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მათემატიკოსთა კავშირი

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Mathematical Union

ბათუმის შოთა რუსთაველის
სახელმწიფო უნივერსიტეტი

Batumi Shota Rustaveli
State University

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საორგანიზაციო კომიტეტი:

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რედაქტორები: გურამ გოგიშვილი, მაია ჯაფოშვილი

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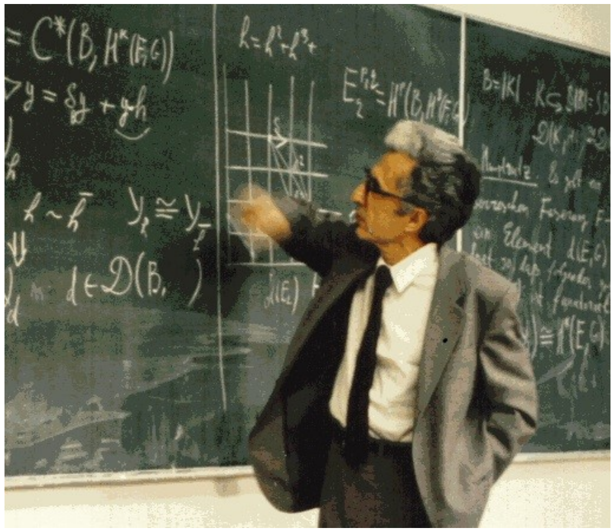
Academician Nodar Berikashvili



Passed away Academician Nodar Berikashvili, internationally recognized famous Georgian mathematician, one of the founder of Georgian Topological School. He was born in 1929. In 1952, after graduating from Tbilisi State university he became an aspirant (PhD student) of the academician George Chogoshvili. Nodar Berikashvili's dissertation (PhD thesis) got the attention of the world famous Muscovite topologists Peter Alexandrov, who invited young Nodar Berikashvili to work at Steklov Mathematical Institute. Nodar Berikashvili got his PhD degree from this institution and worked there till 1959. It is worth mentioning that on the websites of Steklov Institute and of Moscow State University Nodar Berikashvili's name is in the list of their honored scientists.

In 1959 he returned to Georgia and began working at Andrea Razmadze Mathematical Institute of the Georgian Academy of Sciences, in the Department of Geometry and Topology, founded by the academician George Chogoshvili. In 1971 he got the degree of Doctor of Sciences from Steklov Institute. In 2001 he was elected as the member of the Georgian National Academy of Sciences.

The first cycle of his works concerns homology theory of general spaces. Namely, N. Berikashvili continued the approach of his supervisors G. Chogoshvili and P. S. Alexandrov – the duality theory. In Berikashvili's works the duality theory achieved its complete form. For this purpose, Berikashvili constructed several new homology theories, developed axiomatic theory for limits of spectra. These works place N. Berikashvili among the founders of homology theory of general spaces as it is mentioned in the survey papers by Alexandrov and Fedorchuk.

The second cycle concerns the index theory for singular integral equations, the traditional field for the Georgian school of mathematics. The index problem for elliptic differential operators was posed by Israel Gel'fand. In particular, he suggested that it should be possible to express the index of an elliptic operator in topological terms. In 1962 Wolpert proved the coincidence of analytical and topological indexes for 2-spheres. Using topological methods, in 1963 Berikashvili proved the index formula for an arbitrary 2-dimensional manifold. He published this work in the Bulletin of the Georgian Academy of Sciences and reported this result on a conference in Novosibirsk, but here he learned that the solution of the problem in general case was announced by Atiah and Singer. Thus, Berikashvili stopped to work in this direction. Later, Atiah got Fields medal, and in 2001 Atiah and Singer were awarded the Abel Prize for this discovery.

The next cycle of N. Berikashvili's works is an important contribution to one of the most powerful tools of Algebraic Topology – Lere–Serres spectral sequences. Berikashvili's theory of predifferentials essentially strengthens this method and enriches it by a new computational potential.

The predifferential theory is based on the new homotopy invariant, functor D, which uses not only cocycle information from the cochain complex of a space (as the homology functor does), but the so-called twisting cochains as well. Thus, this functor is more informative than the homology functor is. For the large class of spectral sequences, which includes the spectral sequences of fibrations and coverings, N. Berikashvili constructed the so-called "predifferential", an element of the functor D, which determines all differentials and extensions in the corresponding spectral sequence. Thus, it completely reconstructs the limit. This method demonstrates connections between various methods of studying of homology theory of fibrations: the method of spectral sequences of Lere–Serre, the Hirsch method and the Brown's theory of twisted tensor products. Moreover, the predifferential theory develops and generalizes these methods. Namely, the introduced organization in the set of twisting cochains allows to choose a twisting cochain of much simpler form for the calculation of homology of a fibration.

In the 1970-80s the predifferential theory achieved essential refinement and was developed as a powerful tool for the modeling of spaces and fibrations. N. Berikashvili constructed the new versions of the functor D, which determine the multiplicative structure and high rank Steenrod operations.

Afterwards, using his methods, Nodar Berikashvili worked on the central problem of Algebraic Topology, on the problem of homotopy classification, particularly, on the obstruction theory for the section of a fibration. He developed the complete form of the second obstruction problem. His high order obstruction functors seem to be a perspective tool for this important and extremely complicated problem.

Later, Berikashvili's predifferential method was connected with actual notions of physics such as Master equation, flat connections, deformation quantization by various authors, such as Stasheff, Huebschmann, Keller.

Besides his scientific activity, Nodar Berikashvili was deeply involved in the pedagogical activity: he was lecturing in Tbilisi State University, supervising PhD students at Razmadze Mathematical Institute. Representatives of the first generation of the Georgian topological school founded by the academician George Chogoshvili are Nodar Berikashvili and Hvedri Inassaridze; they are respected scientists in various fields of Algebra and Topology.

The academician Nodar Berikashvili's death is a great loss for the worldwide mathematical society.

Georgian Mathematical Union

Academician Vakhtang Kokilashvili

An Academician and an internationally recognized mathematician, Vakhtang Kokilashvili, has passed away recently. This is a heavy loss for the Georgian scientific community.

The fundamental results obtained by Professor V. Kokilashvili play an important role in Function Theory and Functional Analysis. We should emphasize his achievements in the direction of Harmonic Analysis, Function Spaces, Operator Theory, Approximation Theory, Boundary Values Problems of Analytic Functions and Partial Differential Equations of Mathematical Physics. It is especially worth mentioning his contribution in the solution of the two-weighted problem for potential operators. This result was recognized as one of the most significant breakthroughs in the weight theory by leading experts in the area. V. Kokilashvili also obtained the conceptual results in the weighted theory of multiple singular integral operators. During his last years, he fruitfully worked on the problems regarding mapping properties of operators of Harmonic Analysis in non-standard function spaces, as well as the study of boundary value problems for analytical functions in new function spaces. Furthermore, he introduced some new function spaces and successfully investigated the properties of integral operators acting in these spaces.

A scientific group of V. Kokilashvili, which derived important results in the direction of non-standard function spaces, became worldwide recognized in leading scientific centers.

Academician Vakhtang Kokilashvili was born in Tbilisi, 1938, to a family of mathematics teachers. In 1959, he graduated from the Faculty of Mechanics and Mathematics at the Tbilisi State University and started working at A. Razmadze Mathematical Institute of the Georgian Academy of Sciences. Since 1990, he was the head of the department of Mathematical Analysis at the institute. From 1989 to 2006, he was the deputy director of scientific affairs.

In 1997, Vakhtang Kokilashvili was elected as a corresponding member of the Georgian Academy of Science, and he became an academician in 2013. In 1986 he was awarded by the Andrea Razmadze Prize of the Georgian National Academy of Sciences.

Academician V. Kokilashvili is the author of more than 270 scientific works and 9 monographs which have been published in such leading scientific publications as *"Kluwer"*, *"Birkhäuser"*, *"Addition Wesley Longman"*, *"World Scientific"* and others (see the list of monographs [1–9]). His results have been cited by many prominent scientists. Vakhtang Kokilashvili was the chief editor of the international scientific journal *"Transactions of A. Razmadze Mathematical Institute"*, member of the editorial board of the *"Georgian Mathematical Journal"* and other international scientific journals. Vakhtang Kokilashvili participated in the work of many large scientific forums; he was a regional ambassador of the International Congress of Mathematicians (ICM 2014), a participant of ICM held in 1966, 1998, 2002, 2014, 2018, etc.

Along with important scientific research, V. Kokilashvili was engaged in fruitful teaching activities at Tbilisi State University, Georgian Technical University, Black Sea International University, and at Sokhumi branch of the Ivane Javakhishvili Tbilisi State University. Another manifestation of his pedagogical talent was the cycle of televised lessons in mathematics brilliantly presented by him in the 1980s which gained universal approval and recognition in the society. He was the supervisor of ten candidate theses and a scientific consultant of three doctoral theses. V. Kokilasvili was also an active member of the Georgian Mathematical Union.

Vakhtang Kokilashvili's personal qualities were also outstanding. Along with having high professionalism and being a man of principles, he was distinguished by great responsibility, justice, and objectivity. His attentive and benevolent attitude towards colleagues and students earned him their boundless respect and love.

Till the very end of his life, Vakhtang Kokilashvili served the mathematical science tirelessly and fruitfully, with outstanding energy.

Vakhtang Kokilashvili, an academician with outstanding service to the country, a great researcher, and an excellent citizen, will always be remembered, greatly missed, and appreciated by his colleagues, students, disciples, friends and relatives.

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Georgian Mathematical Union

Professor Leonard Mdzinarishvili

The mathematical society suffered a great loss due to the death of Leonard Mdzinarishvili, a recognized researcher of algebraic topology, an outstanding member of the Georgian abstract mathematical school, Professor of the Georgian Technical University.

Leonard Mdzinarishvili's field of scientific interests included: homology theory, homological algebra, category theory, shape theory. Here is a brief overview of the scientific results obtained by Mdzinarishvili, which can be divided into several cycles.

One cycle of the works deals with determining the connection between the Kolmogorov and the Steenrod homology groups. Interest of the problem came from the research in homology theory of the Academician George Choghoshvili. Based on the partitions, Choghoshvili constructed new – Spectral and Projective Homology groups with compact coefficients and by using them for compact metric spaces he proved the isomorphism of the Kolmogorov and the Steenrod homology groups. However, the question remained open for discreet coefficient groups. It was this very problem that was investigated by Mdzinarishvili. His construction and investigation of three different – Strong Projective, Projective and Spectral homology groups with discrete coefficient groups based on different types of coverings – locally finite partitions, complete open coverings, complete closed coverings, locally finite (Kuroš) lattices, played an essential role in solving the problem.

In the second cycle of the works the so-called projective systems in the category of pairs of abelian groups are considered, which are inverse systems of groups in the case when pairs are groups with trivial subgroups. Consequently, the generalized, the so-called projective limit and corresponding satellite functors are defined. The method of computation of satellites are obtained.

In the third cycle of the works the homotopy groups of direct systems of cofibrations and inverse systems of fibrations are studied. In particular, the formula is obtained, which is a generalization of the Milnor and the Cohen–Vogt formulas. The Milnor formula is obtained, in case the inverse sequence of fibrations is degenerated; the Cohen–Vogt formula is obtained if the direct sequence of cofibrations is degenerated.

Mdzinarishvili's role is essential in the development of the Strong homology theory. Under his supervision, the strong homology groups were constructed by Z. Miminoshvili, which are mentioned as 1-height strong homology groups in Mardešić's works. What today is called the strong (not finite height) homology groups was defined a bit later by various authors. However, in Mdzinarishvili's paper, instead of the strong homology groups they were called the total homology groups. In the same paper the so-called fragments of the total homology groups were constructed, which are called the finite height homology groups in the Mardešić's papers. In some sense, the finite height strong homology (fragment of strong homology) groups can be considered as an approximation of the strong (total) homology groups. The essential formulas, which give the connection of these two types of homology groups were announced in the paper by Miminoshvili, whereas the proofs of these formulas appeared for the first time in the paper by Mdzinarishvili.

In the next cycle of the works of professor Mdzinarishvili, the Steendor homology groups are axiomatically described on the category of compact spaces. In particular, co-authored with Professor Hvedri Inasaridze, the partial continuity axiom for a homology theory was introduced, and by its application the uniqueness theorem was obtained. For the category of compact metric spaces, the connections of the proposed axiomatic system and the Milnor's and Berikashvili's axiomatic systems were investigated. In recent years, co-authored with A. Beridze, the connections between Mdzinarishvili–Inasaradze, Berikashvili and Inasaridze axiomatic systems have been investigated on the category of compact spaces. In addition, on the category of polyhedral spaces the singular cohomology theory axiomatically was described by Mdzinarishvili, deferent from Milnor's axiomatic system. Afterwards, co-authored with A. Beridze, additionally two more different axiomatic systems were introduced and the connection between them was investigated.

The investigation of the problem of homology theory leads Mdzinarishvili to the study of commutativity problem of limit and homology functors. Using the derived limit functors, the long exact sequence was obtained by Mdzinarishvili in the category of abelian group, by which the given problem was answered in a specific case. Later, the problem was investigated on the category of modules over principal ideal domain, co-authored with E. Spanier. The following years, co-authored with A. Beridze, this approach was used to study various homology groups. Using the developed techniques and methods of shape theory, the exact homology theory of general topological spaces was axiomatically studied together with A. Beridze and V. Baladze, which coincides with the Steenrod homology groups for compact spaces.

The important part of Mdinarishvili's works deals with the study of different types of homology and colomology groups with topological coefficient groups, the so-called continuous homology and cohomology.

Leonard Mdzinarishvili was the author of 62 scientific papers and several textbooks. He actively participated in international symposia and conferences. His participation in international congresses of world mathematicians should be especially noted. He was a principal investigator of many international grants.

In 1964–1986 Leonard Mdzinarishvili worked as a junior, senior and chief researcher at the Andrea Razmadze Institute of Mathematic in Tbilisi. In 1986–2007 he was the head of the Higher Mathematics Department of the Georgian Technical University; in 2007–2012 – Head of the Algebra-Geometry direction of the Georgian Technical University.

Leonard Mdzinarishvili was an outstanding tennis player, he was the champion of the Soviet Union among juniors as part of Tbilisi and Tbilisi "Dinamo" teams, the champion of the Soviet Union among juniors in doubles and singles. In 1960 he became the champion of Georgia for the first time, and in 1971 he won this title for the second time. He was the member of the national team of the champion of the Soviet Union – "Dinamo". Together with A. Metreveli and T. Kakulia, he won the USSR Cup in 1972. He became the champion of the Soviet Union among students and also in mixed pairs. He was a six-time champion of the Union Academiadas.

The death of Professor Leonard Mdzinarishvili is a great loss both for the Georgian Mathematical Union and for the mathematical community as a whole.

Georgian Mathematical Union

Professor Vladimer Baladze

Doctor of Physical and Mathematical Sciences, Professor Vladimer Baladze is 70 years old.

V. Baladze graduated from the Gegelidzeebi 8-year and Tskhomorisi Secondary schools, continued his studies at the Faculty of Mechanics and Mathematics of the Ivane Javakhishvili Tbilisi State University, and upon graduation became a postgraduate student at Andrea Razmadze Institute of Mathematics.

V. Baladze defended his Candidate and Doctoral theses:

- *"On Dimension-like Functions Based on Distinguished Classes of Spaces and Complexes"*, Tbilisi State University, 1980.
- *"The Resolutions of Topological Spaces and Continuous Maps and their Application in Spectral Homotopy Theory"*, Tbilisi State University, 1993.

V. Baladze's scientific interests include general topology (dimension theory, theory of continuous transformation groups), algebraic topology (homology theory, homotopy theory), geometric topology (theory of retracts, shape theory) and category theory.

Professor V. Baladze has diverse work experience. In 2005–2007, he was the Rector of Batumi Shota Rustaveli State University, and a Deputy Rector in the academic field.

V. . Baladze's pedagogical and research work is primarily related to the Faculty of Mechanics and Mathematics of Ivane Javakhishvili Tbilisi State University, where for years he taught the basic course in topology as well as special courses in algebraic topology, geometric topology and general topology. At the same time, V. Baladze was carrying out active pedagogical at Batumi Shota Rustaveli State University, where in 1991–2004, during one of the harshest periods of our country's history, he conducted the same mathematics courses.

V. Baladze believes that his greatest achievement is conducting the topology course at Tbilisi State University, which was led for several decades by the Academician Giorgi Choghoshvili – the founder of the Georgian Topological School. At Mr. Giorgi's insistence and by the decision of the Algebra-Geometry Department, V. Baladze was assigned to lead the basic topological course for the students of the mathematics specialty of the Mechanics-Mathematics Faculty. The result of V. Baladze's pedagogical work at these two universities is reflected in his own creation of the first Georgian-language textbook on topology.

The book is intended for undergraduate, graduate and doctoral students of mathematics and physics specialties and, in general, for a wide circle of readers wishing to gain systematic knowledge in topology. V. Baladze's main scientific achievement is the development of geometric topology in

Georgia, and the creation of a geometric topological school in Batumi. He is the head of the scientific seminar of the Mathematics Department of Batumi Shota Rustaveli State University: (Co) shape classifications of topological spaces and continuous maps and their algebraic and dimension-like invariants.

The first cycle of V. Baladze's works deals with the modular dimension theory of the class of spaces. It investigates small inductive, large inductive and covering modular dimensions, which are dimension-type functions and play an important role in the research of problems arising in classical dimension theory. The author obtained their main properties and connections with the classical induction and Lebesgue dimensions, the dimension and multiplicity of continuous maps and, as a result, obtained: the axiomatic characterization of the large induction dimension; the generalization of Morita's theorem on the Lebesgue dimensions of spaces and the dimension of continuous maps; the characterization of the dimensions and combinatorial properties reminders of spaces; approximation and factorization theorems of space and map.

The second cycle of V. Baladze's works refers to a new direction of geometric topology – the shape theory.

In the third cycle of V. Baladze's works, which deals with the fiber shape theory, the geometric theory of continuous maps is developed from the point of view of the theory of retracts. The author defines the absolute (neighborhood) retracts and absolute (neighborhood) extensors of different classes of continuous maps, and their main properties are obtained. These properties play an essential role in proving the existence theorems of fiber resolution of continuous maps.

The theorems of the existence of approximations of resolution of any continuous map of topological spaces and continuous map of compact spaces by continuous maps of compact polyhedra are also interesting results of independent significance. Their applications include: fiber shape classification of continuous maps; construction of fiber shape invariant extensions of functors; the definition of functors from the category of continuous maps of spaces into the category of inverse systems of groups, direct systems of groups and long exact sequences of groups; characterization of fiber shape retracts. In addition, using of the obtained results spectral and projective homologies and cohomologies are characterized in the Hu sense without the use of relative groups.

Since 2000, Baladze and his students from Batumi have been systematically investigating an important theory of homology – Alexander–Spanier coholomogy theory. V. Baladze constructed the Alexander–Spanier normal cohomology theory for the category of P-embedding closed pairs of any topological spaces based on the normal coverings and proved the isomorphism between this theory and the Alexandrov–Čech normal cohomology theory. This fact gave his students and L. Mdzinarishvili the opportunity to construct and axiomatically characterize the Alexander–Spanier normal exact homology theory.

Professor V. Baladze is the author of about 100 scientific works and a participant of about 41 international mathematical forums. V. Baladze is also the organizer of about 10 mathematical forums.

Professor V. Baladze was the supervisor of five PhD theses. His students are actively investigating his problems and research topics.

We wish Professor Vladimer Baladze health and further service to the University with his usual devotion.

Georgian Mathematical Union

Abstracts of Plenary Talks

On Computationally Efficient Methods for Testing Multivariate Distributions with Unknown Parameters

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Despite the popularity of classical goodness fit tests such as Pearson's chi-squared and Kolmogorov–Smirnov, their applicability often faces serious challenges in practical applications. For instance, in a binned data regime, low counts may affect the validity of the asymptotic results. Excessively large bins, on the other hand, may lead to loss of power. In the unbinned data regime, tests such as Kolmogorov–Smirnov and Cramer-von Mises do not enjoy distribution-freeness if the models under study are multivariate and/or involve unknown parameters. As a result, one needs to simulate the distribution of the test statistic on a case-by-case basis. In this talk, I will discuss a testing strategy that allows us to overcome these shortcomings and equips experimentalists with a novel tool to perform goodness-of-fit while reducing substantially the computational costs.

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The Maximal Operator in Variable Exponent Stummel Spaces

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In this talk we consider variable exponent Stummel spaces and discuss the boundedness of the Hardy–Littewood maximal operator in such spaces. Our results include also the behaviour of the maximal operator in some vanishing subspaces.

The talk is based on joint work with Humberto Rafeiro.

Tensor Completions of CSA-Groups and Fraisse Limits of Extensions of Centralizers

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The theory of exponential groups goes back to the works of A. Mal'cev and P. Hall, who studied nilpotent exponential groups. In 1960 G. Baumslag constructed a free $\mathbb{Q}$-group $F^{\mathbb{Q}}$ and described its algebraic structure in terms of free products with amalgamation. In the same year R. Lyndon introduced an axiomatic notion of an exponential group and studied free groups $F^{\mathbb{Z}[t]}$ with exponentiation in the ring of polynomials $\mathbb{Z}[t]$ [2]. In particular, he showed that the group $F^{\mathbb{Z}[t]}$, hence every subgroup of it, is fully residually free. In 1996 A. Miasnikov and V. Remeslennikov defined A-completions G^A of an arbitrary group G and described the algebraic structure of the group G^A in the case when G is torsion-free and CSA and the ring A is an arbitrary associative unitary ring with the additive group without 2-torsion [3]. Using these results and Makanin–Razborov's technique on solving equations in free groups O. Kharlampovich and A. Miasnikov proved the converse of the Lyndon's result mentioned above. Namely, they showed that every finitely generated fully residually free group embeds into $F^{\mathbb{Z}[t]}$. This became a cornerstone in the study of algebraic geometry of free groups. Recently, O. Kharlampovich, A. Miasnikov, and R. Sklinos proved that the Lyndon's group $F^{\mathbb{Z}[t]}$ is the Fraisse limit of iterated extensions of centralizers of the free group F, thus bringing a powerful method of Fraisse limits to the subject [1].

In this talk we show that the tensor A-completions G^A of a torsion-free CSA group G by an arbitrary commutative ring A with identity and zero characteristic is a Fraisse limit of the Fraisse class of iterated centralizer A-extensions of the group G. This is a far reaching generalization of the result from [1].

Acknowledgements

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Gromov–Hausdorff Hyperspaces of a Euclidean Space

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Let (M, d) be a metric space. For two non-empty subsets A, $B \subset M$, the Hausdorff distance $d_H(A, B)$ is defined as follows:

$$d_H(A,B) = \max\Big\{\sup_{a\in A} d(a,B),\ \sup_{b\in B} d(b,A)\Big\},$$

where

$$d(x,C) = \inf\big\{d(x,c) \mid c \in C\big\}.$$

The set of all non-empty compact subsets of M is denoted by 2^M and is endowed with the Hausdorff metric d_H. The pair $(2^M, d_H)$ is called the hyperspace of M.

For two compact metric spaces X and Y, their Gromov–Hausdorff distance $d_{GH}(X,Y)$ is defined to be the infimum of all real numbers $r > 0$ such that here exist a metric space (M, d) and isometric embeddings $i : X \hookrightarrow M$ and $j : Y \hookrightarrow M$ with the Hausdorff distance $d_H(i(X), j(Y))$ less than r. It is a useful tool for studying topological properties of families of metric spaces. Clearly, the Gromov–Hausdorff distance between two isometric spaces is zero; it is a metric on the family $\mathbb{GH}$ of isometry classes of compact metric spaces. The metric space $(\mathbb{GH}, \mathrm{d_{GH}})$ is called the Gromov–Hausdorff space.

In this talk we mainly are interested in the subspace $\mathbb{GH}(\mathbb{R}^n)$ of $\mathbb{GH}$ consisting of the classes $[E] \in \mathbb{GH}$ whose representative E is a metric subspace of the standard Euclidean space $\mathbb{R}^n$, $n \geq 1$. $\mathbb{GH}(\mathbb{R}^n)$ is called the Gromov–Hausdorff hyperspace of $\mathbb{R}^n$. One of the main results of this talk asserts that $\mathbb{GH}(\mathbb{R}^n)$ is homeomorphic to the orbit space $2^{\mathbb{R}^n}/E(n)$, where $2^{\mathbb{R}^n}$ is the hyperspace of all non-empty compact subsets of $\mathbb{R}^n$ endowed with the Hausdorff metric, and $E(n)$ is the isometry group of $\mathbb{R}^n$. This is applied to prove that $\mathbb{GH}(\mathbb{R}^n)$ is homeomorphic to the punctured Hilbert cube $[0,1]^{\aleph_0} \setminus \{*\}$.

On a New Principle in Real Analysis and its Application to Some Well Known Equations

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We start with a result of general nature (principle) related to arbitrary smooth real function of one variable. The result presents an evaluation method for the number of zeros of real functions.

Then we apply this principle for studying the number of zeros of solutions of well known equations arising in physics, chemistry, biology, geology, ecology.

We consider a second order ordinary differential equation which widely generalize Schrödinger equation, Emden–Fowler equation, Fisher equation, Kolmogorov–Petrovskii–Piskunov equation, Newell–Whitehead–Segel equation, Zeldovich equation, Van der Pol equation, Chandrasekar equation, generalized Emden–Fowler equation, Sturm–Liouville equation, Bessel equation, Legander equation, Laguerre equation.

Also we consider a generalization of the third order Korteweg–De Vries equation.

A part of them are partial differential equations which we considered in time independent versions; others are ordinary differential equations.

For all these equations we give upper bounds for the number of zeros of their solutions.

As far as we know the number of zeros was studied only for Sturm–Liouville type equations (including the Schrödinger equations) and for other listed above equations the results are brand new.

New Convolutions and Some of Their Applications

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We will introduce new convolutions and analyse some of the consequences they generate. Namely, we will study the solvability of a general class of integral equations whose kernel depends on four different functions, as well as other properties of the corresponding integral operators. Additional properties will appear along the way (among which we highlight, as an example, certain Young-type inequalities and factorizations where the convolutions and integral operators appear).

So, the main technique consists in introducing (eight) new convolutions (weighted by multi-dimensional Hermite functions) and use them as convolutions somehow associated with certain classes of integral equations.

In this talk we will (partially) expose some of the ideas that appear in the works [1–6], carried out together with the co-authors mentioned below.

AMS Subject Classification: 45E10, 33C45, 43A32, 44A20, 44A35, 46E25, 47A05, 47B48.

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Jones Factorization and Rubio de Francia Extrapolation for Matrix Weights

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In the 1990s, Nazarov, Treil and Volberg introduced a generalization of the scalar Muckenhoupt A_p condition to matrix weights. Let W be a $d \times d$ symmetric, positive definite matrix weight function. For $1 < p < \infty$, we say W is in matrix A_p if

$$[W]_{A_p} = \sup_Q \frac{1}{|Q|} \int\limits_Q \bigg(\frac{1}{|Q|} \int\limits_Q |W^{1/p}(x) W^{-1/p}(y)|_{op}^{p'} \, dy \bigg)^{p/p'} dx < \infty,$$

where the supremum is taken over all cubes in $\mathbb{R}^n$. They showed that the Hilbert transform is bounded on $L^p(W)$, the space of vector-valued functions with norm

$$\|f\|_{L^p(W)} = \bigg(\int\limits_{\mathbb{R}^n} |W^{1/p}(x) f(x)|^p \, dx \bigg)^{1/p}.$$

Later, Christ and Goldberg extended this result to all Calderón–Zygmund singular integrals, and a theory of two-weight norm inequalities was introduced by DCU, Isralowitz and Moen.

In the 1990s, Nazarov, Treil and Volberg posed two related problems: extend the Jones factorization theorem and the Rubio de Francia theory of extrapolation, to matrix A_p weights. Since 2011, an important open question has been to prove the A_2 conjecture for matrix weights: it is conjectured that the sharp exponent on the A_p characteristic for matrix weights is the same as for scalar weights, $\max\{1, \frac{1}{p-1}\}$.

In this talk we will discuss the solution of the first two problems. We have proved exact generalizations of factorization and the sharp constant extrapolation theorem that was used to prove the scalar A_2 conjecture. The proofs required the development of a new family of tools for working with convex-set valued functions. This let us generalize the ideas in the convex-body sparse domination theorem of Nazarov, Petermichl, Treil and Volberg. We defined the analog of the Hardy–Littlewood maximal operator on convex-set valued functions, and proved weighted norm inequalities for this operator on $L^p(W)$ (suitably extended to convex-set valued functions). This allowed us to define a Rubio de Francia iteration algorithm, the fundamental tool in the proof of both factorization and extrapolation.

The talk is based on joint work with Marcin Bownik, the University of Oregon.

Overlap Gap Property: a Topological Barrier to Optimizing Over Random Structures

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Many decision and optimization problems over random structures exhibit a gap between the existential and algorithmically achievable values, dubbed as statistics-to-computation gap. Examples include the problem of finding a largest independent set in a random graph, the problem of finding a near ground state in a spin glass model, the problem of finding a satisfying assignment in a random constraint satisfaction problem, and many many more. At the same time, no formal computational hardness of these problems exists which would explain this persistent algorithmic gap.

In the talk we will describe a new approach for establishing an algorithmic intractability for these problems called the overlap gap property. Originating in statistical physics, and specifically in the theory of spin glasses, this is a simple to describe property which

(a) emerges in most models known to exhibit an apparent algorithmic hardness;

(b) is consistent with the hardness/tractability phase transition for many models analyzed to the day;

and, importantly,

(c) allows to mathematically rigorously rule out a large class of algorithms as potential contenders, specifically the algorithms which exhibit noise insensitivity.

We will specifically show how to use this property to obtain stronger than the state of the art lower bounds on the depth of Boolean circuits for solving two of the aforementioned problems: the problem of finding a large independent set in a sparse random graph, and the problem of finding a near ground state of a p-spin model.

Composition Operators on Sobolev Spaces and Ball's Classes

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In this paper we study geometric aspects of Ball's classes in the context of nonlinear elasticity problems. The suggested approach is based on characterization of Ball's classes $A_{q,r}(\Omega)$ in terms of composition operators on Sobolev spaces. This characterization allows us to obtain the volume compression (distortion) estimates of topological mappings of Ball's classes. We prove also that Sobolev homeomorphisms of Ball's classes which possess the Luzin N-property are absolutely continuous with respect to capacity.

Joint with Alexander Ukhlov.

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Differentiation of Integrals: Frontiers and Perspectives

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Arising with the origins of calculus, the topic of differentiation of integrals plays a fundamental role in real variable analysis and finds diverse applications ranging from our understanding of singular integral operators to boundary value problems in partial differential equations. In this expository talk intended for a broad mathematical audience, I will provide a historical overview of the subject of differentation of integrals, highlighting prominent results as well as indicating outstanding open problems in the field. Recent results presented will be joint with Giorgi Oniani, Ioannis Parissis, and Alexander Stokolos.

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Equations and First-Order Sentences in Random Groups

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We prove that a random group, in Gromov's density model with $d < 1/16$, satisfies an $\forall\exists$-sentence σ (in the language of groups) if and only if is true in a nonabelian free group.

Theory of Distribution-Free Testing in Statistics

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Investigating some phenomenon, researcher will develop some theory, or a model, of this phenomenon and will have some data to prove or disprove it.

Any one of diverse statistical procedures used to assess agreement between the data and its theoretical model is called "distribution-free", if its statistical properties do not depend on the model. It may look illogical that the properties of the testing procedure do not depend on what we are testing, but it is possible. Moreover, it is convenient: if the procedure is distribution-free, one does not have to investigate its properties for every new model - to study them for one "representative" model will produce the same results for all models in the equivalence class. Mathematical foundation of distribution-free-ness is of the same nature as in other fields: it is invariance under certain group of transformations. As in other fields, this equivalence is of great importance in statistics: for example, it allowed development of broad theory of rank statistics, and, in slightly different form, of the classical theory of distribution-free goodness of fit tests.

However, this classical theory, in time, accumulated difficulties and became restrictive and somewhat narrow for current applications.

For example, distribution free methods for testing models of stochastic processes, such as Markov chains and diffusion processes, is only in infancy.

We will present an approach, showing how a certain group of unitary operators in functional spaces can be associated with statistics, leading to a broad notion of equivalence of various statistical problems. Looking abstract from one side, the implementation of this new invariance looks practically possible in most of statistical models.

Locally Regular Weighted Lebesgue Spaces

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We present the notion of the class $A_p^{\rm loc}$ of weights. Let $1 < p < \infty$. A weight w belongs to $A_p^{\rm loc}$ if

$$[w]_{A_p^{\rm loc}} \equiv \sup_{Q \in \mathcal{Q},\, |Q| \le 1} m_Q(w) m_Q(w^{-\frac{1}{p-1}})^{p-1} < \infty,$$

where $m_Q(w)$ stands for the average over a cube Q of the weight w. The quantity $[w]_{A_p^{\rm loc}}$ is referred to as the $A_p^{\rm loc}$-*constant*.

We pass this class to variable exponents. We consider local Hardy spaces and Sobolev spaces. We present some applications of this class of weights.

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Real-Variable Theory of Function Spaces Associated with Ball Quasi-Banach Function Spaces

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The concept of ball Quasi-Banach function spaces was introduced in 2017 by Y. Sawano, K.-P. Ho, D. Yang and S. Yang. It is well known that some well-known function spaces, such as Morrey spaces, weighted Lebesgue spaces, mixed-norm Lebesgue spaces, and Orlicz-slice spaces, are ball Quasi-Banach function spaces, but not Quasi-Banach function spaces. In this survey talk, we will introduce some recent developments of the real-variable theory of function spaces associated with ball quasi-Banach function spaces, including the boundedness and the compactness of commutators on ball Banach function spaces, (weak) Hardy spaces associated with ball quasi-Banach function spaces, and Sobolev spaces associated with ball Banach function spaces. This survey talk is based on my several recent joint works with my collaborators.

Abstracts of Sectional Talks

Comparative Analysis of RSA Cryptosystem and Elliptic Curve Cryptography

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Nowadays, technology is developing very fast, so cryptographers are constantly trying to create an improved cryptosystem that is better than the previous one. RSA is a public-key cryptosystem, which is often used as a standard in data security. However, Elliptic Curve Cryptography (ECC), which was created as an alternative to the RSA cryptosystem, is relevant in recent times. For some devices with low computing resources is difficult to perform large-key encryption quickly. Elliptic curve cryptography copes well with this problem. Every public key cryptosystem is constructed on the basis of the complexity of one or more difficult mathematics problems.

The RSA encryption method uses the math problem of factoring [1, 3]. ECC is based on the basis of the elliptic curve discrete logarithm problem [4]. An elliptic curve can be simply illustrated with following equation, whereas, based on a and b values elliptic curve shape is created.

$$y^2 = x^3 + ax + b.$$

This paper evaluates the efficiency of the elliptic curve cryptographic method and the asymmetric RSA algorithm. The conducted comparative analysis showed that elliptic curve cryptography has an advantage in terms of calculation speed over RSA algorithm. The main advantage of ECC system over RSA is that it can provide same security with the smaller key size. For example, for security of 80 bits with ECC can be achieved with key size of 160 bits. For same security level RSA needs to use 1024 bit key size [2, 4]. This huge difference makes ECC better and potential algorithm for the current embedded system.

The future of the technology demands such algorithm that has shorter key size, can maintain high security and consume low computational resources. So, we can predict that ECC could be good alternative for RSA, and will replace it soon.

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Solving Artificial Intelligence Tasks Using Machine Learning Methods

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The purpose of the paper is to develop Machine Learning (ML) methods and use them to solve Artificial Intelligence (AI) tasks in the education process. Machine learning algorithms use historical data as input to predict new output values. Artificial Intelligence has grown into a formidable tool in recent years allowing robots to think and act like humans. A machine learning algorithm allows a pre-designed algorithm to be fed to an "Intelligence Machine" – Robot to produce results. The COVID-19 pandemic has led to an increase in online classes, at this time the lecturer needs more effort to conduct the learning process at a high level. In addition to teaching, student assessment is also important in the learning process. For this, the lecturer additionally needs a lot of resources in correcting student's works. Since this process uses a large information, it is necessary to develop flexible evaluation systems, which will make the process easier for the lecturer and will lead to an objective picture of the assessment. In order to solve this problem, we have developed a student evaluation system using Artificial Intelligence and Machine Learning methods. We used machine learning algorithms and assigned this evaluation process to a "Intelligence Machine" – Robot.

Pell's Equation

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In the school mathematics course we do not often come across equations that need to be solved in the set of whole or natural numbers, although their share is quite high when we talk about school mathematics Olympiads. It is very important for the student to have a rich knowledge in this regard so that he/she can successfully overcome the tasks in front of him/her and, if necessary, create high quality material. One of the important roles in these regards is played by Pell type equations and solid theoretical and practical knowledge of this issue. We tried to present the material to the students and teachers in such a way that it would be clear to them. We would like to add that the mentioned material is also used for the training of the Georgian national team in the preparatory stages for participation in various international Olympiads.

The Pell equation is called the Diophantine equation having the form: $x^2 - dy^2 = 1$, where $x, y \in \mathbb{Z}$ and d are given natural numbers that are not integer squares. In general, the equations of type $x^2 - dy^2 = a$ are called Pell-type equations. Usually, the Diophantine quadratic equation of two variables can be reduced to a Pell type equation. How do we explain them?

As we know, the general solution of a linear Diophantine equation is given as a linear parametric function, however, this is not generally the case with the quadratic Diophantine equation. As we will see later in the case of two variables we will write again, a fairly simple relation describing the general solution of the equation. Here, we can comment, why do we say in the Pell equation that d is not a full square? The reason is simple, so had the left side of the Pell equation been broken down into multiples and its solution would have been less interesting already.

The paper discusses how to solve the Pell type equation, how to write general solutions to such equations (when they exist) and discuss practical tasks, as well as leave a few tasks for the reader to practice and consolidate knowledge.

An Analysis of Positive Solutions to Fractional Differential Equations

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In this study, we have investigated some aspects of the qualitative theory for fractional differential equations. Considering the equation which is equivalent to the Volterra type equation, we have established some new criteria for the asymptotic behavior of positive solutions for the equation.

On Some Properties of the Riesz Potential in the Grand Lebesgue and Grand Sobolev Spaces

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This work considers the Riesz-type potential in non-standard grand-Lebesgue and grand-Sobolev spaces. The classical facts about Lebesgue and Sobolev spaces carry over to this case. The established properties play an important role in the study of the solvability of boundary value problems for an elliptic type equation in Grand Sobolev spaces.

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Blow-Up of Solutions of the Integro-Differential Kelvin–Voight Equation

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In this paper, we study the following initial boundary value problem of determining the pair of functions $(\mathbf{u}(x,t),\ \pi(x,t))$, which satisfy the following integro-differential Kelvin–Voigt equations modified with p-Laplacian diffusion and nonlinear source term

$$\mathbf{u}_t - \operatorname{div}\big(\varkappa\nabla\mathbf{u}_t + \nu|\nabla\mathbf{u}|^{p-2}\nabla\mathbf{u}\big) - \nabla\pi - \int\limits_0^t e^{-(t-\tau)}\Delta\mathbf{u}(x,\tau)\,d\tau = \gamma|\mathbf{u}|^{m-2}\mathbf{u} \ \text{ in } Q_T,$$

$$\operatorname{div}\mathbf{u}(x,t) = 0 \ \text{ in } Q_T,$$

the initial condition

$$\mathbf{u}(x,0) = \mathbf{u}_0(x) \ \text{ in } \Omega,$$

and the Dirihlet boundary condition

$$\mathbf{u}(x,t) = 0 \ \text{ on } \Gamma_T.$$

Here $\mathbf{u}(\mathbf{x},t)$ is the velocity field, $\pi(\mathbf{x},t)$ is the pressure, $\mathbf{u}_0(x)$ is the given initial velocity. The constant ν accounts for the dynamic viscosity, whereas $\varkappa$ is a length scale parameter characterizing the fluid elasticity in the sense that the ratio $\frac{\varkappa}{\nu}$ is a relaxation time scale, i.e. a characteristic time required for a viscoelastic fluid to relax from a deformed state to its equilibrium configuration. The exponents p and m are given positive numbers, such that

$$1 < p, \ \ m < \infty.$$

Acknowledgments

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Algorithm for Finding a Safe Path Based on the Modified Voronoi Diagram Method

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In computer games, algorithms for finding a safe path, determining "enemy" groups, and finding the shortest path to a destination are often used as an artificial intelligence module. All these tasks related to the so-called class of spatial relations are effectively solved by the so-called. through the Voronoi diagram, which allows you to make artificial intelligence decisions effective and real.

Spatial connection is any information that characterizes the relationship of objects in space. For example: the distances between them, the size of areas of influence, the number of objects in one area, and others. Knowledge and analysis of such information is very useful for artificial intelligence. The Voronoi diagram describes the spatial relationship between two adjacent points (objects). But the fact is that with the classical method of Voronoi diagrams, all objects are considered homogeneous, which, obviously, is very far from reality. In addition, information about the surrounding groups of "opponents", the size, the power of the military arsenal, which „comes“ from different sources, is often contradictory.

Based on the modified method of the Voronoi diagram and the theory of fuzzy sets, we have created an algorithm for finding a safe path, taking into account the number of "enemy" groupings, their "armament" and the estimate of their numbers.

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On a Method for Counting Point in Semi-Algebraic Subsets

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The theorems on counting root numbers of real polynomial endomorphism in an arbitrary semi-algebraic set are given and several cases are considered. The estimate of the computational complexity of counting root numbers is obtained. The mentioned facts generalize early results obtained by the author. Particularly the numbers of variables and equations are unrestricted.

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Blow-up of Solutions of a Mixed Problem for Wave Equations with a Nonlinear Transmission Condition and Interior Focusing Source of Variable Order of Growth

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Let $0 \le L_0 < L_1 < L_2$. In the domain of $\bigcup\limits_{i=1}^{2} (L_{i-1}, L_i) \times [0, +\infty)$, consider the mixed problem

$$u_{itt} - (a_i(x)u_{ix})_x = b_i|u_i|^{p_i(x)-2}u_i, \ \ L_{i-1} \le x \le L_i, \ \ t > 0, \ \ i = 1, 2,$$

with boundary conditions

$$u_1(t, L_0) = 0, \quad u_2(t, L_2) = 0, \ \ t > 0,$$

transmission conditions

$$u_1(t, L_1) = u_2(t, L_1) = \varphi(t), \ \ t > 0,$$
$$a_1(L_1)u_{1x}(t, L_1) - a_2(L_1)u_{2x}(t, L_1) + \big[|\varphi'(t)|^{r-2} + d_1\big]\varphi'(t) = d_2|\varphi(t)|^{q-2}\varphi(t), \ \ t > 0$$

and initial conditions

$$u_i(0, x) = u_{i0}(x), \quad u_{it}(0, x) = u_{i1'}(x), \ \ L_{i-1} \le x \le L_i, \ \ i = 1, 2,$$

where $a_i(\,\cdot\,) \in C[0, L]$, $a_i(x) \ge a_{i0} > 0$, $2 \le p_{i1} \le p_i(x) \le p_{i2} < +\infty$, $L_{i-1} \le x \le L_i$, $b_i, d_i \in [0, +\infty)$, $i = 1, 2$, $r, q \in [2, +\infty)$ and $p_i(\,\cdot\,)$, $i = 1, 2$ satisfies the log-Hölder continuity condition (see for example [1, 2] and bibliography in [1, 2]). Theorems on local solvability are proved and various theorems on the blow-up of solutions in finite time are obtained.

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Towards a Grammar for a Georgian CNL

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Controlled natural languages (CNLs) are engineered languages that are based on natural language, but have their vocabulary, syntax, and/or semantics restricted [1]. Our goal is to develop a CNL for Georgian which is not bound to any specific domain, has a clearly defined logical semantics, and capabilities to be adapted to specific areas. In this talk, we discuss challenges and progress towards developing a grammar for such a CNL for Georgian.

Acknowledgements

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An Inverse Problem for Heat Convection System of Kelvin–Voigt Fluids

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In this work, we consider the following inverse source problem for the heat convection system for the incompressible viscoelastic non-isothermal Kelvin–Voigt fluids [1, 2]

$$\mathbf{v}_t + (\mathbf{v}\cdot\nabla)\mathbf{v} - \varkappa\Delta\mathbf{v}_t - \nu\Delta\mathbf{v} + \nabla\pi = \mathbf{g}(\mathbf{x},t)\theta(x,t) + f(t)\mathbf{h}(\mathbf{x},t),\ \ (x,t)\in Q_T, \tag{1}$$

$$\operatorname{div}\mathbf{v}(\mathbf{x},t) = 0,\ \ (x,t)\in Q_T, \tag{2}$$

$$\theta_t + (\mathbf{v}\cdot\nabla)\theta - \lambda\Delta\theta = j(t)\phi(\mathbf{x},t),\ \ (x,t)\in Q_T, \tag{3}$$

$$\mathbf{v}(\mathbf{x},0) = \mathbf{v}_0(\mathbf{x}),\quad \theta(\mathbf{x},0) = \theta_0(\mathbf{x}),\ \ \mathbf{x}\in\Omega, \tag{4}$$

$$\mathbf{v}(\mathbf{x},t) = 0,\quad \theta(\mathbf{x},t) = 0,\ \ (\mathbf{x},t)\in\Gamma_T, \tag{5}$$

$$\int\limits_\Omega \mathbf{v}\sigma(\mathbf{x})\,d\mathbf{x} = e(t),\quad \int\limits_\Omega \theta\eta(\mathbf{x})\,d\mathbf{x} = \delta(t),\ \ t\ge 0,\ \ \text{where}\ \ \sigma(\mathbf{x}) = \omega(\mathbf{x}) - \varkappa\Delta\omega(\mathbf{x}), \tag{6}$$

here Ω is a bounded domain in $\mathbb{R}^d$, $d = 2,3$, with the smooth boundary $\partial\Omega$, and $Q_T = \Omega\times(0,T)$ is a cylinder with lateral $\Gamma_T = \partial\Omega\times[0,T]$.

The investigating inverse problem consists of determining the set of functions $(\mathbf{v}(\mathbf{x},t)$, $\pi(\mathbf{x},t)$, $\theta(\mathbf{x},t)$, $f(t)$, $j(t))$ from (1)–(6) with the given functions $\mathbf{g}$, $\mathbf{h}$, ϕ, $\mathbf{v}_0$, θ_0, ω, $e(t)$, $\delta(t)$, and the constants ν, $\varkappa$, λ.

Here, under suitable assumptions on the data of the problem, we establish the global and local in time existence and uniqueness of solutions to the inverse problem (1)–(6).

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Approximate Solution of Some Boundary Value Problems for Elastic Plates with Voids

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The report considers the Cowin–Nunziato linear model for elastic bodies with voids. From the corresponding three-dimensional basic equations, the two-dimensional equations for plates of constant thickness are derived by the I. Vekua method. The general solution of the system of tension-compression equations is represented by two harmonic functions and the solution of the Helmholtz equation. Based on the constructed general solution, using the method of fundamental solutions, an algorithm for constructing an approximate solution of boundary value problems is presented. This algorithm is used to solve boundary value problems for square plates with circular holes.

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On Selection of "Correct" Machine Learning Algorithm

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Machine learning is a branch of computer science where, using artificial intelligence, machines can automatically learn and improve their methods based on experience [2]. For a machine, the learning process begins with observations or data, examples, direct experience, or instruction to look for patterns in the data and make better decisions in the future based on the examples we give. This allows computers to learn automatically without human intervention and help and adjust their behavior accordingly. Another outstanding feature of machine learning is that it can improve predictions based on experience. Just like we humans do – we consider the success or failure of previous attempts before making the next decision. Many techniques [1, 3, 4], can be used to make decisions in machine learning, such as statistics-based algorithms, automated analytical models, and neural networks. Despite the abundance of similar algorithms, selecting the right algorithm for a particular task is a constant challenge in the field [5].

In the paper, we discuss modern approaches to solving machine learning problems. We have shown in more detail some common methods of trained machine learning and given ways to solve them using the python programming language. In the tasks, we have used examples that are based on well-known data sets and make it clear that depending on the nature of the task, the correctly selected method is of crucial importance.

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On Identification of Different Parts of Speech for Georgian Text Processing

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The integration of statistical/probabilistic methods and syntactic/semantic methods is an ideal solution for increasing the efficiency of natural language processing tasks [2]. Existing natural language processing methods primarily based on processing data presented in widely spoken languages like English [3]. The most famous stemming algorithms like Lovin's and Porter's algorithms were created first for English, and then modified for different languages [6]. However, due to complexity of the Georgian language, it is almost impossible to use them for Georgian [4, 5]. The most difficult part of speech is the verb. Verbs have lots of conjugated forms. For instance Kartu–Verbs [1] contain 16000 verbs, which generates 3 million conjugated verb forms. In total, there are 80 million connections between the characteristics of their collection of verbs. To process text identification of various parts of speach plays an important role.

In the article, we presented our algorithm that can recognize different parts of speech to process text further.

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Comparison of Prediction Models: Bayesian Method and Parallel Data

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The use of the Bayesian method in building models in forecasting is based on Bayesian analysis and is represented by a simple formula

$$P(A \mid B) = \frac{P(A) * P(B \mid A)}{P(B)}\,.$$

The meaning of which is as follows: the probability of occurrence of event A if we have event B is equal. The probability of occurrence of event A multiplied by (the probability of occurrence of event B conditional on the occurrence of event A) divided by the probability of event B.

Later, the rapid development of the speed and capacity of computers led to the Bayesian method Both the theoretical development and the rapid growth of its applications in various ways of life in the fields.

The use of the Bayesian method in the tasks of tactogenic cataclysms is especially noteworthy If there are many models for predicting the event. If we have the opportunity I use many models to predict a single event, and each one predicts a large one When an event occurs with probability, then it can be said that this event is very likely will happen.

The use of the Bayesian method in the tasks of tactogenic cataclysms is especially noteworthy If there are many models for predicting the event. If we have the opportunity I use many models to predict a single event, and each one predicts a large one When an event occurs with probability, then it can be said that this event is very likely will happen.

Unfortunately, small countries have a large number of models for the predecessors of accounting They do not have the means, which is related to large financial support, as a special Installation of hardware as well as receiving and processing of its data in dynamic mode. Therefore, here, as in other areas, parallel data[1,2][hybrid can be used successfully Using a model that selects from these 500 pairs, triplets, etc. (how many models the country will also have the opportunity to use it) will receive a new model, the event of which The probability of occurrence predictions will be very close to using 500 models simultaneously probability

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Convergence of Differentiated General Dirihlet's Integrals

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In 1983 A. A. Zakharov [3] proved the theorem about the convergence of differentiated Fourier series of $f \in L(0, 2\pi)$ function. Taberski [1, 2] considered real-valued Lebesgue locally integrable functions f, such that

$$\lim_{T\to\infty} \frac{1}{T} \int\limits_{T}^{T+c} |f(t)|\,dt = 0; \quad \lim_{T\to\infty} \frac{1}{T} \int\limits_{-T-c}^{-T} |f(t)|\,dt = 0$$

for every fixed $c > 0$. For this class of functions he defined generalized Dirichlet's integrals. Besides, Taberski [1, 2] investigated problems of convergence and $(C, 1)$-summability of these integrals.

In this talk, the analogous of A. A. Zakharov's theorem for the generalized Dirichlet's integrals is proved.

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Integro-Differential Equations with Boundary Conditions and Their Properties

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On the interval $[0,T]$ consider system of linear integro-differential equations

$$\frac{dx}{dt} = A(t)x + \int_0^T K(t,s)x(s)\,ds + f(t), \quad x \in \mathbb{R}^n, \tag{1}$$

$$Bx(0) + Cx(T) = d, \quad d \in \mathbb{R}^n, \tag{2}$$

where $x(t) = \operatorname{colon}(x_1(t), x_2(t), \dots, x_n(t))$ is unknown vector function; the $(n \times n)$ matrix $A(t)$ and n vector function $f(t)$ are continuous on $[0,T]$; the $(n \times n)$ matrix $K(t,s)$ is continuous on $[0,T] \times [0,T]$; the B, C are constant $(n \times n)$ matrices.

The solution of problem $(1), (2)$ is defined as a function $x(t) \in C([0,T], \mathbb{R}^n)$ continuously differentiable on $(0,T)$ and satisfying the integro-differential equation (1) and the boundary condition (2).

Recently, based on the parametrization method [1] and a partition of the interval $[0,T]$ was proposed a method for solving problem $(1), (2)$ in [2]. Necessary and sufficient conditions for solvability, including the unique solvability of problem $(1), (2)$ were obtained in terms of a matrix $Q_{*,*}(h)$ constructed from the fundamental matrix of the differential part of system (1), the matrices in boundary conditions (2), and the resolvent of an auxiliary Fredholm integral equation of the second kind.

In present communication we are also discussed results in [3–5].

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Characterized of σ-Derivations on Some Elementary Operators Algebras

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In 2002, Lu and Li proved the following result [2, Theorem 6]: Let B be a standard operator algebra in a Banach space X containing the identity operator I and let $\delta : B \to B$ be a linear map such that

$$\delta(AB) = \delta(A)B + A\delta(B) \text{ for any pair } A, B \in B$$

with $AB = 0$. Then

$$\delta(AB) = \delta(A)B + A\delta(B) - A\delta(I)B \text{ for all } A, B \in B.$$

If in addition $\delta(I) = 0$, then δ is a derivation. In other words, the result says that an additive map on a standard operator algebra is almost a derivation if it satisfies the expansion formula of derivations on pairs of elements with zero product. Since standard operator algebras involve many idempotents, from this point of view Chebotar, Ke and P.-H.Lee studied maps acting on zero products in the context of prime rings [1]. Tsiu-Kwen Lee [3] posted, let A be a prime ring whose symmetric Martindale quotient ring contains a nontrivial idempotent. Generalized skew derivations of A are characterized by acting on zero products. Precisely, if $g, \delta : A \to A$ are additive maps such that

$$\sigma(x)g(y) + \delta(x)y = 0 \text{ for all } x, y \in A$$

with $xy = 0$, where is an automorphism of A, then both g and δ are characterized as specific generalized -derivations on a nonzero ideal of A.

In this paper we will generalize that theorem from a different point of view. We will study this subject without out the condition that maps acting on zero products via using zero commutative ring. An additive map $d : R \to R$ is called a derivation if the Leibniz's rule $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Also, let σ be an automorphism of a prime ring A. An additive map $\delta : A \to Qml$ is called a σ-derivation if

$$\delta(xy) = \sigma(x)\delta(y) + \delta(x)y \text{ for all } x, y \in A.$$

Furthermore, a ring R is called zero commutative if for $a, b \in R, ab = 0$ implies $ba = 0$ (used the term reversible for what is called zero commutative).

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A Note on the Generalized Hausdorff and Packing Measures of Product Sets in Metric Space

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Let μ and ν be two Borel probability measures on two separable metric spaces $\mathbb{X}$ and $\mathbb{Y}$, respectively. For h, g be two Hausdorff functions and $q \in \mathbb{R}$, we introduce and investigate the generalized pseudo-packing measure $\mathcal{R}_{\mu}^{q,h}$ and the weighted generalized packing measure $\mathcal{Q}_{\mu}^{q,h}$ to give some product inequalities:

$$\mathcal{H}_{\mu\times\nu}^{q,hg}(E\times F) \le \mathcal{H}_{\mu}^{q,h}(E)\,\mathcal{R}_{\nu}^{q,g}(F) \le \mathcal{R}_{\mu\times\nu}^{q,hg}(E\times F)$$

and

$$\mathcal{P}_{\mu\times\nu}^{q,hg}(E\times F) \le \mathcal{Q}_{\mu}^{q,h}(E)\,\mathcal{P}_{\nu}^{q,g}(F)$$

for all $E \subseteq \mathbb{X}$ and $F \subseteq \mathbb{Y}$, where $\mathcal{H}_{\mu}^{q,h}$ and $\mathcal{P}_{\mu}^{q,h}$ is the generalized Hausdorff and packing measures respectively. As an application, we prove that under appropriate geometric conditions, there exists a constant c such that

$$\begin{aligned}
\mathcal{H}_{\mu\times\nu}^{q,hg}(E\times F) &\le c\,\mathcal{H}_{\mu}^{q,h}(E)\,\mathcal{P}_{\nu}^{q,g}(F),\\
\mathcal{H}_{\mu}^{q,h}(E)\,\mathcal{P}_{\nu}^{q,g}(F) &\le c\,\mathcal{P}_{\mu}^{q,hg}(E\times F),\\
\mathcal{P}_{\mu\times\nu}^{q,hg}(E\times F) &\le c\,\mathcal{P}_{\mu}^{q,h}(E)\,\mathcal{P}_{\nu}^{q,g}(F).
\end{aligned}$$

These appropriate inequalities are more refined than well know results since we do no assumptions on μ, ν, h and g.

A Note on Fractal Measures and Cartesian Product Sets

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We are interested on the study of the lower t-dimensional Hewitt–Stromberg measure H^t and the upper t-dimensional Hewitt–Stromberg measure P^t. More precisely, we are trying to answer the question: for E and F two subsets of $\mathbb{R}^d$ and $\mathbb{R}^l$, does there exist a constant γ such that $\Lambda^{s+t}(E\times F)=\gamma\Lambda^s(E)\Lambda^t(F)$? where $\Lambda\in\{\mathsf{H},\mathsf{P}\}$. In fact, such results are far from being true. For example, in [2], it is proved that,

$$\mathsf{b}(E\times F)\geq \mathsf{b}(E)+\mathsf{b}(F)$$

where b stands for the lower Hewitt–Stomberg dimension. However, it is possible to construct two subsets E and F such that $\mathsf{b}(E\times F)>\mathsf{b}(E)+\mathsf{b}(F)$. Similar results were proved for s-dimensional Hausdorff measure $\mathcal{H}^s$ and s-dimensional packing measure $\mathcal{P}^s$ [1, 4, 5]. The reader can see also [3, 6] for various results on this problem.

Our aim is to construct a metric outer measure, denoted H^{*t}, that is comparable to the Hewitt–Stromberg measure H^t but significantly easier to analyse.

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A Relative Vectorial Multifractal Formalism

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In this paper, we give a new vectorial multifractal formalism for which the classical multifractal formalism does not hold. We precisely introduce and study a vectorial multifractal formalism based on the Hewitt–Stromberg measures This formalism is parallel to Peyrière's multifractal formalism which is based on the Hausdorff and packing measures.

On the One Nonparametric Estimate of Poisson Regression Function

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The limiting distribution of the integral square deviation of kernel-type nonparametric estimator of Poisson regression function is established. The test of the hypothesis testing about Poisson regression function is constructed. The question of consistency of the constructed test is studied. The power asymptotic of the constructed test is also studied for certain types of close alternatives.

On a Cauchy Problem with Closed Support of Data and its Discrete Analogue

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We consider the Cauchy problem on a closed data support for one class quasi-linear equations of mixed type [1].

We have proved that the Cauchy problem is well posed. The families of characteristic curves are described and the area of definition of the solution is constructed. To solve the problem, a difference scheme is constructed, the approximation and stability of the scheme is studied. Calculation results are given for concrete examples.

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On Cohomological Realization of the Buchstaber Formal Group Law

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In this talk we give a construction of multiplicative complex-oriented cohomology theory, the coefficient ring of which is the coefficient ring of the Buchstaber formal group low with inverted 2.

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On Perfect and Potentially Convergence Systems

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Dadicated to *Hans Adolph Rademacher German-born American mathematician, known for work in mathematical analysis and number theory (1892–1969)*

An orthonormal sequence $\varphi_n \in H = L_2[0,1]$, $n = 1, 2, \dots$ is called a *convergence system* if for every sequence $(a_n)_{n\in\mathbb{N}} \in l_2$ the series $\sum_n a_n\varphi_n(t)$ converges in $\mathbb{R}$ for almost all $t \in [0,1]$.

Let us call an orthonormal sequence $(\varphi_n)_{n\in\mathbb{N}}$

- a *perfect* convergence system, if for every bijection $\sigma : \mathbb{N} \to \mathbb{N}$ the (rearranged) sequence $(\varphi_{\sigma(n)})_{n\in\mathbb{N}}$ is a convergence system;
- a *potentially* convergence system, if there exists a bijection $\sigma : \mathbb{N} \to \mathbb{N}$ such that the sequence $(\varphi_{\sigma(n)})_{n\in\mathbb{N}}$ is a convergence system.

It is known that:

- The first perfect convergence system was found by Rademacher in 1922.
- The first orthonormal sequence, which **is not** a convergence system was found by Menchov in 1923.
- The trigonometric system **is not** a perfect convergence system. (A brief outline of the proof of this result, conjectured by A. N. Kolmogorov in 1927, was published by Z. Zahorski in 1960; cf. [1, p. 65]).
- A complete orthonormal sequence cannot be a perfect convergence system, see [1, Theorem 1 (p. 65)].

According to [1, p. 51] "…the following problem which goes back to A. N. Kolmogorov remains open": *prove that every orthonormal sequence is potentially convergence system.*

We plan to discuss several recent results related with this problem, which **remains open so far**.

The talk is based on joint work with co-author (Here the name of co-author or co-authors if not indicated above).

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Classification of s-Norms in Orlicz Spaces

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Let (X, Σ, μ) be a σ-finite, non-atomic and complete measurable space. On Orlicz spaces of measurable functions $L^\Phi(X, \Sigma, \mu)$, the classical Orlicz and Luxemburg norm can be defined by use of the Amemiya formula:

$$\|f\| = \inf_{k>0} \frac{1}{k}\,(1 + I_\Phi(kf)) \ \text{ and } \ \|f\| = \inf_{k>0} \frac{1}{k}\, \max\big\{1, I_\Phi(kf)\big\},$$

respectively, where Φ is an Orlicz function and $I_\Phi(f) = \int\limits_X \Phi(f(t))\, d\mu(t)$. Based on this observation in [1], Y. Cui, L. Duan, H. Hudzik and M. Wisla studied geometric properties of Orlicz spaces $L^\Phi(X, \Sigma, \mu)$ such as extreme points and rotundity with more general norm so-called p-Amemiya norms $(1 \leqslant p \leqslant \infty)$ defined by $\|f\|_p = \inf\limits_{k>0} \frac{1}{k}\, s_p(\Phi(kf))$ here s_p is defined by

$$s_p(u) = \begin{cases} (1+u^p)^{1/p}, & \text{if } \ 1 \leqslant p < \infty \\ \max\{1, u\}, & \text{if } \ p = \infty. \end{cases}$$

In [2], M. Wisla generalized p-Amemiya norm $(1 \leqslant p \leqslant \infty)$ via s function. A function $s : [0, \infty] \to [1, \infty]$ is called outer function if it is convex and provides the following inequality $\max\{u, 1\} \leqslant s(u) \leqslant u + 1$ for all $u \geqslant 0$. Using the concept of outer function, s-norms $\|f\| = \inf\limits_{k>0} \frac{1}{k}\, s(I_\Phi(kf))$ are introduced. In this study, we consider about some geometric properties of Orlicz spaces $L^\Phi(X, \Sigma, \mu)$ with respect to s-norms which include Orlicz, Luxemburg and p-Amemiya norm $(1 \leqslant p \leqslant \infty)$ also give some classification of s-norms with respect to $\sigma_s = \sup\{u \geqslant 0 : \ s(u) = 1\}$ in Orlicz spaces $L^\Phi(X, \Sigma, \mu)$.

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Closedness of $\operatorname{Ext} B(L_s^\Phi)$

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Let (X, Σ, μ) be a measure space with non-atomic, σ-finite, complete measurable space. Let Φ be an Orlicz function and L_s^Φ denote Orlicz spaces equipped with s-norms which are more general norms then Orlicz, Luxemburg and p-Amemiya norms. $B(L^\Phi)$ denote the closed unit ball of Orlicz space L^Φ and $\operatorname{Ext} B(L^\Phi)$ denote the set of extreme points of $B(L^\Phi)$.

Let Ω be a compact Hausdorff space, E be a dual of Orlicz space L_s^Φ. The map $N' : \Omega \to E$ that is defined as $(N'\omega)(y) = (Ny)(\omega)$ for $\omega \in \Omega$ and $y \in E$. N is called as nice if $N'(\Omega) \subseteq \operatorname{Ext} B(L_s^\Phi)$. It is known that any compact nice operator N is extremal [1]. For the converse, if the set $\operatorname{Ext} B(L_s^\Phi)$ is closed then extremal operators are nice. In this study we investigate some criteria for the closedness of $\operatorname{Ext} B(L_s^\Phi)$.

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Extreme Points of Unit Ball in Orlicz Spaces Equipped with s-Norms

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Let (X, Σ, μ) be σ-finite, non-atomic and complete measurable space, let Φ be an Orlicz function and $L^{\Phi}(X, \Sigma, \mu)$ be the corresponding Orlicz space. In [1], they defined a family of new norms (called p-Amemiya norms) which covers classical norms such as the Orlicz norm and the Luxemburg norm. Besides defining this family, they presented a criteria for extreme points of unit ball in Orlicz spaces equipped with these norms. On the other hand, in order to present a general and universal method of defining norms in Orlicz spaces, s-norms were introduced by using the concept of outer function in [2]. The family of s-norms is a wider family that includes the p-Amemiya norms, thus also the Orlicz and Luxemburg norms. In this study, we present criteria for extreme points of unit ball in Orlicz spaces equipped with s-norms for two different classes of outer functions. Our study generalizes the results obtained in [1].

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Basic Principles of Functioning and Structure of the Adding Machine Giorgi Nikoladze

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Very few people know the fact that Giorgi Nikoladze, a versatile scientist, engineer, and doctor at the Sorbonne University, was interested in computational technology at some point in his career and that he made some contribution to the creation of the computing machine.

Computing machines and machines were no longer a novelty in the early twentieth century. Giorgi Nikoladze, of course, was familiar with the pre-existing computing devices, the works of previous scientists.

On January 16, 1928. French scientist Maurice Dokan presented to scientists invention of a young Georgian doctor, Giorgi Nikoladze, at the French Academy.

"The machine I am assuming to build is completely automatic: the result of the action is obtained immediately after entering two digits by simply pressing the button", wrote Giorgi Nikoladze.

As Giorgi Nikoladze himself wrote, his calculating machine consisted of two parts. One of them was intended for addition and subtraction, and the other for multiplication-division operation.

Unfortunately, G. Nikoladze could not get a patent and make a car model in France due to lack of funds.

Only after his return to Georgia was made Nikoladze's reporting model. It was exhibited at the Polytechnic Museum in Moscow. Unfortunately, later this model was lost and has not been found to date. Neither the drawings nor the description of the model are left.

This is how the extraordinary invention of the talented Georgian scientist was lost and today in the history of computing technology G. Nikoladze's name is not even mentioned.

Giorgi Nikoladze – Georgian Mathematician and Metallurgist, Inventor of Original Calculating Tool

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Everyone in Georgia knows Giorgi Nikoladze as a great scientist and engineer. He was a mathematician-theorist, metallurgist-practitioner, creator of Georgian scientific terminology, gymnast and mountaineer, founder of Georgian (and Soviet) mountaineering.

The report discusses one of the most interesting and lesser-known aspects of the Georgian scientist's work. During his business trip to Paris, Nikoladze inspected an international technical exhibition, where he paid attention to computing, mechanical computing devices.

He studied the existing works on computing devices at that time, analyzed various types of computing instruments and created a new, original calculating device – the electric arithmeter.

On January 16, 1928, at the French Academy of Sciences, French scientist Maurice D'Ocan introduced a young Georgian doctor to the assembled scientists and introduced his invention. Giorgi Nikoladze Paris prepared the description of his arithmeter (in French, of course) and the corresponding drawings.

He wanted to apply for a patent, as well as to make an arithmeter model, but only after his return to Georgia did it become possible to build a calculation model, which G. After Nikoladze's death he was sent to the Moscow Polytechnic Museum for exhibition. Later this model disappeared without a trace.

This is how the original invention of the talented young Georgian scientist was lost. The Soviet government did not allocated money was allocated for Nikoladze's invention, he only mentioned that "…the invented electric meter is of great scientific interest".

On Some Properties of Small Sets

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Let $(G, \cdot)$ be a arbitrary group and let μ be a nonzero σ-finite G-invariant (more generally, G-quasiinvariant) measure defined on some σ-algebra of subsets of G. We recall that the symbol $I(\mu)$ denotes the σ-ideal of subsets of G, generated by the family of all μ-measure zero sets. Members of $I(\mu)$ are usually called small sets with respect to the given measure μ.

Lemma 2 *Let $(H, \cdot)$ be an uncountable group (commutative or noncommutative) and let μ be a nonzero σ-finite H-invariant measure on H. If*

$$\varphi : G \to H$$

is a surjective homomorphism and there exist a nonzero σ-finite H-left invariant measure $\mu' \supset \mu$ and two sets $X \in I(\mu')$ and $Y \in I(\mu')$ on H such that

$$X \cdot Y \notin dom(\mu'),$$

then there exist measures ν and ν' on G and two sets $X' \in I(\nu')$ and $Y' \in I(\nu')$ on G for which the following relations are satisfied:

(a) $\nu' \supset \nu$;

(b) $X' \cdot Y' \notin dom(\nu')$;

(c) ν *and* ν' *are G-left invariant measures on G.*

About of the method of surjective homomorphisms see, [1, 2].

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The Uniform Subsets and the Problem of Generalized Nonmeasurability

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There is well known properties of uniform sets in the plane connected with the notion of measurability.

- Every uniform set is negligible (see [1, 2]).
- There exist uniform sets which are not absolutely negligible(see [1,2]).
- For any straight line l in the plane, there exists a invariant measure μ on R^2 which extends the standard Lebesgue measure and is such that all uniform sets in direction l belong to $dom(\mu)$ (see [2]).

The goal of the presented talk is to discuss briefly uniform subsets of the Euclidean plane in the context of their nonmeasurability in some generalized sense. In particular, under the assumption that **c** is not measurable in Ulam's sense, we consider the problem of generalized nonmeasurability for Uniform sets.

Acknowledgements

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On Combinatorial Properties of Algebraic Plane Curves

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If we have a curve Z in the plane R^2, then the following question arises at once: is this Z an algebraic curve? We need some set-theoretical concepts which enable us to answer this question in certain nontrivial situations.

For example, there exists a straight line in the plane that intersects our Z in countable many, but not in finitely many points. Then we can definitely assert that Z is not algebraic curve.

In our presentation we discuss more examples, applications of set-theoretical methods by studying algebraic curves in the Euclidean plane and their combinatorial properties.

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Challenges of Problem-Based Learning in Georgian Secondary Schools

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The reform of the Georgian education system includes different stages. The reform policy at the level of general education is clearly visible based on the analysis of the national curricula. The third-generation national curriculum made the general educational schools face new challenges; teachers had to deal with a new dilemma: how to develop the skills of critical thinking, innovative approaches in the student. In the national curriculum of the new generation, great importance is given to problem-based learning, which is particularly important in Mathematics and STEAM subjects.

In the research, we reviewed the principles of PBL at the general education level, using the example of some European countries, we studied what support teachers in different European countries have for their professional development. Based on the example of Georgian secondary schools, we studied what support the Mathematics teachers have in terms of professional development.

The results of the study also displayed that the principles of PBL are often understood and applied in practice by teachers unevenly, which prevents the effectiveness of the method, as well as the achievement of the results set by the national curriculum.

In the final part of the paper, we present recommendations based on international experience and research results for the correct and effective application of project-based learning approach in the teaching of Mathematics at the general education level.

Keywords: project-based learning, critical thinking, creativity, national curriculum, learning outcome.

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Numerical Solution of One Contact Problem with Nonlocal Boundary Conditions for Elliptic Equation

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In this paper, we pose and study a nonlocal contact problem for linear elliptic equations. To solve the stated problem, a method is used that makes it possible to reduce the solution of a nonlocal contact problem to the solution of a sequence of classical boundary value problems.

Then the adjoint problem with nonlocal contact conditions for the elliptic equation is considered, and an iterative method for the numerical solution of the problem is constructed and studied. The iterative method makes it possible to reduce the solution of a non-classical non-local contact problem to the solution of a sequence of classical boundary-value problems. A numerical experiment has been carried out. The results are consistent with the theoretical conclusions and show the effectiveness of the proposed iterative procedure.

Complete Systems of Invariants of a Parametric Figure in the Three Dimensional Minkowski Space

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In this work, "the concept of a parametric figure" is defined in three dimensional Minkowski space. By this concept, the complete systems of invariants of m-point, paths, vector fields, etc. are investigated as a single theory. Using this approach, we obtain the complete systems of invariants of parametric figures and the equivalence conditions of two parametric figures are given in terms of their global invariants.

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Georgian Text Classification by Using Spark

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The 21st century represents an age of information. The data is generated very quickly. Typically, the generated data is unstructured or partially structured. In natural language processing (NLP) tasks, data classification plays a major role (among them, classification/categorization of text data, summarization). This problem is relevant today both in information technology, and in industry and business). The main purpose of text classification is to automate the process of categorizing a text document in one or more different groups. Some examples of text classification are sentiment analysis, topic detection, and language detection. There are various NLP libraries and applications on the market for the data classification (such as spaCy, NLTK, CoreNLP, Spark NLP, and etc.) that are mainly focused on processing English text. Processing Georgian text is one of the most important issues. The difficulties of processing Georgian text are due to the specifics of the language. This document comparatively analyzes the tasks of processing Georgian text using different existing processing methods. The usage of these methods are compared in light of processing Georgian and English texts.

Subgroup Lattices of Hall's W-Power Groups

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We study Hall's W-power nilpotent groups from the lattice standpoint. A relationship between the structure of a W-power group G and the structure of the lattice of its subgroups $L(G)$ is established.

Acknowledgements

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Stochastic Epidemic Model with Limited Treatment

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One of the most ubiquitous epidemic models is the class of SIR models [2]. In such models the population is divided into three categories: susceptible, infected and recovered individuals. The dynamics of the categories are described by differential or stochastic equations.

This study is based on a stochastic SIR epidemic model with limited treatment [1]. Such model takes into account the random fluctuations in the transmission of an infectious disease, as well as limited capacity of medical facilities.

The following equations describe the numbers of susceptible, infected and recovered individuals:

$$\begin{aligned} dS(t) &= \big[S(t)(K - S(t)) - \beta S(t)I(t) - Y(S(t), I(t))\big]\,dt - \varepsilon S(t)I(t)\,dW(t),\\ dI(t) &= \Big[\beta S(t)I(t) - \mu I(t) - \frac{rI(t)}{a + I(t)}\Big]\,dt + \varepsilon S(t)I(t)\,dW(t),\\ dR(t) &= \Big[\frac{rI(t)}{a + I(t)} + Y(S(t), I(t)) - \mu R(t)\Big]\,dt. \end{aligned}$$

Here K, β, ε, μ, r, a are various parameters, $Y(S(t), I(t))$ – vaccination strategy.

The goal is to obtain the vaccination strategy that minimizes the cost functional:

$$\Psi_Y = E\int\limits_0^\tau \Big(m_1 S(t) + m_2 I(t) + rY^2(S(t), I(t))\Big)\,dt.$$

We obtain several properties of the model and present a partial differential equation, such that the optimal vaccination strategy is its unique solution with zero boundary conditions.

Simulation is used to implement the model and obtain the optimal vaccination strategy by finding the numerical solution to the partial differential equation.

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Extreme Values of Potentials of Spherical Designs and the Polarization Problem

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We use interpolation and properties of Gegenbauer polynomials to find points of absolute extremum over the unit sphere S^d in $\mathbb{R}^{d+1}$ of the total potential of certain regular point configurations $\omega_N \subset S^d$. The interaction between points is described by the kernel $K(\mathbf{x}, \mathbf{y}) = f(|\mathbf{x} - \mathbf{y}|^2)$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^{d+1}$. The configuration ω_N is assumed to be

A. a spherical $(2m-1)$-design contained in the union of some m parallel hyperplanes (we find absolute minima),

B. a $2m$-design with m values of the distance between distinct points in ω_N (we find both absolute maxima and absolute minima),

C. an antipodal $(2m-1)$-design with m values of the distance between distinct points in ω_N (we find absolute maxima).

The potential function f is assumed to have a convex derivative $f^{(2m-2)}$ (in cases A and C) or a concave derivative $f^{(2m-1)}$ (in case B).

These results are applied to the problem min-max polarization. It requires finding N-point configurations on the sphere $S^d \subset \mathbb{R}^{d+1}$ with the smallest absolute maximum value over S^d of its total potential (for a fixed N). We show the optimality of any configuration in cases B and C for this problem.

We remark that the problem of finding extreme values of the potential of vertices of a regular N-gon inscribed in S^1 and of a regular simplex, regular cross-polytope, and a cube inscribed in S^d was solved by Stolarsky (1975) and Nikolov and Rafailov (2011, 2013) for Riesz potentials. Each of these configurations falls in one of the cases, A, B, or C. A review of known results on polarization problem as well as on the technique we use (known as linear programming or Delsarte–Yudin bound) can be found, for example, in [1, Chapters 5 and 14].

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Mathematical Language at School and its Functions

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Linguistically, there were no readily-available methods in mathematics, therefore, it has become necessary to create methods of mathematical logic and abstract algebra that became a model for a new science. Mathematical tools are used in linguistics as a kind of mechanism by which the speaker senses the meanings or "thoughts" in his brain and transforms them into the texts.

In order for a teacher to have a good understanding of the essence of mathematical thinking, the patterns of its development and the role of such thinking in the whole system of students' personal development, he/she must have a very good understanding of issues such as: how the mathematical language originated and evolved, what the mathematical language is, where the mathematical language is used and spread, and whether the mathematical language is universal and and why.

Modern mathematics has highly developed sign systems that enable people to reflect the purest thought processes.

Knowledge of mathematical language provides a great opportunity to analyze the whole process of scientific thinking and cognition.

The article considers when the use of mathematical symbols started and when the mathematical symbols were introduced, as well as the role of a distinguished mathematician Francois Viete in creating the algebraic symbols.

Derivation of the Ginzburg–Landau Equation for a Convective Flow in a Vertical Fluid Layer

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Suppose that viscous incompressible fluid is located in the region between two infinite parallel vertical planes. It is assumed that the fluid is chemically reacting so that heat is released as a result of the chemical reaction in accordance with the Arrhenius Law:

$$Q = Q_0 \exp\Big[-\frac{E}{RT}\Big],$$

where E is the activation energy, R is the universal gas constant, Q_0 is a constant and T is the absolute temperature. Internal heat sources generate a steady base flow in the vertical direction. Critical values of the Grashof number Gr_c, wavenumber k_c and phase velocity c_c are obtained using linear stability analysis. It is shown that for small Prandtl numbers ($Pr \to 0$) one can neglect the effect of thermal factors and consider only the Navier-Stokes equations with the given base flow velocity profile $v_0(x)$.

In order to analyze the development of instability in the neighborhood of the critical point (Gr_c, k_c, c_c) under the assumption that the Grashof number is slightly larger than the critical value, namely, $Gr = Gr_c(1+\varepsilon^2)$, the method of multiple scales [1] is used to develop partial differential equation for the amplitude of the most unstable mode. The stream function $\psi(x,\xi,t,\tau)$ is expaned in a power series in ε, where $\xi = \varepsilon(x - c_g t)$ and $\tau = \varepsilon^2 t$ are slow longitudinal coordinate and time (c_g is the group velocity):

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + \cdots .$$

Using solvability condition at order three we obtain the Ginzburg–Landau equation [2] for the amplitude of the most unstable mode in the form

$$\frac{\partial A}{\partial \tau} = \sigma A + \delta\,\frac{\partial^2 A}{\partial \xi^2} + \mu A|A|^2,$$

where σ, δ and μ are complex coefficients that are determined in terms of integrals containing the solution of several linear problems.

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Computational Schemes and Their Numerical Realization for Boundary Value Problems for ODE with Generalized Solutions

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The report presents the results of research on the construction and study of difference schemes for traditional problems of mathematical physics, the accuracy of which is consistent with the smoothness of the generalized solution of the initial differential problem [3].

Another part of the results presented in the report relates to the construction of compact difference schemes of any order of accuracy for nonlinear ODEs based on the theory of exact difference schemes and the application of one-step methods for solving Cauchy problems [1, 2].

For II order ODE, the case of boundary problems with Sturm–Liouville conditions is studied, when the operator is either monotonic or satisfies the Picard–Lettenmeyer–Schauder conditions [4]. In the work also the computation schemes for ODE systems [5] are built and the issues arising during numerical implementation are investigated.

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Fefferman–Stein Theory on Lie Groups of Polynomial Growth: Geometric Aspects and the Role of the Symmetries

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Oscillating Fourier multipliers on the torus and on R^n play a fundamental role in analysis and PDE and in the setting of Lie groups are still a subject of intensive research. The classical results by Fefferman and Stein in this direction (published between 1970 and 1972 in Acta Math.) have consolidated a fundamental theory for the harmonic analysis of these operators, even, in a more general setting that contains Calderón–Zygmund singular integrals of convolution type. In this talk, we present some recent results that extend the Fefferman and Stein theory of oscillating Fourier multipliers to arbitrary Lie groups of polynomial growth. We also present the L^p-theory of pseudo-differential operators on graded/compact Lie groups.

This talk is based on the joint works with Michael Ruzhansky.

Global Pseudo-Differential Operators on the Lie Group $G = (-1, 1)^n$

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In this work we characterise the Hörmander classes $\mathbb{S}^m_{\rho,\delta}(G, \text{Hör})$ on the open manifold $G = (-1, 1)^n$. We show that by endowing the open manifold $G = (-1, 1)^n$ with a group structure, the corresponding global Fourier analysis on the group allows to define a global notion of symbol on the phase space $G \times \mathbb{R}^n$. Then, the class of pseudo-differential operators associated to the global Hörmander classes $\mathbb{S}^m_{\rho,\delta}(G \times \mathbb{R}^n)$ recover the Hörmander classes $\mathbb{S}^m_{\rho,\delta}(G, \text{loc})$ defined by local coordinate systems. The analytic and qualitative properties from the classes $\mathbb{S}^m_{\rho,\delta}(G \times \mathbb{R}^n)$ are presented in terms of the corresponding global symbols. In particular, $\mathbb{L}^p$-Fefferman type estimates and Calderón–Vaillancourt theorems are analysed.

Claim Risk Model Estimation for Non-Life Insurance

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It's necessary risk models to be estimated by the insurance company in order to predict the number of the claim and prevent losses and insurer bankrupty in the future. In this paper, we discuss the estimation of risk model claims. We assumed that the frequency of claims follow a Poisson distribution, while a number of claims assumed to follow a Gamma distribution. The estimation of parameters of the distribution of the frequency and amount of claims are made by using Bayesian methods. Furthermore, the estimator distribution of frequency and amount of claims are used to estimate the aggregate risk models as well as the value of the mean and variance. we discuss the maximum likelihood and Bayesian estimation methods. Maximum Likelihood is one of the most commonly methods used for performing an estimation of parameters.

In the analysis of non-life insurance, the risk distribution model of loss is an important concern for insurance companies. The risk distribution model is very useful in determining the premium to be paid by the insured to the insurer. We discuss the collective risk model and estimate Parameters of claim amount model and parameters of claim frequency model. In this paper the main aim is to study the results and discussions that include: analyzed specific data, an estimation of the claims frequency distribution model and the estimation of the claims amount distribution model using Easyfit software. To solve the mentioned problem, using the Software is quite an interesting, modern and easy-to-understand method from the point of view of visualization.

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In the European Union with the Georgian and Abkhazian Languages – the Concept of Development of Georgian and Abkhazian Language Technologies in Georgia

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At the presentation on the bases of "Unified program (strategy) of the official language" (Published in 2021), "Organic Law of Georgia on Official Language" (Published in 2015) and, also, on the base of our publications [1–5] we will overview the Concept of Development of Georgian and Abkhazian Language Technologies in Georgia. The concept is elaborated within the Scientific-Educational Center for Cultural Protection and Technological Development of Georgian State Languages at the Georgian Technical University together with collaboration Aleksandre Maskharashvili, Lasha Abzianidze, Temur Kutsia, Besik Dundua, Mikheil Rukhaia, and David Kurtskhalia.

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Bipolar and Three-Pole Political Mathematical Models

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Over the past few decades, mathematical modeling of social processes, such as information warfares, language globalization, assimilation of peoples, political conflicts, etc. has been of particular interest [1–4].

Currently, the ongoing military events related to the Russian invasion of Ukraine are directly related to Russia's desire to change the natural course of development of geopolitical influence on the world policy of the three main powers and associations (USA, China, EU). As you know, their economic components in world GDP (Gross Domestic Product) are respectively 23,9%, 15,9%, 24%. For comparison, Russia's contribution to world GDP is 1,9%, less than India, Brazil, etc.

This paper proposes new nonlinear mathematical models describing both a bipolar (USA, China) and a three-pole (USA, China, EU) system of real influence on world politics. Mathematical models are described by two-dimensional and three-dimensional nonlinear dynamic systems with variable coefficients and corresponding initial conditions characterizing the current state of influence of the main world actors. The models consider the conditions for rationing solutions, which imply a complete redistribution of world influence in the case of a bipolar world between the United States and China, in the case of a three-pole one-between the United States, China and the EU, and the contribution of other countries is considered insignificant.

In the mathematical model of the bipolar arrangement of the world, exact analytical solutions are obtained in quadratures, showing the dynamics of changes in the influence of these two powers on world politics, i.e. changes in their relative contribution to the redistribution of world influence.

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On Investigation and Numerical Solution of One Nonlinear Fourth-Order Integro-Differential Model

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A system of fourth-order nonlinear integro-differential equations is considered. The second-order analogous models partially are derived, on one hand, from the description of real diffusion processes and on the other hand, in the generalization of well-known equations and systems of equations, the study of which devoted many scientific papers (see, for instance, [2, 3, 4] and references therein). Such type higher order models are also studied in some other works (see, for instance, [1, 3, 5] and references therein). In our research uniqueness and stability of the solution of the initial-boundary value problem for fourth-order models are studied. The approximate algorithm based on the finite-difference scheme is constructed and corresponding numerical experiments are fulfilled.

Acknowledgements

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About One Functional Inequality

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The study of differential properties of generalized solutions of boundary value problems for the poly-harmonic equation $(-\Delta)^m u = f$ in the neighborhood of non-regular boundary points of two-dimensional domain in often is based on integral estimates of the solutions [1–4].

These estimates, in turn, can be obtained by positiveness on the space $\overset{\circ}{H}{}^m([0,\omega])$ of the functional

$$F(z) = (-1)^m \int\limits_0^\omega z(\varphi) L(a, \partial\varphi) z(\varphi)\, d\varphi,$$

where L is linear ordinary differential operator defined in the following form:

$$L(a, \partial\varphi) : z(\varphi) \longmapsto r^{-a+m+1} \Delta^{2m} (r^{a+m-1} z(\varphi)),$$

$\partial\varphi = \frac{d}{d\varphi}$, $\omega \in (0, 2\pi]$, $a = const > 0$ and (r, φ) are polar coordinates on the plane.

This work is devoted to prove that the functional $F(z)$ is positive for any $a \in [0, \delta(\omega))$, where $\delta(\omega)$ is a smallest of the positive real parts of the poles of resolvent of $L(\delta, \partial\varphi)$ operator on the space $H^{2m}([0,\omega]) \cap \overset{\circ}{H}{}^m([0,\omega])$. The poles are solutions of the following equation: $w(\delta, \omega) = 0$, where w(δ, ω) equal to value when $\varphi = \omega/2$ of multiplication of Wronskians of the following function systems

$$\big\{ \sin(\delta + m - 2k + 1)\varphi \big\}_{k=1}^m, \quad \big\{ \cos(\delta + m - 2k + 1)\varphi \big\}_{k=1}^m.$$

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Idempotents and Radical Classes of Rings

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A (Kurosh–Amitsur) *radical class* of rings is a non-empty homomorphically closed class $\mathcal{R}$ such that

(i) $\forall\, A$, $\sum\{I \triangleleft A:\ I \in \mathcal{R}\} \in \mathcal{R}$;

(ii) $\forall\, A$, $\mathcal{R}(A/\mathcal{R}(A)) = 0$.

Examples

Jacobson radical class: the class $\mathcal{J}$ of *quasiregular* rings $[(\forall\, a)(\exists\, b)(a+b+ab=0)]$.

Nil radical class: class of *nil* rings $[(\forall\, a)(\exists\, n)(a^n=0)]$.

Idempotent radical class: class of rings A for which $A^2 = A$.

Clearly if $I \triangleleft A$ and $I \in \mathcal{R}$, then $I \subseteq \mathcal{R}(A)$. The analogous statements for left ideals and subrings are not true in general, but much work has been done on radical classes for which they *are* true.

Our concern is with *corners*, subrings of the form eAe, where e is an idempotent in a ring A, and it is instructive to compare radical theoretic results involving corners with analogous results involving other types of subrings. We call a radical class $\mathcal{R}$

- *corner-hereditary* if it satisfies $e^2 = e \in A \in \mathcal{R} \Longrightarrow eAe \in \mathcal{R}$;
- *very corner-hereditary* if it satisfies $e^2 = e \in A \Longrightarrow \mathcal{R}(eAe) = eAe \cap \mathcal{R}(A)$;
- *corner-strict* if it satisfies $e^2 = e \in A \& eAe \in \mathcal{R} \Longrightarrow eAe \subseteq \mathcal{R}(A)$.

Very corner-hereditary radical classes are corner-hereditary but not conversely in general. The radical class of strongly regular rings is corner-hereditary but not very corner-hereditary. $\mathcal{J}$ is very corner hereditary; $\mathcal{N}$ is corner-hereditary, but whether it is very corner-hereditary or not is equivalent to the *Köthe Problem*. If $\mathcal{R}$ satisfies the condition

$$e^2 = e \in A \& \mathcal{R}(A) = 0 \Longrightarrow \mathcal{R}(eAe) = 0$$

then $\mathcal{R}$ is corner-strict but the converse is false. This parallels known results for left strong radical classes and left ideals (cf. also strict radical classes and subrings).

Every radical class is hereditary for the corners corresponding to *central* idempotents as these are direct summands and hence homomorphic images. Less obviously, all radical classes are hereditary for the corners corresponding to *left semicentral* idempotents (those for which $Ae - eAe$). Nevertheless these idempotents are relevant to some other constructions which yield radical classes. For certain properties (P) of idempotents, the class of rings A for which every non-zero homomorphic image has a non-zero idempotent with (P) is a radical class. When (P) is the property of simply being an idempotent, we get the Brown–McCoy radical, while for central idempotents we get the lower radical class defined by the class of rings with identity.

Thermo-Elastic and Thermo-Piezo-Elastic Interaction Crack Type Problems with Regard of the Microrotation

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We investigated interaction boundary-transmission problems of pseudo-oscillations between thermo-elastic and thermo-piezo-elastic bodies taking into account microrotations. The model under consideration is based on the Green–Haghdi theory of thermo-piezo-electricity without energy dissipation. This theory permits propagation of thermal waves only with finite speed. Using the potential theory and boundary pseudodifferential equations method, we prove existence and uniqueness of solutions, and analyze their asymptotic properties near the interface crack adge, we obtain optimal regularity of solutions.

Study of Some Energy Characteristics of Air Flow in the Rioni River Valley

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The strong wind regime and statistical characteristics of the Imereti region were researched according to the data of the Kutaisi Meteorological Station. For the period 1960–2021, the wind speeds are divided into intervals of $5\,m/s$, and for each interval the wind speed recurrence rate is studied by months. The paper presents the percentage distribution of wind speed gradations and the change in their average values over the years and months. It has been determined that in terms of energy, the main range of wind speed for the Kutaisi region is 16–20 m/s. It should also be mentioned that the wind values at intervals of 20–25 m/s in summer are minimal compared to the wind values in other seasons. But from an energy point of view, this is less important because a wind speed interval of 16–20 m/s ensures maximum efficiency of wind energy use. Thus, from an energy point of view, speeds of such magnitude are essential, which ensure the automatic mode of the wind farms and are an important basis for the development of wind power plants in western Georgia.

One Economical Method of Solving the Multidimensional Non-Stationary Problem for the System of Hyperbolic Equations

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In the present work, the one economical difference scheme for solving the multidimensional non-stationary problem is considered for the system of hyperbolic equations with mixed derivatives.

The proposed scheme is a factorized difference scheme, constructed using the method of regularization. The absolute stability and convergence are proved for this difference scheme. The computational algorithm for implementing this scheme on the parallel system is considered.

The proposed method can be successfully used for the numerical solution of dynamic problems of elasticity theory.

Modeling Sensitivity of the Caucasus Glaciers to Regional Warming

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This paper proposes a simple mathematical model of the dynamics of change in the thickness of glaciers in the Caucasus, based on the integration of nonlinearly generated differential equations. To some extent the model takes into account the change in the mass balance of the glacier due to the direct solar radiation. A scheme similar to the Lax–Wendroff scheme is used to numerically solve the nonlinear PDE. Some typical problems inherent in mathematical and numerical modeling of glaciers are discussed. For the first time, the process of melting of some glaciers in the Caucasus has been assessed using mathematical modeling. Some simulation results are presented and analyzed.

The Planning Principles of Topical Unit-Matrix in Mathematics on the Elementary Level

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An on-going educational reform in Georgia aims at raising quality of education in schools so that they ensure a proper upbringing of competitive mindful citizens. A goal of National Curriculum III is to provide three categories of knowledge to students. While previous generations of Curriculum focused on factual mastering and the advancement of related skills and procedures, National Curriculum III has introduced purpose concepts, great ideas, and long-term goals through which students will develop advanced understanding skills. With this regard, it is essential that a teacher properly plans topical unit-matrix.

Present work highlights principles of matrix planning for elementary level, and proposes complete matrix for grade 5 in geometry and space understanding by focusing on where the teacher starts completing the matrix and what sorts of exercises could be used to do complex assignments in each phase.

Optimal Control Problem for One Contact Problem with Nonlocal Boundary Conditions

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In this paper considers a nonlocal contact boundary value problem for partial differential equations of elliptic type. A linear optimal control problem with an integral performance criterion is posed. For a contact nonlocal boundary value problem, a necessary and sufficient optimality condition is obtained. To study the adjoint problem, an iterative process has been constructed that makes it possible to reduce the solution of the original problem to the solution of a sequence of Dirichlet problems. In the Sobolev space, the existence and uniqueness of a generalized solution of the conjugate problem is proved. On the basis of necessary and sufficient optimality conditions, a numerical algorithm for solving a linear optimal control problem is proposed.

Construction of Mutually Unambiguous Linear Transformations for Solving Edge Problems of Dirichlet–Neumann, Excited by Gaussian Random Fields in the Euclidean Space R^n

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The proposed work consists of two parts. These theses cover only the first part. The second part will be published later.

Let $\{\Omega, \gamma, p\}$ a fixed probabilistic space. In a measurable Euclidean space (R^n, γ_R^n), where γ_R^n denotes σ the algebra of measurable subsets of the Euclidean space R^n, two boundary problems of Dirichle Neuman are considered. It is assumed that Q – some open connected area from the n-dimensional Euclidean space R^n, $Q \subset R^n$ with a homogeneous and isotropic medium. These Dirichlet Neuman problems are considered in the region Q for elliptic differential equations of the same order of 2k and excited by the same Gaussian field $\xi(x, \omega)$, $x \in R^n$ with a nuclear correlation operator R_ξ, and the task of existence and uniqueness is raised with a probability of equal to the unit of their solution $u(x, \omega)$ and $\nu(x, \omega)$, accordingly. For this, another Dirichlet Neuman problem is considered for the main differential order of 2k, for which the Green function $G_0(x, y)$ si built in space R^n, which in space $L_2(Q)$ – in the space of real functions integrated with ins square as Lebesba. As the kernel, the integral operator of Hilbert Schmidt $\mathbf{G_0}$ will generate. Application, further, this operator $\mathbf{G_0}$ to the differential equations considered in two boundary value problems and, given some properties of the Green function, we reduce these differential equations of boundary value problems with two linear transformations

$$\mathfrak{L}_1(u(x, \omega)) = \xi(x, \omega) \ \text{ and } \ \mathfrak{L}_2(\nu(x, \omega)) = \xi(x, \omega)$$

with the same source of perturbation $\xi(x, \omega)$, where converted fields $u(x, \omega)$ and $\nu(x, \omega)$, if $\mathfrak{L}_1[\,\cdot\,]$, $\mathfrak{L}_2[\,\cdot\,]$ mutually unequivocal transformations are the only solution of the regional-neimane boundary problems under consideration, and since these transformations are linear, it is obvious that their transformed quantities $u(x, \omega)$ and $\nu(x, \omega)$ are also gaussian fields, so the original field $\xi(x, \omega)$, is Gaussian. Further, as is known from the theory of random functions and random processes, random fields $u(x, \omega)$ and $\nu(x, \omega)$ generate probabilistic measures μ_u and μ_ν to σ algebra γ_R^n and, accordingly, in this case, Gaussian measures.

The second part of this work is fully devoted to the establishment of conditions that ensure absolute continuity $(\mu_u \| |\mu_\nu)$ and measures μ_u and μ_ν $(\mu_u \sim \mu_\nu)$ and strict mathematical methods of evidence of some of the statements discussed in the first part of this work and will also be calculated by the corresponding density of Radon–Nicodem.

Epistemic Łukasiewicz Logic of Partial Knowledge

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We offer a new logic, called Epistemic Łukasiewicz logic of partial knowledge that is represented as multimodal epistemic Ł ukasiewicz logic $\text{KŁ}_P(n)$ with n knowledge operators $\Box_i$ $(1 \leq i \leq n)$ interpreted in a non-archimedean monadic MV-algebra. We choose knowledge operators, which can be estimated by some grading (different kinds of knowledge): absolute knowledge or partial knowledge. We consider a very special type of partial knowledge. Actually, we take infinitesimal elements (the radical) of perfect MV-algebras as a range of this estimation. The choice of infinitesimal elements seems suitable for actual situations like a measure of partial information.

Grothendieck Group of a Locally Finite Category

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Let R be a Dedekind domain, F be its field of fractions. For a prime ideal $\mathfrak{p} \subset R$ we denote by $R_{\mathfrak{p}}$ the $\mathfrak{p}$-adic completion of R and by $F_{\mathfrak{p}}$ is field of fractions.

An *R-category* is, by definition, a category $\mathbf{A}$ such that all sets of morphisms $\mathbf{A}(A, B)$ are R-modules and the composition of morphisms is R-bilinear. We call such a category *locally finite* if all R-modules $\mathbf{A}(A, B)$ are finitely generated. We always suppose that the category $\mathbf{A}$ is additive and *Karoubian*, i.e. all its idempotents split. By $F \otimes \mathbf{A}$ we denote the category with the same objects as $\mathbf{A}$ but with the sets of morpisms $F \bigotimes\limits_R \mathbf{A}(A, B)$. As this category is not necessary Karoubian, we consider the category $F\mathbf{A}$ obtained from it by splitting all idempotents. The notations $R_{\mathfrak{p}}\mathbf{A}$ and $F_{\mathfrak{p}}\mathbf{A}$ have analogous meaning.

The *Grothendieck group* $K_0(\mathbf{A})$ of the category $\mathbf{A}$ is the abelian group generated by the symbols $[A]$, where $A \in \operatorname{Ob} \mathbf{A}$, with the relations $[A] = [B] + [C]$ when $A \simeq B \oplus C$. We denote by $\overline{K_0}(R_{\mathfrak{p}}\mathbf{A})$ the kernel of the natural map $K_0(R_{\mathfrak{p}}\mathbf{A}) \to K_0(F_{\mathfrak{p}}\mathbf{A})$. Note that $K_0(F\mathbf{A})$ and $K_0(R_{\mathfrak{p}}\mathbf{A})$ are free groups generated by the symbols $[A]$, where A runs through isomorphisms classes of indecomposable objects of the corresponding categories.

We suppose that the locally finite category $\mathbf{A}$ satisfies the *Max-condition*, namely: For every object $U \in \operatorname{Ob} F\mathbf{A}$ there is an object $C_U \in \operatorname{Ob} \mathbf{A}$ such that $C_U \simeq U$ in the category $F\mathbf{A}$ and $E_U = \operatorname{End}(C_U)/\operatorname{nil}(C_U)$ is a maximal order in the semisimple F-algebra $F \bigotimes\limits_R E(U)$ [3], where $\operatorname{nil}(C_U)$ denotes the nil-radical of the ring $\operatorname{End}(C_U)$.

Theorem *Let $\mathbf{A}$ be a locally finite R-category satisfying the Max-condition. Then*

$$K_0(\mathbf{A}) \simeq K_0(F\mathbf{A}) \oplus \Big(\bigoplus_{\mathfrak{p}} \overline{K}_0(R_p\mathbf{A}) \Big) \oplus \operatorname{Cl}(\mathbf{A}),$$

where

$$\operatorname{Cl}(\mathbf{A}) = \bigoplus_U \operatorname{Cl}(E_U).$$

Here $\mathfrak{p}$ runs through the maximal ideals of R, U runs through isomorphism classes of the category $F\mathbf{A}$ and $\operatorname{Cl}(E_U)$ denotes the group of the ideal classes of the maximal order E_U [3].

For details and proofs see [1]. As an immediate consquence of this result, we obtain the well-known Freyd's theorem about the Grothendieck group of the category of finite CW-complexes [2].

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Mixed and Transmission Type Boundary Value Problems for the Helmholtz Equation on a Surface with Lipschitz Boundary

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We have studied the solution of a transmission type boundary value problem for the anisotropic Helmholtz equation in the non-classical setting on a surface with Lipschitz boundary. At first the problem is reduced by the localization method (cf. [1]) to several model problems at flat angles consisting of two and three beams emerging from point 0. Mixed type (Dirichlet–Neumann) boundary conditions on the boundary beams and transmission condition on the interface beam are given. We apply the potential method and reduce the boundary problem to the system of boundary integral equations, which represent a system of Mellin-type convolution equations in the Bessel potential space on the half-axis. By using the resent results on such equations, obtained in cf. [2], we write symbol of equations and formulate criteria for solvability (Fredholmness) of such systems of equations in the Bessel potential spaces.

For the model angle the mixed type BVP were investigated in [3]. We obtain necessary and sufficient conditions for the solvability of the model BVP for two angles with the interface along a beam. The final result is the solvability criteria for the mixed type BVP for the anisotropic Helmholtz equation in the non-classical setting on a surface with interface and with Lipschitz boundary.

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On Functional (Co)Homology Groups of Completely Regular Spaces and Applications

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In the report, using the set of functionally open finite covers [2] of completely regular spaces, for each integer number $n \geq 0$ there are constructed Čech type functional homology functor $\check{H}^F_n(-,-;G) : \mathbf{Top^2_{cr}} \to \mathbf{Ab}$ and functional cohomology functor $\hat{H}^n_F(-,-;G) : \mathbf{Top^2_{cr}} \to \mathbf{Ab}$ from the category of pairs of completely regular spaces and their completely closed subspaces to the category of abelian groups, defined Bokshtein–Nowak type functional coefficient of cyclicity $\eta^F_G(X)$, large and small functional cohomological dimensions $D_F(X;G)$ and $d_F(X;G)$ of completely regular space X and proved the equalities

$$\check{H}^F_n(X,A;G) = \check{H}_n(\beta X,\beta A;G), \quad \hat{H}^n_F(X,A;G) = \hat{H}^n(\beta X,\beta A;G),$$
$$\eta^F_G(X) = \eta_G(\beta X), \quad D_F(X;G) = D(\beta X;G), \quad d_F(X;G) = d(\beta X;G),$$

where $\check{H}_n(\beta X,\beta A;G)$, $\hat{H}^n(\beta X,\beta A;G)$ [1], $\eta_G(\beta X)$, $D_F(X;G)$ and $d_F(X;G)$ are Čech homology group, Čech cohomology group, Bokshtein–Nowak coefficient of cyclisity, large and small cohomological dimensions of Stone–Čech compactifications of pair $(X,A) \in \mathrm{ob}(\mathbf{Top^2_{cr}})$ and completely regular space X, respectively.

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Solving Symbolic Equations with Multiple Similarity Relations

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Similarity relations are reflexive, symmetric, and transitive fuzzy relations. They help to make approximate inferences, replacing the notion of equality. Similarity-based symbolic equations solving has been quite intensively investigated, as a core computational method for approximate reasoning and declarative programming.

In many practical situations, one needs to deal with several similarities between the objects from the same set, where examples about building online fashion compatibility representation and understanding visual similarities are considered in the context of learning image embeddings. Multiple similarities pose challenges to equation solving, since we can not rely on the transitivity property anymore. Existing methods for solving symbolic equations with fuzzy proximity relations (reflexive, symmetric, non-transitive relations) do not provide a solution that would adequately reflect particularities of dealing with multiple similarities.

In this talk we discuss solving symbolic equations over several similarity relations, instead of a single one. We developed a constraint solving algorithm for multiple similarity relations and showed that the algorithm is terminated, sound, and complete. This research has been published in [1].

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Probabilistic Unranked Predicate Logic

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Probability theory deals with the challenges posed by uncertainty, while logic is more used for reasoning with perfect knowledge. Probabilistic logic combines capability of probability and logic. It gives expressive and flexible platform to model and reason problems coming from with Artificial Intelligence (AI).

Unranked predicate logic is an variant of predicate logic with function symbols having flexible arity. Such an extension brings flexibility and expressiveness in the language to model and reason with unstructured data.

In this talk we propose probabilistic extension of unranked predicate logic. In particular, we discuss syntax, semantic and inference mechanism of the extended formalism – probabilistic unranked logic.

Sumability of a Double Trigonometric Fourier Series by Riemann's R^2 Method

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According to Ch. Fefferman's theorem [1], there exists a 2π periodic with respect to every variable and continuous at every point of the plane function of two variables whose double trigonometric Fourier series has no even one point of rectangular convergence. This implies that the methods of summability of double series are more important than those in the case of one-dimensional series.

The present report states that any double trigonometric Fourier series is almost everywhere summable by the iterated methods and by Riemann's R^2 method.

Theorem *For any summable on a square* $[-\pi,\pi]^2$ *function* f *and its corresponding double trigonometric Fourier series*

$$\sum_{m,n=-\infty}^{+\infty} c_{mn} e^{i(mx+ny)}$$

the following equalities

$$\lim_{k\to 0}\lim_{h\to 0}\sum_{m,n=-\infty}^{+\infty} c_{mn}^{i(mx+ny)}\Big(\frac{\sin mh}{mh}\Big)^2\Big(\frac{\sin nk}{nk}\Big)^2=f(x,y),$$

$$\lim_{h\to 0}\lim_{k\to 0}\sum_{m,n=-\infty}^{+\infty} c_{mn}^{i(mx+ny)}\Big(\frac{\sin mh}{mh}\Big)^2\Big(\frac{\sin nk}{nk}\Big)^2=f(x,y),$$

$$\lim_{(h,k)\to(0,0)}\sum_{m,n=-\infty}^{+\infty} c_{mn}^{i(mx+ny)}\Big(\frac{\sin mh}{mh}\Big)^2\Big(\frac{\sin nk}{nk}\Big)^2=f(x,y)$$

are fulfilled at almost all points $(x,y)\in[-\pi,\pi]^2$.

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Localized Boundary-Domain Integral Equations Method of Nonhomogeneous Isotropic Couple-Stress Elasticity Theory for Dirichlet and Neumann Pseudo-Oscillation Problems

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The paper deals with the three-dimensional Dirichlet and Neumann boundary value pseudo-oscillation problems (BVPs) of the couple-stress elasticity theory for isotropic inhomogeneous solids and develops the generalized potential method based on the localized parametrix method. Using Green's integral representation formula and properties of the localized layer and volume potentials we reduce the Dirichlet and Neumann BVPs to the localized boundary-domain integral equations (LBDIE) systems. The equivalence between the Dirichlet, Neumann BVPs and the corresponding LBDIE systems is studied. We establish that the obtained localized boundary-domain integral operators belong to the Boutet de Monvel algebra and with the help of the Wiener–Hopf factorization method we investigate corresponding Fredholm properties and prove invertibility of the localized operators in appropriate Sobolev function spaces.

Theoretical Modeling, Numerical Simulation and Experimental Investigation of the Nonlinear Processes in the Near Earth Space Plasma

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The intensification and further evolution of internal-gravity wave (IGW) structures in the dissipative ionosphere in the presence of a nonuniform zonal wind (a sheared flow) is investigated. The efficiency of the linear amplification of IGW structures in their interaction with a nonuniform zonal wind is analyzed. When there are sheared flows, the operators of linear problems are non-self-conjugate and the corresponding eigenfunctions are nonorthogonal, so the canonical modal approach is poorly suited for studying such motions and it is necessary to utilize the so-called non-modal mathematical analysis. It is shown that, in the linear evolutionary stage, IGW efficiently extract energy from the sheared flow, thereby substantially increasing their amplitude and, accordingly, energy (by several orders). As the shear instability develops and the perturbation amplitude grows, a nonlinear self-localization mechanism comes into play and the process ends with the self-organization of nonlinear, highly localized, solitary IG vortex structures. The system thus acquires a new degree of freedom, thereby providing a new way for the perturbation to evolve in a medium with a sheared flow. Depending on the shape of the sheared flow velocity profile, nonlinear structures can be either purely monopole vortices or vortex streets against the background of the zonal wind. The accumulation of such vortices can lead to a strongly turbulent state in an ionospheric medium.

As far as the terrestrial magnetosheath is a highly turbulent medium, with a high level of magnetic1field fluctuations throughout a broad range of scales. These often include an inertial range where a magnetohydrodynamic turbulent cascade is observed. The multifractal properties of the turbulent cascade, strictly related to intermittency, are observed here during the transition from quasi-parallel to quasi-perpendicular magnetic field with respect to the bow-shock normal. The different multifractal behavior in the two regions is analyzed. A standard coarse-graining technique has been used to evaluate the generalized dimensions D_q and the corresponding multifractal spectrum $f(\alpha)$. A p-model fit provided a quantitative measure of multifractality and intermittency, to be compared with standard indicators: the width of the multifractal spectrum, the peak of the kurtosis, and its scaling exponent. Results show a clear transition and sharp differences in the intermittency properties for the two regions.

Keywords: Magnetosheath, Turbulence, Multifractal.

Regional Climate Modeling: Challenges, and Prospects in Georgia

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Atmosphere-Ocean Global Climate Models are considered the most advanced tools to make climate change projections. Their output can be used as the basis of climate change risk and vulnerability assessments for large countries and regions and for the establishment of general adaptation and mitigation strategies. However, most AOGCM climate change simulations are performed at resolutions (> 100 km) too coarse to adequately resolve the complex topography, coastlines, and weather systems required understand climate change impacts for some smaller regions, such as Georgian. As a result, regional climate models (RCMs) are often implemented to zoom into a specific region and dynamically downscale AOGCM output to enhance the AOGCM information to a level sufficient for impacts purposes and policymakers. In Georgia there are very few climate modeling studies at a high spatial resolution, and the highest special resolution so far is 20 km. Given Georgia's complex topography, coastlines, hydrology, and weather, even a 20 km resolution is still quite coarse for climate change assessments. Thus, today one of the most important challenges for Georgia is the development of a reliable high resolution (< 20 km) regional climate model (RegCM4) customized specifically for the Georgia region.

Under the project: "Assessing the Projected Impacts of Climate Change in Georgia Using a Very High-Resolution Climate Model" financed by Shota Rustaveli National science foundation of Georgia, we are developing 12 km resolution model for Georgia. This study is very significant from both scientific and societal perspectives, as it will

(1) provide the highest resolution climate information to date for the Georgian region using the latest climate change projections;

(2) enhance the field of climate change not only in Georgia but also internationally;

(3) provide quality impact level information and assessments important for local stakeholders and regional and global policymakers.

The talk is based on joint work with co-authors: Elizbara Elizbarashvili, Tamar Khuntselia, George Mikuchadze, Magda Tsintsadze, Tsezari Mshvenieradze.

Acknowledgements

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Character Degree Graphs of Rational Groups

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Let G be a finite group and $\operatorname{Irr}(G)$ be the set of all irreducible complex characters of G. The set of all irreducible complex character degrees of G is denoted by $\operatorname{cd}(G)$ so that

$$\operatorname{cd}(G) = \big\{\chi(1) : \ \chi \in \operatorname{Irr}(G)\big\}.$$

Let $\rho(G)$ be the set of all primes that divide some irreducible character degrees in $\operatorname{cd}(G)$. The character degree graph of G, denoted by $\triangle(G)$, is the graph whose vertex set is $\rho(G)$. Two distinct vertices p, q in $\rho(G)$ are connected by an edge iff there exists at least one degree $a \in \operatorname{cd}(G)$ such that pq divides a. Manz, Willems and Wolf proved in [2] that the character degree graph $\triangle(G)$ of a finite group G has at most three connected components and if G is solvable, then $\triangle(G)$ has at most two connected components. Pálfy showed in [5] that each connected component of the character degree graph of a solvable group must be a complete graph. In [3], Lewis has given a complete classification of solvable groups whose character degree graphs are disconnected. Then in [4], Lewis and White have discribed the structure of nonsolvable groups whose character degree graphs are disconnected. A finite group all of whose complex character values are rational is called a rational group. Motivated by these results, in [1], we classify all rational groups whose character degree graphs are disconnected. Throughout this talk, I will mention that character degree graphs of rational groups.

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Spectral Properties of Two Classes of Toeplitz Operators on H^p, $1 < p < \infty$

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We consider two classes of Toeplitz operators on H^p, $1 < p < \infty$: Toeplitz operators with unimodular symbols and Toeplitz operators whose spectra satisfy a specific geometric condition. We give some inclusions for their spectrum and some estimates for their resolvents. Using obtained results, we show the existence of nontrivial invariant subspaces of these types of Toeplitz operators. This result gives a partially answer to the question of which type operators on a Banach space has a nontrivial invariant subspace.

Asymptotic Properties and Approximate Solution of Initial-Boundary Value Problem for One Nonlinear Partial Differential Model

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A one-dimensional system of nonlinear partial differential equations is considered. Such type models are based on the well-known Maxwell system and are studied in many works (see, for instance, [1–5] and references therein). The asymptotic behavior of solution for initial-boundary value problem as time variable tends to infinity is studied. The question of linear stability of the stationary solution of the system and the possibility of the Hopf-type bifurcation is investigated. Finite difference scheme is constructed and corresponding numerical experiments are carried out.s

Acknowledgements

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About Teaching English and Georgian Conjugation in Georgian Schools

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During the presentation I will overview main differences between Georgian and Indo-European Conjugations [1-7] on the basis of K. Pkhakadze's linguo-logical approaches developed in his Logical Grammar of Georgian Language finally elaborated within the framework of FR/362/4-105/12 project *"Foundations of Logical Grammar of Georgian Language and Its Application in Information Technology"*. After, I will attempt to prove that, in order to improve the teaching process of English and Georgian languages in Georgian schools, it will be beneficial to teach Georgian conjugation in the way it is presented in K. Pkhakadze's Logical Grammar of Georgian Language.

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On the Algorithm of an Approximate Solution and Numerical Computations for the J. Ball Nonlinear Integro-Differential Equation

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An initial-boundary value problem is posed for the J. Ball integro-differential equation, which describes the dynamic state of a beam (see [1]). A physical model that J. Ball uses in the article [1] is taken from the handbook of Engineering Mechanics written by E. Mettler (see [2]). For this model, he wrote the corresponding initial-boundary value problem for the integro-differential equation of beam [1]. The presented article is a direct continuation of the articles [3, 4] that consider the construction of algorithms and their corresponding numerical computations for the approximate solution of nonlinear integro-differential equations of the Timoshenko type. In particular, in this work, it is considered an initial-boundary value problem for the J. Ball integro-differential equation, which describes the dynamic state of a beam [1]. The solution is approximated utilizing the Galerkin method, stable symmetrical difference scheme and the Jacobi iteration method. In the articles [3, 4] the algorithm is approved by tests. This paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam are represented in the tables. Issues of the initial-boundary value problem of the iron beam are studied for the following meanings of parameters: spatial, temporal, mathematical algorithm and physical nature of the beam.

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Oscillation-Free Momentum Optimization Algorithms

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In the last decade, the field of application of artificial neural networks and its methods, in particular, numerical algorithms of unconditional minimization, have been continuously expanding. The result of the research carried out in this direction is the intensive use of new variants of momentum minimization algorithms in artificial neural networks.

The presented work develops an approach that allows us to create modified, faster momentum algorithms based on exited momentum minimization algorithms. The obtained modification monotonically decreases the objective function and does not allow it to oscillate. Such an approach has already been implemented (see [1]) for the classical momentum algorithm, i.e. the Heavy Ball algorithm of the Polyak. Every time during the execution of the Heavy Ball algorithm (the same classical momentum or HB for short) the value of the function starts to increase, the modified Heavy Ball (MHB) will restart itself, what (at least on the first step) will guarantee to decrease the value of the function. The second difference is that each restart uses line search to determine the learning rate.

Experiments on the selection of the descent direction in the minimization algorithm led us to the idea of adding restarts and line searches to the Nesterov's accelerated gradient (the same NAG. See [2, 3]), in order to obtain an oscillation-free, very fast momentum method.

To determine the efficiency of the new algorithm, numerical experiments were conducted both on standard optimization test functions and on single-layer neural networks for several popular datasets. Comparisons were made with the best unconstrained minimization algorithms – lcg and lbgfs, as well as with ADAM, which is widely used in neural networks (see [3]). In [1] it was shown that MHB is competitive with lcg and lbgfs. Recent experiments have shown that the modified Nesterov accelerated gradient method is even more promising. Especially on neural networks.

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Building Skills of Reasoning in Mathematics

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Thinking and speech are closely related but are not the same, so there can be no complete correspondence between reasoning and a sentence. One reasoning can be expressed in different sentences, but there are sentences that do not express any reasoning, because reasoning is characterized by the meaning of trueness (erroneousness), so its expression can only be given by statement. Just what is the reasoning up to? In the practice of thinking and speech, we often use various affirmations and denials, such as "the bases of a trapezoid are parallel", "there is no triangle in Euclidean geometry with two right angles", and so on. Through such constructions, the presence or absence of a certain sign in a specific subject, the different states of this subject, the relations between the subjects are fixed in the consciousness, and such an affirmation or denial is called reasoning.

Reasoning has a more complex formula than concept. This happens because reasoning is made up of concepts; it contains at least two concepts, the concept is an element of reasoning. Reasoning is characterized by qualities that have cognitive significance. If we set reasoning against a fragment of reality reflected therein, we will be able to assess it as true or false; so, reasoning along with the concept is the structural basis of the cognitive process.

The article discusses the issue of the role of reasoning in mathematics. The reasoning and notion are compared, examples of argumentation of truth by method of contradiction assumption are provided.

Effecective Solution of Dirichlet and Neumann BVP of the Theory of Thermoelasticity of Microstretch Materials with Microtemperatures and Microdilatations for a Ball

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We consider the system of differential equations of statics of the theory of thermoelastisity of microstretch materials with microtemperatures and microdilatations in the case of isotropic homogeneous bodies. By means of general representation formulas for solutions, which contains four harmonic and seven metaharmonic functions, we construct an explicit solutions in the form of absolutely and uniformly convergent series of Dirichlet and Neumann boundary value problems for a ball.

On the Moments of Probability Distributions

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Investigating the Subgaussian random elements with values in Banach spaces and analysing the results of [1] and [2], R. Fukuda [3] came to a result, which is improved in our Theorem stated below.

Let ξ be a random variable and $\mathbb{E}$ be the symbol of the mathematical expectation.

Theorem *Let $p > q > 0$ and for some $C \geq 1$:*

$$\big\{\mathbb{E}|\xi|^p\big\}^{\frac{1}{p}} \leq C\big\{\mathbb{E}|\xi|^q\big\}^{\frac{1}{q}} < \infty.$$

Then for any r, s, $0 < r, s \leq p$, we have

$$\big\{\mathbb{E}|\xi|^r\big\}^{1/r} \leq C^{\beta}\big\{\mathbb{E}|\xi|^s\big\}^{1/s},$$

where

$$\beta = \begin{cases} 0, & \text{if } 0 < r \leq s \leq p, \\ 1, & \text{if } q \leq s < r \leq p, \\ \dfrac{q(p-s)}{s(p-q)}\,, & \text{if } 0 < s < q < r \leq p, \\ \dfrac{p(q-s)}{s(p-q)}\,, & \text{if } 0 < s < r \leq q. \end{cases}$$

Note that applying Kahan's inequality, Fukuda in [3] as a constant C^{β} for $r = p$ and $s = 1$ obtained the expression

$$C^{1+\frac{pq}{p-q}} \cdot q \cdot B^{-1}\Big(\frac{1}{q}\,, \frac{p}{p-q} + 1\Big),$$

where $B(\,\cdot\,,\,\cdot\,)$ is a beta function. As our calculations show the constant C^{β} obtained by us improves Fukuda's constant.

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On a Condition for the Affinity of the Sum Range of the Series in a Normed Space

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The talk is about the Chobanyan–Pecherski condition which is a sufficient condition for the affinity of the sum range of a series in a normed space. This condition is automatically fulfilled for the null-sequences in finite dimensional normed spaces. In this paper we describe some classes of null-sequences in the infinite dimensional normed spaces l_p, $1 \leq p < \infty$, satisfying the mentioned condition.

On the Development of Creative Skills in Teaching Mathematics

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One of the most important aspects of teaching mathematics in the process of school education is the formation and development students' critical, and creative thinking. When studying the real world, understanding the role of abstract ideas and perceiving the necessity of proving the visibly evident facts, while studying the abstract world, researching completely new, sometimes "paradoxical" realities, essentially expands the horizons, deepens innovative thinking, promotes the search and discovery of original, bold ideas and approaches.

Establishment of positional systems of numbers, expansion of the concept of number are one of the current examples of this direction. The discovery of analytic geometry was followed by radical changes in the development of mathematics, and this should be presented with clear examples. Getting to know the seemingly completely unusual non-Euclidean world beyond the so natural Euclidean geometry for the student can become a stimulating factor for original ideas. This is due, for example, to a new understanding of some geometrical objects – it becomes possible to draw many parallel lines to a point out of a line in these geometries, or to prove the absence of such a parallel line. Familiarity with completely unusual, unexpected properties of an infinite set in contrast to the obvious properties of a finite set can serve as an excellent example of deepening critical perception, when the concept of the number of elements of a set loses its meaning and the concept of the cardinality of a set, based on one-to-one correspondence of sets, is introduced. Proving that a proper subset of an infinite set can "have more elements" than the set itself, proving equivalence: of the set $\mathbb{N}$ of natural numbers and the set $\mathbb{Q}$ of rational numbers; $(0;1)$ interval and any $(a;b)$ interval even the set $\mathbb{R}$ of real numbers; a completely unexpected equivalence of $(0;1)$ interval and the $(0;1)\times(0;1)$ square and other unusual examples. The use of both analytical and geometric methods in proofs will strengthen the perception.

The talk emphasizes the outstanding importance of such facts and methods of their proving in the creative intellectual formation of students at the secondary and higher schools levels, describes the teaching methods of such material.

New Approach to Mathematical Genetics. Mathematical Patterns of Fetal Development-Special Mathematical Setting for the Decisive Endpoints in the Growth and Development of the Fetus

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Theory of information define the genetic matrix as a typical insular developing information system, also is the biological cheep of information. Now it is proved that the temporal expression profiles of circadian genes in uterus and embryo are existing. The intrauterine growth of the fetus must have the oscillate mode, or otherwise – intermittent irregular development. The endpoints between each of developing quantum's are the extremely decisive for the physiological systems regulation quality.

The aim of the study was to determine the basic decisive endpoints in fetal growth during pregnancy using General Theory of Ontogenesis Periodization, elaborated and published in 1990 by **Prof. Nickoloz Gogoberidze†**.

Methods: Using the basic principles of mathematical modeling and General Theory of Ontogenesis Periodization by N. Gogoberidze, the special formula was proposed for determination the decisive endpoints of gestation. The basic equation of ontogenesis periodization is as the following:

$$T_0 = F_0[Z(k)] = a\Big[\frac{Z(k)}{2^k}\Big]^2, \quad \frac{Z(k)}{2^2} < 1,$$
$$T_1 = F_1[Z(k)] = b\Big[\frac{Z(k)}{2^k}\Big]^2 - 1, \quad \frac{Z(k)}{2^2} \geq 1,$$

where a is a mathematical expectancy of daily continuance of pregnancy, b is a mathematical expectancy of monthly or yearly continuance of pregnancy, T_0 is a quantity of days after fertilization, T_1 is a quantity of years after birth.

The definitive formula for determination the decisive endpoints for fetal growth is the following: $t = 274/64 \times X^2$. The decisive endpoints (t) are in case, where $X = 0, 1, 2, 3, 4, 5, 6, 7$. Also $X = 8$ is the day of birth.

Results: Using evaluated formulas the decisive endpoints in fetal health were determined as: *5th day. 18th day. 39th day. 69th day. 107th day. 154th day. 210th day. The 274th day* – is the time of birth.

All cases are +/ − 1 day correct and the first day of fertilization is named as a *0 day.*

Conclution: A new era begins now in the development of medicine and medical genetics. For the first time in the world using General Theory of Ontogenesis Periodization, was determined the 8 basic permanent existing decisive endpoints in fetal development, most special for fetal health. By touching each of named endpoints arise the increased risk of fetus damaging, and the danger increased in case of internal/external factors influence.

Elaborated determination of decisive endpoints of human gestation is not empiric and is based on the method of mathematical modeling.

A real opportunity opens now to create new mathematical models in medicine.

About One Approximate Method for Solving an Abstract Evolutionary Problem

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We study the semi-discrete schemes for the following evolutionary problem in the Hilbert space H:

$$\frac{du(t)}{dt} + Au(t) = f(t), \quad t \in]0,T], \tag{1}$$

$$u(0) = u_0, \tag{2}$$

where A is the self-adjoint positive definite operator in H with domain of definition $D(A)$; $f(t)$ is a continuously differentiable function with values in H; u_0 is a given vector from H; $u(t)$ is the unknown function.

On the interval $[0,T]$, we define the grid $t_k = k\tau$, $k = 0,1,\dots,n$, with the step $\tau = T/n$. Now applying the perturbation method, we obtain the following algorithm for an approximate solution of problem $(1),(2)$:

$$\frac{u_k^{(0)} - u_{k-1}^{(0)}}{\tau} + Au_k^{(0)} = f_k, \quad u_0^{(0)} = u_0, \ \ k = 1,\dots,n,$$

$$\frac{u_k^{(1)} - u_{k-1}^{(1)}}{\tau} + Au_k^{(1)} = -\frac{1}{2}\,\frac{\Delta^2 u_{k-2}^{(0)}}{\tau^2}\,, \ \ k = 2,\dots,n,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\frac{u_k^{(p)} - u_{k-1}^{(p)}}{\tau} + Au_k^{(p)} = -\sum_{i=2}^{p+1} \frac{1}{i}\,\frac{\Delta^i u_{k-i}^{(p+1-i)}}{\tau^i}\,, \ \ k = p+1\dots,n,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\frac{u_k^{(m-1)} - u_{k-1}^{(m-1)}}{\tau} + Au_k^{(m-1)} = -\sum_{i=2}^{m} \frac{1}{i}\,\frac{\Delta^i u_{k-i}^{(m-i)}}{\tau^i}\,, \ \ k = m,\dots,n,$$

An approximate solution of problem $(1),(2)$ is defined by the formula

$$v_k = \sum_{i=0}^{m-1} \tau^i u_k^{(i)}, \qquad k = m,\dots,n.$$

The following theorem is formulated.

Theorem *Let A be a self-adjoint positive-definite operator in the space H and let the solution of problem $(1),(2)$ be a smooth enough function. Then, if*

$$\|u(t_k) - v_k\| = O(\tau^m), \ \ k = 1,\dots,m+1,$$

the estimate

$$\|u(t_k) - v_k\| = O(\tau^m), \ \ k = m+2,\dots,n.$$

is true.

Intelligent System for the Safe Consumption of Domestic Gas

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We all know that the use of natural gas for household purposes, despite its usefulness and convenience, has a very significant disadvantage. This is a gas leak and accumulation that can lead to explosion, collapse and fire. Injuries and casualties are also possible.

The problem of safe consumption of household gas by the population should be solved with a systemic approach, which provides for the separation of the constituent components of the problem.

One important factor emerged from the study and analysis of the issues: it is insufficient to equip individual apartments with gas leak detectors and detectors in apartment buildings. It is necessary to equip all apartments in the building with "smart security" devices and integrate them into a single intelligent monitoring and management system. Only with such an approach is it possible to protect the population and ensure their safety.

If it turns out that at least one apartment in a multi-storey building does not have an alarm and detector with a shut-off valve control function, then leakage and accumulation of harmful explosive and poisonous gases are expected in such a residential building.

There is another issue that needs to be resolved. This is a necessity for the establishment of a municipal service-service for the maintenance of gas safety devices and monitoring systems.

Set Relations and Set Systems Induced by Some Families of Integral Domains

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Starting from the problem of extending subdomains of a given integral domain U to suitable subdomains of $\mathbb{K}_U$ satisfying some given properties, we introduce and investigate the collection of the *extensible subdomains of* U, providing nontrivial examples and counterexamples. Due to the aforementioned properties, we are led to outline a general theory for *finitary simplicial complexes* and also to introduce other collections of subdomains whose interactions have been studied by means of some *reduction techniques* deriving from the decomposition of U through the image and the kernel of a suitable class of endomorphisms.

The talk is based on two recent works, written together with Dr. Chiaselotti and Prof. Oliverio.

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Sums of Squares, Positive Definiteness and Moment Problems in Several Variables: a Case for Algebra Extensions

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Many problems of mathematical physics/mathematical analysis are problems about sums of positive operators. In this talk, we show how attention to the algebra, and its extensions, plays a fundamental role in these problems. Several representation theorems on the moment problem in several variables will be given, new results on the moment completion problem will be discussed and some open questions will be posed.

On the Differentiation of Integrals with Respect to Translation Invariant Density Bases

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We investigate properties of density differentiation bases. It's known that a convex translation invariant density basis B and its Busemann–Feller extension B_{BF} have similar properties, in particular, as it was proved by G. Oniani [1], the Busemann–Feller extension differentiates the same class of non-negative functions as the basis B. We prove the following theorem which shows the optimality of the convexity condition in this result.

Theorem *For every $n \geq 2$ there exists a homothecy invariant basis B in $\mathbb{R}^n$ consisting of star-shaped sets which differentiates the integral of an arbitrary summable function on $\mathbb{R}^n$ but its Busemann–Feller extension B_{BF} is not a density basis.*

Note that in the work of P. Hagelstein and I. Parisis [2] it is given an example of a one-dimensional translation invariant density basis whose Busemann–Feller extension is not a density basis.

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Numerical Solutions for the Nonlinear Partial Fractional Differential Equations Using the Triple Laplace Transform

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In this paper, a new algorithm is proposed to solve linear and nonlinear partial differential equations in three dimensions. Moreover, some examples are provided to verify the performance of the proposed algorithm. This method presents a wide applicability to solve nonlinear fractional partial differential equations in the sense of conformable derivatives.

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Company Financial Information Flowsintegrated Model

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A component of any kind of business activity is the economic and financial information that is generated and circulated within the company. The degree of management of the company largely depends on the relevance, accuracy and timeliness of this information. Obtaining large volumes and multifaceted information, processing and obtaining new information without computer technologies and mathematical models now is, if not impossible, a very difficult task. To overcome this difficulty, the paper develops:

(1) financial accounting and reporting module;
(2) financial analysis module;
(3) financial variable sensitivity estimation module;
(4) equity pricing module;
(5) investment project evaluation module;
(6) module for forecasting needed variables.

In the paper, these modules are implemented in Excel – VBA programming language and Presented in a unified format of financial-information interface.

On the Solvability of a Mixed Problem for One Class of Second-Order Nonlinear Hyperbolic Systems

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For one class of second-order hyperbolic systems, a mixed problem with Dirichlet and Poincare boundary conditions is studied. In the linear case, under certain conditions on the data of the problem, an explicit representation of the solution of the problem is given. If these conditions are violated, the problem posed may have an infinite number of linearly independent solutions. In the nonlinear case, depending on the nature of the nonlinearity presented in the system, an a priori estimate is obtained, on the basis of which the solvability of this problem is proved. The question of the uniqueness of the solution of the stated problem is also considered. Separately, cases are singled out when the growth rate of nonlinearity in the system is greater than one.

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One Approach for Testing Asymmetrical Hypotheses

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Constrained Bayesian method (CBM) is used for testing asymmetrical hypotheses. The direct application of all statements of CBM allows us to make decisions on the desired levels of reliability. There is proven that mixed directional false discovery rates (mdFDR) are restricted on the desired levels at the suitable choice of restriction levels at different statements of CBM. The computation of concrete examples confirms the correctness of theoretical results for different statements of CBM.

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Initial Boundary Value Problem for Noncharacteristic Degenerate Hyperbolic Equations

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In the study of the mixed Cauchy problem in a cylindrical domain, the lateral boundary conditions are usually local boundary conditions of the Dirichlet type or periodic boundary conditions.

In [1], the boundary condition for the Newton (volume) potential was found, which is a new integro-differential self-adjoint boundary condition for the Laplace equation. In this paper, we study the mixed Cauchy problem for one class of noncharacteristic degenerate hyperbolic equations using this boundary condition. Unlike other works devoted to this topic, where solutions of the mixed Cauchy problem with different lateral boundary conditions of the problems under consideration are obtained in weighted spaces, in this paper, all solutions of the mixed Cauchy problems under consideration are obtained in classical Sobolev spaces. Indeed, in a cylindrical domain $D = \Omega \times (0,T)$ with $\Omega \subset R^n$ we consider a mixed Cauchy problem with a potential lateral boundary condition for the following noncharacteristic degenerated equation

$$Lu = u_{tt} - k(t)\Delta_x u(x,t) = f(x,t),$$

where $k(t) \geq 0$. As in the case for strictly hyperbolic equations, we first establish that $u \in W_2^1(D)$ and $u \in W_2^2(D)$ under the assumptions

$$\Big\|\frac{f}{k}\Big\|_{L_2(\Omega)}(t) < \infty \ \text{ and } \ \Big\|\frac{grad_x f}{k}\Big\|_{L_2(\Omega)}(t) < \infty \ \text{ for every } \ t \in [0,T],$$

respectively.

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Half Plane Diffraction Problems on Triangular Lattice

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We investigate thin-slit diffraction problems for two-dimensional lattice waves. The peculiar structure allows us to consider problems on the semi-infinite triangular lattice, consequently, we study Dirichlet problems for the two-dimensional discrete Helmholtz equation in a half-plane. In view of the existence and uniqueness of the solution, we provide new results for the real wave number $k \in (0,3) \setminus \{2\sqrt{2}\}$ without passing to the complex wave number and derive an exact representation formula for solutions. For this purpose, we use the notion of the radiating solution. Finally, we propose a method for numerical calculation. The efficiency of our approach is demonstrated in an example related to the propagation of wave fronts in metamaterials through two small openings.

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Modelling Rayleigh and Rayleigh-Type Waves by Reduced Scalar Partial Differential Equations

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Asymptotic formulations for Rayleigh and Rayleigh-type elastic waves have been studied within the last decade, see [1] and references therein, generalising the approach of surface waves of arbitrary profile [3] to forced boundary value problems. In case of Rayleigh, Stoneley and Schölte waves the model includes a hyperbolic equation on the surface/interface, with the right hand side involving the loading causing the excitation of the wave in question. At the same time, decaying behaviour away from the surface/interface is modelled by the elliptic equations.

The current contribution allows formulating a hyperbolic equation at an arbitrary depth, with the right hand side containing the pseudo-differential operator applied to the prescribed loading. As a result, the general vector problem of elasticity reduces to a scalar problem for a hyperbolic operator at a given depth, which acts as a parameter of the problem [2]. The approach is illustrated for the classical Lamb's problem.

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Fundamental Concepts of Mathematics and Algorithms

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Researches of the leading countries of the world prove that today programming, directions of informatics or information technologies are among the most renting professions. There directions are developing and widening with great speed correspondingly. Students of these specialties must study many different subjects. Besides the special subjects they have to study such basic subjects as Mathematics.

Each no special subject causes a question why it is necessary to study it? And University has a purpose to raise the knowledge just in this direction. Modern education requires strict reasoned purposes and corresponding to this profession studying. Without knowledge of certain subjects to raise the specialist of informatics one cannot imagine. That is why Mathematics is corresponding to the profession and must contain corresponding questions.

Students must feel necessity of studying such subjects and connection with their specialty. There are fundamental concepts of Mathematics on which are based basic questions of Informatics. Such concepts are function, limit, sequence, derivative, integral and many questions of algebra and Discrete Mathematics. Supply of such questions in programming in sense of their usage will increase motivation of studying which simplify to study it.

On Consistency and Large Deviations of Empirical Estimates for Stochastic Programming Problems

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One of the most known methods of solving of stochastic programming problems consists in consideration an empirical criterion function instead of the first one. Then asymptotical properties of optimum points of the empirical function are investigated. The report deals with stochastic programming problems where the uncertainty factor is a stationary random process with discrete or continuous time, but observations for constructing an empirical function depend on time. It is supposed that the process satisfies the strong mixing condition. Consistency of empirical estimates for the problems is proved. Large deviations are estimated for the case when observations do not depend on time and the process satisfies the first hypothesis of hypermixing [1].

The first criterion function supposed to be continuous and convex, and it should satisfy some sharpness condition at the minimum point. Some restrictions on moments of the observations function and its right and left derivatives take place.

For large deviations estimating it is supposed that both the right and left derivatives of observations function at a minimum point of the former criterion function are bounded.

So for a stationary random sequence $\{\xi_i,\ i \in \mathbb{N}\}$ and continuous convex on the second argument function $h : \mathbb{R} \times X \times Y \to \mathbb{R}$, where it is some metric space (Y, ρ); $X = [a; b] \subset \mathbb{R}$; one has the problem

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} h(i, x, \xi_i) \longrightarrow \min, \ \ x \in X.$$

When one deals with a stationary process $\{\xi(t),\ t \in \mathbb{R}\}$ then

$$F_T(x) = \frac{1}{T} \int\limits_0^T h(t, x, \xi(t))\, dt \longrightarrow \min, \ \ x \in X.$$

For formulating and proving the results some facts from the convex analysis [2] are used.

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Some Remarks on Equidecomposability of Sets

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The present report is devoted to various definitions of equidecomposability of sets [1]. The connection between finitely equidecomposable and countable equidecomposable sets will be shown. In particular:

(a) if X and Y are finitely equidecomposable, then they are also countable equidecomposable;

(b) in R^n there exist two sets X and Y, with $\lambda_n(X) > 0$ and $\lambda_n(Y) = 0$, which are not countable equidecomposable under than of all affine translations of R^n;

(c) in R^n there exist two sets X and Y such that $card(X) = card(Y) = c$ and X is not countably equidecomposable with Y.

Acknowledgements

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Finite Topology and the Full Transitivity of a Cotorsion Hull

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Using the finite topology defined in the ring of endomorphisms of an Abelian group, it is shown how the question of full transitivity can be determined for a cotorsion hull of a separable primary group with unbounded basic subgroup.

About the New Spreading Model of the SARS-CoV-2 Virus and Security Management Issues

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New mathematical and computer models of the spread of the SARS-Cov-2 virus have been proposed, taking into account the protocol for fighting the epidemic adopted by the Georgian authorities. The task is to control the fight against the epidemic taking into account vaccination with temporary immunity.

A computational experiment conducted on a computer model built on the basis of a mathematical model of the spread of the SARS-Cov-2 virus in the form of a system of ordinary differential equations with constant coefficients allows us to conclude that by choosing the value of the parameters, we can ensure that the number of infected citizens does not exceed a level at which the economy does not need a lockdown, and the prognosis for recovery of those infected with the SARS-CoV-2 virus is favorable.

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On Almost Invariant Sets and Their Measurable Hulls

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Let E be a basic space, which is equipped with a transformation group G and $X \subset E$. We say that X is almost G-invariant (with respect to μ measure), if for each transformation $g \in G$ we have the equality

$$(\forall\, g)(g \in G \Longrightarrow \mu(g(X) \vartriangle X) = 0)$$

(see [1, 2]).

Theorem *If a set X is not almost invariant, but its measurable hull is almost invariant, then X is non-measurable set.*

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On the Novel Exact Solutions of Navier–Stokes Equations

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We consider 2D and 3D incompressible unsteady fluid flow over the rhombus and over the prism with the rhomboidal cross-section correspondingly. The velocity components of the flow satisfy the nonlinear Navier–Stokes equations (NSE) with the suitable initial-boundary conditions and with the specific pressure. The exact solutions of NSE are obtained in some specific cases [1–10]. We supposed that near sharp edges the velocity components are non-smooth and by the methods of mathematical physics we have obtained novel exact solutions of NSE.

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The Non-Abelian Exterior Product and Low-Dimensional Homology of Leibniz Algebras

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Leibniz algebras were first defined by Bloh [1] as a non skew-symmetric analogue of Lie algebras, but they became very popular when Loday rediscovered them in [7], mainly due to the development of a new, Leibniz (co)homology theory for Lie algebras. Since then, many authors have been studying them obtaining very relevant algebraic results and due to their relations with Physics and Geometry. Among them, many results of Lie algebras have been extended to the Leibniz case.

The notions of non-abelian tensor and exterior products were introduced in groups by Brown and Loday [2], as tools in homotopy theory, but they can give us nice information about central extensions and (co)homology. They were extended to the Lie case by Ellis [4]. Later, the notion of non-abelian tensor product was extended to Leibniz algebras by Gnedbaye [5]. The exception was exterior product in Leibniz case, which was not specified, as we believe, by technical difficulties.

In this talk we present construction of the non-abelian exterior product of Leibniz algebras, with applications in low dimensional Leibniz homology and in the study of capability properties of Leibniz algebras. The results of this research are presented in the papers [3] and [6].

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Influence of Chiral Asymmetry on Phase Structure of the Two- and Three- Color Quark Matter

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Dense baryonic matter, which can exist in neutron stars or be even observed in heavy-ion collision experiments, is characterised by not only μ_B chemical potential. In the real case there appears an additional isospin chemical potential μ_I in the system. Moreover, since nuclear matter is usually under the influence of extremely strong external magnetic fields, chiral asymmetry of the medium can also be observed.

In heavy-ion collisions due to large temperatures and non-trivial gluon configurations chiral imbalance μ_5 may appear. Therefore, it would be interesting to clarify the situation with the dual symmetries of the phase diagram of this system in the most general case.

In this work (based on [1] and [2]) the influence of chiral chemical potential μ_5 on the phenomenon of diquark condensation and phase structure of dense quark matter altogether is contemplated in the framework of effective 2- and 3- color and 2-flavor Nambu–Jona-Lasinio model. The nonzero values of baryon μ_B, isospin μ_I and chiral isospin μ_{I5} chemical potentials are also taken into account. We show that the duality relations between diquark condensation, charged pion condensation and chiral symmetry breaking phenomena, found in the case of zero μ_5, are also valid for any value of $\mu_5 \neq 0$. In terms of dualities and the influence on the phase diagram, chiral imbalance μ_5 stands alone from other chemical potentials.

Indeed, in comparison with other chemical potentials, μ_5 has two interesting features.

(i) In the region of moderate values of μ_B, μ_I and μ_{I5} it manifests itself as a *universal catalyst*, since it enhances just the phase that is realized in the system at $\mu_5 = 0$.

(ii) In the second regime, when several other chemical potentials reach rather large values, one could observe a rather complicated and rich phase structure, and chiral chemical potential μ_5 can be a factor that not so much catalyzes as *triggers* rather peculiar phases.

It turns out that the full $(\mu_B, \mu_I, \mu_{I5}, \mu_5)$-phase diagram of the model is interconnected by the dualities and possesses a very high symmetry.

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Machine Translation and Human Capabilities

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In the current age of globalization, the need for simplifying communication between speakers of different languages increases. A large part of the communication is done through machine translation. This raises a question: Can a computer translate as well as a human can? This work is an attempt to answer this question. It presents the analysis of machine translation software solutions and shows real-life problems.

Today, machine translation is integrated function on such websites as Facebook and Twitter as well as web browsers like Google Chrome. It is obvious that this method of translation is very widespread nowadays; however, this does not mean that translation done by humans lost its purpose and need.

There are several factors that stop machine translation from dominating, which includes: context analysis, understanding new slang, reading handwritten text, natural translation in spite of grammatical differences between languages, maintaining rhymes while translating poems and songs, idioms, deciphering badly written text (such as without proper spelling or punctuation), listening to voice recordings, live translation, translation of less widespread languages, and more.

Our goal is to discover the positive and negative sides of machine translation as well as its current state and future potential. With this in mind, we compare text translated by machines to results of human translation.

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Multiple Linear Regression Model

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Let (E, S) be a measurable space with a given family of probability measures $\{\mu_i, i \in I\}$.

The following definitions are taken [1, 2].

Definition 1 An object $\{E, S, \mu_i, i \in I\}$ is called a statistical structure.

Definition 2 A statistical structure $\{E, S, \mu_i, i \in I\}$ is called orthogonal (singular) if a family of probability measures $\{\mu_i, i \in I\}$ consists of pairwise singular measures (i.e. $\mu_i \perp \mu_j$, $\forall i \neq j$).

Let I be the set of parameters and let $B(I)$ be a σ-algebra of subsets of I which contains all finite subsets of I.

Let H be the set of hypotheses and let $B(H)$ be a σ-algebra of subsets of H which contains all finite subsets of H.

Definition 3 We will say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimator for parameters if there exists at least one measurable mapping $f : (E, S) \to (I, B(I))$ such that

$$\mu_i\big(\{x : \ f(x) = i\}\big) = 1, \ \ \forall \ i \in I.$$

Definition 4 We will say that the statistical structure $\{E, S, \mu_h, h \in I\}$ admits a consistent criterion for hypothesis testing if there exists at least one measurable mapping $\delta : (E, S) \to (H, B(H))$ such that

$$\mu_h\big(\{x : \ \delta(x) = i\}\big) = 1, \ \ \forall\, h \in H.$$

Let $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + u$ be a multiple linear regression model, where $u \sim N(0, \sigma^2)$ is a normally distributed random variable with unknown variance and $\beta_0, \beta_1, \dots, \beta_k$ are unknown parameters. That is, the Gaussian statistical structure

$$\big\{E, S, \mu_u, \mu_{\beta_i}, \ i = 0, 1, \dots, k\big\}$$

is given.

Theorem 1 *The Gaussian statistical structure $\{E, S, \mu_\beta, \beta \in I\}$ $(card\, I = \chi_0)$ admits a consistent criterion for hypothesis testing if and only if the statistical structure is an orthogonal statistical structure.*

Theorem 2 *In order for the Gaussian statistical structure $\{E, S, \mu_\beta, \beta \in I\}$ $(card\, I = \chi_0)$ to admit a consistent estimator for parameters, it is necessary and sufficient that the statistical structure be orthogonal.*

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On Construction and Properties of Some Lévy-Type Processes

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We consider an integro-differential operator

$$Lf(x) = b(x)\cdot\nabla f(x) + \int\limits_{\mathbb{R}^d\setminus\{0\}} \big(f(x+u) - f(x) - \nabla f(x)\cdot u 1_{|u|\le 1}\big)\, N(x,du),$$

defined on the space $C^2_\infty(\mathbb{R}^d)$ of twice continuously differentiable functions with vanishing at infinity derivatives. In this talk we discuss the methodology of the construction of strong Markov process X as a unique solution to the martingale problem for $(L, C^2_\infty(\mathbb{R}^d))$. By this we mean, that for any $f\in C^2_\infty(\mathbb{R}^d)$ the process

$$f(X_t) - \int\limits_0^t Lf(X_s)\,ds$$

is the martingale with respect to the natural filtration of X. We discuss the distribution properties of the process.

The talk is based on the joint work with A. Kulik and R. Schilling. There will be the related talk by Y. Mokanu, based on [4].

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The Numerical Solution of the External Dirichlet Generalized Harmonic Problem for a Sphere by the Method of Probabilistic Solution

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In the present paper the algorithm for the numerical solution of the external Dirichlet generalized harmonic problem for a sphere by the method of probabilistic solution (MPS)is given. Under a generalized problem is meant the problem when a boundary function has a finite number of first kind discontinuity curves. The algorithm consists of the following main stages:

(1) transition from an infinite domain to a finite domain by an inversion;

(2) consideration of the Dirichlet a new generalized harmonic problem on the basis of Kelvin's theorem for the obtained finite domain;

(3) application of the MPS for the numerical solution a new problem which in turn is based on a computer simulation of the Wiener process;

(4) finding the probabilistic solution of the posed generalized problem at any fixed points of the infinite domain by the solution of the new problem.

For illustration numerical examples are considered and results are presented.

The Inequalities for Trigonometric Polynomials and Entire Functions of Finite Order in Generalized Weighted Grand Lebesgue Spaces

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The goal of our talk is to present Bernstein and Nikol'skii type inequalities for trigonometric polynomials and entire function of finite order in generalized weighted grand Lebesgue spaces. The later inequalities we apply to the approximation problems in approximable subspaces of above mentioned non-separable Banach function spaces.

Stability of a Convective Flow due to Internal Heat Sources in a Magnetic Field

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Consider a flow of a conducting viscous incompressible fluid in a channel between two parallel vertical planes. The study is motivated by one of the factors that has to be taken into account in liquid blanket design of thermonuclear reactors [1]: internal heating of the flow due to neutron irradiation. In order to explore the effect of non-uniform heating we assume that the density Q of the internal heat sources is given by

$$Q = Q_0\, e^{-\alpha x}, \tag{1}$$

where α is a given constant and x is the transverse coordinate. Linear stability of the flow is investigated for two values of the Prandtl number representing different liquids that are currently proposed for the blankets:

(a) $Pr = 0.01$ (representing liquid metals);

(b) $Pr = 20$ (representing FliBe [2]).

The system of magnetohydrodynamic equations in the Boussinesq approximation is used to analyze the problem. Base flow in the vertical direction is generated due to (1). The channel is assumed to be closed so that the total fluid flux through the cross-section is equal to zero. The corresponding boundary value problem for the determination of the base flow is solved analytically. The inductionless approximation is used in the paper (an induced magnetic field due to flow of the fluid is assumed to be negligible). Calculations show that the increase in α leads to more asymetric temperature distribution.

Linear stability analysis is performed using the method of normal modes. Chebyshev collocation method is used to discretize the linearized problem. Calculations performed for $Pr = 0.01$ and small Hartmann numbers Ha (characterizing the intensity of the imposed magnetic field) show that the main instability mechanism is associated with the shear of the base flow. Thermal factors are more important for larger Ha and especially for $Pr = 20$. It is shown that the marginal stability curves can have a cusp or even closed loops as the Prandtl number grows. Calculations show that instability for $Pr = 20$ is developed in the form of thermal running waves propagating downstream with sufficiently large phase velocity. It is shown that for both Prandtl numbers considered the increase in α destabilizes the flow while the flow is stabilized for larger Ha.

Acknowledgements

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Thermal Convection in an Annulus: a Linear Stability Analysis

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The problem considered in the paper is relevant to the analysis of factors that influence biomass thermal conversion [1]. Biomass energy is considered nowadays as a promising source of renewable energy.

Suppose that viscous fluid is located in the annular region between two infinitely long vertical cylinders of radii R_1 and R_2 ($R_1 < R_2$). There exists a steady convective flow in the vertical direction caused by two factors: (a) internal heat generation and (b) temperature difference between the walls of the cylinders. Internal heating of the flow occurs due to chemical reaction that takes place in the fluid.

The base flow is determined numerically as the solution of the nonlinear boundary value problem for the system of ordinary differential equations. Linear stability analysis [2] is conducted using the method of small perturbations. Both axisymmetric and asymmetric perturbations with respect to the angular coordinate are considered. The linearized boundary value problem is solved numerically using collocation method. Chebyshev polynomials of the first kind are chosen as the base functions. Flow stability depends on the relative importance of the four dimensionless parameters that characterize the flow: the Grashof number Gr depending on the temperature difference between the walls, the Frank–Kamenetskii parameter F characterizing the intensity of the chemical reaction, the radius ratio $R = R_1/R_2$ and the Prandtl number Pr characterizing the properties of the fluid.

Numerical calculations show that there are several destabilizing factors: the increase in F and Pr destabilize the flow. In addition, the increase of the temperature difference between the walls acts as an additional destabilizing factor. It is shown that instability is axisymmetric for large R and the first asymmetric mode is the most unstable for the values of R in the range $0 < R \leq 0.2$. Note that instability is desirable in this case (in contrast with applications in aerodynamics) since it enhances mixing and may lead to more efficient conversion process.

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Poroelastic Medium with Non-Penetrating Crack Driven by Hydraulic Fracture

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A new class of unilateral variational models appearing in the theory of poroelasticity is introduced and studied. A poroelastic medium consists of solid phase and pores saturated with a Newtonian fluid. The medium contains a fluid-driven crack, which is subjected to non-penetration between the opposite crack faces. The fully coupled poroelastic system includes elliptic-parabolic governing equations under the unilateral constraint. Well-posedness of the corresponding variational inequality is established based on the Rothe semi-discretization in time, after subsequent passing time step to zero. The NLCP-formulation of non-penetration conditions is given which is useful for a semi-smooth Newton solution strategy.

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About some Numerical Schemes Constructed for the Calculation of Engineering Details with Cracks

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The report will address the issues of effectively calculating engineering details weakened by cracks using the example of a bearing-type mechanical device.

In the form of a proper mathematical model, a system of solid bodies with specific configuration and areas of rupture (cracks) distribution (propagation), which have a common surface (contact area) of interaction within certain limits, is considered. Numerical schemes have been constructed and processed to determine the real picture of voltage distribution in the mentioned environment.

In particular, for the corresponding singular integral equations of the loads acting along the cracks, the issues of constructing new schemes based on the approximation of the singular operator are studied, in terms of significantly simplifying the corresponding numerical realization process. As for the equation with a weak singularity representing the loads acting directly in the contact area, effective quadratic formulas have also been obtained for it in the case of different elastic characteristics of interacting bodies.

On the basis of the processed algorithms, a computing software package based on "Wolfram Mathematica" was created, which can be used to solve practically important contact problems of engineering mechanics.

Towards Similarity-Based Set Unification

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Reasoning with incomplete, imperfect information is very common in human communication. For such problems, exact equality/equivalence is replaced by its approximation. This kind of reasoning is a highly nontrivial task and remains an important issue in applications of artificial intelligence.

Modeling the incomplete and imprecise information is achieved using so called proximity relations, which are reflexive and symmetric, but not necessarily transitive relations. When we have transitivity, we get so called similarity relation, i.e. fuzzy equivalence relation.

While similarity-based unification and crisp set unifications are separately well-studied techniques, their combinations has attracted less attention. In this talk we define similarity-based crisp set unification problem and discuss possible solutions.

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European e-Infrastructure Initiatives (GEANT, EOSC) for Research and Education in Georgia

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The short introduction of European e-infrastructure initiatives, mainly European research and education network GÉANT and European Open Science Cloud EOSC are presented. Involvement of Georgian Research and Educational Networking Association GRENA in these initiatives are discussed. The status of network and computing infrastructures and available services for research and education community of Georgia are presented.

GRENA provides IT services using its own infrastructure and different technologies to more than 130 organisations in Georgia. GRENA owns fibre-optic-based network infrastructure connecting Georgian research and education institutions to the Pan-European research and education network GEANT.

GRENA runs a data centre with main services such as federated cloud, virtualisation, e-learning, etc. GRENA's cloud infrastructure https://www.gcloud.ge/ is integrated in the European Open Science Cloud EOSC. GRENA also supported the integration of developed resources at Ivane Beritashvili Experimental Biomedicine Centre – Electroencephalography pattern study https://eeghub.ge/ and Tbilisi State University National Science Library of Georgia – digital repository of Georgian scientific works https://openscience.ge/.

The main services provided by GRENA are following:

- GÉANT and internet connectivity;
- VPN – Virtual Private Network;
- eduroam – education roaming;
- eduGAIN – authentication and authorization;
- Cloud and Virtualization;
- Cybersecurity: CERT-GE – Computer Emergency Response Team (TI member), DDoS protection, UTM – Unified Threat Management, IDS – Intrusion Detection System, e-mail security.

Currently development of e-infrastructures and services are performed in framework of four European Commission projects briefly described in presentation.

Regarding the Limit Distribution of the Chain m Dependent Sum of a Sequence

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Let $\{\xi_n, X_n\}_{n\geq 1}$ be a two component stationary sequence in narrow sense defined on the (Ω, F, P) probability space. The determining $\{\xi_n\}_{n\geq 1}$ sequence is a finite, stationary, ergodic Markov chain with 1 class of ergodicity (may contain cyclic subclasses). $\{X_n\}_{n\geq 1}$ $(X_n : \Omega \to R^k)$ is a chain m dependent sequence of vectors[1].

Using the Berstein sectioning method it is proven, that the limit distribution of the

$$S_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (X_i - EX_1)$$

sum is normal

$$P_{S_n} \xrightarrow{W} \Phi_{0, R^{(0)} + \sum\limits_{i=1}^{m} [R^{(i)} + (R^{(i)})'] + T},$$

where

$$R^{(l)} = E\Big\{ \big[X_1 - E(X_1|\xi_1)\big] \cdot \big[X_{1+l} - E(X_{1+l}|\xi_{1+l})\big]' \mid \overline{\xi}_{1,1+l} \Big\}$$

while $\overline{\xi}_{1,1+l} = (\xi_1, \xi_2, \dots, \xi_{1+l})$ fixed trajectory of the chain. The T matrix is represented by the characteristics of the chain.

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On Stationary Probabilities of Multi Channel Retrial Queueing Systems with Thresholds

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The work is devoted to the promising direction of the stochastic systems theory associated with research of retrial queueing systems. Retrial queues are characterized by the basic assumption that a customer, who cannot get service, leaves the service area and goes to the "orbit", but after some random period of time returns to the system and tries to get service again.

Retrial queues arise naturally in our daily activities, in phone systems and computer networks, in the field of data transmission systems. They are widely used in designing of computer networks, in studying of stochastic information processing networks, modern mobile communication systems, etc. Within the framework of those models, qualitative characteristics of the stochastic system performance can be evaluated and optimal controlled problems can be set and solved.

The main model under consideration is a Markov model of a multi channel $[M|M|m|m+n]^{\{\infty\}}$ – retrial system. The input flow in the system has a variable rate dependent on the number of calls in the orbit. The rate is controlled by a threshold strategy. There are no restrictions on the capacity of the orbit.

We study the stationary regime for such a system. An efficient algorithm for calculating stationary probabilities are obtained. For threshold control strategies, the optimization problem of the total income of the system is formulated and solved. For the system with one server and one place in the queue, direct formulas for stationary probabilities are obtained. In this case, we study the rate of convergence of the stationary distribution of a finite system to the stationary distribution of an infinite one under the threshold strategy.

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Modeling of Health and Mortality Functions Based on Data for the Population of Ukraine

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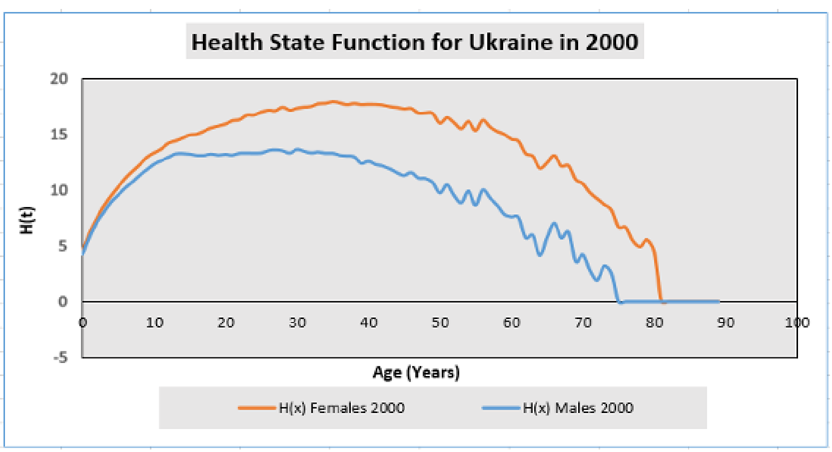
The state of human health is a decisive factor in human life and can be viewed as a stochastic variable, as it is closely related to various factors both from the environment and external factors and information contained in genes. The characteristics of a person's physical state fluctuate around average values, and the deviation is a consequence of the probabilistic stochastic nature of the process. Death occurs when the trajectory of the stochastic process that describes health first crosses the zero line, representing zero vitality or zero health. One of the approaches to modeling health status of the population is proposed in [1], [2]. It uses life expectancy limits based on stochastic mortality modeling and the application of the critically low first achievement theory. The model builds health and mortality functions, their parameters can be estimated on the data of demographic tables.

Based on the data of the population of Ukraine, we estimate parameters and build the model of the health and mortality functions. The results of the study are presented for comparing the intervals of the maximum value of the health status function and the age of the zero value of the health status function – the age of death. We can see, that the state of health of women is significantly higher than the one of men, this also applies to the age of zero value of the health. This difference is especially noticeable in 2004, where the age of zero health status is 83 years for women and 69 years for men. In the case of women, the age with the maximum state of health is considered to be from 30 to 45 years, but for men, this indicator is from 20 to 35 years. It can also be concluded that the maximum value of the health function does not affect the age of the zero health value.



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On Riesz Basis Property for $n \times n$ Dirac Type Operators. Application to Timoshenko Beam Model

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In this talk we continue our investigation [1, 2] of the spectral properties of non-selfadjoint boundary value problems (BVP) for the following first order system of ODE

$$L(Q)y = -iB(x)^{-1}\big(y' + Q(x)y\big) = \lambda y, \quad y = \operatorname{col}(y_1, \dots, y_n), \ \ x \in [0, \ell], \tag{1}$$

on a finite interval $[0, \ell]$. Here $Q \in L^1([0, \ell]; \mathbb{C}^{n\times n})$ is a potential matrix and $B = B^* \in L^1([0, \ell]; \mathbb{R}^{n\times n})$ is a diagonal "weight"-matrix. If $n = 2m$ and $B \equiv \operatorname{diag}(-I_m, I_m)$, equation (1) is equivalent to Dirac equation of order n.

We show the existence of triangular transformation operators for such equation under additional conditions on the entries of the matrix function $B(\cdot)$. Here we apply this result to study direct spectral properties of the BVP associated with equation (1) subject to the general boundary conditions

$$U(y) = Cy(0) + Dy(\ell) = 0, \ \ \operatorname{rank}(C \ D) = n.$$

As a first application of this result, we show that the deviation of the characteristic determinants of this BVP and the unperturbed BVP (with $Q = 0$) is a Fourier transform of some summable function explicitly expressed via kernels of the transformation operators. In turn, this representation yields asymptotic behavior of the spectrum in the case of regular boundary conditions. Namely,

$$\lambda_m = \lambda_m^0 + o(1) \ \text{ as } \ m \to \infty, \quad \sigma(L(Q)) = \{\lambda_m\}_{m\in\mathbb{Z}}, \quad \sigma(L(0)) = \{\lambda_m^0\}_{m\in\mathbb{Z}}.$$

Further, we prove that the system of root vectors of the above BVP with *strictly regular* boundary conditions forms a **Riesz basis** in a certain weighted L^2-space.

The main results are applied to establish asymptotic behavior of eigenvalues and the Riesz basis property for the dynamic generator of the damped Timoshenko beam model.

The talk is based on a joint preprint [3] with Mark Malamud.

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Summability of Differentiated Sequence of the Fourier Series with Respect of System of Generalized Spherical Functions by (C, α) Method

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By I. M. Gel'fand and Z. Ya. Shapiro was considered the system of generalized spherical functions and were presented the extension of a given function on the surface of a sphere $U_m(\vartheta, \varphi)$ functions to the system of generalized spherical functions where ϑ, φ are spherical coordinates.

We have studied the summability of the sequence obtained by r-times differentialization of this sequence by the (C, α) method where the Laplace operator on the fieldsphere is applied as the differential operator.

Is determine the generalized Laplace $\Delta U_m(\vartheta, \varphi)$ and $\widetilde{\Delta}(\vartheta, \varphi)$ operators (derivatives) for given by $U_m(\vartheta, \varphi)$ function on the surface of the sphere, and is determined sufficient conditions for summability by the r-times differentiated sequence by (C, α) method. In particular is proved:

Theorem 1 *If for any non-negative integers exist for r, then the sequence obtained by r-times differentiation of the $U_m(\vartheta, \varphi)$ Fourier series of the $\Delta^r U_m(\vartheta, \varphi)$function is summable by the (C, α) method to the $\Delta^r U_m(\vartheta, \varphi)$ function when $\alpha > 2r + 1$.*

Theorem 2 *If for any non-negative integers exist $\widetilde{\Delta}^r U_m(\vartheta, \varphi)$, then the obtained as result of r-times differentiation of the Fourier series of the $U_m(\vartheta, \varphi)$ function is summedable by the (C, α) method to the $\widetilde{\Delta}^r U_m(\vartheta, \varphi)$ function when $\alpha > 2r + 2$.*

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Risk Assessment by Using Fuzzy Mathematical Expectation

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Receiving the benefits of the alternative is determined by the unintended consequences that really arise in the future. the connection between the benefit and the negative outcome is called fuzzy. Let us denote the distribution of probabilities by $p(m)$ and the outcome of benefit that are received from alternative by $f(m)$, then the probable value of receiving benefit can be written as:

$$Ef = \int\limits_M f(m)p(m)\,dm.$$

Ability–probability distribution is used to represent fuzzy risk

$$\Pi_{M,P} = \big\{\pi_m(p) \mid \ m \in M,\ p \in P\big\},$$

where M and P are the space and the probability of an unfavorable outcome, respectively. In this case, expected value of receiving benefit should be fuzzy number, that replaces the exact value of Ef bin previous equation. It depends on fuzzy distribution of probability $p(m)$, which determined by $\pi_m(p)$. We can calculate fuzzy mathematical expectation by formula:

$$\widetilde{E}f = \int\limits_M f(m)\widetilde{p}(m)\,dm.$$

Then, according to its α-levels by the fuzzy multiplication formula, we can get the value of the fuzzy mathematical expectation

$$\widetilde{E}f = \bigcup_{\alpha\in[0,1]} \alpha E_\alpha f.$$

By sorting the value of mathematical expectations that fit all alternatives, we can find out which alternative is the best. Operators that turns $\widetilde{p}m$ into $\widetilde{E}f$ is linear. Therefore, when we bring alternatives in order according to the center of gravity, it is possible to avoid calculating the α level. In this case, we first calculate the center of gravity $\widetilde{p}(m)$, which equals to

$$c(m) = \frac{\int\limits_p p\pi_m p(m)\,dm}{\int\limits_p \pi_m(p)\,dp}\,.$$

Then, for a $\widetilde{p}(m)$-based order, the value of mathematical expectation is:

$$Ef = \int\limits_M f(m)c(m)\,dm.$$

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On One Intersubject Relationship of Mathematics and Physics

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Assessing the importance and usefulness of identifying interdisciplinary connections in the learning process as a whole, in this work we present some manifestations of interdisciplinary connections between mathematics and physics. No secret that mathematics plays special role in this regard, contributing to the substantiation of various physical phenomena and laws. Despite this circumstance, the interdisciplinary connection between mathematics and physics is of a dual nature. On the one hand, mathematical knowledge is used to study physical problems and questions, on the other hand, there is a feedback when physical laws fill the area of application of mathematical knowledge, make teaching mathematical "abstract" material more objective and interesting. In this work, on the example of specific multilevel problems, we will identify possible innovative applications of the formula of determining the center of mass of a body in solving problems of covering areas or bypass encountered in combinatorial mathematics. As an example, we mention two such problems of combinatorics:

Problem 1 *We took a square of checkered paper 8x8 in size, cut off two cells from it (lower left and upper right). Is completely covering of the resulting figure with "dominos – rectangles"* 1×2 *possible?*

Problem 2 *Is it possible to cut a square of 8x8 cells with a corner cell cut out into* 1×3 *columns?*

Usually this kind of problems are solved by coloring of the area. Proposing a physical approach for solving the above-mentioned combinatorial problems, we firstly represent the initial range as a homogeneous plate. The idea of the proposed approach is based on the following intuitively clear properties of the center of mass that have a simple mechanical meaning:

1. Any system consisting of a finite number of material points has a center of mass, moreover, a single one.
2. The center of mass of two material points is located on the segment connecting these points, and its position is determined by the Archimedes' rule of the lever.
3. If, in a system consisting of a finite number of material points, several material points are marked and the masses of all marked points are transferred to their center of mass, then the position of the center of mass of the entire system will not change from this.

Having conveniently divided the initial range into parts and determined the center of mass of the resulting system according to the above, we will try to answer the question formulated in the problem. That's the whole "theory" of the physical approach. It should be especially noted that the proposed physical solution is impressive not only in terms of highlighting interdisciplinary connections, but also in terms of applicability. In particular, although in the considered problems, traditional typical solutions are creative and heuristic by their nature, but still many years of teaching experience shows that most students experience difficulties in solving such problems in the correct choice of colors, as well as, in a reasonable choice of principles for coloring a given areas, therefore they cannot solve such problems, while the proposed physical solution, both by its nature and by its mechanisms of application, is quite simple, therefore it can be an effective and easy-to-use tool for solving problems of covering areas.

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On the Use of Certain Informative-Communicative Technologies in the Learning Process

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The speed of spreading digital technologies in today's world is extraordinarily rapid. Developing of the appropriate infrastructure and getting involving in the informative space of the world is considered a foreground task in our country.

As high technologies radically change the living conditions it is essential that during the learning process students develop the skills the modern world requires from them. IT plays an important part in the development, also stimulating motivation.

However, today the teacher's personal initiative plays a considerable part in creating diverse and interactive educational course. With the aim numerous useful programs and resources actively and effectively used in the educational space of the leading countries have been created throughout the world. The interesting, powerful, available and consecutively renewed resource is the interactive and dynamic program GeoGebra created only about 10 years ago is one of the resources.

During the report I'll be reviewing:

- Short history of developing the program and its achievements.
- Effectiveness of its use and advantages towards other programs.
- Presenting many of its potentials and discussing the usage, the original methodological findings and the expected success.
- Discuss about Geogebra classroom- modified virtual platform, adapted to the online teaching and experience obtained during the working process, not only with students, but with teachers as well.
- Overview an activity, created with "STEAM" project: about the determination of fluid level in tank.
- Latest ressources and activities created by myself.

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About the Organization of Distance Learning

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Many things have been said and written regarding the arrangement of the learning process. The remote forms of teaching have not gone unnoticed from the method specialists. The following forms of teaching have been used for many years by didactic organizations focused on different user groups. One of the main principles of remote learning processes was the voluntary involvement of the user, which ensured the involvement and activity of the trainee.

The situation has changed drastically during the world pandemic, when the traditional learning process at all levels of education has been forcefully replaced by a remote form. Society was unprepared for such change. In technologically and materially less provided countries, the first stage of distance education practically failed. It became necessary to introduce new teaching technologies in the shortest possible time, to create electronic resources adapted to national educational standards and to develop effective strategies for their use.

The author shares her own experience of organizing the distance learning process in elementary school classes (students' age 10-12 years); Discusses in detail the methods of using standard electronic resources, the issues of creating original learning resources for teaching a specific topic, organizing and managing synchronous and asynchronous teaching formats.

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Maximal Equivariant Compactifications

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Let G be a locally compact group. Then for every G-space X the maximal G-proximity β_G can be characterized by the maximal topological proximity β as follows:

$$A\overline{\beta}_G B \iff \exists\, V \in N_e \;\; VA\,\overline{\beta}\,VB.$$

Here, $\beta_G : X \to \beta_G X$ is the maximal G-compactification of X (which is an embedding for locally compact G by a well-known result of J. de Vries [1]), V is a neighborhood of e and $A\overline{\beta}_G B$ means that the closures of A and B do not meet in $\beta_G X$.

Note that the local compactness of G is essential. This theorem comes as a corollary of a general result about maximal $\mathcal{U}$-uniform G-compactifications for a useful wide class of uniform structures $\mathcal{U}$ on G-spaces for not necessarily locally compact groups G. It helps, in particular, to derive the following result. Let $(\mathbb{U}_1, d)$ be the Urysohn sphere and $G = \mathrm{Iso}(\mathbb{U}_1, d)$ is its isometry group with the pointwise topology. Then for every pair of subsets A, B in $\mathbb{U}_1$, we have

$$A\overline{\beta}_G B \iff \exists\, V \in N_e \;\; d(VA, VB) > 0.$$

Here we use joint results with T. Ibarlucia from [2]. More generally, the same is true for any $\aleph_0$-categorical metric G-structure (M, d), where $G := \mathrm{Aut}(M)$ is its automorphism group. More details can be found in [3].

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Orderable Groups and Semigroup Compactifications

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This project is dedicated to Eli Glasner on the occasion of his 75th birthday. Our aim is to find some new links between linear (circular) orderability of groups and topological dynamics. We suggest natural analogs of the concept of algebraic orderability for *topological* groups involving order-preserving actions on compact spaces and the corresponding enveloping semigroups in the sense of R. Ellis.

This approach leads to several natural questions. Some of them might be useful also for discrete (countable) orderable groups. We study the following questions:

Question *Which topological groups can be embedded into the topological group $H_+(K)$ of all circular (linear) order-preserving homeomorphisms of K, endowed with compact-open topology, for some circularly (resp., linearly) ordered compact space K.*

Question *Which topological groups G admit proper linearly (circularly) order compact right topological semigroup compactification $G \hookrightarrow S$? When such S is:*

(a) *metrizable?*

(b) *hereditarily separable? c) first countable?*

Weighted Extrapolation in Grand Morrey Spaces Beyond the Muckenhoupt Range

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Rubio de Francia's extrapolation in weighted grand Morrey spaces with weights beyond the Muckenhoupt range is established. Based on this result, the boundedness of maximal and Calderón–Zygmund operators, and commutators of singular integrals in weighted grand Morrey spaces for appropriate class of weights is obtained. The problems are studies for spaces and operators defined on quasi-metric measure spaces with doubling measure but the results are new even for special cases of space of homogeneous type.

Similar problem for another weighted grand Morrey spaces for Muckenhoupt class of weights was studied in [1].

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Brief Overview of the First Results of the PhD Thesis "Formalism and Applications of Georgian Language Processing by Machine Learning Methods"

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Within framework of doctoral program "Informatics" of Georgian Technical University doctoral thesis "Formalism and Applications of Georgian Language Processing by Machine Learning Methods" (Doctoral Student – B. Mikaberidze, Scientific Supervisor – Professor K. Pkhakadze, Director of the Scientific-Educational Center for Cultural Protection and Technological Development of Georgian State Languages) was launched in 2020. The direct aim of the thesis is to develop the formalism of processing Georgian language by machine learning methods [1]. The thesis is based on the Pkhakadze's Logical Grammar of Georgian Language developed [2, 3]. This project is a sub-project of the long-term project "Technological Alphabet of Georgian Language" [2, 3], which, in turn, is aimed to construction computer system, which will have almost complete knowledge of Georgian Language. Thus, at the presentation, the first results of the doctoral thesis will be overviewed briefly.

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Estimating the Density Function of Relative Distribution Via Frequency Moments

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In this talk we introduce new method of recovering the density function of the relative distribution and the quantile density function given the sequence of frequency moments. The uniform and L_1-rate of convergence of proposed constructions are established. Questions about approximating and estimating the conditional distribution and corresponding conditional quantile function are discussed as well. The estimation errors are investigated by means of simulation study.

The talk is based on joint work with Denys Pommeret.

On Ergodicity of Lévy-Type Processes on $\mathbb{R}$

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The aim of this work is to give sufficient conditions for ergodicity of the certain class of Lévy-type processes on $\mathbb{R}$ in terms of its drift and Lévy-type kernel. For this purpose the Foster–Lyapunov approach is used.

We consider a process X, whose infinitesimal generator defined on the test functions is of the following form

$$Lf(x) = a(x)f'(x) + \int\limits_{\mathbb{R}\setminus\{0\}} \big(f(x+u) - f(x) - uf'(x)\mathbb{I}_{|u|\le 1}\big)\nu(x,du), \quad f \in C^2_\infty(\mathbb{R}),$$

where $a : \mathbb{R} \to \mathbb{R}$ is (in general, unbounded) drift and the kernel $\nu(x, du)$ satisfies

$$\int\limits_{\mathbb{R}\setminus\{0\}} \min\big(1, |u|^2\big)\, \nu(x, du) < \infty \ \text{ for all } \ x \in \mathbb{R}.$$

We also assume that for every x the measure $\nu(x, \cdot\,)$ is symmetric in the sense that $\nu(x, A) = \nu(x, -A), A \in \mathcal{B}(\mathbb{R}), \{0\} \notin A$, and its tails $N(x, u) := \nu(x, (-\infty, u])$, $u < 0$, satisfy the following inequalities

$$\lambda^{-\delta(z)} \le \liminf_{x\to\infty} \frac{N(z, \lambda x)}{N(z, x)} \le \limsup_{x\to\infty} \frac{N(z, \lambda x)}{N(z, x)} \le \lambda^{-\sigma(z)}$$

for all $\lambda \ge 1$, $|x| \ge 1$ and for some bounded $0 < \sigma \le \sigma(z) \le \delta(z) \le \delta < \infty$.

We prove that under the above conditions together with some drift assumptions the respective process is ergodic, i.e.

$$\|P_t(x, \cdot\,) - \pi(\,\cdot\,)\|_{TV} \le C_{erg}(x) r(t), \quad t \to \infty,$$

where $r(t) \to 0$ as $t \to \infty$ and $\|\cdot\|_{TV}$ is the total variation norm. The function $r(t)$ can be explicitly calculated.

The talk is based on the joint paper with V. Knopova [1].

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Characterizing Linear Maps on Standard Operator Algebras

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In this talk, we evaluate linear maps F and G on standard operator algebras and in special case, we get an appropriate representation for them.

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The Dispersion Problem with a Measure Type Singular Potential

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This paper is devoted to the spectral analysis of the Schrödinger operator in E_3 with a complex-valued potential, which is a generalized function of a certain form, corresponding to a singular interaction characterized by a measure-type potential.

Let us define operator L in $L_2(E_3)$:

$$D(L) = \big\{\psi \in L_2 \mid \ \psi \in C \cap L_\infty - \Delta\psi + q\psi \in L_2\big\}, \ \ L\psi = -\Delta\psi + q\psi \in L_2.$$

Denote by $H = L_2(E_3, \alpha)$ the Hilbert space of complex-valued measurable functions $f(x)$ with the norm

$$\|f\|_H^2 = \int\limits_{E_3} |f(x)|^2 \, d\mu(x), \ \ d\mu(x) = e^{-\alpha|x|}\, dx.$$

Definition Solution $\psi(x,k,\omega)$ from H of the integral equation

$$\psi(x,k,\omega) = e^{ik(x,\omega)} - \int\limits_{E_3} \frac{e^{ik|x-y|}}{4\pi|x-y|}\, \psi(y,k,\omega)\, d\sigma(y) \tag{1}$$

is called the solution of the scattering theory problem (s.t.p.) for the equation

$$L\psi = \lambda\psi,$$

where $k^2 = \lambda$ ω arbitrary unit vector from E_3.

Let us consider the homogeneous equation

$$f(x) = -\int\limits_{E_3} G(x,y,k) f(y)\, d\sigma(y), \tag{2}$$

corresponding to the equation of the (s.t.p.) (1). The following statement has been proved:

Theorem *Let* $\operatorname{Im} k > -\alpha/2$ *and* $f(x) \in C \cap L_\infty$ *is a solution of the homogeneous equation* (2). *Then* $f(x) \in H$ *and* $Lf = \lambda f$ *in the sense of generalized functions, where* $k^2 = \lambda$.

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Using GeoGebra as an Interactive Methodology in the Educational Process

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Modern students live in the era where all information is accessible for them: smartphones, tablets, I-pads, computers have been widely used in our daily life and have the ability to adopt new technologies efficiently. The capacity of teacher is changing in teaching of information technologies day by day. So, it is crucial to has computer literacies and new ones should be integrated in the teaching and learning process.

The team of GeoGebra creates such a perfect, interactive and educational environment. GeoGebra can be used in every math lesson. It is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one engine. Teachers can use the capabilities of this program to improve the teaching process of their students.

In the mathematics lesson, GeoGebra's innovative proficiencies can be used to solve various problem-solving tasks, and it can also be used in the research-based teaching process, which increases the motivation of students and promotes the generalization of knowledge. GeoGebra software is used to solve various problems, which is a new approach in the modern education system.

The teacher should plan the learning process in such a way as to show students the usefulness of using the program, which will increase their motivation. When it comes to ways of achievement, the teacher should evaluate not only the final result, but also encourage the students' efforts. It should also be considered that the teacher sets realistic and obtainable goals for the students. The teacher should promote the development of metacognitive skills in students. It is important that they are aptitude to self-assess, analyze their own thought process and understand their significance.

An eye-catching visual materials have the ability to arouse an interest and awake motivation in learning process. As a result, it helps the teachers to provide visual aids. Graphics are a very effective tool for making a presentation that can draw students' attention.

In my presentation, GeoGebra resources will be presented, most of which are uploaded on the official GeoGebra page and can be used by any teacher. The process of making some resources is recorded and broadcasted as public ones on YouTube.

Stochastic Integral Representation of a Class of Non-Smooth Brownian Functionals

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We consider one class of Brownian functionals, which includes non-smooth functionals (therefore, it is impossible to use the well-known Clark–Ocone formula ([1]), depending on the trajectory, and we propose a method for obtaining a constructive stochastic integral representation. In addition, the class under consideration also includes functionals for which even the conditional mathematical expectation is not stochastically smooth and, therefore, neither the Glonti–Purtukhia generalization of the Clark–Ocone formula ([2]) is applicable to them. In particular, we study the functional of integral type $\int_0^T u_s(\omega)\,ds$, with $u_s(\omega)\notin D_{1,2}$ (where $D_{1,2}$ is the closure of the class of smooth random variables with respect to the Sobolev-type norm) but $E[u_s(\omega)|\Im_t^B]\in D_{1,2}$.

Let u_s be a bounded integrable process adapted to the flow of σ-algebras $\Im_s^B$. Denote

$$F(t,T)=\int_t^T u_s\,ds \ \text{ and } \ F:=F(0,T)=\int_0^T u_s\,ds.$$

Suppose that $F(t,T)$ admits a decomposition

$$F(t,T)=F_1(t,T)+F_2(t,T),$$

where $F_1(t,T)$ is a continuous process of finite variation, adapted to the flow of σ-algebras $\Im_t^B$ with $F_1(0,T)=0$ (if such a decomposition does not exist, then we assume that $F_1(t,T):=0$).

Theorem 1 *If the function $V(t,x)=E[F_2(t,T)|B_t=x]$ satisfies the requirements of the Itô formula, then the following stochastic integral representation is fulfilled*

$$F=EF+\int_0^T V_x'(t,B_t)\,dB_t \ \ (P\text{-a.s.}).$$

Theorem 2 *Let $u_s\in D_{1,2}$ for almost all s. Then the Clark–Ocone representation for the functional $F=\int_0^T u_s\,ds$ follows from the Theorem 1.*

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On an Alternative Approach for Mixed Boundary Value Problems for the Lamé System

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We consider a special approach to investigate a mixed boundary value problem (BVP) for the Lamé system of elasticity in the case of three-dimensional bounded domain $\Omega \subset \mathbb{R}^3$, when the boundary surface $S = \partial\Omega$ is divided into two disjoint parts, S_D and S_N, where the Dirichlet and Neumann type boundary conditions are prescribed respectively for the displacement vector and stress vector. Our approach is based on the potential method. We look for a solution to the mixed boundary value problem in the form of linear combination of the single layer and double layer potentials with densities supported respectively on the Dirichlet and Neumann parts of the boundary. This approach reduces the mixed BVP under consideration to a system of pseudodifferential equations which do not contain neither extensions of the Dirichlet or Neumann data, nor the Steklov–Poincaré type operator. Moreover, the right hand sides of the resulting pseudodifferential system are vectors coinciding with the Dirichlet and Neumann data of the problem under consideration. The corresponding pseudodifferential matrix operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies the unconditional unique solvability of the mixed BVP in the Sobolev space $[W_2^1(\Omega)]^3$ and representability of solutions in the form of linear combination of the single layer and double layer potentials with densities supported respectively on the Dirichlet and Neumann parts of the boundary. Using a special structure of the obtained pseudodifferential matrix operator, it is also shown that the operator is invertible in the L_p-based Besov spaces with $\frac{4}{3} < p < 4$, which under appropriate boundary data implies C^α-Hölder continuity of the solution to the mixed BVP in the closed domain $\overline{\Omega}$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.

On Equivalence of Bernoulli's and Cauchy's Inequalities

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We plan to discuss the equivalence between the Bernoulli inequality:

$$(1+h)^n \geq 1+nh, \;\; h>-1, \;\; n=1,2,\ldots$$

and AM-GM inequality.

The talk is based on [1].

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Generalization of SUSY Intertwining Relations: New Exact Solutions of Fokker–Planck Equation

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It is commonly known that the Fokker-Planck equation is exactly solvable only for some particular systems, usually with time-independent drift coefficients. To extend the class of solvable problems, we use the intertwining relations of SUSY Quantum Mechanics but in new – asymmetric – form. It turns out that this form is just useful for solution of Fokker–Planck equation. As usual, intertwining provides a partnership between two different systems both described by Fokker–Planck equation. Due to the use of an asymmetric kind of intertwining relations with a suitable ansatz, we managed to obtain a new class of analytically solvable models. What is important, this approach allows us to deal with the drift coefficients depending on both variables, x, and t. An illustrating example of the proposed construction is given explicitly.

The Problem of Construction of Equi–Strong Holes

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The construction of equi–strong holes for a problem of plane elasticity theory with a doubly connected domain is considered. The solvability of these problems provides stress optimal distribution at the boundary by selecting the appropriate boundary. For each external loading on the plate's outer boundary, appropriate construction for its equi–strong holes are built and tangential normal stresses are determined on it. Using the methods of complex analysis, the unknown equi–strong part of the boundary and a stressed state of the body are defined.Computer mathematical systems [2] MATLAB and [1] Mathcad are used for calculations and constructions.

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An Overview of the First Theoretical Results of the Doctoral Thesis "Georgian-Mathematical Two-Way Automatic Translator"

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Within the framework of the doctoral program "Informatics" of the Georgian Technical University, doctoral thesis "Georgian-Mathematical Automatic Translator" (Doctoral Student – N. Okroshiashvili, Scientific Supervisor – Professor K. Pkhakadze, Director of Scientific-Educational Center for Cultural Protection and Technological Development of Georgian State Languages at the Georgian technical University) was launched in 2019 [1]. The thesis is based on Pkhakadze's translation methods developed on the base of his Logical Grammar of Georgian Language [2, 3]. This project is a sub-project of the long-term project "Technological Alphabet of the Georgian Language" of the above-mentioned center [4], which, in turn, is aimed to the complete technology support of the Georgian Language [5]. Thus, at the presentation, the first theoretical results of the doctoral thesis will be overviewed.

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On Sets of Divergence of Fourier Series in Systems of Characters for Compact Abelian Groups

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For a class of systems of characters of compact abelian groups and homogeneous Banach spaces B satisfying some additional conditions of regularity the alternative is proved: either the Fourier series of every function from B converges almost everywhere or there exists a function in B whose Fourier series diverges everywhere. It is proved also that the classes of the divergence sets for Fourier series of functions from the mentioned spaces are closed with respect to at most countable unions and contain all zero measure sets. As corollaries, there are obtained a number of known and new results on everywhere divergent Fourier series with respect to the trigonomertic, Walsh and Vilenkin systems and their rearrangements. One of the application implies a positive answer to a problem of T. Lukashenko concerning the existence of divergent Vilenkin–Fourier series. The results of the talk are published in the paper [1].

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Dynamical Contact Problems for a Viscoelastic Half-Space with an Elastic Inclusion

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The dynamical contact problem for a viscoelastic half-space ($-\infty < x, z < \infty$, $y > 0$) which is reinforced by an elastic inclusion in the form of strip ($0 \leq y \leq b$, $-\infty < z < \infty$) lying in the plane $x = 0$ is investigated. The outer border of the inclusion is under the action of uniformly distributed shearing harmonic (acting along the oz axis) load of intensity $\tau_0 e^{-ikt}\delta(y)$, where $\delta(y)$ is the Dirac function, k is oscillation frequency, t is time. In the linear theory of viscoelasticity for Kelvin–Voigt materials only displacement component $\omega = \omega(x, y, t)$ and tangential stresses components

$$\tau_{yz} = G\,\frac{\partial \omega}{\partial y} + G_0\,\frac{\partial \dot{\omega}}{\partial y}\,, \quad \tau_{xz} = G\,\frac{\partial \omega}{\partial x} + G_0\,\frac{\partial \dot{\omega}}{\partial x}$$

are other than zero (anti-plane deformation), where G and G_0 are the elastic and viscoelastic shear modulus, respectively, the dot means a derivative with respect to the variable t. The problem is equivalent to the boundary value problem

$$G\Delta\omega + G_0\Delta\dot{\omega} = \rho\ddot{\omega}, \quad |x| < \infty, \ \ y > 0, \quad \frac{\partial \omega(x,0,t)}{\partial y} + \frac{\partial \dot{\omega}(x,0,t)}{\partial y} = 0$$

(these equations are satisfied everywhere, except the domain occupied by the inclusion). ρ is the material density of the semi-space. When passing through the inclusion, the tangential stress has discontinuities

$$\langle \tau_{xz}(0,y,t) \rangle = \mu(y,t), \ \ 0 < y < b; \quad \mu(y,t) \equiv 0, \ \ y \geq b$$

and the displacement of inclusion points $\omega^{(1)}(0, y, t)$ satisfies the condition

$$\frac{\partial^2 \omega^{(1)}(y,t)}{\partial y^2} - \frac{\rho_0}{E_0}\,\ddot{\omega}^{(1)}(y,t) = -\frac{1}{E_0 h_0}\,\big(\mu(y,t) + \tau_0 e^{-ikt}\delta(y)\big), \ \ 0 < y < b, \ \ t > 0,$$

where $\mu(y, t)$ is an unknown contact stress at the point y at the time t, acting onto the inclusion along the surface of its contact with a semi-space, ρ_0, E_0 and h_0 are the material density, elasticity modulus and thickness of the inclusion, respectively. It is required to find a fields of stresses and displacement. The solution of the problem is reduced to the integral equation. Using the method of orthogonal polynomial, the integral equation is reduced to an infinite system of linear algebraic equations. The quasi-completely regularity of the obtained system is proved.

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On the Optimal Control of Stochastic Dynamic Systems with Fractional White Noise

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Let B_t^H be a fractional Wiener process with Hurst parameter $H \in (\frac{1}{2}, 1)$. Consider the equation

$$\xi(t) = \xi_0 + \int_0^t a(x, \xi, u)\, dx + \int_0^t b(x)\, dB_x^H \tag{1}$$

where $u : [0,1] \to \widetilde{U}$ is a control that doesn't depend on the future, b is a continuous bounded non-negative function, $t \in [0,1]$. The existence of a weak solution of this equation was studied in [2].

Let U be the class of all controls for which exists a weak solution of (1). Let the functional $a(t, \xi, u(t, \xi))$ satisfy certain conditions. We define the cost of control as

$$F(u) = E \int_0^1 f\big(t, \xi^u(t), u(t, \xi^u(t))\big)\, dt,$$

where $\xi^u(t)$ is a weak solution of (1) corresponding to the control $u = u(t, \xi^u(t))$. The task of optimizing the control of the solution of (1) is to minimize the control cost F.

The main result is that under some conditions on the functionals $a(x, t, u)$ and $b(t)$ there is a control $u^* \in U$ such that

$$F(u^*) = \inf_{u \in U} F(u).$$

The proof of this statement is given in [1]. The obtained outcome can be used in solving problems of stochastic systems control in financial mathematics, hydrology, biology and many other areas.

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Sumability of a Double Trigonometric Fourier Series by Riemann's R^2 Method

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According to Ch. Fefferman's theorem [1], there exists a 2π periodic with respect to every variable and continuous at every point of the plane function of two variables whose double trigonometric Fourier series has no even one point of rectangular convergence. This implies that the methods of summability of double series are more important than those in the case of one-dimensional series.

The present report states that any double trigonometric Fourier series is almost everywhere summable by the iterated methods and by Riemann's R^2 method.

Theorem *For any summable on a square $[-\pi,\pi]^2$ function f and its corresponding double trigonometric Fourier series*

$$\sum_{m,n=-\infty}^{+\infty} c_{mn} e^{i(mx+ny)}$$

the following equalities

$$\lim_{k\to 0}\lim_{h\to 0}\sum_{m,n=-\infty}^{+\infty} c_{mn}^{i(mx+ny)}\Big(\frac{\sin mh}{mh}\Big)^2\Big(\frac{\sin nk}{nk}\Big)^2 = f(x,y),$$

$$\lim_{h\to 0}\lim_{k\to 0}\sum_{m,n=-\infty}^{+\infty} c_{mn}^{i(mx+ny)}\Big(\frac{\sin mh}{mh}\Big)^2\Big(\frac{\sin nk}{nk}\Big)^2 = f(x,y),$$

$$\lim_{(h,k)\to(0,0)}\sum_{m,n=-\infty}^{+\infty} c_{mn}^{i(mx+ny)}\Big(\frac{\sin mh}{mh}\Big)^2\Big(\frac{\sin nk}{nk}\Big)^2 = f(x,y)$$

are fulfilled at almost all points $(x,y)\in[-\pi,\pi]^2$.

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A Jacobi–Cardano Iteration Process for Solution of a Timoshenko Nonlinear System of Discrete Equations

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A boundary value problem for a nonlinear system of ordinary differential equations that describes the static symmetric displacement of a plate is considered. For construction of an approximate solution of the problem the Green functions, the Galerkin method and the Jacobi–Cardano iteration process are used. The condition for convergence of the iteration process is established and its error is estimated.

Interaction Problems on Periodic Hypersurfaces for Dirac Operators on $\mathbb{R}^n$

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We consider the Dirac operators with singular potentials

$$D_{\boldsymbol{A},\Phi,m,\Gamma\delta_\Sigma} = \mathfrak{D}_{\boldsymbol{A},\Phi,m} + \Gamma\delta_\Sigma \tag{1}$$

where

$$\mathfrak{D}_{\boldsymbol{A},\Phi,m} = \sum_{j=1}^{n} \alpha_j(-i\partial_{x_j} + A_j) + \alpha_{n+1}m + \Phi I_N \tag{2}$$

is a Dirac operator on $\mathbb{R}^n$ with variable magnetic and electrostatic potentials $\boldsymbol{A} = (A_1, \dots, A_n)$, Φ, and the variable mass m. In formula (2) α_j are the $N \times N$ Dirac matrices, that is $\alpha_j\alpha_k + \alpha_k\alpha_j = 2\delta_{jk}I_N$, I_N is the unit $N \times N$ matrix, $N = 2^{[(n+1)/2]}$, $\Gamma\delta_\Sigma$ is a singular delta-potential supported on C^2-hypersurface $\Sigma \subset \mathbb{R}^n$ periodic with respect to the action of a lattice $\mathbb{G}$ on $\mathbb{R}^n$.

We consider the self-adjointnes and discretness of the spectrum of unbounded in $L^2(\mathbb{T}, \mathbb{C}^N)$ operators associated with the formal Dirac operator (1) on the torus $\mathbb{T} = \mathbb{R}^n/\mathbb{G}$.

We study the band-gap structure of the spectrum of interaction problems associated with the formal Dirac operator (1) on $\mathbb{R}^n$ with $\mathbb{G}$-periodic regular and singular potentials.

We also consider the Fredholm property and the essential spectrum of these interaction problems associated with non-periodic regular and singular potentials supported on $\mathbb{G}$-periodic smooth hypersurfaces in $\mathbb{R}^n$.

On Low Dimensional Regular Cohomologies of Biparabolic Subalgebras

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The dimensions of zero and first regular cohomologies of a biparabolic subalgebra P of some simple Lie algebra are calculated. Namely, it is proved that if S and T are subsets of simple roots such as

$$P = H \oplus L^{R_+^S} \oplus L^{R_-^T}$$

(see [1]) where H is a Cartan subalgebra and R^S and R^T are the positive (negative) roots generated by S (by T respectively) then the dimension d_0 of the center of P is equal to the number of simple roots which is not contained in $S \cup T$. If $n = a_0 + \cdots + a_r = b_0 + \cdots + b_s$ are partitions of n and P is the corresponding biparabolic subalgebra of $sl(n)$, then the dimension of outer derivations of P is equal to $(r + s - d_0)d_0$.

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A Combination of a Three-Layer Semi-Discrete Scheme and Legendre–Galerkin Spectral Approximation for the Nonlinear Dynamic Kirchhoff String Equation

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In this work, the initial-boundary value problem is considered for the following nonlinear dynamic Kirchhoff string equation

$$u_{tt}(x,t)-\bigg(\alpha(t)+\beta\int\limits_{-1}^{1}\left[u_{\xi}(\xi,t)\right]^2\,\mathrm{d}x\bigg)u_{xx}(x,t)=f(x,t),\ \ (x,t)\in(-1,1)\times(0,T]\,.$$

Here $\alpha(t)\geq \mathrm{c}_0>0$ with $\alpha(t)$ is a continuously differentiable function, $\beta>0$ and $f(x,t)$ is a continuous function. $u(x,t)$ is unknown function. For solving this problem approximately, a symmetric three-layer semi-discrete scheme with respect to the temporal variable is applied, in which the value of a nonlinear term is taken at the middle node point (see: [1–3]). This approach allows us to find numerical solutions per temporal step by inverting the linear operators. In other words, applying this scheme, a linear ordinary differential equations system is obtained. The local convergence of the scheme is proved. The results of numerical computations using this scheme for different test problems are given (see [3]) for which the Legendre–Galerkin spectral approximation is applied with respect to the spatial variable.

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On the Sum of Powers of Integer Numbers

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The sufficient conditions for the equation $\sum\limits_{i=1}^{m} x_i^n = b$ (with $n,m \geq 2$, $n,m \in \mathbb{N}$, and a nonnegative integer b) to have no nonnegative integer solution are given, for each of the following cases: $3^k \mid n$, $2 \cdot 5^k \mid n$, $\frac{(p-1)}{2} \mid n$ ($k \in \mathbb{N}$, p is a prime with $p > 3$). Besides, the sufficient conditions for the equation $\sum\limits_{i=1}^{m} x_i^n = b(c^s)^n$ (with $n,m \geq 2$, $n,m \in \mathbb{N}$, and nonnegative integers b and c) to have no nonnegative integer/natural solution, for arbitrary nonnegative integer s, are given, for each of the following cases: $2^k \mid n$, $2 \cdot 3^k \mid n$, $4 \cdot 5^k \mid n$, $(p-1) \mid n$ ($k \in \mathbb{N}$, p is a prime with $p \geq 3$).

Moreover, we consider the equation $x_1^n + x_2^n + \dots + x_m^n = z^n$ with $n,m \in \mathbb{N}$, $n,m \geq 2$. Namely, for each of the particular cases where $2^k \mid n$, $2 \cdot 3^k \mid n$, $4 \cdot 5^k \mid n$, $(p-1) \mid n$ ($k \in \mathbb{N}$, p is a prime with $p \geq 3$), we find the upper bounds of m's, for which the following condition is satisfied: if $(x_1, x_2, \dots, x_m, z)$ is this equation's solution with coprime components, then one has resp. $2 \dagger z$, $3 \dagger z$, $5 \dagger z$, $p \dagger z$, and, moreover, precisely one of the integers $x_1, x_2, \dots, x_m$ is not divisible by resp. 2, 3, 5, p. Further, for each of the particular cases where $3^k \mid n$, $5^k \mid n$, $\frac{p-1}{2} \mid n$ ($k \in \mathbb{N}$, p is a prime with $p > 3$), we find the upper bounds of even m's, for which the following condition is satisfied: if $(x_1, x_2, \dots, x_m, z)$ is this equation's solution with natural components, then at least one of them is divisible by resp. 3, 5, p.

Applying the explicit formula for the sum of powers of natural numbers, found in our paper [2], we obtain the following formula:

$$\sum_{x=1}^{m} x^n = (-1)^n \sum_{i=1}^{n} a_i m(m+1)\dots(m+i),$$

where

$$a_1 = -\frac{1}{2}\,, \ \ a_i = \frac{1}{i+1} \sum_{k=1}^{i} \frac{(-1)^k k^n}{k!(i-k)!} \ \ (i = 2,3,\dots,n-1), \ \ a_n = \frac{(-1)^n}{n+1} \ \ (n \in \mathbb{N}).$$

It implies that the polynomial $\varphi(m) = \sum\limits_{x=1}^{m} x^n$ (over a variable m) is divisible not only by m, as it follows from the Kudryavtsev's formula for $\varphi(m)$ (in which some Bernoulli numbers present) [1], but also by $(m+1)$, for any $n \in \mathbb{N}$.

The method to find the lower estimates of values of the Hardy function $G(n)$ appearing in Waring's problem, for the certain set of values of n, is given. Applying it, we show that

$$G(2^k l) \geq 4 \cdot 2^k \ \text{ for } \ k \geq 2, \quad G(2 \cdot 3^k l) \geq 3^{k+1}, \ \ G(4 \cdot 5^k l) \geq 5^{k+1} \ \ (k, l \in \mathbb{N}).$$

All the results of this work are obtained employing only the elementary methods.

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Fuzzy Expert System to Model the Breast Cancer Diagnosis and its Prognosis

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Nowadays decision-making applications with complex issues and knowledge involving imprecision and uncertainty are very much important. Therefore, soft computing approaches including Artificial Neural networks or Fuzzy Inference systems have been used widely to model expert behaviour. The fuzzy soft computing applications are rapidly emerging in several fields including medical diagnosis and prognosis too. There are several technology-oriented studies reported for breast cancer diagnosis, and some studies have also been done for the prognosis of breast cancer. However, breast cancer prognosis suffers from uncertain and imprecise input factors and incompleteness of knowledge of the problem as well as diagnosis. This study presents a fuzzy expert system for the diagnosis and prognosis of breast cancer, which is capable enough to record the ambiguity and imprecision prevalent in the interpretation of breast cancer. In this work, Mamdani fuzzy inference model has been used as it is more intuitive and has high interpretability when it comes to interacting with experts during the prognosis process. The advantage of this model is to predict the risk of developing breast cancer in females of the age group 20-50 years. Furthermore, the fuzzy model is evaluated on a real dataset of 'All India Institute of Medical Sciences, New Delhi. In this work, we have considered six input parameters i.e. Age, First menstrual cycle, First pregnancy age, Body mass index, Smoking, and Radiation exposure. This approach is promising for the prediction of developing the risk of breast cancer and early diagnosis of cancer, therefore this may improve the survival rate.

Keywords: fuzzy logic, breast cancer, diagnosis, prognosis.

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Infinitely Many Solutions for Fractional Hamiltonian System

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In this paper, we are concerned with the existence of infinitely many solutions for the following fractional Hamiltonian system

$$\begin{cases} {}_tD^{\alpha}_{\infty}({}_{-\infty}D^{\alpha}_t u)(t) + L(t)u(t) = \nabla W(t, u(t)), \quad t \in \mathbb{R}, \\ u \in H^{\alpha}(\mathbb{R}), \end{cases} \tag{1}$$

where ${}_{-\infty}D^{\alpha}_t$ and ${}_tD^{\alpha}_{\infty}$ are left and right Liouville–Weyl fractional derivatives of order $\frac{1}{2} < \alpha < 1$ on the whole axis respectively, $L \in C(\mathbb{R}, \mathbb{R}^{N^2})$ is a symmetric matrix valued function unnecessary coercive and $W(t, x) \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$. The novelty of this paper is that, assuming that L is bounded from below and unnecessarily coercive at infinity, and W is only locally defined near the origin with respect to the second variable, we show that (1) possesses infinitely many solutions via a variant Symmetric Mountain Pass Theorem.

About Continuity of Solution on the Initial Data for a Neutral Differential Equation

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The neutral differential equation is a mathematical model of such system whose behavior at a given moment depends on the velocity and state of the system in the past. Many real processes are described by neutral differential equations [1, 3].

In the present work is considered the quasi-linear neutral differential equation with the two types controls

$$\dot{x}(t)=A\big(t,x(t),x(t-\tau),v(t)\big)\dot{x}(t-\tau)+B\big(t,x(t),x(t-\tau),u(t)\big),\ \ t\in[t_0,t_1]$$

and with the initial condition

$$x(t)=\varphi(t),\ \ t<t_0,\ \ x(t_0)=x_0,$$

where the control function $v(t)$ is piecewise-continuous and the control function $u(t)$ is measurable.

The theorem about continuity of solution on the initial data is proved. Here, under the initial data we mean the collection of delay parameter τ, initial function φ, initial vector x_0 and control functions $v(t)$ and $u(t)$. Analogous theorems for the quasi-linear neutral differential equation without controls, where

$$A\big(t,x(t),x(t-\tau),v(t)\big)\equiv A(t)$$

are proved in [2, 3].

Acknowledgments

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Minimality Conditions Equivalent to the Finitude of Fermat and Mersenne Primes

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It is still open whether there exist infinitely many Fermat primes or infinitely many composite Fermat numbers. The same question concerning the Mersenne numbers is also unsolved. Extending some results from [1], we characterize the Fermat primes and the Mersenne primes in terms of topological minimality. This is done by showing, among other things, that if $\mathbb{F}$ is a subfield of a local field of characteristic $\neq 2$, then the special upper triangular group $SUT(n,\mathbb{F})$ is minimal precisely when the special linear group $SL(n,\mathbb{F})$ is. We provide criteria for the minimality (and total minimality) of $SL(n,\mathbb{F})$ and $SUT(n,\mathbb{F})$, where $\mathbb{F}$ is a subfield of $\mathbb{C}$.

Let $\mathcal{F}_\pi$ and $\mathcal{F}_c$ be the set of Fermat primes and the set of composite Fermat numbers, respectively. As our main result, we prove that the following conditions are equivalent for $\mathcal{A} \in \{\mathcal{F}_\pi, \mathcal{F}_c\}$:

- $\mathcal{A}$ is finite;
- $\prod\limits_{F_n\in\mathcal{A}} SL(F_n-1,\mathbb{Q}(i))$ is minimal, where $\mathbb{Q}(i)$ is the Gaussian rational field;
- $\prod\limits_{F_n\in\mathcal{A}} SUT(F_n-1,\mathbb{Q}(i))$ is minimal.

Similarly, denote by $\mathcal{M}_\pi$ and $\mathcal{M}_c$ the set of Mersenne primes and the set of composite Mersenne numbers, respectively, and let $\mathcal{B} \in \{\mathcal{M}_\pi, \mathcal{M}_c\}$. Then the following conditions are equivalent:

- $\mathcal{B}$ is finite;
- $\prod\limits_{M_p\in\mathcal{B}} SL(M_p+1,\mathbb{Q}(i))$ is minimal;
- $\prod\limits_{M_p\in\mathcal{B}} SUT(M_p+1,\mathbb{Q}(i))$ is minimal.

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On an Initial-Boundary-Value Problems for the Time-Fractional Diffusion Equation

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In this paper the initial-boundary and the initial problems for the time-fractional diffusion equations are studied. The existence and uniqueness of the solution have been established using the Fourier methods. The convergence of the solutions was evidenced using the estimate of Kilbas–Saigo function, by Parseval's identity and by Plancherel theorem.

On Finding a Proper Number and Proper Vector-Function for First Plane Interior Homogeneous Boundary Value Problem of Stationary Oscillations with Regard to Moment Strains

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In the paper by variation method it is stated that in the case of a finite simply connected plane domain a minimal proper number of the first hemogeneous boundary value problem, of stationary oscillations with regard to moment strains (when on the boundary domain the displacement vector and the rotation are equal to zero) is equal to the minimum value of the specially constructed functional, minimizing vector-function of the functional represent a proper vector-function of the problem.

On the Solution of the Roben BVP of Thermo-Electro-Magneto Elasticity for Half Space

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Let $\mathbb{R}^3$ be divided by some plane into two half-spaces. Assume that these half-spaces are

$$\mathbb{R}^3_1 := \big\{x \mid \ x=(x_1,x_2,x_3)\in\mathbb{R}^3,\ x_3>0\big\},\quad \mathbb{R}^3_2 := \big\{x \mid \ x=(x_1,x_2,x_3)\in\mathbb{R}^3,\ x_3<0\big\}.$$

We investigate the following Roben type boundary value problem of the thermo-electro-magneto-elasticity for a half-space.

Roben Problem Find a solution vector $U=(u,\varphi,\psi,\vartheta)^T\in[C^1(\overline{\mathbb{R}^3_j})]^6\cap[C^2(\mathbb{R}^3_j)]^6$, $j=1,2$, to the system of equations

$$A(\partial)U=0 \ \text{ in } \mathbb{R}^3_j,\ \ j=1,2, \tag{1}$$

satisfying the Roben type boundary condition

$$\{\mathcal{T}(\partial,n)U+aU\}^{\pm}=F \ \text{ on } \ S=\partial\mathbb{R}^3_1=\partial\mathbb{R}^3_2, \tag{2}$$

where $A(\partial)=[A_{pq}(\partial)]_{6\times 6}$ is the matrix differential operator of statics in the theory of thermo-electro-magneto-elasticity and $\mathcal{T}(\partial,n)$ is the corresponding generalized stress operator [1], a is a positive constant. We require that $F\in\mathring{C}^\infty(\mathbb{R}^2)$.

Theorem 1 *The Roben boundary value problems* (1), (2) *have at most one solution* $U=(u,\varphi,\psi,\theta)^T$ *in the space* $[C^1(\overline{\mathbb{R}^3_j})]^6\cap[C^2(\mathbb{R}^3_j)]^6$, $j=1,2$, *provided*

$$\theta(x)=\mathcal{O}(|x|^{-1}) \ \text{ and } \ \partial^\alpha\widetilde{U}(x)=\mathcal{O}(|x|^{-1-|\alpha|}\ln|x|) \ \text{ as } \ |x|\to\infty$$

for arbitrary multi-index $\alpha=(\alpha_1,\alpha_2,\alpha_3)$. *Here* $\widetilde{U}=(u,\varphi,\psi)^T$.

Theorem 2 *Let* $F\in\mathring{C}^\infty(\mathbb{R}^2)$ *and* $\int\limits_{\mathbb{R}^2}F(\widetilde{x})\widetilde{x}^\beta\,d\widetilde{x}=0$ *for arbitrary multi-index* $\beta=(\beta_1,\beta_2)$, $|\beta|=0,1$. *Then the unique solutions of the boundary value problems* (1), (2) *can be represented in the form*

$$U(x)=\mathcal{F}^{-1}_{\widetilde{\xi}\to\widetilde{x}}\Big[\Phi^{(-)}(\widetilde{\xi},x_3)\big[\mathcal{T}(-i\xi,n)\Phi^{(-)}(\widetilde{\xi},0)+a\Phi^{(-)}(\widetilde{\xi},0)\big]^{-1}\widehat{F}(\widetilde{\xi})\Big],\ \ x_3>0,$$

or

$$U(x)=\mathcal{F}^{-1}_{\widetilde{\xi}\to\widetilde{x}}\Big[\Phi^{(+)}(\widetilde{\xi},x_3)\big[\mathcal{T}(-i\xi,n)\Phi^{(+)}(\widetilde{\xi},0)+a\Phi^{(+)}(\widetilde{\xi},0)\big]^{-1}\widehat{F}(\widetilde{\xi})\Big],\ \ x_3<0.$$

Here $\mathcal{F}^{-1}$ *denotes the inverse generalized Fourier transform and* $\Phi^{(\pm)}$ *are matrices constructed by the symbol matrix of the operator* $A(\partial)$ [2].

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Real and Complex Analysis on Tangential Limit of Bliashke–Djerbashyan Canonical Product

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If exist $f:\{z:\ |z|<1,\ z\in C\}\to C$ function and exist finite limit, when $z\to e^{i\theta}$ so that

$$z\in R(m,\theta,\gamma)=\big\{z:\ 1-|z|\geq m|\arg(ze^{-i\theta)}|,\ z\in D\big\},$$

where $-\pi<\arg ze^{-i\theta}\leq\pi$, m is arbitrary and γ is fixed positive number T_γ is tangential limit and is denoted by

$$T_\gamma\lim_{z\to e^{i\theta}}f(z).$$

Theorem 1 *Let* $0<|a_n|\leq|a_{n+1}|<1$,

$$1,\ \lim_{n=1}^{+\infty}|a_n|=1,\ \ |z|<1,$$

$$\sum_{n=1}^{+\infty}\big(1-|a_n|\big)^p=+\infty,\quad \sum_{n=1}^{+\infty}\big(1-|a_n|\big)^{p+1}=+\infty,$$

where p is natural number, Blaschke–Djerbashyan canonical product

$$B_{p+1}(z,(a_n))=\prod_{n=1}\Big(1-\frac{1-|a_n|^2}{1-\overline{a_n}z}\Big)\exp\bigg(\sum_{k=1}^{p}\frac{1}{k}\Big(\frac{1-|a_n|^2}{1-\overline{a_n}z}\Big)\bigg)$$

has T_γ tangential limit in the $e^{i\theta}$ point and

$$T_\gamma\lim_{z\to e^{i\theta}}B_{p+1}(z,(a_n))=B_{p+1}(e^{i\theta},(a_n))\neq 0,+\infty.$$

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D-Independent Topological Groups

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A topological group G with $|G| > 1$ is called *d-independent* if for every subgroup S of G with $|S| < 2^\omega$, one can find a countable dense subgroup H of G such that $S \cap H = \{e\}$. Therefore, d-independent groups are separable and have cardinality at least 2^ω. Our main result is a purely algebraic characterization of d-independence in the class of compact metrizable abelian groups. We prove that a compact metrizable abelian group G is d-independent if and only if for every integer $m \geq 1$, either $|mG| = 2^\omega$ or $|mG| = 1$. This characterization implies that a compact metrizable abelian group G is d-independent if and only if it is *maximally fragmentable* in the sense of [1] if and only if G is an M*-group*, as defined by D. Dikranjan and D. Shakhmatov in [2].

Also, we present a characterization of separable metrizable d-independent topological abelian groups and show that products of separable topological groups can often be d-independent, even if the factors fail to be d-independent.

This is a joint work with E. Márquez Rodríguez.

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Mixed Type Boundary-Transmission Problems with Interior Cracks of the Thermo-Piezo-Electricity Theory Without Energy Dissipation

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In the paper, we study mixed type interaction problem of pseudo-oscillations between thermo-elastic and thermo-electro-elastic bodies with interior cracks. The model under consideration is based on the Green–Haghdi theory of thermo-piezo-electricity without energy dissipation. This theory permits propagation of thermal waves only with finite speed. Using the potential theory and boundary pseudodifferential equations method, we prove existence and uniqueness of solutions, and analyze their smoothness.

Some Problems of Convergence of General Fourier Series

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S. Banach proved that good differential properties of function do not guarantee the a.e. convergence of the Fourier series of this function with respect to general orthonormal systems (ONS). On the other hand it is very well known that a sufficient condition for the a.e. convergence of an orthonormal series is given by the Menshov–Rademacher Theorem.

The talk we devoted deals with sequence of positive numbers (d_n) such that multiplying the Fourier coefficients $(C_n(f))$ of functions with bounded variation by these numbers one obtains a.e. convergent series of the form $\sum\limits_{n=1}^{\infty} d_n C_n(f)\varphi_n(x)$. It is established that the resulting conditions are best possible.

The talk is based on joint work with co-author.

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Approximate Solution of a Timoshenko Dynamic Beam Equation

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Galerkin and difference methods are used to solve a nonlinear integro-differential Timoshenko dynamic beam equation. The resulting algebraic system of cubic equations is solved by the iterative method. The iteration process error is estimated.

Optimal Management of Systems with Reconfigurable Structure

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Complex systems designed with multifunctional elements (MFE) have many options for the distribution of functions between MFE and many ways of functioning. High efficiency of the systems is reached by optimal distribution of functions among elements. At the same time in the case of any element failure, the management of system maneuvering (system structure reconfiguration) problem appears [1]. To solve appeared problem, we use probability matrix $P(F_a) = \{p_j(f_i)\}$, $i \in [1,n]$, $j \in [1,m]$, at the stage of system creation as well as during it's functional phase. This matrix shows how many functions can be completed by the MFE and at what probability. The system of $n > m \geq k_i \geq 1$ class are discussed in the paper. In this class n is number of elements, m–number of functions, k_i-number of functional resources of each MFE. The condition of system failure is defined using by logical-probability methods [2]. The condition of complex system successful performance is defined by using following function:

$$Y\big\{a_i(f_j),\ i \in [1,n],\ j \in [1,m]\big\} = \bigcup_{q=1}^{N_S} S_q,$$

where

$$S_q = a_{i_1}(f_{j_1})\,\&\,a_{i_2}(f_{j_2}),\&\ldots\&,a_{i_m}(f_{j_m}),\ \ i_1 \neq i_2 \neq \cdots \neq i_n,\ \ j_1 \neq j_2 \neq \cdots \neq j_m.$$

To optimally distribute functions among MFEs, we have to replace logical elements with probability elements of P(Fa) matrix and solve following optimization problem.

$$\Phi = \prod_{i=1}^{n}\prod_{j=1}^{m} p_i(f_j)x \longrightarrow \max \sum_{i=1}^{n} x_{ij} = 1,\ \ j \in [1,m];\quad \sum_{j=1}^{m} x_{ij} = 1,\ \ i \in [1,n].$$

In case $n > m$ of during the calculations we will select m elements from n, with optimal distribution of functions in a way that system will be able to perform F function with maximum efficiency. In case $n = m$, above mentions model will help us to optimally distribute functions among elements. In the process of the system performance in case ith MFE partially loses the ability to perform jth function, $p_j(f_i)$ element will be equaled to zero in the $P(Fa)$ matrix and optimization problem will be solved again. As a result, new distribution of functions will be conducted among elements (optimal inter-replacement of elements). This will ensure the continuation of successful performance of the system.

Keywords: Complex System, Multifunctional Element, Reconfigurable Structure, Optimal Management, Probability Matrix.

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On Canonical Commutation Relation for Creation and Annihilation Operators in the Strict Quantum Frechet–Hilbert Space of States

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For a particle moving in $\mathbb{R}$, let quantum Hilbert space be $L^2(\mathbb{R})$. Let

$$C\psi = \Big(-\frac{d}{dx} + \frac{x}{2}\Big)\psi, \ \ \psi \in D(C) \subset L^2(\mathbb{R})$$

and

$$A\psi = \Big(\frac{d}{dx} + \frac{x}{2}\Big)\psi, \ \ \psi \in D(A) \subset L^2(\mathbb{R})$$

are unbounded creation and annihilation operators in the space $L^2(\mathbb{R})$. It is well-known that these operators do not commute, but satisfy the relation $AC-CA=I$, where I is identity operator. This relation is known as the canonical commutation relation. We extend these operators from Hilbert space to strict Frechet–Hilbert space $L^2_{loc}(\mathbb{R})$. It is easy to prove that extended operators C^* and A^* also satisfies relation $C^*A^* - A^*C^* = I$, that is the generalization of canonical commutation relation in the Frechet–Hilbert space $L^2_{loc}(\mathbb{R})$. As well as some relations for these operators are proved.

The considered of the space $L^2_{loc}(\mathbb{R})$ instead of the space $L^2(\mathbb{R})$ essentially extend the space of states, which is achieved by weakening the topology of the space $L^2(\mathbb{R})$. In [3] we have studied in detail the topological and geometrical properties of these spaces. In [2], we have extending some results about self-adjoint operators for Frechet–Hilbert spaces that contains the nuclear Frechet and countable Hilbert spaces. Indeed, we generalize the theorems von Neuman, Hellinger–Teplitz, Fridrichs and Ritz's for self-adjoint operators in these spaces. In [1] the problem of extension of self-adjoint operators from Hilbert spaces to the strict Frechet–Hilbert spaces was considered.

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On Exact Solution of One Task of Magnetic Hydrodynamics

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In the article is considered the nonstationary flow of a viscous incompressible conductive fluid in between two planar parallel plates at existence of transversal magnetic fluid. Due the Laplace transformations are obtained the expressions for velocity of fluid and magnetic field [1–4].

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On Boundedness of the Higher Dimensional Hilbert Operator in Rearrangement Invariant Spaces

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The talk is based on a recently published joint work [3] with F. Sukochev and D. Zanin where we obtained a weak $(1, 1)$ type estimate for a higher dimensional Hilbert operator answering an open question by A. Os kowski [1]. This result together results in [2] allow us to investigate boundedness of the Hilbert operator in rearrangement invariant quasi-Banach spaces.

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Canonical Commutation Relation for Orbital Operators Corresponding to Creation and Annihilation Operators

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In this article the orbits of creation C, annihilation A and numerical operators N at the states of quantum Hilbert spaces $L^2(\mathbb{R})$ are created. As well the Hilbert space of finite orbits $D(C^n)$, $D(A^n)$, $D(N^n)$ and the Frechet–Hilbert space $D(C^\infty)$, $D(A^\infty)$, $D(N^\infty)$ of all orbits for these operators are created and the orbital operators corresponding to these operators in the spaces of orbits are defined and studied. These space and operators for Hamiltonian of quantum harmonic oscillator was recently studied in [1]. Generalization of well-known canonical commutation relations for orbital operators corresponding to creation and annihilation operators are established. The following theorems are proved:

Theorem 1 *For the commutator* $[A_n, C_n] = A_nC_n - C_nA_n$ *the following is hold. If*

$$(\varphi_0, \varphi_1, \cdots, \varphi_n) \in D([A_n, C_n]) = D(A_nC_n) \cap D(C_nA_n),$$

then

$$[A_n, C_n](\varphi_0, \varphi_1, \dots, \varphi_n) = (\varphi_0, \varphi_1, \dots, \varphi_n).$$

Theorem 2 *For the commutator* $[A^\infty, C^\infty] = A^\infty C^\infty - C^\infty A^\infty$*, the following is hold. If*

$$(\varphi_0, \dots, \varphi_n, \dots) \in D([A^\infty, C^\infty]) = D(A^\infty C^\infty) \cap D(C^\infty A^\infty),$$

then

$$[A^\infty, C^\infty](\varphi_0, \dots, \varphi_n, \dots) = (\varphi_0, \dots, \varphi_n, \dots).$$

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Introduction of Mathematical Notions in the School Course of Mathematics

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In this paper we give some recommendations on moderate application of mathematical accuracy when introducing new notions.

q-Laplace Transforms for the Product of Basic Analogue for H-Functions for Two Variables and the General Class of q-Polynomials

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The q-Laplace transforms for the product of basic analogue of H-function of two variables and the general class of q-polynomials has been evaluated in the present paper. Applications involving the basic analogues of Fox's H-function have also been established.

Key words: q-Laplace transforms, q-analogues of Fox's H-function of two variables, general class of q-polynomials.

On Left Hereditary Left Perfect Right Coherent Rings

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In the present work, "ring" means an associative ring with identity, and "module" means a left module over the ring.

The criterion for a ring to be left perfect and right coherent, found by S. U. Chase in 1960 [1], is well-known. It requires that the direct product of an arbitrary (small) family of projective modules be projective. In the present work several more criteria are given, for the case of a left hereditary ring. In particular, we prove that such a ring is left perfect and right coherent if and only if any module over it can be represented as the direct sum of a stable module and a projective module (recall that a module is said to be stable if it has no nonzero projective direct summand). Such a representation is unique up to an isomorphism and functorial.

It is shown that the pair $(\mathit{Stable\ modules}, (\mathit{Stable\ modules})^{\perp})$ is a pre-torsion theory if a ring is left hereditary. Moreover, such a ring is left perfect and right coherent if and only if the class $(\mathit{Stable\ modules})^{\perp}$ coincides with the class of projective modules.

It is proved that, for a left hereditary ring, the pair

$$(\mathit{Stable\ modules}, \mathit{Projective\ modules})$$

is a torsion theory if and only if the injective envelope of the ring, viewed as a module over itself, is projective. In this case the modules that are divisible torsionfree with respect to this torsion theory are precisely the projective injective modules.

Note that rings of the latter kind (i.e. left hereditary rings the injective envelopes of which, viewed as modules over themselves, are projective) were characterized by R. R. Colby and E. A. Rutter in 70's of the past century [2]. They proved that these rings are precisely the finite direct products of complete blocked triangular matrix rings over division rings. Recently, in our joint paper with A. Martsinkovsky [3], another characterization of rings of this kind in terms of the stable category was given. In the present work, we study some other properties of this category, namely, the existence of cokernels and injective envelopes in it.

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The Role of Developmental Assessment in Improving Student Achievement

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The main goal of third generation national curriculum is to assess the student. The purpose of student assessment is to manage the quality of teaching and learning. A developmental role plays a special role in this regard. It is through developmental assessment that the student's progress and development is promoted. At this time not only the learning process improves but also the teaching process, the teacher makes the teaching adapted to the needs of the students. The student receives benefits for quality developmental assessment. It is important for students to understand the purposes of formative assessment, namely that making mistakes is acceptable – even expected – and that they will not receive grades based on their answers, but will receive descriptive, constructive feedback and, consequently, tailored teaching needs.

The paper presents various assessment developmental techniques developed in the elementary mathematics lesson, such as rapid "pulse checking" in order for the teacher to assess the students' understanding of the learning content and to guide the learning process accordingly.

Mini-Symposium: Topology in Georgia – Abstracts of Talks

On Nadiradze's Work On Formal Group Laws

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We give an overview of Nadiradze's doctoral dissertation and the current state of the art.

One aspect is a constriction of a commutative complex oriented cohomology, the coefficient ring of which is the coefficient ring of the Buchstaber formal group low with inverted 2.

Theorem *There exists a commutative complex oriented cohomology which is module over* $MU_*[1/2]$ *and has the scalar ring* $MU_*[1/2]$ *modulo* J^e_{SU}*, the ideal generated by any set of polynomial generators of* $MSU_*[1/2]$ *of degree* ≥ 10 *viewed as elements in* $MU_*[1/2]$ *by forgetful injection map. The latter is identical to the scalar ring of the universal Buchstaber formal group law localized away from 2.*

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Topology in Batumi

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The report presents a brief overview of the results of topologists working in Batumi starting from the 1950s. In particular, it deals with the researches of Givi Khukhunaishvili, Dursun Baladze, Shalva Bakhtadze, Onise Surmanidze, Vladimer Baladze (the author himself) and his students – Lela Turmanidze, Maia Dzadzamia, Anzor Beridze, Ruslan Tsinaridze and Pridon Dumbadze.

George Chogoshvili's Contribution to the Development of Homology Theory

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The greater part of G. Chogoshvili's mathematical works belong to Algebraic Topology, in particular to Homology Theory. The brief description of some of the main results will be given and their role in the development of Homology Theory discussed.

Leonard Mdzinarishvili and Strong Homology Theory

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The strong homology theory is an exact homology theory defined on the category $\mathbb{K}_{Top}$ pairs of general topological spaces. It is known that on the category $\mathbb{K}_C$ of compact pairs this theory satisfies the Universal Coefficient Formula [3, Theorem 21.15]. Therefore, it is isomorphic to the Steenrod homology theory for pairs of compact topological spaces [1]. On the other hand, the study of the exact homology theories on the category $\mathbb{K}_C$ started in Georgia from the beginning of 1940s by George Chogoshvili [2] and his school. For general topological spaces an exact homology theory was constructed in [4]; in particular, it is mentioned that the so called strong homology groups were defined by Z. Miminoshvili (PhD student of Mdzinarishvili). According to Mardesic [3] these groups are to be called 1-height homology groups [3, § 17.1, p. 333]). What today is called the strong (not finite height) homology groups was defined a bit later. However, in the paper [5] these homology groups are called the total homology groups (see bibliographic notes of § 17 in [3]). Note that, in some sense the finite height homology groups can be considered as an approximation of the strong homology groups [3]. In the paper [5] these groups are simply called fragments of the total homology groups. The essential formulas, which give the connection of strong (total) homology and finite height (fragment) homology groups are the sequences:

$$\cdots \longrightarrow \overline{H}^{(r-2)}_{n+1}(\mathbf{C}) \longrightarrow \varprojlim{}^r H_{n+r}(\mathbf{C}) \longrightarrow \overline{H}^{(r)}_n(\mathbf{C}) \longrightarrow \overline{H}^{(r-1)}_n(\mathbf{C}) \longrightarrow \varprojlim{}^{r+1} H_{n+r}(\mathbf{C}) \longrightarrow \dots,$$

$$0 \longrightarrow \varprojlim{}^1 \overline{H}^{(*)}_{n+1}(\mathbf{C}) \longrightarrow \overline{H}_n(\mathbf{C}) \longrightarrow \varprojlim \overline{H}^{(*)}_n(\mathbf{C}) \longrightarrow 0.$$

The announcement about these formulas was in [6], but the proofs for the first time appeared in [5] (see bibliographic notes of § 17 in [3]).

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Fine Topologies, Everett–Ulam's Problem for Simple Extensions and Other Applications of Bitopologies

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The reasons connected with applications of bitopologies (τ_1, τ_2) on a set X in potential theory, real analysis, set-point topology, etc., require to consider the cases, when $\tau_1 \subset \tau_2$ or τ_1 and τ_2 are connected by $<_S$, $<_C$ and $<_N$-relations (see, for example [1, 2]). In particular, to establish properties of τ_i are used the properties of τ_j, $i \neq j$, $i, j \in \{1, 2\}$ or pairwise properties of (τ_1, τ_2) (or their combinations).

Following Brelot [3], a convex cone Φ of lower semicontinuous functions f, defined of a topological space (X, τ_1), determines a new topology τ_2 on X, finer than τ_1, making all functions from Φ continuous. The obtained bitopological space, denoted by $(X, \tau_1 <_\Phi \tau_2)$, is called a bitopological space in potential theory and is studied in [1]. There naturally arised and was later answered the question which can treat a space $(X, \tau_1 <_\Phi \tau_2)$ as a space, for which the generalized version of the classic Kadec Renorming Theorem [4] is true.

Furthermore, Everett and Ulam posed the problem: when and how can a new topology τ_2 be constructed on (X, τ_1) such a way that $H(X, \tau_1) = H(X, \tau_2)$, where $H(X, \tau_i)$, $i \in \{1, 2\}$, is the class of all homeomorphisms of (X, τ_i) onto itself [5]. The bitopological solution of this problem is also given for a simple extension $\tau_2 = \tau_1(A)$, $A \,\overline{\in}\, \tau_1$ [6], of the topology τ_1.

Finally, together with consideration of bitopological essence of Baire-like properties in real analysis [7], and transmission of $<_S$, $<_C$ and $<_N$-relations from bitopology (τ_1, τ_2) to the bitopology $(\tau_1(A), \tau_2(A))$, $A \,\overline{\in}\, \tau_2$, a new equivalence relation on a special subfamily of the lattice $\mathrm{LT}(X)$ of all topologies on a set X is introduced, for which if one member of an equivalence class is connected, then all members of this class are also connected.

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George Chogoshvili. History and Unasked Questions

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George Chogoshvili (1914–1998) is widely known as the founder of Topology School in Georgia, but, as many of us know, his role in the history of Georgian mathematics is actually even greater. This talk is an attempt to describe it, and to propose some not-exactly-mathematical questions that could be asked a long time ago, including those about Chogoshvili's first twelve publications listed below. We will also recall some facts of his scientific biography, which includes:

1939: PhD ("Candidate of Science") at Moscow University (supervisor P. Alexandrov);

1945: DSc at Moscow University;

1956: Corresponding Member of the Georgian Academy of Sciences;

1960: (Full) Member of the Georgian Academy of Sciences.

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Survey of Mathematical Work of Dito Pataraia

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Dimitri Pataraia was one of the brightest minds and kindest souls that I had luck to meet in my life.

To give him proper credit I should speak not so much about his contribution to mathematics but about some far more important matters.

He has worked in K-theory, combinatorics, intuitionistic logic, homotopy theory, doing uniquely illuminating work, some (very little) of which is widely recognized, some other well known but yet unpublished, and still other only known to very few people.

I will try to explain his original construction of fundamental groups, his masterpiece now known under the name of Witt–Bourbaki–Pataraia fixed point theorem, and his work on the Pitts problem in topos theory.

Pataraia's construction of the fundamental group is very elegant, although it requires some technical detail which I will try to provide.

His constructive proof of the fixed point theorem for dcpo's is astonishingly clever, short and simple, so I will be able to tell this proof in full.

His work on the Pitts problem, although completed, is unfortunately still unpublished. It consists of two major steps, of which the first one was later reproved by Johnstone using alternative methods (theory of allegories). But the second and the most hard step is very technical, full exposition on paper would take about 40 pages, so I certainly cannot tell any details of it, but I will try to explain the main idea.

Nodar Berikashvili and His School

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Nodar Berikashvili, internationally recognized famous Georgian mathematician, was born in 1929. In 1952 he become an aspirant of academician George Chogoshvili. Nodar Berikashvili's dissertation got the attention of world famous Moscowian topologists Pavel Alexandrov, who invited yang Nodar Berikashvili to work in Steklov Mathematical Institute. Nodar Berikashvili got his PhD degree from this institution and worked there till 1959. Interesting to mention that on the site of Steklov Institute, as well as on the site of Moscow State University Nodar Berikashvili's name is in the list of their honored scientists.

In 1959 he returns to Georgia and starts to work in A. Razmadze Mathematical Institute of Georgian Academy of Sciences. In 1971 he becomes the degree of Doctor of Sciences from Steklov Institute, and in 2001 he was elected as the member of Georgian Academy of Sciences.

Besides his scientific activity Noder Berikashvili was deeply involved in the pedagogical activity, he was lecturing in Tbilisi State University, supervising PhD students in Razmadze Institute. Representatives of Georgian topological school, which was founded by academician George Chogoshvili and his first generation students, Nodar Berikashvili and Hvedri Inassaridze, are respected scientists in various fields of Algebra and topology.

Here is the short review of Nodar Berikashvili's scientific works.

The first cycle of his works concerns with homology theory of general spaces. Namely N. Berikashvili continued the approach of his supervisors G. Chogoshvili and P. S. Alexandrov, the duality theory. In Berikashvili's works the duality theory achieved it's complete form. For this purpose Berikashvili has constructed some new homology theories, developed axiomatic theory for limits of spectra. Those works place N. Berikashvili among founders of homology theory of general spaces.

The second cycle concerns with the index theory for singular integral equations. Using topological methods, in 1963 N. Berikashvili has proved the index formula for an arbitrary 2-dimensional manifold, which earlier was proven by Wolpert for 2-spheres. Approximately in the same time the famous Atiah–Singer theorem was proven, which generalizes these results for general case and for which Atiyah and Singer got Fields medal and Abel prize.

The next cycle of N. Berikashvilis works is an important contribution in the one of most powerful tools of algebraic topology – Lere–Serres spectral sequences. The Berikashvili's theory of predifferentials essentially strengthens this method, enriches it by a new computational potential and got many applications and generalizations. His students T. Beitrishvili, T. Kadeishvili, S. Saneblidze, L. Klelaia, M. Mikiashvili were involved in this activity.

On Generalized Apollonius Pursuit Problem

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We give an analytic solution of a generalized Apollonius pursuit problem with positive radius of intercept. It will be shown that, in this setting, an analog of the classical Apollonius circle is given by a convex compact component of a real algebraic curve of fourth order given by an explicit equation. This yields an algebraic equation for the optimal intercept time and an explicit formula for the optimal intercept direction. An analogous solution will be given for a version of Apollonius pursuit problem with lifeline.

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