviashvili earlier (unpublished), and for Henon map of degree 2 was proved in [2].

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DRAFT

Bounded Invertibility of the Sturm–Liouville Operator with Negative Parameter

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The Sturm–Liouville operator $L = -\frac{d^2}{dx^2} + q(x)$, $x \in R$ is one of the main operators of modern quantum mechanics and theoretical physics. It is known that many fundamental results have been obtained for the Sturm–Liouville operator L. Among them, for example, are questions about the existence of a resolvent, separability (coercive estimate), various weight estimates, estimates of intermediate derivatives of functions from the domain of definition of an operator, estimates of eigenvalues and singular numbers (s-numbers). At present, there are various generalizations of the above results for elliptic operators.

For general differential operators, the solution of such problem as a whole is far from complete. In particular, as far as we know, there was no result until now showing the existence of the resolvent and coercivity, as well as the discreteness of the spectrum of a hyperbolic type operator in an infinite domain with increasing and oscillating coefficients.

It is easy to see that the study of some classes of differential operators of hyperbolic type defined in the space $L_2(R^2)$, using the Fourier method, can be reduced to the study of the Sturm–Liouville operator with a negative parameter:

$$L_t = -\frac{d^2}{dx^2} + \big(- t^2 + itb(x) + q(x)\big),$$

where t is a parameter $(-\infty < t < \infty)$, $i^2 = -1$.

Hence, it is easy to see that we get $-t^2 \to -\infty$ when $|t| \to \infty$ for the operator L_t. Consequently, a completely different situation arises here compared to the Sturm–Liouville operator

$$L = -\frac{d^2}{dx^2} + q(x),$$

and in particular, the methods worked out for the Sturm–Liouville operator L turn out to be little adapted when studying the Sturm–Liouville operator L_t with a negative parameter. All these issues indicate the relevance and novelty of this research.

This research is devoted to study the existence of the resolvent of the Sturm–Liouville operator with a negative parameter.

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DRAFT

Spectral Analysis of One Three-Dimensional Biharmonic Operator

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Consider the differential expression

$$l(u) = \Delta^2 u + \sum_{k=1}^{3} \alpha_k \cdot \delta(x - x^{(k)}) u,$$

where Δ – three-dimensional Laplace operator, $\delta(x)$ – Dirac function, $x^{(k)} \in \mathbb{R}^3$, $k = 1, 2, 3$ – are arbitrary fixed points, $\alpha_k \in \mathbb{R} = (-\infty; +\infty)$, $k = 1, 2, 3$.

In $L_2(\mathbb{R}^3)$ we define the operator

$$A = \Delta^2 + \sum_{k=1}^{3} \alpha_k \cdot \delta(x - x^{(k)})$$

with domain $D(A) = \{u \in W_2^2(\mathbb{R}^3) : \ l(u) \in L_2(\mathbb{R}^3)\}$.

In [1], the self-adjointness of the operator A in the space $L_2(\mathbb{R}^3)$ was proved.

Theorem. *The largest number of different negative eigenvalues of the operator A is three. The number of eigenvalues is equal to the number of negative numbers α_1, α_2, α_3. In particular, if $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $\alpha_3 \geq 0$, then the operator A has no eigenvalues.*

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DRAFT

On Clark–Ocone-Type Formulas for Nonsmooth Functionals

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As is known, the famous Clark formula (1970) is the inverse statement of the following important property of the Ito stochastic integral: if ψ is square-integrable $\Im_t^W := \sigma\{W_s:\ 0 \le s \le t\}$-adapted random process, then process $M_t = \int\limits_0^t \psi(s,\omega)\,dW_s(\omega)$ is a martingale with respect to the filtration $\{\Im_t^W\}_{t\ge 0}$. When the functional F belongs to the Hilbert space $D_{2,1}$ (where $D_{2,1}$ denotes the space of square integrable functionals having the first order stochastic derivative) Ocone [2] proved that the integrand in the Clark representation is $E[D_tF \mid \Im_t^W]$ (where D_tF denotes the stochastic (Malliavin) derivative of F). Subsequently, the formula obtained by Ocone was called the Clark–Ocone formula.

It has turned out that the requirement of smoothness of functional can be weakened by the requirement of smoothness only of its conditional mathematical expectation. We (with Prof. O. Glonti, see [1]) generalized the Clark–Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method of finding of integrand. Here we consider functionals that did not satisfy even these weakened conditions. Such functionals include, for example, the Lebesgue means of stochastically nonsmooth square integrable processes.

Theorem. *If $f(t,x)$ is a deterministic, measurable, bounded function then the following stochastic integral representation is fulfilled*

$$\int\limits_0^T f(t,W_t)\,dt = \int\limits_0^T E[f(t,W_t)]\,dt + \int\limits_0^T \bigg\{ \int\limits_t^T \bigg[\int\limits_{-\infty}^{\infty} f(s,y)\,\frac{y-W_t}{s-t}\,\varphi\Big(\frac{y-W_t}{\sqrt{s-t}}\Big)\,dy \bigg]\,ds \bigg\}\,dW_t,$$

where φ is the density function of the standard normal distribution.

Remark. It should be noted that the result of theorem is especially interesting for stochastically non-smooth $f(t,W_t)$, although it is also useful for smooth $f(t,W_t)$.

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DRAFT

Generalized Multi-Field Mixed Dynamical Problems for Composed Elastic Structures

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We investigate multi-field mixed dynamical problems for complex multi-layer elastic anisotropic structures containing interfacial cracks when in different adjacent components of the composed body different refined models of elasticity theory are considered. In particular, we analyse the case when we have the generalized thermo-electro-magneto elasticity model (GTEME model) in one region of the composed body and the generalized thermo-elasticity model (GTE model) in another adjacent region. Both models are associated with Green–Lindsay's model [1], [2]. This type of mechanical problem mathematically is described by systems of partial differential equations with appropriate boundary-transmission and initial conditions. In the GTEME model part we have six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function), while in the GTE model part we have four dimensional unknown physical field (three components of the displacement vector and temperature distribution function). The diversity in dimensions of the interacting physical fields complicates mathematical formulation and analysis of the corresponding initial-boundary-transmission problems. We apply the Laplace transform technique, the potential method and the theory of pseudodifferential equations to prove uniqueness and existence theorems of solutions to different type basic and mixed initial-boundary-transmission problems in appropriate Sobolev spaces. We analyse the smoothness properties of solutions and establish asymptotic behaviour of the first derivatives near the exceptional curves (the crack edges and curves where different boundary conditions collide). It is shown that smoothness of solutions and stress singularity exponents at the exceptional curves essentially depend on the material parameters of the composed body. Moreover, we describe an efficient algorithm for evaluating the stress singularity exponents.

This is a joint work with Tengiz Buchukuri and Otar Chkadua.

Acknowledgment

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FR-18-126).

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DRAFT

gs-Essential Submodules

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In this work, every ring has unity and every module is unital left module. Let M be an R-module and N be a submodule of M. If $L = 0$ for every $L \ll_g M$ with $N \cap L = 0$, then N is called a g-small essential (briefly, gs-essential) submodule of M and denoted by $N \trianglelefteq_{gs} M$. In this work, some properties of these submodules are investigated.

Key words. essential submodules, small submodules, g-small submodules, s-essential submodules.

2020 Mathematics Subject Classification. 16D10, 16D80.

Results

Proposition 1. *Let M be an R-module. Then every essential submodule of M is gs-essential submodule of M.*

Proposition 2. *Let M be an R-module. Then every gs-essential submodule of M is s-essential submodule of M.*

Lemma 1. *Let $f : M \to N$ be an R-module homomorphism. If $K \trianglelefteq_{gs} N$, then $f^{-1}(K) \trianglelefteq_{gs} M$.*

Proposition 3. Let M be an R-module and $K \leq N \leq M$. If $N/K \trianglelefteq_{gs} M/K$, then $N \trianglelefteq_{gs} M$.

Lemma 2. Let M be an R-module, $N_1 \leq L_1 \leq M$ and $N_2 \leq L_2 \leq M$. If $N_1 \trianglelefteq_{gs} L_1$ and $N_2 \trianglelefteq_{gs} L_2$, then $N_1 \cap N_2 \trianglelefteq_{gs} L_1 \cap L_2$.

Proposition 4. *Let M be an R-module and $N_1, N_2 \leq M$. If $N_1 \trianglelefteq_{gs} M$ and $N_2 \trianglelefteq_{gs} M$, then $N_1 \cap N_2 \trianglelefteq_{gs} M$.*

Proposition 5. *Let M be an R-module and $N, T \leq M$. If $N \trianglelefteq_{gs} M$, then $N \cap T \trianglelefteq_{gs} T$.*

Proposition 6. *Let M be an R-module and $K \leq L \leq M$. If $K \trianglelefteq_{gs} M$, then $K \trianglelefteq_{gs} L \trianglelefteq_{gs} M$.*

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DRAFT

On $\oplus - e$-Supplemented Modules

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In this work, every ring has unity and every module is unital left module. Let M be an R-module. If every essential submodule of M has a supplement that is a direct summand of M, then M is called a $\oplus - e$-supplemented module (see also [2]). In this work, some properties of these modules are investigated.

Key words. essential submodules, small submodules, supplemented modules, $\oplus$-supplemented modules.

2010 Mathematics Subject Classification: 16D10, 16D80.

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Results

Lemma. *Let $M = M_1 \oplus M_2$. If M_1 and M_2 are $\oplus - e$-supplemented, then M is also $\oplus - e$-supplemented.*

Corollary 1. *The finite direct sum of $\oplus - e$-supplemented modules is $\oplus - e$-supplemented.*

Corollary 2. *Let M be a $\oplus - e$-supplemented module. Then $M^{(\Lambda)}$ is $\oplus - e$-supplemented for every finite index set Λ.*

Proposition. *Let M be a $\oplus - e$-supplemented module. If every e-supplement submodule in M is a direct summand of M, then every direct summand of M is $\oplus - e$-supplemented.*

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DRAFT

A Concrete Quantum Channel

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Construction and testing of pre-conditioners of Toeplitz/block Toeplitz matrices have been done in [6] using Korovkin's classic theorem of approximations. Later the map implementing pre-conditioner discussed in [4] was observed to be a Completely Positive Map and strucure lead to an abstract formulation and Korovkin-type theorems in a non commutative setting. Now, interestingly enough [7, Lemma 2.1, p. 311] the properties listed are of an abstract quantum channel. In this short lecture this view point is discussed by computing related quantities such Kraus representation, Channel capacity and fidelity of the quantum channel induced by the above 'Pre Conditioner.

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DRAFT

Preliminary Overview of the Aims and Methods of the Doctoral Thesis "Georgian-Mathematical Automatic Multilingual Semantic Translator"

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Within the framework of the doctoral program "Informatics" of the Georgian Technical University, doctoral thesis "Georgian-Mathematical Automatic Multilingual Semantic Translator" (Doctoral Student – N. Okroshiashvili, Supervisor – K. Pkhakadze, Director of the Scientific-Educational Center for Georgian Language Technology of the Georgian technical University) was launched in 2019. The doctoral thesis is mainly based on the Pkhkadze's translation methodology developed within the framework of the two-year FR/362/4-105/12 project "Foundations of Logical Grammar of Georgian Language and Its Application in Information Technology" (project supervisor – K. Pkhakadze, project was funded by Shota Rustaveli Georgian National Scientific Foundation) [1], [3], which is a sub-project of the long-term project "Technological Alphabet of Georgian Language" [2] of the Center for the Georgian Language Technology of the Georgian Technical University. At the same time, the doctoral thesis is aimed to build the first experimental versions of the Georgian-mathematical multilingual semantic translators, that makes this research as an important part of the long-term project "Georgian Language Technological Alphabet", which was launched in 2012 and which is aimed to the complete technology support of the Georgian language [4]. Thus, considering all of above, at the presentation the aims and methods of doctoral thesis will be reviewed in detailed.

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DRAFT

sQ_1-Degrees of Computably Enumerable Sets

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We say that a set A is sQ_1-reducible to a set B (in symbols: $A \leqslant_{sQ_1} B$), if there exist computable functions f and g such that, the following three conditions are satisfied:

(i) $(\forall\, x)(x \in A \Longleftrightarrow W_{f(x)} \subseteq B)$,

(ii) $(\forall\, x)(\forall\, y)(y \in W_{f(x)} \Longrightarrow y \leq g(x))$,

(iii) $(\forall\, x)(\forall\, y)(x \neq y \Longrightarrow W_{f(x)} \cap W_{f(y)} = \varnothing)$.

This relation generates the sQ_1-degrees.

Our notation and terminology are standard, and can be found e.g., in [1], [2].

In this talk we will present the following results.

Theorem 1. *Given any c.e. sQ_1-degree a such that $o_{sQ_1} <_{sQ_1} a <_{sQ_1} o'_{sQ_1}$, there exist infinitely many pairwise sQ-incomparable c.e. sQ-degrees $\{c_i\}_{i\in\omega}$ such that*

$$(\forall\, i)\ \big(A <_{sQ_1} C_i <_{sQ_1} B\big).$$

Theorem 2. *If A is a maximal set and B is a non-maximal hyperhypersimple set, then either $A|_{sQ_1}B$, or there exist a non-maximal hyperhypersimple set C and a maximal set D such that*

$$A <_{sQ_1} D <_{sQ_1} C <_{sQ_1} B.$$

Corollary. *There exist infinite collections of sQ_1-degrees $\{a_i\}_{i\in\omega}$ and $\{b_i\}_{i\in\omega}$ such that for every i, j*

(1) $a_i <_{sQ_1} a_{i+1}$, $b_{j+1} <_{sQ_1} b_j$, *and* $a_i <_{sQ_1} b_j$;

(2) *every c.e. set in a_i is a maximal set;*

(3) *every c.e. set in b_j is a non-maximal hyperhypersimple set.*

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DRAFT

On Sjölin–Soria–Antonov Type Extrapolation for Locally Compact Groups and a.e. Convergence of Vilenkin–Fourier Series

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Sjölin–Soria–Antonov type extrapolation theorem for locally compact σ-compact non-discrete Hausdorff groups is proved. Applying this result it is shown that the Fourier series with respect to the Vilenkin orthonormal systems on the Vilenkin groups of bounded type converge almost everywhere for functions from the class $L\log^+ L\log^+\log^+\log^+ L$.

The results of the talk have been published in [1].

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Maximal Regularity of Second-Order Differential Equations and its Applications

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We study the following differential equation:

$$-s(x)(\rho(x)y')' + r(x)y' + q(x)y = f(x), \tag{1}$$

where $x \in R = (-\infty, +\infty)$ and $f \in L_p = L_p(R)$, $1 < p < +\infty$. We assume that s, ρ, and r are continuously differentiable functions, and q is a continuous function.

Let $ly = -s(\rho y')' + ry' + qy$ is the operator with $D(l) = C_0^{(2)}(R)$. By L we denote the closure of l in L_p. The function $y \in D(L)$ such that $Ly = f$ is called the solution of equation (1).

The purpose of this work is to find some conditions for the coefficients s, ρ, r and q such that for any $f \in L_p$ there exists a unique solution y of the equation (1) and the following estimate holds:

$$\big\| - s(x)(\rho(x)y')'\big\|_p + \big\|ry'\|_p + \big\|(1 + |q|)y\big\|_p \leq C\|f\|_p, \tag{1}$$

where $\| \cdot \|_p$ is the norm in L_p.

Here we assume that s and ρ are positive, and they can go to zero at infinity. The function r grows rapidly and does not obey the function q. These functions can fluctuate quickly. Further, using inequality (2), we give criteria for the compactness of the resolvent L^{-1} and the operator $m(x)\frac{d}{dx}L^{-1}$, where m is an unbounded function. We have obtained some important estimates for the Kolmogorov widths of the sets of solutions and their derivatives.

The equation (1) and its multidimensional generalizations with unbounded coefficients have used in stochastic analysis, biology and financial mathematics (see, for example, [1], [2]). For this reason, interest in these equations has considerably grown in recent years. A number of researches were devoted to the case that the coefficient r are controlled by q. Without the dominating potential q, the case that r growth at most as $|x|\ln(1 + |x|)$ were considered by Lunardi and Vespri (1997), Metafune (2001), Prüss, Rhandi and Schnaubelt (2006), Hieber and others (2009).

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DRAFT

On the Approximate Solution Using the Collocation Method of the Singular Integral Equation Containing the Fixed Singularity

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Anti-plane problems of the elasticity theory, composed for the orthotropic plane, weakened by cracks, when a crack reaches the interface, are reduced to the following integral equation containing a fixed singularity to the unknown characteristic function of the crack disclosure

$$\int_0^1 \Big(\frac{1}{t-x} - \frac{a}{t+x}\Big)\rho(t)\,\mathrm{d}t = 2\pi f(x), \quad x \in [0,1], \tag{1}$$

where the unknown function $\rho(t) \in H^*([0,1])$, the parameter dependent on the elasticity constants of the materials $a \in [0,1]$, the right-hand side $f(x) \in H_\mu[0,1]$, $0 < \mu \le 1$ (see: [2], [3]). The range of singularity of the solution at the point $t = 0$ is also dependent on the elasticity constants of the materials $\alpha \in (0,1)$. However at the point, $t = 1$ The range of singularity of the solution is constant, namely, $\beta = \frac{1}{2}$. The equation (1) is solved by the collocation method, in particular, the method of discrete singularities [1]. The corresponding algorithms are designed and implemented. The results of numerical computations are presented.

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The Exactness of an Algorithm for a Nonlinear Static Beam Equation

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We consider a Kirchhoff type nonlinear beam equation [1]–[3]

$$u^{\mathrm{IV}}(x)-a\bigg(\int_0^L(u'(\xi))^2\,d\xi\bigg)u''(x)=f(x),\ \ 0<x<L, \tag{1}$$

with the conditions

$$u(0)=u(L)=0,\quad u''(0)=u''(L)=0. \tag{2}$$

Here $a(\lambda)\geq const>0$, $0\leq\lambda<\infty$, and $f(x)$, $0<x<L$, are the known functions, $u(x)$, $0\leq x\leq L$, is the function we want to define and L is some constant. An approximate solution of problem (1), (2) is written in the form of a finite series

$$u_n(x)=\sum_{i=1}^n u_{ni}\sin\frac{i\pi x}{L}\,,$$

where the coefficients u_{ni} are defined by the Galerkin method from the system whose vector form is as follows

$$\boldsymbol{\varphi}(\boldsymbol{u}_n)=\mathbf{0}. \tag{3}$$

Here $\boldsymbol{\varphi}(\boldsymbol{u}_n)=(\varphi_i(\boldsymbol{u}_n))_{i=1}^n$, $\boldsymbol{u}_n=(u_{ni})_{i=1}^n$, $\mathbf{0}$ is the n-dimensional zero vector,

$$\varphi_i(\boldsymbol{u}_n)=\bigg[\Big(\frac{i\pi}{L}\Big)^4+\Big(\frac{i\pi}{L}\Big)^2a\Big(\frac{L}{2}\sum_{j=1}^n\Big(\frac{j\pi}{L}\Big)^2u_{nj}^2\Big)\bigg]u_{ni}-\frac{2}{L}\int_0^L f(x)\sin\frac{i\pi x}{L}\,dx,$$

$i=1,2,\dots,n$. To solve nonlinear system (3) we apply the Newton iteration process $\boldsymbol{u}_{n,k+1}=\boldsymbol{u}_{n,k}-J^{-1}(\boldsymbol{u}_{n,k})\,\boldsymbol{\varphi}(\boldsymbol{u}_{n,k})$, $k=0,1,\dots$, where $\boldsymbol{u}_{n,k}=(u_{ni,k})_{i=1}^n$ is k-th iteration approximation of $\boldsymbol{u}_n$ and $J(\boldsymbol{u}_{n,k})=(\frac{\partial\varphi_i(\boldsymbol{u}_{n,k})}{\partial u_{nj,k}})_{i,j=1}^n$ is the Jacobi matrix.

A theorem on the total error of the algorithm is proved.

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DRAFT

Logic, Language, Artificial Intelligence and Objectives of Protecting the State Languages of Georgia from Danger of Digital Extinction

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At the presentation we will briefly present the research aims of the interdisciplinary scientific field of logic, language and artificial intelligence [2], [3]. At the same time, on the basis of the here mentioned, we will prove the direct connection of this interdisciplinary scientific field with problems of creation the technological alphabets of different natural languages, in other words, with problems of creation the computer systems, which will have almost complete knowledge of the different natural languages [1].

Taking into account the necessity of creation of the technological alphabets of the Georgian and Abkhazian languages equipped with the translation skills, at the presentation we will prove the necessity of rapid development of the interdisciplinary scientific field of logic, language and artificial intelligence in Georgia and, also, we will prove that for this purpose there is a obligatory needing of establishment "Research Institute for Cultural Defense and Technological Development of Georgian State Languages" with this conditional name [4]. – All this is conditioned with aims and obligations of the Georgian state to protect Georgian and Abkhazian languages from danger of digital extinction [1], [4].

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Computer Modeling of Political Conflicts in Case of Variable Demographic Factors of Sides and Variable Parameters of Models

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In this work are given the computer modeling of four nonlinear mathematical models of conflict resolution through economic cooperation between two politically opposed sides: conflict resolution model in case of the process of economic cooperation free from political pressure; conflict resolution model in case of political pressure on the both sides interfere with the process of economic cooperation; conflict resolution model in case of the government of both sides encourage the process of economic cooperation; conflict resolution model in case of the government of the first side interferes, and government of the second promotes cooperation [1]–[3].

Computer modeling is performed in Matlab software environment for exponential and trigonometric functions of variables coefficients, when the derivatives of the coefficients are significant functions. Computer simulations have established and graphically shown the relationships between the coefficients of the variable demographic factor, aggression, cooperation, coercion to aggression and cooperation, as well as their factor coefficients and the conflict resolution period at different interval for model consideration.

In all four cases considered, the initial conditions are taken as different numbers of the citizens of the first and second sides at the initial moment of time, as well as the different numbers of the citizens of the sides who are wishing or already being in economic cooperation.

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DRAFT

On the Duality Principle on Polish Groups

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Let P be sentence formulated only by using the notion of Haar null sets, meagre sets and purely set-theoretical notions. We say that the duality principle between the measure and category is valid with respect to the sentence P if the sentence P is equivalent to the sentence P^* obtained from the sentence P by interchanging in it the notions of the above small sets.

Theorem. *Let P be a sentence defined by:*

> *For every two Polish groups G_1 and G_2 and for every Haar null set $Y \subset G_1$ we have $(\forall X)(X \subseteq G_2 \Longrightarrow Y \times X)$ is Haar null in $G_1 \times G_2$.*

Then the duality principle between the measure and category is valid with respect to the sentence P.

Acknowledgment

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Extended Eigenoperators of the Differentiation Operator: Hypercyclicty, Supercyclicity, Hypercyclic and Supercyclic Subspaces

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Let D be the differentiation operator on the sapce of entire functions $H(\mathbb{D})$ endowed with the compact-open topology. A continuous linear operator X is and extended eigenoperator of D if $X \neq 0$ and $DX = \lambda XD$. We characterize when X is hypercyclic, supercyclic and it has a hypercyclic (supercyclic) subspace.

Joint work with I. Bensaid, M. GonzÁlez, and F. Romero.

On the Number of Integer Non-Negative Solutions of Some Diophantine Equations

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Applying the elements of combinatorics and properties of the Kronecker function, the number $P(b)$ of integer non-negative solutions of a linear Diophantine equation $\sum\limits_{i=1}^{n} a_i x_i = b$ with natural coefficients is represented via $P(r), P(r+M), \dots, P(r+(s-1)M)$ in the case where $s \neq 0$. Here M is the least common multiple of the numbers $a_1, a_2, \dots, a_n$; r is the remainder of b modulo M and

$$s = \left[n - \frac{\sum\limits_{i=1}^{n} a_i + r}{M} \right].$$

Note that s takes quite small values in some particular cases. For that reason in these cases the finding $P(b)$ by our method is simpler than finding it by the well-known standard methods.

Also, the recurrent formulas are derived to calculate $P(b)$, for any non-negative integer b, which, in particular, are used in finding $P(r), P(r+M), \dots, P(r+(s-1)M)$. For the case where $s = 0$ and $a_1, a_2, \dots, a_n$ are coprime, the explicit formula for $P(b)$ is proved:

$$P(b) = \frac{M^{n-1}}{a_1 a_2 \cdots a_n}\, C^{n-1}_{[\frac{b}{M}]+n-1}.$$

Moreover, the problem of finding the number $P'(b)$ of integer non-negative solutions of a non-linear Diophantine equation $\sum\limits_{i=1}^{m} x_i^n = b$ with even degree $n = 2^k l$, where $k, l, b \in \mathbf{N}$ and $m \geq 2$ is considered.

It is proved that if $m < 2^{k+1}$ and $s \in \mathbf{N}$, then

$$P'\Big(\sum_{i=1}^{m} x_i^n = b\Big) = P'\Big(\sum_{i=1}^{m} x_i^n = b \cdot (2^s)^n\Big).$$

This formula in particular implies that if there is at least one natural number which can not be written in the form $\sum\limits_{i=1}^{m} x_i^n$ with $m < 2^{k+1}$, then there are infinitely many naturals with this property. Another consequence of the above-mentioned formula is that if $m < 2^{k+1}$, then the equation $\sum\limits_{i=1}^{m} x_i^n = (2^s)^n$ $(s \in \mathbf{N})$ does not have a solution $(x_1, x_2, \dots, x_m)$ with natural $x_1, x_2, \dots, x_m$.

It is also proved that

$$P'\Big(\sum_{i=1}^{m} x_i^n = b\Big) = 0$$

if $m < 2^{k+2} - 1$, $k \geqslant 2$ and $r > m$, where r is the remainder of b modulo 2^{k+2}. This implies that $G(2^k) \geqslant 4 \cdot 2^k - 1$ for $k \geqslant 2$, where $G(2^k)$ denotes the smallest number m, for which almost all natural numbers (i.e. all natural numbers, except for a finite number of naturals) can be written in the form $\sum\limits_{i=1}^{m} x_i^{2^k}$.

DRAFT

The Importance of Basic Algorithmic Structures in the Process of Teaching Algorithms

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The presented paper refers to the pedagogical and methodological experience accumulated over the years with students in the field of teaching informatics, in particular algorithms. As it is known, the teaching of informatics includes information, algorithm, and computer. All three of these components are important and require special attention.

In this article, we will discuss some issues of teaching algorithms. In particular, teaching the basics of algorithmization to students who have not yet learned to program. By teaching them the basics of algorithmization, they should be given the basis for programming.

In this regard, the student needs to develop an algorithmic approach to solving the posed problem. Students must realize that the algorithm is a language understandable to man, subject to certain principles. The student should be able to determine the way to solve the task and, in general, write the algorithm in such a way that it is easy for the user to program it, i.e. to write in any programming language.

For novice programmers, the essence of the algorithm is most evident in the algorithm recorded through the block diagram. Block diagrams can be programmed with less effort in this or that programming language. In this regard, special attention is paid to giving a structural face to the algorithm.

The paper provides relevant examples that illustrate the importance of a structural approach to building an algorithm. Particular attention is paid to the methods that are necessary to build a proper algorithm. It would be more convenient to translate any such algorithms into a programming language.

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DRAFT

On the Generalized Lifeguard Problem

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To solve this main problem we use reconcept of the T-rescue range of lifeguard which consists of the points H_+ in which lifeguard can reach from $A(0, a)$ within time T. Let us denote by $B(A, T)$ the boundary of $R(A, T)$ which consists of the points which lifeguard can reach from $A(0, -a)$ along a minimal time path exactly in time T. A closely related notion is the T-rescuer's area of swimmer, $L_T(B)$, consisting of all points in H_- from which point $B(0, b)$ can be reached within time T. obviously, this set is the union of t-levels of minimal rescue time for $0 < t < T$.

Knowledge of these sets makes the situation more visual and may be used in planning safe beach structures. To determine them we use the analogy between the optimal path of lifeguard and refraction of light, which enables us to use the concept of refraction wavefront from geometric optics. Since both settings are based on the refraction formula the following results and their proofs are mathematically rigorous. The crucial observation is that the boundary $B(a, T)$ of $R(a, T)$ coincides with refraction wavefront with coefficient which is known to be a straight line segment.

DRAFT

About One Method, Providing Necessary and Sufficient Conditions for Equivalence of Linearly Connected Two Gaussian Measures in the Hilbert Space

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The paper considers two Gaussian processes that are solutions to different linear evolutionary differential equations in Hilbert space, excited by the same Gaussian process. In terms of the linear operators of the equations themselves and the characteristics of a given exciting Gaussian process, the Hilbert–Schmidt operator is constructed in an explicit form, which is the main criterion of Yu. L. Daletsky [1], which provides a necessary and sufficient condition for the equivalence of probability measures generated by Gaussian processes that are solutions of the considered linear evolutionary differential equations, since the exciting process itself is Gaussian, and the equation is linear.

Therefore, the measures generated by the solutions of these equations are also Gaussian. Applying now to these two Gaussian measures the theorem of Yu. L. Daletsky for this case, using our calculated Hilbert–Schmidt operator, we prove a theorem on the equivalence of Gaussian measures generated by solutions of the linear evolutionary differential equations under consideration and in explicit form in terms of the known coefficients of differential equations and in explicit form and in terms of known coefficients of differential equations the density Radon–Nikodim.

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DRAFT

The Adhesive Contact Problems for a Piecewise-Homogeneous Plate

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A piecewise-homogeneous elastic orthotropic plate, reinforced with a finite inclusion $(E_1, \nu_1, h_1(x))$ of the wedge-shaped, which meets the interface at a right angle and is loaded with tangential $\tau_0(x)$ and normal $p_0(x)$ forces is considered. The contact between the plate and inclusion is realized by a thin glue layer with width h_0 and Lame's constants λ, μ_0. The contact conditions has the form

$$u_1(x) - u_2(x,0) = k_0\tau(x), \quad v_1(x) - v_2(x,0) = -m_0 p(x), \ \ 0 < x < 1,$$

where $u_2(x,y)$, $v_2(x,y)$ are displacement components of the plate points, $u_1(x)$, $v_1(x)$ displacements of the inclusion points along the contact line. $k_0 := h_0/\mu_0$, $m_0 := h_0/(\lambda_0 + 2\mu_0)$. According to the equilibrium equation of inclusion elements and Hooke's law we have:

$$\frac{du_1(x)}{dx} = \frac{1}{E(x)} \int\limits_0^x \big[\tau(t) - \tau_0(t)\big]\,dt, \quad E(x) = \frac{E_1(x)h_1(x)}{1-\nu_1^2}\,,$$

$$\frac{d^2}{dx^2}\, D(x)\, \frac{d^2 v_1(x)}{dx^2} = p_0(x) - p(x), \quad D(x) = \frac{E_1(x)h_1^3(x)}{1-\nu_1^2}\,, \ \ 0 < x < 1$$

and the equilibrium condition of the inclusion has the form

$$\int\limits_0^1 \big[\tau(t) - \tau_0(t)\big]\,dt = 0, \quad \int\limits_0^1 \big[p(t) - p_0(t)\big]\,dt = 0, \quad \int\limits_0^1 t\big[p(t) - p_0(t)\big]\,dt = 0.$$

By using methods of the theory of analytic function, the problem is reduced to a singular integro-differential equations on a finite interval. Using an integral transformation a Riemann problem is obtained, the solution of which is presented in explicit form. The tangential $\tau(x)$ and normal $p(x)$ contact stresses along the contact line are determined and the asymptotic behavior of the contact stresses in the neighborhood of singular points is established.

DRAFT

On the Duffin–Schaeffer Conjecture

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In this talk we discuss about the Duffin–Schaeffer conjecture. It is a conjecture proposed by R. J. Duffin and A. C. Shaefer in 1941. It states that if $f : N \to R^+$ is a real valued function, then for all real α, inequality

$$\Big|\alpha - \frac{p}{q}\Big| < \frac{f(q)}{q}$$

has infinitely many solutions in co-prime integers p, q with $q > 0$ if and only if

$$\sum_{q=1}^{\infty} f(q)\,\frac{\varphi(q)}{q} = \infty,$$

where $\varphi(q)$ is the Euler function.

In 2019, the Duffin–Schaeffer conjecture was proved by Dimitris Koukoulupulos and James Maynard.

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DRAFT

Convergence and Summability of the One- and Two-Dimensional Vilenkin–Fourier Series in the Martingale Hardy Spaces

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Unlike the classical theory of Fourier series which deals with decomposition of a function into sinusoidal waves, the Vilenkin (Walsh) functions are rectangular waves. Such waves have already been used frequently in theory of signal transmission, multiplexing, filtering, image enhancement, codic theory, digital signal processing, pattern recognition.

This lecture is devoted to review theory of martingale Hardy spaces. We derive necessary and sufficient conditions for the modulus of continuity such that partial sums with respect to one- and two-dimensional Vilenkin–Fourier series converge in norm. Moreover, we also present some strong convergence theorems of partial sums of the one- and two-dimensional Vilenkin systems.

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DRAFT

On Some Versions of Sylvester's Problem

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In the talk two versions of Sylvester's well-known problem (see [1]–[3]) for the Euclidean plane are considered. Also, several variants of this problem for higher-dimensional Euclidean spaces are presented. One problem related to the original version of Sylvester's problem is posed. A solution of this problem is obtained as one of the consequences of the following statement:

Theorem. *Let $n \geq 5$ be an arbitrary, fixed natural number and a and b be any two fixed positive real numbers. There exists a subset S of the Euclidean plane α, such that* $\operatorname{card}(S) = n$ *and for every straight line $L_{U,V}$ passing through arbitrary two distinct points U and V of S there exists a point $W_{L_{U,V}}$ of S such that the Euclidean $d(W_{L_{U,V}}, L_{U,V})$ distance between $\{W_{L_{U,V}}\}$ and $L_{U,V}$ satisfies the following inequality: $d(W_{L_{U,V}}, L_{U,V}) < a$. Also, for any straight line $L \subset \alpha$ there exists a point Z_L of S such that the Euclidean $d(Z_L, L)$ distance between $\{Z_L\}$ and L satisfies the following inequality: $d(Z_L, L) > b$.*

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DRAFT

Unification in τ-Logic

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Unification is a process to identify two expressions by replacing variables of each expression by other expressions. More specifically, if we have two terms s and t, the aim is to find a substitution σ (mapping from variables to terms), such that $s\sigma$ and $t\sigma$ are identical terms. In this case, we say the substitution is an unifier of s and t.

In this talk we discuss unification problem in τ-logic [1, 2, 3]. In particular, we define notions of substitution, unifier, and most general unifier. We construct a sound and complete algorithm, which takes as an input s and t terms of τ-logic and returns a substitution σ such that $s\sigma = t\sigma$. We proved that, the algorithm always terminates and computed σ is a most general unifier for s and t.

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Boundary-Transmission Problems of the Thermo-Piezo-Electricity Theory without Energy Dissipation

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In the paper, we study Dirichlet, Neumann and mixed type interaction problems of pseudo-oscillations between thermo-elastic and thermo-piezo-elastic bodies. The model under consideration is based on the Green–Haghdi theory of thermo-piezo-electricity without energy dissipation (see [2]). This theory permits propagation of thermal waves only with finite speed.

Using the potential theory and boundary integral equations method, we prove existence and uniqueness of solutions, and analyze their smoothness (in the direction see [1]).

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DRAFT

On Quaternion Polynomial Equations

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The structure of the root set of quaternion polynomial equations, in particular their algebraic and topological characteristics, is investigated. It is proved that a non-commutative determinant-based resultant [2] provides a method for excluding one of the unknowns from a system of equations consisting of two two-known quaternion polynomials.

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DRAFT

Comparative Analysis of Some Classical and Modern Stochastic Models

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Structural control of complex systems is a problem of a strategic nature in terms of content, and its solution requires the development of appropriate, specific methods and tools.

Carrying out structural control means ensuring the maintenance of the existing structure of a controlled object – its elements, including connections and their modes of operation. Otherwise it is a compensation for structural disturbances (change of the system structure as a result of the failure of its elements).

Stochastic service models play a crucial role in the structural control of a complex technical system. These models describe structural control processes (namely diagnostics, redundancy control and maintenance). On the other hand, as systems become more complex, new types of models need to be built and explored. A comparative analysis of classical models of stochastic services and modern types of models is important in this regard.

The present paper discusses the relationship between the classical problem of structural control of a complex technical system and the problem of modern multi-unit redundant system, which is an important component of overall system optimization. There is the problem statement, the solution of which will be reduced to the use of the classical models.

DRAFT

Mixed Boundary Value Problems for the Laplace Equation

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We consider a new approaches to investigate mixed boundary value problems for the Laplace equation in the case of three-dimensional bounded domain $\Omega \subset \mathbb{R}^3$, when the smooth boundary surface $\partial\Omega = S$ is divided into two disjoint parts, S_D and S_N, where the Dirichlet and Neumann type boundary conditions are prescribed respectively. With the help of the theory of pseudodifferential equations the uniqueness and existence theorems are proved for mixed boundary value problems and the solutions are represented by a linear combination of single and double layers potentials.

This type of mixed boundary value problems are studied in the scientific literature by using the potential methods (see, e.g., [1], [2]). In contrast to the existing approaches, our alternative approach has two essential advantages, on the one hand, it does not require extension of the given boundary data to the whole surface and, on the other hand, the representation of a solution doesn't contain the Steklov-Poincaré type operator, which contains the inverse operator of the single layer boundary operator, which is not available explicitly in general for arbitrary surface.

We reduce the mixed BVP under consideration to the boundary integral (pseudodifferential) equations generated by the limiting values of he single and double layer potentials and show the invertibility of the corresponding pseudodifferential operator in appropriate Bessel potential and Sobolev–Slobodetski spaces. This fact will play a crucial role in the process of construction of efficient algorithms for numerical solutions of the mixed BVPs.

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DRAFT

Right Units and Idempotent Elements of a Complete Semigroups of a Binary Relations Defined by Semilattices of unions $\Sigma_3(X,7)$

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In this article, we study the right units and the idempotent of the complete semigroup of binary relations defined by X-semilattice unions.

Let

$$D=\{\breve{D},Z_1,Z_2,Z_3,Z_4,Z_5,Z_6\}$$

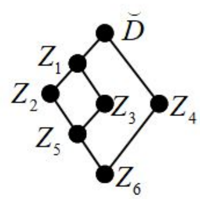
be some $X-$semilattice of unions and

$$C(D)=\{P_0,P_1,P_2,P_3,P_4,P_5,P_6\}$$

be the family of sets of pairwise nonintersecting subsets of the set X (the set $\varnothing$ can be repeat several time). If φ is a mapping of the semilattice D on the family of sets $C(D)$ which satisfies the condition

$$\varphi=\begin{pmatrix}\breve{D} & Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6\\ P_0 & P_1 & P_2 & P_3 & P_4 & P_5 & P_6\end{pmatrix}.$$

Then formal equalities of a given semilattice has a form:



$$\begin{cases}\breve{D}=P_0\cup P_1\cup P_2\cup P_3\cup P_4\cup P_5\cup P_6,\\ Z_1=P_0\cup P_2\cup P_3\cup P_4\cup P_\xi\cup P_6,\\ Z_2=P_0\cup P_3\cup P_4\cup P_5\cup P_6,\\ Z_3=P_0\cup P_2\cup P_4\cup P_5\cup P_6,\\ Z_4=P_0\cup P_1\cup P_2\cup P_3\cup P_5\cup P_6,\\ Z_5=P_0\cup P_4\cup P_6,\\ Z_6=P_0,\end{cases}$$

where P_0, P_5, P_6 are the completeness sources and P_1, P_2, P_3, P_4 are basic sources.

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Logical-Probabilistic Methods of Structural Analysis of Multifunctional Elements

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One of the most efficient way to increase reliability of complex systems, is creating them based on interchangeable multifunctional elements (MFE). To achieve this goal, it is important to evaluate the structural criteria of MFEs based on weight, importance and contribution in system reliability.

Evaluation of these criteria are managed by logical-probability methods based on computer modeling. For calculating weight of each functional recourse (x_i structural element) we use orthogonalization algorithm, which will transform the logical function $y(x_1, x_2 ... x_n)$ of the condition of working ability into orthogonal disjunctive normal form (ODNF). ODNF gives probability model of weight evaluation of MFE. To calculate the contribution of each function we use recurrent algorithm (tabular method) which transforms logical function into probability function. The contribution of each functional resource in the reliability of the system is calculated by multiplying the probability of flawless work by its importance.

Every criteria of the role and importance of the functional recourse in the reliability of the system have different sensitivity and informational load. For example, $g(x_i)$ weight of x_i structural element in the system $y(x_1, x_2, \dots, x_n)$ depends on the location of this element in the system structure. $M(x_i)$ importance of x_i element in the reliability of the system depends not only on the location but also on the reliability of every other element (except the reliability of current x_i element). $W(x_i)$ contribution in the $y(x_1, x_2, \dots, x_n)$ system reliability depends on location of this element as well as workability condition and reliability of every other element including x_i element.

Definition of role and importance of every functional resource (structural element), helps us to find week points in the structure of the system, calculation of level of multifunctionality of the elements, finding optimal reserves, creation of the interchangeability strategy, in the formation of demand for reliability characteristics and etc.

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DRAFT

A Generalization of the Canonical Commutative Relation in the Quantum Fréchet–Hilbert Space

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For a particle moving in $\mathbb{R}$, let quantum Hilbert space be $L^2(\mathbb{R})$. Let $X(\psi)(x) = x\psi(x)$ be position operator and

$$P(\psi)(x) = -ih\, d/dx(\psi)(x)$$

be momentum operator in quantum mechanics, where i is complex number and h is Plank's constant.

It is well-known that the position and momentum operators do not commute, but satisfy the relation $XP - PX = ihI$, where I is identity operator. This relation is known as the canonical commutation relation. These operators are unbounded essentially selfadjoint operators in the space $L^2(\mathbb{R})$ and are called operators of observable physical quantities.

In [1], these notions of position and momentum operator in the quantum Frechet–Hilbert space $L^2_{loc}(\mathbb{R})$ are generalized. The considered of the space $L^2_{loc}(\mathbb{R})$ instead of the space $L^2(\mathbb{R})$ essentially extend the space of states, which is achieved by weakening the topology of the space $L^2(\mathbb{R})$. The symmetry but not selfadjointness of extension of momentum operator P in the Fréchet–Hilbert space $L^2_{loc}(\mathbb{R})$ is proved. The selfadjointness of the extension of position operator and continuity in this space is proved, because the Theorem Hellinger–Teoplitz is generalized for Fréchet–Hilbert spaces [2]. More detailed, properties of Fréchet–Hilbert spaces that is represent as strict projective limit of the sequence of Hilbert spaces, is given in [3].

In this article the canonical commutation relations between this operators in the Fréchet–Hilbert space $L^2_{loc}(\mathbb{R})$ are generalized. As well the canonical commutation relations between extensions of creation and annihilation operators in the Fréchet–Hilbert space $L^2_{loc}(\mathbb{R})$ are also generalized.

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DRAFT

Convergence and Summability Fejér and T Means with Respect to the One-dimensional Vilenkin–Fourier Series in Martingale Hardy Spaces

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Unlike the classical theory of Fourier series which deals with decomposition of a function into sinusoidal waves, the Vilenkin (Walsh) functions are rectangular waves. Such waves have already been used frequently in theory of signal transmission, multiplexing, filtering, image enhancement, codic theory, digital signal processing, pattern recognition.

This presentation is devoted to review theory of martingale Hardy spaces. In particular we discuss some new (H_p, L_p) type inequalities of maximal operators of T means with respect to the Vilenkin systems with monotone coefficients. We show that these inequalities are the best possible in a special sense. We also apply these inequalities to prove strong convergence theorems of such T means. Moreover, we derive necessary and sufficient conditions for the modulus of continuity such that partial sums with respect to the one-dimensional Vilenkin–Fourier series converge in norm.

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DRAFT

About the Concept of Orbital Quantum Mechanics

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The von Neiman's mathematical model of quantum mechanics based on the theory of quantum Hilbert space H and the unbounded self-adjoint operators operating in it has long since ceased to be sufficient; in many cases do not provide the possibility for the necessary observations and calculations. Therefore, the basic concepts are represented by the methods of the theory of generalized function, in which the Frćhet spaces and their strong conjugates occupy an essential place [2]. Therefore, it was necessary to develop computational methods in the Fréchet space.

The elements ψ of quantum Hilbert space H are called states. As well the Hamiltonian of the quantum harmonic oscillator, the impulse, location, creation, annihilation and numerical operator are described by unbounded self-adjoint operators in quantum Hilbert space. These operators $A : H \to H$ are observable quantities. Each of the considered operator A create n-finite orbits $\operatorname{orb}_n(A,\psi) = \{\psi, A\psi, \dots, A^n\psi\}$ [3] and orbits $\operatorname{orb}(A,\psi) = \{\psi, A\psi, \dots, A^n\psi, \dots\}$. The space of n orbits $\operatorname{orb}_n(A,\psi)$ (resp. $\operatorname{orb}(A,\psi)$) of the elements ψ of the quantum Hilbert space is denoted by $D(A^n)$ (resp. $D(A^\infty)$) and is called the Hilbert space of n-orbits (resp. the space of all orbits). These operators also form n-finite orbital operators $A_n : D(A^n) \to D(A^n)$ [3] and the orbital operator $A^\infty : D(A^\infty) \to D(A^\infty)$ [1]. $D(A^\infty)$ is a well-known space and after the introduction of orbital operator A^∞, it acquired new content. We have generalized the theory of self-adjoint operators in Fréchet spaces and proved that the operator $\mathcal{H}^\infty$ for Hamiltonian $\mathcal{H}$ of the quantum harmonic oscillator is topological homomorphism onto the Fréchet space $D(\mathcal{H}^\infty)$.

We consider the equation $\mathcal{H}u = f$ containing the operator $\mathcal{H}$ in the space $D(\mathcal{H}^n)$ (resp. in the space $D(\mathcal{H}^\infty)$) that has the form $\mathcal{H}_n(\operatorname{orb}(\mathcal{H},u)) = \operatorname{orb}_n(\mathcal{H},f)$ (resp. $\mathcal{H}^\infty(\operatorname{orb}(\mathcal{H},u)) = \operatorname{orb}(\mathcal{H},f)$). For the obtained equations, a linear spline central algorithm is constructed in the Hilbert space $D(\mathcal{H}^n)$ (resp. in the Fréchet space $D(\mathcal{H}^\infty)$. The mathematical model of orbital quantum mechanics significantly expands classical quantum mechanics.

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DRAFT

The Satisfaction of Boundary Conditions on the Surfaces and an Extension of Complex Analysis for Some Nonlinear DEs

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The basic problem of satisfaction of boundary conditions is considered when the generalized stress vector is given on the surfaces of elastic plates and shells. This problem has so far remained open for both refined theories in a wide sense and hierarchic type models. In the linear case, it was formulated by Vekua for hierarchic models too. In the nonlinear case, bending and compression-expansion processes do not split and in this context the exact structure is presented for the system of differential equations of von Kármán–Mindlin–Reisner type, constructed without using a variety of ad hoc assumptions since one of the two relations of this system in the classical form is the compatibility condition, but not the equilibrium equation. In this paper, a unity mathematical theory is elaborated in both linear and nonlinear cases for anisotropic inhomogeneous elastic thin-walled structures. The theory approximately satisfies the corresponding system of partial differential equations and the boundary conditions on the face of surfaces of such structures. The problem is investigated and solved for hierarchic models too. The obtained results broaden the sphere of applications of complex analysis methods. The classical theory of finding a general solution of partial differential equations of complex analysis (which in the linear case was thoroughly developed in the works of Goursat, Weyl, Walsh, Bergman, Kolosov, Muskhelishvili, Bers, Vekua and sufficient many others) is extended to the solution of basic nonlinear differential equations containing the nonlinear summand: a composition of Laplace and Monge–Ampère operators [1].

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DRAFT

Novelties in the New National Curriculum for Mathematics and Ways to Overcome Them

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Teaching of mathematics in secondary schools in Georgia takes places according to the new national study plan. The standard update process is currently underway. The novelty is that each mandatory topic is accompanied by essential questions and enduring understandings. In the present work the connection between the introduction of mathematical notions and national curriculum innovations is offered.

DRAFT

A Class of the Extended Multi-Index Bessel–Maitland Functions and it's Properties

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Here, we present the extended multi-index Bessel–Maitland function (EMBMF) defined using the extended beta function and investigate its several properties including, integral representation, derivatives, beta transform and Mellin transform. The relationships of the said function (EMBMF) with the Leguerre polynomial and Whittakar function respectively, are also elaborated. Further, image formulas for the extended multi-index Bessel Maitland function are also investigated using fractional calculus operators. All the derived results generalize plentiful well-known results and can be used to derive a number of pertinent results in the theory of special functions.

DRAFT

On the Existence of Group Terms in Some Protomodular Varieties of Universal Algebras

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The notion of a protomodular category was introduced by Bourn [1] as an abstract setting where certain properties of groups remain valid. There is a syntactical characterization of protomodular varieties (Bourn and Janelidze [2]). It requires the existence of a term θ of an arbitrarily high arity $(n+1)$, binary terms α_i and 0-ary terms e_i $(i = 1, 2, \dots, n)$ that satisfy certain identities (these terms are called protomodular).

The aim of the present work is to give protomodular analogs of the well-known criterion for the existence of a group term (i.e. a binary term that satisfies the group identities for some unary and 0-ary terms) in the algebraic theory of a variety (recall that this criterion asserts that the latter theory contains a group term if and only if it contains an associative Mal'cev term and a 0-ary term). To this end the notions of a consociative term, a consociative protomodular algebra, and of a right-cancellable protomodular algebra are introduced. Moreover, for a protomodular variety $\mathbb{V}$, a certain functor from the category of consociative $\mathbb{V}$-algebras with $e_1 = e_2 = \dots = e_n$ to the category of groups is constructed. Also the translation group functor from the category of right-cancellable $\mathbb{V}$-algebras to the category of groups is constructed.

It is proved that, for an arbitrary variety $\mathbb{V}$ of universal algebras, the following conditions are equivalent:

(i) the algebraic theory of $\mathbb{V}$ contains a group term;

(ii) the algebraic theory of $\mathbb{V}$ contains protomodular terms θ, α_i, and e_i $(i = 1, 2, \dots, n)$ such that θ is consociative and the identities $e_1 = e_2 = \dots = e_n$ are satisfied;

(iii) the algebraic theory of $\mathbb{V}$ contains protomodular terms with respect to which all $\mathbb{V}$-algebras are right-cancellable.

The consociative algebras with $e_1 = e_2 = \dots = e_n$ (resp. the right-cancellable algebras) from the simplest protomodular varieties $\mathbb{V}_n$ are characterized as groups with additional structures. The right-cancellable algebras from such varieties are also characterized as sets with principal group actions.

The results of this work are presented in the papers [3] and [4].

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DRAFT

Index

DRAFT

DRAFT

DRAFT