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Symmetry and Symmetry Breaking for Optimizers of Functional Inequalities

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In this talk I will present a series of results on the symmetry properties of optimizers of functional inequalities which are invariant under a certain symmetry group. The symmetry issue is of big importance in many of the applications of those inequalities, and also in the study of many physical systems for which knowing when the symmetry is broken is of the utmost importance.

The results presented in this talk are mainly theoretical, showing in which cases one can prove symmetry and symmetry breaking, and by which methods. But in order to understand some of the results, some numerical computations can be very useful and therefore I will also explain the numerics done for this purpose and what kind of new results they suggest.

Recent Developments in Operator Inequalities

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Whenever we see a result concerning real or complex numbers (commutative case), a nice question is to ask ourselves whether it is true for bounded linear operators on a Hilbert space or their norms (noncommutative case).

In this talk, we investigate some analogies and differences between numbers and operators. We focus on inequalities and describe some methods by which we may extend a numerical inequality to operator inequalities. For instance, we examine some inequalities involving unitarily invariant norms, operator mean inequalities, Cauchy-Schwarz type inequalities as well as inequalities concerning with operator convex and operator monotone functions.

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Some Application of Mathematics to Structural Biology

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Macromolecules play role in all aspects of organisms. Knowledge of there 3D structures is key for understanding how they work. Understanding their mechanisms of action is an important step in fixing them when they go wrong, and therefore in designing drugs to cure a range of illnesses. There are three main experimental techniques to study these structures: Macromolecular X-ray crystallography (MX), nuclear magnetic resonance (NMR) and single particle Electron Microscopy (EM). Usually the data produced by these techniques are very noisy and the number of parameters of the model to be estimated is very large, often exceeding 1000000. The problem is how to extract optimal information from such noisy data? Extracted information should also be consistent with prior knowledge about the macromolecules. Analysis of such large and noisy data and derivation of biologically useful models with large number of parameters require state-of-art statistical and computational tools. Solving such problems can naturally be formulated in one of several ways such as Bayesian statistics, regularization of ill-posed problems.

Our approach for solving this type of problems has several components [1]: 1) analysis and organisation of prior chemical and structural knowledge in machine readable from [2]; 2) designing of prior probability distribution encapsulating such information; 3) designing the likelihood function that links experimental data with the parameters of the atomic models to be derived1; 4) designing posterior probability distribution that combines prior knowledge and experimental data1; 5) optimisation of large system accounting for the fact often the problem is often ill-conditioned1; 6) producing the possible images that is used for manual and/or automatic critique and revision of the model. By its nature the problem is highly non-linear and therefore large number of iteration is needed for full convergence. To aid faster solution of the problem we also develop graphical tools [3] for manual inspection of model vs experimental data as well as prior knowledge.

Our approach to regularized de-blurring, that is essentially a solution of inverse problem with large number of parameters in the presence of noise and when blurring function is approximately known, has similarity to Tikhonov type regularisations and it is solved approximately using Fourier methods for one particular type of blurring function - position independent blurring [1]. Application of developed tools to EM model interpretation will also be presented [4].

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Categories in Geometry and Physics: Mirror Symmetry, D-Branes and Landau-Ginzburg Models

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Triangulated and derived categories naturally appear in different fields of mathematics: algebra, geometry, topology and etc. Recently, they have gained importance in such branch of physics as string theory,where these categories appear as categories of supersymmetric D-branes insigma-models and Landau-Ginzburg models. Triangulated categories are natural and powerful invariants of the corresponding geometrical structures, allowing to compare seemingly incomparable objects from different fields of mathematics and physics. The talk will be an attempt on the one hand to make some review, and on the other hand - to introduce new, having natural applications to geometry and physics results from the theory of triangulated and derived categories.

Supersymmetric Gauge Theories and the Quantisation of Integrable Systems

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I review the recent developments about the relation between supersymmetric vacua and quantum integrability. From the quantum integrability side this relation includes various spin chains, as well as many well-known quantum many body systems like elliptic Calogero-Moser system and generalizations. From the gauge theory side one has the supersymmetric gauge theory with four (and eight) supercharges in the space-time background which is a product of a d-dimensional torus, or a two dimensional cigar with Omega-deformation, and a flat space (with the total dimension of space-time being 2, 3, 4 or 5). The gauge theory perspective provides the exact energy spectrum of the corresponding quantum integrable system. Key notions, usually appearing in the topic of quantum integrability, such as Baxter equation, Yang-Yang function, Bethe equation, spectral curve, Yangian, quantum affine algebra, quantum elliptic algebra, q-characters - all acquire meaning in these gauge theories.

On Bott-Chern and Chern-Simons Characteristic Forms

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The refinement of the Chern-Weil theory for Hermitian holomorphic vector bundles naturally leads to secondary characteristic forms, introduced by R. Bott and S. S. Chern in 1964. These forms play an important role in the arithmetic intersection theory and geometric stability. In the smooth category, secondary forms were introduced by S. S. Chern and J. H. Simons in 1974, and have numerous applications in mathematics and theoretical physics. In this lecture I will review the old and recent results on secondary characteristic forms. In particular, we will discuss the 'double descent' construction, associated with the Chern-Weil theory for Hermitian vector bundles, and the method for computing characteristic forms 'explicitly'.

Small Gaps Between Primes: the GPY Method and Recent Advancements Over it

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In the works with D. A. Goldston and J. Pintz (GPY), the use of short divisor sums has led to strong results concerning the existence of small gaps between primes. The results depend on the information about the distribution of primes in arithmetic progressions, specifically on the range where the estimate of the Bombieri-Vinogradov Theorem is taken to hold. Let p_n denote the n-th prime. We obtained, unconditionally, $\liminf_{n\to\infty} \frac{p_{n+1}-p_n}{\log p_n} = 0$. In fact, we have the stronger quantitative result $\liminf_{n\to\infty} \frac{p_{n+1}-p_n}{\sqrt{\log p_n}\,(\log\log p_n)^2} < \infty$. Furthermore for any fixed positive integer ν, it is shown that $\liminf_{n\to\infty} \frac{p_{n+\nu}-p_n}{\log p_n} \le e^{-\gamma}(\sqrt{\nu}-1)^2$, along with a generalization for small differences between primes in arithmetic progressions where the modulus of the progression can be taken to be as large as $(\log\log p_n)^A$ with arbitrary $A > 0$. Assuming that the estimate of the Bombieri-Vinogradov theorem holds with any level beyond the known level $\frac{1}{2}$, i.e. conditionally, the method establishes the existence of bounded gaps between consecutive primes. Another result is that given any arbitrarily small but fixed $\eta > 0$, a positive proportion of all gaps between consecutive primes are comprised of gaps which are smaller than η times the average gap (on this last matter a variety of quantitative results, some unconditional and some conditional, are obtained).

The corresponding situation for E_2-numbers q_n, numbers which are the product of two distinct primes, have been studied by S. W. Graham and the three mentioned researchers (GGPY) with the unconditional result that $\liminf_{n\to\infty}(q_{n+r} - q_n) \leq C(r)$ for certain constants $C(r)$, in particular $C(1) = 6$. The methods and results in this work also yielded stronger variants of the Erdös-Mirsky conjecture. For example, it is shown that there are infinitely many integers n which simultaneously satisfy $d(n) = d(n+1) = 24$; $\Omega(n) = \Omega(n+1) = 5$; $\omega(n) = \omega(n+1) = 4$ (here $d(n)$; $\Omega(n)$; $\omega(n)$ denote respectively the number of positive integer divisors of n, the number of primes dividing n counted with multiplicity, and the number of distinct prime divisors of n). In 2013 there has been two further breakthroughs leading to unconditional proofs of the existence of bounded gaps between primes, both advancing from the GPY works. First, Zhang succeeded in combining the method of GPY with improved versions of special instances of the Bombieri-Vinogradov type of results of Bombieri, Friedlander, Iwaniec and Fouvry in the 1980's. This necessitated the use of deep results from the theory of Kloosterman sums, and bounds derived from Deligne's proof of the Riemann Hypothesis over finite fields. Zhang obtained that there exist infinitely many gaps between primes of size $< 7 \cdot 10^7$. Zhang didn't try to find the smallest bound possible, but a Polymath project initiated by T. Tao brought the bound from Zhang's method down to 4680. Maynard, upon modifying the weights used in our works (and also almost simultaneously T. Tao), brought the bound down to 600. Upon optimizations undertaken by another Polymath project the bound is now at 252. Maynard's method is considerably more elementary than Zhang's. Whereas in the GPY works the level of distribution of primes is crucial (unconditional results are obtained from level $\frac{1}{2}$; below level $\frac{1}{2}$ the method gives nothing; assuming a level $> \frac{1}{2}$ gives bounded gaps), Maynard's method reveals that any fixed positive level of distribution of primes leads to the existence of bounded gaps between primes and furthermore it produces not only pairs but arbitrarily long finite blocks of primes.

In my talk I shall try to give a presentation of the main ideas involved in these works.

Abstracts of Invited Talks by Young Mathematicians (under 40)

On a Two-Weighted Inequality for Certain Sublinear Operator in Weighted Musielak-Orlicz Spaces

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Let B represents sublinear operator satisfying that for any $f \in L_1(R^n)$ with compact support and $x \notin \operatorname{supp} f$

$$|\widetilde{B}f(x)| \le C \int\limits_{R^n} \frac{|f(y)|}{|x-y|^{n-s}}\,dy, \quad 0 < s < n, \tag{1}$$

where $C > 0$ is independent of f and x. Note that the condition (1) was introduced in [3] and was developed in [2].

In this paper we prove a sufficient conditions on general weights ensuring the validity of the two-weight strong type inequalities for sublinear operator satisfy condition (1) acting boundedly in weighted Musielak-Orlicz spaces. In the proof of obtained result used the boundedness of for multidimensional Hardy type operator acting from usual weighted Lebesgue spaces to weighted Musielak-Orlicz spaces (see [1]).

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Twistor Approach for Harmonic 2-Spheres in a Loop Space

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There is a motivation for studying harmonic 2-spheres in a loop space ΩG, where gauge group G is a compact Lie group, namely, there is a conjecture, that a parameter space of based harmonic maps into loop space of a degree k is in a bijective correspondence with a parameter space of k-Yang-Mills connections over S^4 with a group G modulo based gauge transformations. The Atiyah theorem states the bijective correspondence between a parameter space of based holomorphic 2-spheres in a loop space of a degree k and a space of k-instantons over S^4 with a gauge group G modulo based gauge transformations. So, our variant is in some sense realification of this theorem.

Harmonic maps from Riemannian manifold M into Riemannian manifold N are extremal points of a functional $E(\phi) = \int\limits_M |d\phi(p)|^2 vol_g$, where ϕ is varying over all smooth maps between M and N with finite value of $E(\phi)$, where vol_g is a standard volume form on N. In a case of $\mathbb{R}^n$ as a target manifold we have smooth functions, which are in a kernel of a Laplacian operator, as harmonic maps, as usual.

Concerning a loop space of a Lie group G, it is known that a loop space may be isometrically embedded into a Hilbert-Schmidt Grassmannian (infinite dimensional counterpart of Grassmannian manifold, where a vector space is replaced with a Hilbert space), so the task of these harmonic maps' studying comes to harmonic 2-spheres in a Hilbert-Schmidt Grassmannian (a Kähler Hilbert manifold) investigation.

It is known as well, that a flag manifold is a twistor manifold for a Grassmannian manifold (Eels, Salamon). A pseudo-complex manifold $J(N)$ is a twistor manifold for a Riemannian manifold N, if it is smoothly fibred over N and for every Riemann surface M and every pseudo-holomorphic map $\psi : M \to J(N)$ its projection $\phi = \pi \cdot \psi$ to N is a harmonic map.

For $M = \mathbb{C}P^1$ the inverse statement is correct as well, I mean, that for every harmonic map $\phi : \mathbb{C}P^1 \to G_r(\mathbb{C}^n)$ there exists a flag manifold (with some set of indices of intermediate dimensions $\vec{r}$) ans a pseudo-holomorphic map $\psi : \mathbb{C}P^1 \to F_{\vec{r}}(\mathbb{C}^n)$, that $\pi \cdot \psi = \phi$ - initial harmonic map.

This approach allows us to investigate harmonic 2-spheres in a loop space using its embedding into infinite-dimensional Grassmannian and twistor bundle technique for quite explicit understanding how to construct it from pseudo-complex curves in an infinite-dimensional flag manifold.

The Birch and Swinnerton-Dyer Conjecture: p-Adic VS Complex

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The celebrated conjecture Birch and Swinnerton-Dyer, one of Clay Millennium Problems, predicts the size of the group of rational points on an elliptic curve E (called the Mordell-Weil group of E) in terms of its Hasse-Weil L-function $L(E, s)$, which is a complex analytic object. In mid-80s Mazur, Tate and Teitelbaum formulated a p-adic version of this conjecture which seems more approachable via Iwasawa theoretic techniques. One then would like to compare the p-adic version to the original conjecture. This has been achieved in a recent work of mine so as to allow (using results of Kato, Skinner and Venerucci) to prove the following statement: The Mordell-Weil group of E has rank one if and only if the entire function $L(E, s)$ has a simple zero at $s = 1$.

Mixed Boundary Value Problem on Hypersurfaces

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The purpose of the present paper is to investigate the mixed Dirichlet-Neumann boundary value problems for the "anisotropic" Laplace-Beltrami equation $\mathrm{div}_{\mathscr{C}}(A\nabla_{\mathscr{C}}\varphi) = f$ on a smooth hypersurface $\mathscr{C}$ with the boundary $\Gamma = \partial\mathscr{C}$ in $\mathbb{R}^n$. $A(x)$ is an $n\times n$ bounded measurable positive definite matrix function. The boundary is decomposed into two non-intersecting connected parts $\Gamma = \Gamma_D \cup \Gamma_N$ and on the part Γ_D the Dirichlet boundary conditions while on Γ_N the Neumann boundary condition are prescribed. The unique solvability of the mixed BVP is proved, based upon the Green formulae and Lax-Milgram Lemma.

We also prove the invertibility of the perturbed operator in the Bessel potential spaces $\mathrm{div}_{\mathscr{S}}(A\nabla_{\mathscr{S}})+\mathscr{H}\,I\ :\ \mathbb{H}^s_p(\mathscr{S}) \to \mathbb{H}^{s-2}_p(\mathscr{S})$ for a smooth hypersurface $\mathscr{S}$ without boundary for arbitrary $1 < p < \infty$ and $-\infty < s < \infty$, provided $\mathscr{H}$ is smooth function, has non-negative real part $\mathrm{Re}\,\mathscr{H}(t) \geqslant 0$ for all $t \in \mathscr{S}$ and $\mathrm{mes\,supp\,Re}\,\mathscr{H} \neq 0$. Further the existence of the fundamental solution to $\mathrm{div}_{\mathscr{S}}(A\nabla_{\mathscr{S}})$ is proved, which is interpreted as the invertibility of this operator in the setting $\mathbb{H}^s_{p,\#}(\mathscr{S}) \to \mathbb{H}^{s-2}_{p,\#}(\mathscr{S})$, where $\mathbb{H}^s_{p,\#}(\mathscr{S})$ is a subspace of the Bessel potential space and consists of functions with mean value zero.

Loday Symbol, Countou-Carrère Symbol, and Reciprocity

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The talk is based on a joint work with D. Osipov. We recall a symbol in algebraic K-groups constructed by Loday, its basic properties, and a relation between differential forms and tangent space to K-theory provided by the Loday symbol. This allows to give an explicit formula for higher Countou-Carrère symbol defined in terms of the delooping of K-theory. Finally, we state a reciprocity law for the higher Countou-Carrère symbol.

Fixed Point Theory for Generalized Nonexpansive Mappings

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Let C be a nonempty subset of a Banach space X. A mapping $T : C \to X$ is said to be nonexpansive whenever $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in X$. A Banach space X is said to have the fixed point property for nonexpansive mappings (FPP, in short) provided that every nonexpansive mapping $T : C \to C$ has a fixed point, where C is an arbitrary nonempty, closed, convex, bounded subset of X.

In 1965, Kirk proved that every reflexive Banach space with normal structure has the FPP. Since then, fixed point theory of nonexpansive mappings has been developed in several directions. On one side, many authors investigated sufficient conditions on X that imply that X has FPP. On another side, some authors have studied the existence of fixed points for some generalized nonexpansive mappings.

In this talk, we introduce a new class of generalized nonexpansive mappings and study the existence of fixed points.

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On Basis Properties of Root Functions of a Nonselfadjoint Boundary Value Problem

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The boundary value problem is considered which occurs in the theory of small transversal vibrations of a inhomogeneous string. The ends of the string assumed to be fixed and the midpoint of the string is damped by a pointwise force. The problem is reduced to a spectral problem for a linear operator pencil on a direct sum of two Banach spaces. The spectrum of the pencil can be presented as a union of two subsequences. The asymptotic behavior of eigenvalues and eigenfunctions are obtained, as well as of its Green function. These leads us to study the completeness of root functions in Lebesgue spaces. The uniform boundedness of Riesz projections of the problem also investigated and as a consequence the basisness of root functions of the problem is studied.

Some Property of Fourier Series with Respect by Orthogonal Systems

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We will discuss about unconditional and absolute convergence of Fourier series with respect by orthogonal systems.

Definition. The basis $\{\varphi_n(x)\}_{n=1}^{\infty}$ of $C[0,1]$ space is called has (D^{∞}) property if for any measurable set $E \subset [0,1]$, $|E| > 0$ and condensation point x_0 there exists a continuous function $f_0(x)$ such that Fourier series any bounded function $g(x)$, which coincides with f_0 on set E, absolutely diverges in point x_0.

Theorem 1. *The Haar system have* (D^{∞}) *property.*

Theorem 2. *The Franklin system have* (D^{∞}) *property.*

From M. Grigoryan and T. Grigoryan paper follow that the Faber-Schauder system haven't (D^{∞}) property.

On Compromise Solutions and Scalarization in Multiobjective Optimization

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Consider a general multiobjective optimization problem (MOP) as: $\min\limits_{\mathbf{x}\in X} f(\mathbf{x})$. The feasible set is $X \subseteq R^n$ and $f : R^n \to R^m$ is the objective function. The natural ordering cone is defined as $R^m_{\geqq} = \{\mathbf{x} \in R^m : \ x_j \geq 0, \ \forall\, j = 1, 2, \ldots, m\}$. A feasible solution $\widehat{\mathbf{x}} \in X$ is called an efficient solution to (MOP) if $(f(\widehat{\mathbf{x}}) - R^m_{\geqq}) \cap f(X) = \{f(\widehat{\mathbf{x}})\}$.

Definition 1 ([4]). An efficient solution $\widehat{\mathbf{x}} \in X$ is called a properly efficient solution to (MOP), if there is a real number $M > 0$ such that for all $i \in \{1, 2, \ldots, m\}$ and $\mathbf{x} \in X$ with $f_i(\mathbf{x}) < f_i(\widehat{\mathbf{x}})$ there exists $j \in \{1, 2, \ldots, m\}$ such that $f_j(\mathbf{x}) > f_j(\widehat{\mathbf{x}})$ and $\frac{f_i(\widehat{\mathbf{x}}) - f_i(\mathbf{x})}{f_j(\mathbf{x}) - f_j(\widehat{\mathbf{x}})} \leq M$.

Definition 2 ([2]). The point $\mathbf{y}^I = (y_1^I, \ldots, y_m^I)$ in which $y_i^I = \min\limits_{\mathbf{x}\in X} f_i(\mathbf{x})$, is said the ideal point of (MOP). The point $\mathbf{y^U} = \mathbf{y^I} - \boldsymbol{\alpha}$ for some $\boldsymbol{\alpha} > \mathbf{0}$, is called an utopia point.

One of the popular measure functions, which has been widely used in the literature, is defined by $d(\boldsymbol{\lambda}, \mathbf{y}) = \|\boldsymbol{\lambda} \odot \mathbf{y}\|_p$ for each $(\boldsymbol{\lambda}, \mathbf{y}) \in R^m \times R^m$, in which p is a positive integer, $\boldsymbol{\lambda} \odot \mathbf{y} = (\lambda_1 y_1, \lambda_2 y_2, \ldots, \lambda_m y_m)$ and $\|\boldsymbol{\lambda} \odot \mathbf{y}\|_p = \big(\sum_{j=1}^{m} |\lambda_j y_j|^p\big)^{\frac{1}{p}}$. Considering a vector $\boldsymbol{\lambda} > 0$, the set of best approximations of the ideal point measured by $\| \cdot \|_p$ is defined by

$$A(\boldsymbol{\lambda}, p, Y) = \Big\{ \bar{\mathbf{y}} \in Y : \ \|\boldsymbol{\lambda} \odot (\bar{\mathbf{y}} - \mathbf{y}^U)\|_p = \min_{\mathbf{y} \in Y} \|\boldsymbol{\lambda} \odot (\mathbf{y} - \mathbf{y}^U)\|_p \Big\},$$

in which $Y = f(X)$. Now, the set of best approximations of $\mathbf{y}^I$ considering all positive weights (compromise solutions) is defined by

$$A(Y) = \bigcup_{\boldsymbol{\lambda} \in \Lambda^0} \bigcup_{1 \leq p < \infty} A(\boldsymbol{\lambda}, p, Y),$$

where

$$\Lambda^0 = \Big\{ \boldsymbol{\lambda} \in R^m : \ \sum_{j=1}^{m} \lambda_j = 1, \ \lambda_j > 0, \ \forall\, j = 1, 2, \ldots, m \Big\}.$$

In this talk, some important connections between the members of $A(Y)$ and proper efficient solutions (without $R^m_{\geq}$-closedness) are discussed. In the second part of the talk, some scalarization problems are considered. In the literature it was proved that, under convexity assumption, the set of properly efficient points is empty when some scalarization problem(s) is (are) unbounded. In this talk, some connections between the proper efficient solutions, compromise solutions and the solutions of scalarization problems are discussed.

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On the Cesàro Means of Walsh-Fourier Series

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This lecture is devoted to review Cesàro means (C,α) $(0<\alpha\leq 1)$ on the Hardy spaces H_p, when $0<\alpha\leq 1/(1+\alpha)$.

In the talk will be presented the boundedness of subsequences of Fejér means $(C,1)$ of Walsh-Fourier series on the Hardy spaces, when $0<p\leq 1/2$.

Moreover, by applying above mentioned results we derive necessary and sufficient conditions for the modulus of continuity of Hardy spaces, for which Fejér means $(C,1)$ convergence on the Hardy spaces H_p, when $0<p\leq 1/2$.

We also present strong convergence theorem of Cesàro means on the Hardy spaces H_p, when $0<\alpha\leq 1/(1+\alpha)$.

Isometric Representations of Abelian Semigroups

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In the paper [2] Coburn proved that all semiunitary representations of the semigroup of nonnegative integers by isometric operators generate canonically isomorphic C^*-algebras. Later a similar result for semigroups with Archimedean order and total order have been proved by Douglas [3] and Murphy [6], respectively. A simple example of a semigroup with non-total order provides the semigroup $\mathbb{Z}_+\setminus\{1\}$, which originally was discussed by Murphy [6]. Later, Jang [5] pointed out two representations of this semigroup that generate canonically non-isomorphic C^*-algebras. Vittadello [8] studied all isometric representations of the numerical semigroups under certain condition.

Here we investigate isometric representations of Abelian semigroups and C^*-algebras generated by them. We show that this representations can be divided into two classes: inverse representations (the isometric representations that can be extended up to representations of an inverse semigroup) and non-inverse representations.

We show that all isometric representations of Abelian cancellative semigroup are inverse if and only if the natural order on semigroup is a total order. A description of all irreducible inverse representations of a semigroup without total order ($\mathbb{Z}_+\setminus\{1\}$) is given. Also, we introduce the class of non-inverse representations (β-representations) of this semigroup and study his properties.

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Free Group Actions on Products of Spheres

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Every finite group G can act freely on a product of spheres. Hence given a finite group G we could define $n(G)$ as the minimum integer n such that G acts freely on a product of n spheres. In this talk, I will first discuss some known lower bounds on $n(G)$ in terms of subgroup data for G. Then I will give a list of methods for constructing free group actions on products of spheres. Finally I will discuss how one can use these methods to obtain some upper bounds on $n(G)$.

Abstracts of Participants' Talks

Two Different Approaches in Prediction of Infinite Variance ARMA(1,1)

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Economic time series have error or innovation terms with infinite variance, i.e. their distribution is α-stable with $\alpha < 2$. Classical methods in prediction of time series, are based on the normality assumption of innovations distribution. In 2009, Mohammadi and Mohammadpour obtain best linear prediction for α-stable processes based on projection theory. In this study, we compare their method with minimum dispersion method presented by Brockwell and Cline (1985) in prediction of ARMA(1,1) with infinite variance. According to results of simulation, for all $0 < \alpha < 2$, projection method performs better than minimum dispersion method in the sense of absolute mean error.

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Solving Volterra Integro-Differential Equations by Using an Expansion Method

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Consider the second kind Volterra integro-differential equation of the form

$$a_1(s)x'(s) + a_0(s)x(s) - \lambda \int_0^s k(s,t)x(t)\,dt = y(s), \quad 0 \le s \le 1, \tag{1}$$

$$x(0) = x_0 \tag{2}$$

where the parameter λ and the functions $k(s,t), a_0(s), a_1(s), y(s)$ and the constant x_0 are given, and $x(s)$ is the solution to be determined. We assume that (1) has a unique solution.

Alternative approximate solution procedures have also been proposed for solving such integro-differential equations [1-3]. These procedures transform the integral equation to a linear ordinary differential equation that can be solved. But in these methods we need to specified more boundary conditions.

Here, we present a Taylor-series expansion method for solving equation (1) with smooth or weakly singular kernels. This method use Taylor-series approximation method for integro-differential equation and transform the integro-differential equation to a linear differential equation. Boundary conditions for differential equation produce in easy way. This method give an approximate simple and closed form solution for integro-differential equation.

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On the Characterization of Finite Moufang Loops by their Non-Commuting Graph

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The non-commuting graph associated to a non-abelian group G, $\Gamma(G)$, is a graph with vertex set $G \setminus Z(G)$ where distinct non-central elements x and y of G are joined by an edge if and only if $xy = yx$. The non-commuting graph of a non-abelian finite group has received some attention in existing literature. Recently, many authors have studied the non-commuting graph associated to a non-abelian group. In particular the authors put forward the following conjectures [1, 6, 4]:

Conjecture 1. Let G and H be two non-abelian finite groups such that $\Gamma(G) = \Gamma(H)$. Then $|G| = |H|$.

Conjecture 2 (AAM's Conjecture). Let P be a finite non-abelian simple group and G be a group such that $\Gamma(G) = \Gamma(P)$. Then $G \cong P$.

Some authors have proved the first conjecture for some classes of groups (specially for all finite simple groups and non-abelian nilpotent groups with irregular isomorphic non-commuting graphs) but in [5], Moghaddamfar has shown that it is not true in general with some counterexamples to this conjecture. On the other hand, Solomon and Woldar proved the second conjecture, in [7].

In this paper, we will define the same concept for a finite non-commutative Moufang loop M and try to characterise some finite non-commutative Moufang loops with their non-commuting graph. Also, we will obtain some results related to the non-commuting graph of a finite non-abelian Moufang loop. Finally, we give a conjecture stating that the above result is true for all finite simple Moufang loops.

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Existence Results for Non-Linear Fractional Integro-Differential Equation with Non-Local Boundary Conditions

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In this paper, we shall establish sufficient and necessary conditions for the existence of solutions for a first order boundary value problem for fractional differential equations including integral term in right hand side of the equation

$$
\begin{gathered}
{}^cD^\alpha y(t) = f(t, y(s)) + \int\limits_0^t g(t, s, y(s))\, ds, \quad t \in [0, T], \\
ay(0) + by(T) = c,
\end{gathered}
$$

where ${}^cD^\alpha$ is the Caputo fractional derivative, and a, b, c are real constants. This will be accomplished by Banach and Kransnoselskii fixed-point theorems.

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On Boundary Value Problems of the theory of Generalized Analytic Vectors

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Dedicated to Prof. G. Manjavidze's 90th Anniversary

Prof. G. Manjavidze has made a great contribution to the mathematical science. In particular, he obtained marked results for boundary value problems of the theory of generalized analytic functions and vectors. In this talk old and new about boundary value problems of the theory of generalized analytic vectors are presented.

On a Problem of Optimal Control Described by a Difference Analogue of Barbashin Type Integro-Differential Equation

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Problem of minimization of the functional

$$S(u)=\sum_{x=x_0}^{x_1}\varphi(z(t_1,x),\,x) \tag{1}$$

is considered under the following constraints

$$u(t,x)\in U,\quad t\in T=\{t_0,t_0+1,\dots,t_1-1\},\quad x\in X=\{x_0,x_0+1,\dots,x_1\}, \tag{2}$$

$$z(t+1,x)=\sum_{x=x_0}^{x_1}K\big(t,x,s,z(t,s)\big)+f\big(t,x,z(t,x),u(t,x)\big),\quad (t,x)\in T\times X, \tag{3}$$

$$z(t_0,x)=a(x),\quad x\in X.$$

Here $u(t,x)$ is an n-dimensional vector of status, $u(t,x)$ is a vector of control impact, U is a given non-empty set.

It is assumed that the data of the problem (1)-(3) satisfy usual smoothness requirements as in [1].

In the report we expose some necessary conditions of first order optimality and is investigated a singular case in the sense of Pontrjagin (see [3]).

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Global Existence, Asymptotic Behavior and Blow-up of Solutions to Cauchy Problem of the System of Klein-Gordon Equations with Damping Terms

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Consider the Cauchy problem for a system of Klein-Gordon equations

$$\left.\begin{aligned} u_{1tt}-\Delta u_1+m_1u_1+\alpha_1u_{1t}&=|u_1|^{p_1-1}|u_2|^{p_2+1}u_1,\\ u_{2tt}-\Delta u_2+m_2u_2+\alpha_2u_{2t}&=|u_1|^{p_1+1}|u_2|^{p_2-1}u_2,\end{aligned}\right\}\tag{1}$$

$$u_i(0,x)=u_{i0}(0,x),\quad u_{it}(0,x)=u_{i1}(0,x),\quad x\in R^n,\ \ i=1,2,$$

where $(t,x)\in R^+\times R^n$, $a_j>0, m_j>0$, $j=1,2$,

$$p_j\geq 1,\ \ j=1,2\ \text{ if }\ n\geq 2,$$

and

$$1 \leq p_1 + p_2 \leq \frac{2}{n-2} \text{ if } n \geq 3.$$

The system (1) defines a model for the interaction of two fields with the masses m_1, m_2. We investigate the qualitative characteristics of the family of the potential wells, the existence and absence of global solutions, the question of the instability of standing waves, and behavior of energy norms of solutions for large times. When $p = q$, this problem was studied in [1]. The similar problem was studied in [2] for the Klein-Gordon equation.

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Counting the Number of Some Kind of Dominating Sets of Graphs

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Let $G = (V, E)$ be a simple graph. A set $S \subseteq V$ is a dominating set of G if every vertex in $V \backslash S$ is adjacent to at least one vertex in S. A dominating set $S \subseteq V(G)$ is a weakly connected dominating set in G if the subgraph $G[S]_w = (N_G[S], E_w)$ weakly induced by S is connected, where E_w is the set of all edges having at least one vertex in S. In this presentation we consider some certain graphs and study the number of their dominating sets and their weakly connected dominating sets.

Sturm-Liouville Problems with Eigenparameter Dependent Boundary Conditions

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Sturm-Liouville problems with one boundary condition depending on spectral parameter are studied:

$$
\begin{gathered}
-y'' + q(x)y = \lambda y, \quad 0 < x < 1, \\
y'(0)\sin\beta = y(0)\cos\beta, \quad 0 \le \beta < \pi, \\
y'(1) = (a\lambda^2 + b\lambda + c)y(1),
\end{gathered}
$$

where λ is the spectral parameter, $q(x)$ is a real valued and continuous function on the interval $[0,1]$, and a, b, c are reals.

Eigenvalue parameter appears in the second boundary condition in two forms: linear and quadratic. It was shown in [1] that in both cases the following four cases are possible.

A. all the eigenvalues are real and simple;

B. all the eigenvalues are real and all, except on double, are simple;

C. all the eigenvalues are real and all, except on triple, are simple;

D. all the eigenvalues are simple and all, except two conjugate complex, are non real;

The cases A and D are not difficult to study. But in cases B and C the associated functions are involved and there are many combinations for the choices of functions in the system of eigenvectors. The basis properties of the system of eigen and associated functions have been completely studied. Many examples have been given as an application of theoretical results.

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Numerical Solution of the Interrelated Differential Equation of Motion in Phonon Engineering

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In physics and electronics, the quantized energies of elastic vibrations of crystals are called phonons. Similar to electrons, phonons are characterized by their dispersion $\omega(q)$, where ω is an angular frequency and q is a wave vector of a phonon. In order to find the phonon dispersion, the equation of motion for elastic vibration should be solved. In general the equation of motion for the elastic vibrations can be written as:

$$\rho \frac{\partial^2 U_m}{\partial t^2} = \frac{\partial \sigma_{mi}}{\partial x_i}, \quad m, i = x, y, z, \tag{1}$$

where $\vec{U}(U_x, U_y, U_z)$ is the displacement vector in three dimensions, ρ is the mass density of the materials, σ_{mi} is the elastic stress tensor and is equal to $\sigma_{mi} = C_{mikj} S_{kj}$ with S_{kj} being the strain tensor and C_{mikj} being the fourth-order tensor. Because of symmetry of C_{mikj}, they could adopt to the two-index notations C_{ij}. Since the considered structure is a multi-layer heterostructure with layer growth direction along the z-axis (non-uniform along the z-axis) and is uniform in the (x, y) plane the material parameters such as and C_{ij} $(i, j = 1, \ldots, 6)$ depend on the z coordinate and so we look for a solution of equation (1) in the following form of sinusoidal traveling waves subjected to appropriate boundary conditions.

$$U_i(x, z, t) = U_i(z) e^{(\omega t - qx)}, \quad i = x, y, z, \tag{2}$$

where u_i are the amplitudes of the displacement vector components. Substituting equation (2) for $i = y$ in equation (1), one can turn the partial differential equation (1) into an ordinary second order differential equation as below

$$-\rho(z)\omega^2 u_y(z) = c_{44}(z) \frac{d^2 u_y(z)}{dz^2} + \frac{dc_{44}(z)}{dz} \frac{du_y(z)}{dz} - c_{66}(z) q^2 u_y(z), \tag{3}$$

with the initial value that deduced from force equilibrium in surfaces. Substituting equation (2) for $i = x, z$ in equation (1), one can obtain two interrelated equations which are given as following

$$-\rho\omega^2 u_x(z) = -q^2 c_{11} u_x(z) + c_{44} \frac{d^2 u_x(z)}{dz^2} + q(c_{11} + c_{44}) \frac{du'_z(z)}{dz} + \\ + \frac{dc_{44}}{dz} \Big(\frac{du_x(z)}{dz} + q u'_z(z) \Big), \tag{4}$$

$$-\rho\omega^2 u'_z(z) = -q^2 c_{44} u'_z(z) + c_{33}\,\frac{d^2 u'_z(z)}{dz^2} + \frac{dc_{33}}{dz}\,\frac{du'_z(z)}{dz} - \\ - q\Big[(c_{44}+c_{13})\,\frac{du'_z(z)}{dz} + \frac{dc_{13}}{dz} u_x(z)\Big], \tag{5}$$

with the boundary conditions on the outer surfaces where $u'_z = -iu_z$. Equations (4) and (5) are interrelated and could not be solved easily. Using the finite difference method for solving equation (3), it could be converted to an eigenvalue problem. In this paper a new numerical method has been presented for solving the equations (4), (5) with them boundary conditions.

Some Results of the Theory of Exponential R-Groups

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This paper is devoted to the study of groups from the category $\mathfrak{M}$ of R-power groups. We examine the problems on the commutation of the tensor completion with basic group operations and on the exactness of the tensor completion. Moreover, we introduce the notion of a variety and obtain a description of abelian varieties and some results on nilpotent varieties of A-groups. We prove the hypothesis on irreducible coordinate groups of algebraic sets for the nilpotent R-groups of nilpotency class 2, where R is a Euclidean ring. We state that the analog to the Lyndon result for the free groups holds in this case, whereas the analog to the Myasnikov–Kharlampovich result fails.The paper is dedicated to partial R-power groups which are embeddable to their A-tensor completions. The free R-groups and free R-products are described with usual group-theoretical free constructions.

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Lifting Hilbert's Fourth Problem to Three Dimensions

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One of the results obtained within the theory of "Combinatorial Integral Geometry" (CIG, see author's John Wiley 1982 monograph [1]) about 40 years ago was a theorem now considered by many specialists as a solution of Hilbert's Fourth Problem in the planar case. In its original formulation, Hilbert's fourth problem asks to construct and study the geometries in which the straight line segment is the shortest connection between two points. CIG came to the mentioned planar result by construction of a "combinatorial valuation" in the space of lines followed by a measure continuation procedure. We report now on two new results obtained by "lifting" the same methodology to dimension 3, where we separately consider a) combinatorics of planes in $\mathbb{R}^3$ and b) combinatorics of lines in $\mathbb{R}^3$. (The results on a) are now in print.)

First we recall the planar theorem. The combinatorial valuation in question lives on the class of so-called Buffon sets in G, that is (up to equivalence), on the minimal ring containing all sets $[\nu]=$ lines $g \in G$ that separate the end-points of the needle ν (a line segment in $\mathbb{R}^3$). The "combinatorial valuation" $\Psi_F(A)$ (where A is a Buffon set) first constructed within CIG depends on a needle function $F(\nu)$ assumed continuous and additive along every line $g \in G$. CIG's planar result states: $\Psi_F(A)$ *happens to be a "bundleless" locally finite measure in the space G if and only if $F(\nu)$ happens to be continuous linearly additive pseudometric.*

Turning to a) and b), we note, that in both cases a wedge W is defined to be a "rechtangur" subset of some unit radius cylinder in $\mathbb{R}^3$; the axis of the cylinder can be arbitrary, and so we get "the space of wedges". We consider continuous and additive "wedge functions" $X(W)$ (case a)) or $Y(W)$ (case b)), defined on that space; yet the additional conditions that specify "wedge metrics" for the two cases are different.

Case a). In the space $\mathbb{E}=$ planes in $\mathbb{R}^3$ the ring of "Buffon sets" has been defined (up to equivalence) in [1] essentially as the minimal ring containing all sets $[\nu]=$ planes $e \in \mathbb{E}$ that separate the end-points of the needle ν ($\nu =$ a line segment in $\mathbb{R}^3$). The "combinatorial valuation" $\Psi_X(A)$ (where $A \in \mathbb{E}$ is Buffon) constructed within CIG depends on a wedge function $X(W)$ assumed continuous and additive on every unit radius cylinder. A continuous and additive $X(W)$ is called a *wedge metric a)* if $X(W)$ satisfies ***Tetrahedral Inequalities***, i.e. if for every tetrahedron Θ and any marking of the vertices of Θ by numbers 1, 2, 3, 4 we have

$$\Psi_F(A_1) \geq 0 \ \text{ and } \ \Psi_F(A_2) \geq 0,$$

where A_1 = planes that separate 1 from 2, 3 and 4; A_2 = planes that separate 1 and 2 from 3 and 4.

Theorem 1. $\Psi_F(A)$ *happens to be a "bundleless" locally finite measure in the space* G *if and only if* $X(\nu)$ *happens to be continuous linearly additive wedge metric* a).

Case b). In the space Γ = lines in $\mathbb{R}^3$ the ring of "Buffon sets" have been defined (up to equivalence) in [1] as the minimal ring containing all sets $[\pi]$= lines $\gamma \in \Gamma$ that "pierce" the "plate" π (π = a planar polygon "hanging" in $\mathbb{R}^3$, triangular plates are enough). The "combinatorial valuation" $\Psi_Y(A)$ (where $A \in \mathbb{E}$ is Buffon) constructed within CIG depends on a wedge function $Y(W)$ assumed continuous and additive on every unit radius cylinder. A continuous and additive $Y(W)$ is called a *wedge metric b)* if $X(W)$ satisfies the ***Two-Plate Inequalitiy***, that is, for every two plates π_1 and π_2

$$\Psi_Y\big([\pi_1] \cap [\pi_2]\big) \geq 0.$$

Theorem 2. $\Psi_Y(A)$ *happens to be a locally finite measure in the space* Γ *if and only if* $Y(W)$ *happens to be continuous linearly additive wedge metric* b).

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A Search for a Fragment Space: Organization and use of Prior Structural Knowledge

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Last ten years have seen rapid advances in all aspects of structural biology that resulted in an increase of the number of 3D structures of biomacromolecules. According to the PDB - http://www.pdb.org/, more than 100000 macromolecular structures are known. These data can answer many questions in biology and they pose new challenges. Solution to these problems would allow development of approaches to such problems as analysis of noisy X-ray crystallographic (MX) and Electron Microscopic data. Most proteins share similar polypeptide chain patterns. For instance, the numbers of different folds are few thousands. Furthermore, almost all proteins contain the same short polypeptide chains secondary structures. Regular update of the classifications is time consuming, and they lag behind structural information. Classified patterns then can be used to tackle such problems as the derivation of biologically relevant information from noisy and limited experimental data. The main goal of this research is an automatic classification of short polypeptide patterns. Analysis and classification of polypeptide fragments is done in two stages: 1) Using Procrustes distances between fragments and equivalence class identification algorithm divide all fragments into class of patterns, verify validity of classes and reclassify if necessary; 2) Using clusterization algorithms find classes of patterns, analysis each class and find variability within and between classes. Result of the first stage is vast amount data with classes and patterns in it and enter this data to clustering algorithm is ineffective owing to time consuming computation. For solving this problem each pattern on classes entered to computation to select member of class according to superposition principles and Procrustes distances. Only after decreasing class identification algorithm data applied clusterization algorithms. Distances between fragments were calculated using procrustes analysis. Fragment lengths were chosen to be 7. Only backbone atoms CA, C, N, O were used in calculations. This type two stage algorithm allows reduction of the complexity of the problem.

Acknowledgment

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Generalized Fredholm Operator in Locally m-Convex H^*-Algebras

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Let $(E, (|\cdot|_\lambda)_{\lambda\in\Lambda})$ be a locally m-convex H^*-algebra and X be a Hilbert E-module. An operator $F \in B_E(X)$ is said to be a generalized Fredholm operator if $\pi(F)$ is an invertible element in the generalized Calkin algebra $C_E(X) = \frac{B_E(X)}{K_E(X)}$. In this talk we are going to prove a generalization of Atkinson's theorem as follows "F is a generalized Fredholm operator if and only if the image of F is a closed submodule and both dim ker F and dim ker F^* are finite".

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A Representation for Convex Bodies

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We denote by $\mathbf{R}^n$ $(d \geq 2)$ the Euclidean n-dimensional space and $\mathbf{S}^{n-1}$ the unit sphere in $\mathbf{R}^n$. The class of convex bodies (nonempty compact convex sets) $\boldsymbol{K}$ in $\mathbf{R}^n$ we denote by $\mathscr{K}$ and the class of centrally symmetric convex bodies (so called the *centred* bodies) denote by $\mathscr{K}_o$.

The most useful analytic description of compact convex sets is by the support function. The support function of $\mathbf{K}$ is positively homogeneous and convex. It is well known that a convex body $\boldsymbol{K} \in \mathscr{K}$ is determined uniquely by its support function.

It is known that (Blaschke [1], Schneider and Wiel [2]) *the support function* $H(\boldsymbol{K},\cdot)$ *of a sufficiently smooth centrally symmetric convex body* $\boldsymbol{K}\in\mathscr{K}_o$ *has the following representation (we assume that the center of* $\boldsymbol{K}$ *is the origin of* $\mathbf{R}^n$*)*

$$H(\boldsymbol{K},u)=\int\limits_{\mathbf{S}^{n-1}}|\langle u,\nu\rangle|\,m(d\nu)\ \text{ for all }\ u\in\mathbf{S}^{n-1}. \tag{1}$$

with a signed even Borel measure m *on* $\mathbf{S}^{n-1}$*. Such bodies (whose support functions have the integral representation (1) with a signed even measure) are called* generalised zonoids. *In the case then* m *is a positive even Borel measure* $\boldsymbol{K}$ *is called a zonoid.*

Also it is known ([2]): *The generalised zonoids are dense in the class of centrally symmetric convex bodies.*

In [3] was extended the representation for the support function of centrally symmetric convex bodies to a larger class of arbitrary convex bodies by considering the transform of the integral (1) into the integral over the hemisphere involving a non- even measure on $\mathbf{S}^{n-1}$. In [3] was discussed some questions on unique determination of convex bodies.

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g-Tridiagonal Majorization and Circulant Majorization on $\mathbf{M}_{n,m}$

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Let $\mathbf{M}_{n,m}$ be the set of all $n\times m$ real matrices. An square matrix D is called tridiagonal g-doubly stochastic if it is tridiagonal and $De=e=D^te$, where $e=(1,1,\ldots,1)^t$. For $A,B\in\mathbf{M}_{n,m}$, it is said that B is g-tridiagonal majorized by A (written $B\prec_{gt}A$) if there exists an n×n tridiagonal g-doubly stochastic matrix D such that $B=DA$. An square matrix D is called circulant g-doubly stochastic if it is circulant and $De = e = D^t e$, where $e = (1, 1, \dots, 1)^t$. For $A, B \in \mathbf{M}_{n,m}$, it is said that B is g-circulant majorized by A (written $B \prec_{gc} A$) if there exists an n×n circulant g-doubly stochastic matrix D such that $B = DA$. In this paper we investigate some properties of tridiagonally and circulant majorization on $\mathbf{M}_{n,m}$.

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On Induced Hyperspace Topological Transformation Semigroups

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By a topological transformation semigroup (X, S, π) or simply (X, S) we mean a topological space X, a discrete topological semigroup S with identity e, and continuous map $\pi : X \times S \to X$ ($\pi(x, s) = xs$, $x \in X$, $s \in S$) such that $xe = x$ and $x(st) = (xs)t$ for all $x \in X$ and $s, t \in S$.

In topological space X by $\mathscr{P}_0(X)$ we mean the collection of all nonempty subsets of X. Consider $\mathscr{P}_0(X)$ under Vietoris topology. It is evident that if (X, S) is a topological transformation semigroup, then $(\mathscr{P}_0(X), S)$ is a topological transformation semigroup too, where $As := \{xs : \ x \in A\}$ (for $A \in \mathscr{P}_0(X)$ and $s \in S$), we call it induced hyperspace topological transformation semigroup. For nonzero cardinal number α, let:

1. $K(X)$ =the collection of all nonempty compact subsets of X,
2. $\mathscr{P}_0^{<\alpha}(X) = \{A \in \mathscr{P}_0(X) : \ \mathrm{card}\,(A) < \alpha\}$.

The induced transformation semigroups $(\mathscr{K}(X), S)$ and $(\mathscr{P}_0^{<n}(X), S)$ for compact Hausdorff (resp. metric) X and finite n has been studied in several texts, in this talk we pay special attention to other invariant subspaces of $(\mathscr{P}_0(X), S)$ like $\mathscr{P}_0^{<\omega}(X)$ and $\mathscr{P}_0^{<c}(X)$ where ω is the least infinite cardinal number and c is the least uncountable cardinal number (using CH).

Various Versions of Zagreb Indices Under Some Local Graph Operators

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A graph invariant (also known as structural descriptor/measure or topological index) is any function on a graph that does not depend on a labeling of its vertices. Several hundreds of different invariants have been employed to date with various degrees of success in QSAR/QSPR studies [1]. The Zagreb indices are among the oldest topological indices and were introduced by Gutman and Trinajstić in 1972 [2]. These indices have since been used to study molecular complexity, chirality, ZE-isomerism and hetero-systems. The first and second Zagreb indices of a simple graph G are denoted by $M_1(G)$ and $M_2(G)$, respectively and defined as:

$$M_1(G) = \sum_{u \in V(G)} deg_G(u)^2 \text{ and } M_2(G) = \sum_{uv \in E(G)} deg_G(u) deg_G(v),$$

where $deg_G(u)$ denotes the degree of the vertex u of G.

The first Zagreb index can also be expressed as a sum over edges of G:

$$M_1(G) = \sum_{uv \in E(G)} (deg_G(u) + deg_G(v)).$$

It is well known that many graphs of general and in particular of chemical interest arise from simpler graphs via various graph operators. It is, hence, important to understand how certain invariants of such composite graphs are related to the corresponding invariants of their components. In this paper, we study the behavior of various versions of Zagreb indices under some local graph operators.

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On the Reduction and Convergence of Payoffs for Partially Observable Stationary Sequences

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We consider the following discrete model of partially observable stationary wide sense random sequence $(\theta,\xi)=(\theta_n,\xi_n)$, $n=0,1,\dots$, where θ is nonobservable random sequence and ξ is the observable random sequence:

$$\theta_{n+1}=-b_1\theta_n+\xi_1\eta_1(n+1),$$
$$\xi_{n+1}=(b_2-b_1)\theta_n-b_2\xi_n+\varepsilon_1\eta_1(n+1)+\varepsilon\varepsilon_2\eta_2(n+1),$$

where $|b_1|<1$, $|b_2|<1$, ε, ε_1, ε_2 are small parameters, $\eta_1(n)$, $\eta_2(n)$ are independent standard normal random variables, $n=0,1,\dots,$. For the gain function $g(n,x)=f_0(n)+f_1(n)x+f_2(n)x^2$ we consider the optimal stopping problem with incomplete data with payoffs:

$$s^0=\sup_{\tau\in\mathfrak{M}^\theta}Eg(\tau,\theta_\tau),\quad s^{\varepsilon,\varepsilon_1,\varepsilon_2}=\sup_{\tau\in\mathfrak{M}^\xi}Eg(\tau,\theta_\tau).$$

The problem of optimal stopping of θ with incomplete data is reduced to the optimal stopping with complete data and the convergence of payoffs is proved when the small parameters tend to zero.

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A Prym Torelli Theorem for General Prym-Canonical Curves of Genus $g=7$

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Throughout the lecture we live on the field of complex numbers. The foci technique would be reconsidered for general Prym-Canonical curves, in this lecture. We will apply this technique to the family of linear subspaces contained in the projectivized tangent cones of a Prym-Theta divisor of a Prym-Canonical curve. We denote this family by Λ. The theorem:

Theorem 1. *For any line bundle in the double locus of the Prym-Theta divisor of the Prym-Canonical curve C, there exists a surface $Y \subset \widetilde{\alpha}^{-1}(L)$ such that through any point $\overline{p}$ of C it passes 2-dimensional family of Λ_s's parameterized by Y, through $\overline{p}$, is a product of our research which implies that the focal varieties of the family Λ are singular varieties.*

In the genus $g = 7$ case the focal varieties would be curves of degree three in projective plane. In this case we will describe the focal curves. The main result of this description is the following theorem.

Theorem 2. Let C be a general curve of genus $g = 7$ and $f : \overline{C} \to C$ an etale double covering of C. Then C can be reconstructed uniquely from the family Λ. Indeed the curve C is a component of closure of singular locus of focal curves of the family Λ.

This reproves the generic injectivity of the Prym-Torelli map, in the genus $g = 7$ case.

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The (co)Shape and (co)Homological Properties of Continuous Maps

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One of the original ideas of geometric topology, in particular (co)shape theory, is to describe the properties of continuous maps of general topological spaces by us-ing the expansions of continuous maps into direct and inverse systems of continuous maps between the spaces which behave well locally.

The purpose of this report is to investigate continuous maps from the standpoint of geometric topology and algebraic topology. Using a direct system approach and an inverse system approach of continuous maps, we study the (co)shape and (co)homological properties of continuous maps. Applications of the obtained results include the constructions of long exact sequences of continuous maps for the (co)homology pro-groups and (co)homology inj-groups, spectral Čech (co)homology groups and spectral singular (co)homology groups.

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On the Generalized Hermitian and Skew-Hermitian Splitting Iterative Method for Image Restoration

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In [1] Bai et al. proposed Hermitian and skew-Hermitian splitting (HSS) iterative method to solve the linear systems $Ax = b$. They splitted the cofficeint matrix A as $A = H + S$ where H and S are Hermitian and skew-Hermitian part of A, respectively. In [2] Benzi proposed the generalization of the HSS (GHSS) method to solve linear systems. He splitted the Hermitian part of coefficient matrix as $H = G + K$ where G and K are semi-definite matrices. Moreover, Lv et al. presented a special HSS (SHSS) method for image restoration problem in [3]. They introduced the following equivalent linear system to restore images:

$$\underbrace{\begin{bmatrix} I & A \\ -A^T & \mu^2 I \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} e \\ f \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} g \\ 0 \end{bmatrix}}_{b}, \qquad (1)$$

where T is $2n^2 \times 2n^2$ non-Hermitian positive definite matrix, A and I are $n^2 \times n^2$ blurring and identity matrix, respectively, and μ is regularization parameter. Moreover, the n^2-dimensional vectors f and g are the true image and degraded image, respectively, and the auxiliary variable, e, represents the additive noise.

In our study, we present the following splitting for the Hermitian part of the coefficient matrix T in Eq. (1):

$$H = \beta \begin{bmatrix} (1-\mu^2)I & O \\ O & \mu^2 I \end{bmatrix} + \begin{bmatrix} (1-\beta(1-\mu^2))I & O \\ O & (1-\beta)\mu^2 I \end{bmatrix} = G + K, \qquad (2)$$

where β is a positive parameter and O is $n^2 \times n^2$ zero matrix. The proposed splitting is applied to implement the GHSS method for image restoration problem. The convergence of the method is investigated. Moreover, a special case of the proposed method is also presented for image restoration problem. Two numerical examples are given to show the efficiency and accuracy of the GHSS method and compare proposed method with the SHHS method.

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Geometric Representation of Groups in Pseudo-Finite Fields

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A pseudo-finite field is perfect pseudo-algebraically closed (PAC) field which has $\hat{\mathbb{Z}}$, the profinite cyclic group, as absolute Galois group. Pseudo-finite fields exist and they can be realized for example as ultraproducts of finite fields. A group G is geometrically represented in a theory if there are models M_0 and M of T substructures A, B of M such that $M_0 \leq A \leq B \leq M$ and $Aut(B/A)$ is isomorphic to G. Let T be a complete theory of pseudo-finite fields. We show with Ehud Hrushovski that, geometric representation of a group whose order is divisible by p in T heavily depends on the p'th roots of unity in models of T. As a consequence it follows that abelian groups can be geometrically represented in certain theories of pseudo-finite fields. Yet, the question that "what kind of groups were represented geometrically in a theory of pseudo-finite fields" remained open. I will lay out a background and talk on the possible answers, and ways to attack to the aforementioned question. This is work in progress with Zoe Chatzidakis.

Bases in Morrey-Lebesgue and Morrey-Hardy Spaces

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In this paper we consider Banach spaces of functions of Morrey-Lebesgue and Morrey-Hardy. Basicity of exponential system and its parts in these spaces is proved.

Let us consider the space $\overline{L}^{p,\alpha}$, $0 < \alpha \leq 1$, $1 < p < +\infty$, and the system of exponentials $\{e^{int}\}_{n \in Z}$.

We will prove the basicity in a usual way, i.e. using a basicity criterion (see, e.g., [1-3]). We have

$$\|e^{i\,nt}\|_{L^{p,\alpha}} = \left[\sup_{I \subset [-\pi,\pi]} \frac{1}{|I|^{1-\alpha}} \int_I |e^{i\,nt}|^p \, dt \right]^{\frac{1}{p}} = (2\pi)^{\frac{\alpha}{p}}.$$

As a result, a basicity criterion implies the validity of

Theorem 1. *System of exponentials* $\{e^{i\,nt}\}_{n \in Z}$ *forms a basis* (*Riesz basis when* $p = 2$) *for* $\overline{L}^{p,\alpha}$ *when* $1 < p < +\infty$, $0 < \alpha \leq 1$.

In a similar way we can prove

Theorem 2. *Systems* $\{e^{i\,nt}\}_{n \in Z_+}$; $\{e^{-i\,nt}\}_{n \in N}$ ($\{z^n\}_{n \in Z_+}$; $\{z^{-n}\}_{n \in N}$) *form bases* (*Riesz bases when* $p = 2$) *for spaces* $\overline{L}^{p,\alpha}_+$; ${}_{-1}\overline{L}^{p,\alpha}_-$ ($\overline{H}^{p,\alpha}_+$; ${}_{-1}\overline{H}^{p,\alpha}_-$), *respectively.*

The case $p = 2$ directly follows from the Hilbertness of considered spaces and the orthogonality of systems in these spaces.

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Frame Properties of the Part of a System of Exponents with Degenerate Coefficient in Hardy Classes

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Part of a system of exponents with degenerate coefficient is considered in this work. Frame properties (basicity, an atomic decomposition) of this system in Hardy classes are studied in case when the coefficient may not satisfy the Muckenhoupt condition.

Some considerations regarding frame sequences in Banach spaces are introduced. The parts of a system of exponent with degenerate coefficient corresponding to positive and negative values of the index are considered. We prove that if the coefficients satisfy the Muckenhoupt condition, then these parts form bases for the corresponding Hardy classes of analytic functions. It is proved that if the degenerate coefficient doesn't satisfy the Muckenhoupt condition, then these systems have finite defects.

More details can be found in [1-5].

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On One Mixed Characteristic Problem

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Let γ be an arc of continuously differentiable curve given in the explicit form

$$\gamma : y = f(x), \quad 0 < a \leq x \leq b,$$

where the function f is a strictly monotone with $f(a) = 0$.

Mixed characteristic problem. *Find a regular hyperbolic solution $u(x, y)$ of equation*

$$x^2(u_y^4 u_{xx} - u_{yy}) = cuu_y^4, \;\; c = const,$$

and simultaneously, the domain of its propagation if the curve γ is characteristic along this solution and the solution itself satisfies the conditions

$$u(a, 0) = \tau, \quad u_x(a, 0) = \nu.$$

We plan to discuss the solution of the formulated problem by the method of characteristics and reduction to the Cauchy problem.

Laplace-Beltrami Equation on Hypersurfaces and Γ-Convergence

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Partial differential equations on surfaces in the Euclidean space and corresponding boundary value problems (BVPs), encounter rather often in applications. For example: heat conduction by a thin conductive surface or deformation a of thin elastic surface are governed by some differential equations on these surfaces.

To rigorously derive equations which govern the above mentioned processes we need a calculus of tangential partial differential operators on a hypersurface (i.e., a surfaces in the Euclidean space $\mathbb{R}^n$ of co-dimension 1). There are known many approaches to this problem, but the main task is to find the one which gives simplest results. We suggest a calculus of partial differential operators on a hypersurface based on Günter's and Stoke's tangential derivatives.

We will expose basics of this calculus and demonstrate how mixed boundary value problem (mixed BVP) for the Laplace equation in thin layer domain around a mid hypersurface $\mathscr{C}$ converges in the sense of Γ-convergence to an appropriate Dirichlet BVP for the Laplace-Beltrami equation of the mid-hypersurface $\mathscr{C}$ when the thickness of the layer diminishes to 0.

Infinite Determinantal Measures

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Infinite determinantal measures introduced in the talk are first examples if an explicit description of sigma-finite measures on the infinite-dimensional space of configurations. They are obtained as inductive limits of determinantal measures on an exhausting family of subsets of the phase space. Alternatively, an infinite determinantal measure can be described as a product of a determinantal point process and a convergent, but not integrable, multiplicative functional. The main result of the talk gives an explicit description for the ergodic decomposition of infinite Pickrell measures on the spaces of infinite complex matrices in terms of infinite determinantal measures obtained by finite-rank perturbations of Bessel point processes.

Spectral Stability of the Dirichlet-Laplace Operator in Plane Domains

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We prove that the eigenvalues problem for the Dirichlet-Laplace operator in bounded simply connected plane domains $\Omega \subset C$ can be reduced by conformal transformations to

the weighted eigenvalues problem for Dirichlet-Laplace operator defined on Sobolev spaces with conformal (hyperbolic) weights in the unit disc D [1, 2]. It permits us to estimates variation of eigenvalues of Dirichlet-Laplace operators in terms of energy type integrals for a large class of conformal (regular) domains that includes all quasidiscs, i.e. images of the unit disc under a quasiconformal homeomorphism of the plane onto itself. Let us remark that Hausdorf dimension of quasidiscs boundaries can be any number $s \in [1, 2)$.

It is known that in a bounded plane domain $\Omega \subset C$ the spectrum of the Dirichlet-Laplace operator is discrete and can be written in the form

$$0 < \lambda_1[\Omega] \leq \lambda_2[\Omega] \leq \cdots \leq \lambda_n[\Omega] \leq \cdots .$$

We call a bounded simply connected plane domain $\Omega \subset C$ a conformal regular domain if there exists a conformal mapping $\varphi : D \to \Omega$ of the unit disc D onto Ω of the Sobolev class $L^{1,p}(D)$ for some $p > 2$. Note that any conformal regular domain has finite geodesic diameter.

Let, for $2 < p \leq \infty$, $\tau > 0$, $G_{p,\tau}$ be the set of all conformal mappings φ of the unit disc D of the Sobolev class $L^{1,p}(D)$ such that

$$\|\nabla\varphi\|_{L^{1,p}(D)} \leq \tau.$$

The main result is

Theorem. *For any $2 < p \leq \infty, \tau > 0$ there exists $B_{p,\tau} > 0$ such that for any $\varphi_1, \varphi_2 \in G_{p,\tau}$ and for any $n \in N$*

$$\left|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]\right| \leq c_n B_{p,\tau} \left\|\varphi_1 - \varphi_2 \mid L^{1,2}(D)\right\|, \tag{1}$$

where $\Omega_1 = \varphi_1(D)$, $\Omega_2 = \varphi_2(D)$. Here $\varphi_1 : D \to \Omega_1$ and $\varphi_2 : D \to \Omega_2$ are conformal homeomorphisms.

Results were announced in [3].

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The Birch and Swinnerton-Dyer Conjecture: p-Adicvs Complex

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The celebrated conjecture Birch and Swinnerton-Dyer, one of Clay Millennium Problems, predicts the size of the group of rational points on an elliptic curve E (called the Mordell-Weil group of E) in terms of its Hasse-Weil L-function $L(E, s)$, which is a complex analytic object. In mid-80s Mazur, Tate and Teitelbaum formulated a p-adic version of this conjecture which seems more approachable via Iwasawa theoretic techniques. One then would like to compare the p-adic version to the original conjecture. This has been achieved in a recent work of mine so as to allow (using results of Kato, Skinner and Venerucci) to prove the following statement: The Mordell-Weil group of E has rank one if and only if the entire function $L(E, s)$ has a simple zero at $s = 1$.

Subgroups of Some Discrete Groups and Some Useful Methods

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Discrete groups have been studied since the end of 19th century after different geometries had been defined. The most popular example is the well-known modular group. In 1936, E. Hecke defined generalisations of the modular group in his study with Dirichlet series. These groups are named after him as Hecke groups. Since then, several authors studied on several versions of Hecke groups and because of different underlying fields, they have very nice algebraic, combinatoric and number theoretic properties.

Here we shall give a brief idea of Hecke groups and their relation with the modular group and obtain their normal subgroups by means of the Riemann-Hurwitz formula, Reidemeister-Schreier method, permutation method and regular map theory. The topic is in the intersection of algebra and complex analysis.

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Crack Impedance-Dirichlet Boundary Value Problems of Diffraction in a Half-Plane

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We study two wave diffraction problems modeled by the Helmholtz equation in a half-plane with a crack characterized by Dirichlet and impedance boundary conditions. The existence and uniqueness of solutions is proved by an appropriate combination of general operator theory, Fredholm theory, potential theory and boundary integral equation methods. This combination of methods leads also to integral representations of solutions. Moreover, in Sobolev spaces, a range of smoothness parameters is obtained in which the solutions of the problems are valid.

On L-Soft Merotopies

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In 1965, Katetov introduced merotopic spaces by axiomatizing the concept of collection of sets containing arbitrary small members called a micromeric collection. Nearness spaces are those merotopic spaces for which a relationship exists between the near collection and the closure operator. In 1974, Herrlich introduced the concept of nearness spaces by axiomatizing the concept of nearness of arbitrary collection of subsets of X. It is a generalization of the concept of two sets being near. In 1999, Molodtsov proposed a completely new concept called soft set theory to model uncertainty, which associates a set with a set of parameters. Later, Maji et al. introduced the concept of fuzzy soft set which combines fuzzy sets and soft sets. All over the globe, (fuzzy) soft set theory is a topic of interest for many authors working in diverse areas due to its rich potential for applications in several directions. This study is devoted to describe L-soft merotopic (nearness) spaces and examine some of their properties. Also, the notion of L-approach soft merotopy is defined.

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Weakly Independent Random Elements, Gaussian Case

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As it is known, the random vectors ξ_1 and ξ_2 are independent if and only if for any linear functionals f and g , the random variables $\langle \xi_1, f\rangle$ and $\langle \xi_2, g\rangle$ are independent. If we weaken this condition and demand independence of the random variables $\langle \xi_1, f\rangle$ and $\langle \xi_2, f\rangle$ for all linear functionals f , we will receive weakly independence. Weakly independent random elements keep a multitude of behavior of the independent random elements (see[1]). The probability laws, where appears the distributions of partial sums of independent random elements (the weak law of large numbers, central limit theorem, convergence of the sums of independent X-valued random elements by probability, by distributions and so on) are right also for corresponding weakly independent X-valued random elements. In finite dimensional case, as all fix coordinate of vectors are independent, all of probability laws, true for independent real valued random variables are valid for all coordinates of vectors (strong law of large numbers, almost sure convergence of sums of independent random variables and so on), therefore they are valid for weakly independent random vectors, but in infinite dimensional case it seems sufficiently difficult to solve this problem for arbitrary weak second order random elements, but we consider the case, when the random elements are Gaussian. We want to consider Levi's type inequality in this case to solve these problems.

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New Proofs of Some Results on BMO Martingales Using BSDEs

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The aim of this paper is to prove some results on BMO martingales using the BSDE technique.

It is well known that if M is a BMO martingale, then the mapping $\phi : \mathscr{L}(P) \ni X \longrightarrow \widetilde{X} = \langle X, M\rangle - X \in \mathscr{L}(\widetilde{P})$ is an isomorphism of $BMO(P)$ onto $BMO(\widetilde{P})$, where $d\widetilde{P} = \mathscr{E}_T(M)dP$. E. g., it was proved by Kazamaki that the inequality

$$\|\widetilde{X}\|_{BMO(\widetilde{P})} \leq C_{Kaz}(\widetilde{M}) \cdot \|X\|_{BMO(P)}$$

is valid for all $X \in BMO(P)$, where the constant $C_{Kaz}(\widetilde{M}) > 0$ is independent of X but depends on the martingale M. Using the properties of a suitable BSDE we prove this inequality with a constant $C(\widetilde{M})$ which we express as a linear function of the $BMO(\widetilde{P})$ norm of $\widetilde{M} = \langle M\rangle - M$ and which is less than $C_{Kaz}(\widetilde{M})$ for all values of this norm.

Using properties of BSDEs we also prove the well known equivalence between BMO property, Muckenhoupt and reverse Hölder conditions (Doleans-Dade and Meyer, Kazamaki) and obtain BMO norm estimates in terms of reverse Hölder and Muckenhaupt constants.

Nonlinear Mathematical Model of Bilateral Assimilation Taking Into Account Demographic Factor

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In work the new nonlinear mathematical model describing assimilation of the people (population) with some less widespread language by two states with two various widespread languages, taking into account demographic factor is offered. In model three subjects are considered: the population and government institutions with the widespread first language, influencing by means of state and administrative resources on the third population with some less widespread language for the purpose of their assimilation; the population and government institutions with the widespread second language, influencing by means of state and administrative resources on the third population with some less widespread language for the purpose of their assimilation; the third population (probably small state formation, an autonomy), exposed to bilateral assimilation from two rather powerful states. Earlier by us it was shown that in case of zero demographic factor of all three subjects, the population with less widespread language completely assimilates the states with two various widespread languages, and the result of assimilation (redistribution of the assimilated population) is connected with initial quantities, technological and economic capabilities of the assimilating states. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model “predator - the victim”, thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilator. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model “a predator - the victim”, with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found.

Localized Boundary-Domain Integral Equations Approach for Problems of the Theory of Piezo-Elasticity for Inhomogeneous Solids

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We consider the three-dimensional Dirichlet and Robin boundary-value problems (BVPs) of piezo-elasticity for anisotropic inhomogeneous solids and using the localized parametrix approach develop the generalized potential method. Using Green's representation formula and properties of the localized layer and volume potentials we reduce the Dirichlet and Robin BVPs to the localized boundary-domain integral equations (LBDIE) systems.

First we establish the equivalence between the original boundary value problems and the corresponding LBDIE systems. Afterwards, we establish that the localized boundary-domain integral operators obtained belong to the Boutet de Monvel algebra and with the help of the Vishik-Eskin theory, based on the factorization method (Wiener-Hopf method), we investigate corresponding Fredholm properties and prove invertibility of the localized operators in appropriate function spaces.

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C_λ-Rate Sequence Space Defined by a Modulus Function

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Recall that a C_λ method is obtained by deleting a set of rows from the Cesáro matrix C_1. The purpose of this article is to introduce a new class of sequence space using a modulus function f, namely C_λ-rate sequence space. It is denoted by $C_\lambda(f, p, \pi)$, and study some inclusion relations and topological properties of this space.

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Hypergroups Over the Group and Applications

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Let G be a group, H be a subgroup of G, M be a complementary set (transversal) to H in G. Then one can to define a system of four mappings $\Omega = (\Phi, \Psi, \Xi, \Lambda)$, which satisfy the following properties.

(P1) The mapping $\Xi : M \times M \to M$, $\Xi(a, b) = [a, b]$ is a binary operation such that (M, Ξ) is a right quasigroup with a left neutral element o.

(P2) The mapping $\Phi : M \times H \to M$, $\Phi(a, \alpha) = a^\alpha$ is an action of the group H on M.

(P3) The mapping $\Psi : M \times H \to H$, $\Psi(a, \alpha) = {}^a\alpha$ sends the subset $\{o\} \times H$ on H.

(P4) The mappings of Ω, where $\Lambda : M \times M \to H$, $\Lambda(a, b) = (a, b)$, are connected by relations:

(A1) ${}^a(\alpha \cdot \beta) = {}^a\alpha \cdot {}^{a^\alpha}\beta$;

(A2) $[a, b]^\alpha = [a^{{}^b\alpha}, b^\alpha]$;

(A3) $(a, b) \cdot {}^{[a,b]}\alpha = {}^a({}^b\alpha) \cdot (a^{{}^b\alpha}, b^\alpha)$;

(A4) $[[a, b], c] = [a^{(b,c)}, [b, c]]$;

(A5) $(a, b) \cdot ([a, b], c) = {}^a(b, c) \cdot (a^{(b,c)}, [b, c])$.

We call a pair (M, H), consisting of a set M and a group H, together with a system of structural mappings $\Omega = (\Phi, \Psi, \Xi, \Lambda)$, satisfying the conditions P1-P4, *a hypergroup over the group*.

In our talk some results on hypergroups over the group and their applications are proved.

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Local Hardy-Littlewood Maximal Operator in Variable Lebesgue Spaces

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We investigate the class $B^{loc}(R^n)$ of exponents $p(\cdot)$ for which the local Hardy-Littlewood maximal operator is bounded in variable exponent Lebesgue spaces $L^{p(\cdot)}(R^n)$. Littlewood-Paley square function characterization of $L^{p(\cdot)}(R^n)$ spaces with the above class of exponent are also obtained.

Some Fixed Point Results in Boolean Metric Spaces

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Fixed points of Boolean functions have numerous applications in the theory of error-correcting codes, to switching and relationship between the consistency of a Boolean equation, cryptography, convergence of some recursive parallel array processes in Boolean arrays and many others. In the cryptography fixed point of Boolean functions is a criterion to design the ciphers that underestand, a good cipher must no any fixed point ([2, 7]).

Subrahmanyam ([5, 6])introduced the notion of Boolean and normed algebra vector spaces and he studied the basis and convergence in a normed Boolean vector space over a σ-complete Boolean algebra. Rao and Pant ([4]) obtained some fixed and common fixed point theorems for asymptotically regular maps on finite dimensional normed Boolean vector spaces. In this article we obtain some fixed point and common fixed point theorems in finite ordered sets and normed Boolean vector spaces.

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Equivariant Infinitesimal Deformations of Algebraic Threefolds with an Action of an Algebraic Torus of Complexity 1

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A construction of normal affine algebraic varieties over $\mathbb{C}$ with an action of an algebraic torus was given in [1]. Fix a torus $T = (\mathbb{C}^*)^2$. Let $M = \mathfrak{X}(T)$ be its character lattice, $N = \operatorname{Hom}_{\mathbb{Z}}(M, \mathbb{Z})$, $M_{\mathbb{Q}} = M \otimes_{\mathbb{Z}} \mathbb{Q}$, and $N_{\mathbb{Q}} = N \otimes_{\mathbb{Z}} \mathbb{Q}$. Then $N_{\mathbb{Q}} = \operatorname{Hom}_{\mathbb{Q}}(M_{\mathbb{Q}}, \mathbb{Q})$.

A *polyhedron* $\Delta \subseteq N_{\mathbb{Q}}$ is a nonempty (maybe unbounded) intersection of finitely many closed *affine* half-spaces of $N_{\mathbb{Q}}$. The *tail cone* $\operatorname{tail}(\Delta)$ is the set of all vectors $v \in N_{\mathbb{Q}}$ such that for all $a \in \Delta$ one has $a + v \in \Delta$. Fix a polyhedral *cone* $\sigma \subseteq N_{\mathbb{Q}}$, i.e. an intersection of finitely many *vector* half-spaces of $N_{\mathbb{Q}}$. Suppose that σ contains no lines, and that its linear span is the whole $N_{\mathbb{Q}}$. A *polyhedral divisor* with tail cone σ on $\mathbf{P}^1$ is a formal finite linear combination $\mathscr{D} = \sum p_i \Delta_i$, where $p_i \in \mathbf{P}^1$, and Δ_i are polyhedra with $\operatorname{tail}(\Delta_i) = \sigma$. Suppose that all vertices of all polyhedra Δ_i are lattice points. Then $\mathscr{D}$ is called *proper* if the Minkowski sum of all Δ_i's is strictly contained in σ. From now on, $\mathscr{D}$ is proper. As explained in [1], $\mathscr{D}$ gives rise to a normal affine threefold X with a faithful action of T.

For a general reference on deformations (see [2]). Denote $S = \operatorname{Spec}(\mathbb{C}[\varepsilon]/(\varepsilon^2))$, and let $s \in S$ be the (unique) closed point of S. An *infinitesimal deformation* of X is a triple (Y, f, ι), where Y is a scheme of finite type over $\mathbb{C}$, $f \colon Y \to S$ is a flat morphism, and $\iota \colon X \to f^{-1}(s)$ is an isomorphism. Two infinitesimal deformations (Y, f, ι) and (Y', f', ι')

are called equivalent if there exists an isomorphism $g\colon Y \to Y'$ such that $f'g = f$ and $g|_{f^{-1}(s)}\iota = \iota'$. Denote the set of the equivalence classes of all infinitesimal deformations of X by $T^1(X)$. An infinitesimal deformation (Y, f, ι) of X is called *equivariant* if T acts on Y, and f is T-invariant, i.e., if the actions of T and of $\mathbb{C}[\varepsilon]/(\varepsilon^2)$ on $\mathbb{C}[Y]$ commute.

One can (see [2]) introduce a vector space structure on $T^1(X)$ with good functorial properties. Since T acts on X, $T^1(X)$ becomes graded, and its zeroth graded component (denote it by $T^1(X)_0$) consists exactly of all equivariant infinitesimal deformations of X.

Recall that X was obtained from a proper polyhedral divisor $\mathscr{D}$ on $\mathbf{P}^1$. For each of the polyhedra Δ_i we have, the lattice points split its boundary into minimal segments (which themselves do not contain lattice points inside anymore). Denote the number of these segments by k_i. We call a point p_i essential if $k_i > 0$, i.e. Δ_i cannot be written as $\sigma + v$, where $v \in M$. Let r be the number of essential points. Our goal is to find $\dim T^1(X)_0$.

Theorem (see [3] for a proof). $\dim T^1(X)_0 = \max(0, r-3) + \sum\limits_{p_i \in \mathbf{P}^1 essential} (k_i - 1)$.

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Mathematical Behavior of a Modified Two-Component Shallow Water Equations

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The modelling of the propagation of waves on the surface of water plays significant role in applied mathematics and physics. There are a considerable number of previous works on shallow water wave equations [1,2,3,4]. In this work, we consider a two-component generalization of a shallow water equation. The purpose of this work is to study the well posedness and blow-up Cauchy problem for a two-component generalization of shallow water equation.

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On T-Neighborhood on Univalent Functions

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Let A be the class of analytic function f in the open unit disk $U = \{z : |z| < 1\}$ with the normalization conditions $f(0) = f^{'}(0) - 1 = 0$. If $f(z) = z + \sum\limits_{n=2}^{\infty} a_n z^n$ and $\delta > 0$ are given, then the T_δ-neighborhood of the function f is defined as

$$TN_\delta(f) = \left\{ g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in A : \ \sum_{n=2}^{\infty} T_n |a_n - b_n| \leq \delta \right\},$$

where $T = \{T_n\}_{n=2}^{\infty}$ is a sequence of positive numbers. In the present paper we investigate some problems concerning T_δ-neighbrhoods of functions in various classes of analytic functions with $T = \{\frac{2-n}{n^2}\}_{n=2}^{\infty}$. We also find bounds for $\delta_T^*(A, B)$ defined by

$$\delta_T^*(A, B) = \inf \left\{ \delta > 0 : \ B \subset TN_\delta(f) \text{ for all } f \in A \right\},$$

where A, B are given subsets of A.

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Homomorphism on Local Top Spaces

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Local top spaces as a generalization of local Lie groups [1] are defined in the following manner.

Definition. A smooth manifold H is called a local top space if there exists

- a set $e(H) \subset H$, the identity elements;
- a smooth product map $\mu : U \to H$ defined on an open subset $(e(H) \times H) \cup (H \times e(H)) \subset U \subset (H \times H)$;
- a smooth inversion map $i : V \to H$ defined on an open subset $e(H) \subset V \subset H$ such that $V \times i(V) \subset U$, and $i(V) \times V \subset U$,

all satisfying the following properties:

(i) Identity: For each $x \in H$ there is a unique element $e(x)$ such that $\mu(e(x), x) = x = \mu(x, e(x))$.

(ii) Inverse: $\mu(i(x), x) = \mu(x, i(x)) = e(x)$ for all $x \in V$.

(iii) Associativity: If (x, y), (y, z), $(\mu(x, y), z)$ and $(x, \mu(y, z))$ all belongs to U, then $\mu(\mu(x, y), z) = \mu(x, \mu(y, z))$.

(iv) $\mu(e(x), e(y)) = e(\mu(x, y))$ for each $x, y \in H$.

(v) $e : H \to H$ is a smooth map.

In this talk the following theorems about local top spaces are proved.

Theorem 1. *Suppose L and M are connected m-dimensional local Lie groups, and θ, η denote their respective right invariant Maurer-Cartan coframes. If a map $\phi : L \to M$ satisfies $\phi^*(\eta) = \theta$ and and $\phi(e) = \widetilde{e}$, then ϕ defines a local group homeomorphism from L onto its image.*

Theorem 2. *Suppose that H is a connected local top spaces with finite number of identities, L is a connected local Lie group and θ, η denote their left invariant Maurer-Cartan coframe. If a map $\phi : H \to L$ satisfies $\phi^*(\eta) = \theta$ and $\phi(e(t)) = e$, then ϕ defines a local top space homeomorphism.*

Theorem 3. *Let H be a local top space with finite number of identities. Then there is a covering top space $\widetilde{H}$ of H which is also a local covering of an open subset M of a global Lie group G.*

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A New Causal Ladder in General Relativity

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Let I^+ and J^+ be the chronological and causal relations on the spacetime M [1, 2].

A continuous function $t : M \to R$ is a time function if it strictly increases on every causal curve.

Definition ([2]). The spacetime (M, g) is stably causal if it admits a time function.

Definition ([1]). A space time is causally continuous if for every $x \in M$, $K^{\pm}(x) = \overline{I^{\pm}(x)}$.

Definition ([3]). A space time is causally easy if and only if it is strongly causal and $\overline{J^+}$ is transitive.

Every causally continuous spacetime is causally easy but the converse is not true. In addition stable causality implies causal easiness but the converse is not true. In this talk we define a new time function which may help us to define a new causal ladder between causally easy and causally continuous.

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Action of Fuzzy Top Spaces on Fuzzy Manifolds

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In this talk the notion of fuzzy top space and the action of a fuzzy top space on a fuzzy manifold is introduced. Some examples of fuzzy top spaces is given and some theorems about them is proved.

Definition 1. A fuzzy top space T is a C^1- fuzzy manifold T which is also a top space and the mappings:

$$m : (T \times T, \tau \times \tau) \longrightarrow (T, \tau), \quad (x, y) \mapsto xy,$$
$$i : (T, \tau) \longrightarrow (T, \tau), \quad x \mapsto x^{-1},$$

are fuzzy differentiable.

Suppose that a group G and two sets Λ and I are given. If $p : I \times \Lambda \to G$ is a mapping, then $\Lambda \times G \times I$ with the product $(\lambda, g, i)(\lambda_1, g_1, i_1) = (\lambda, gp(i, \lambda_1)g_1, i_1)$ is a MG-group, which is called Rees matrix semigroup denoted by $M(G, I, \Lambda, p)$.

Theorem 1. *If I and Λ are fuzzy manifolds, G is a fuzzy Lie group and $p : I \times \Lambda \to G$ is a fuzzy differentiable map then $M(G, I, \Lambda, p)$ is a fuzzy top space too.*

Definition 2. A fuzzy left action of a top space T on the C^1-fuzzy top space M is a mapping $\phi : T \times M \to M$, which satisfies:

(i) $\phi(e(t), t) = t$.

(ii) $\phi(x, \phi(y, z)) = \phi(xy, z)$.

Definition 3. Let $\phi : G \times M \to M$ be a fuzzy left action. A subset S is called a fuzzy G- invariant subset of M, if $\phi(G \times S) \subseteq S$.

Theorem 2. *Consider ϕ be a fuzzy left action of a fuzzy top space T on the manifold M. Let M' be a fuzzy regular C^1-submanifold of M which is invariant under ϕ, then T acts naturally on M'.*

Theorem 3. *Let $\sim$ be an equivalence relation on C^1-fuzzy manifold M. If $\sim$ is preserved by a fuzzy left action ϕ of G on M the G acts naturally on $M/\sim$.*

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Some Computational Aspects of Novel Matrix Spectral Factorization Algorithm

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Spectral factorization is the process by which a positive matrix-valued function $S(t)$, on the unit circle in the complex plane, is expressed in the form $S(t) = S^+(t)(S^+(t))^*$, where $S^+(z)$, $|z| < 1$, is a certain analytical matrix function with boundary values $S^+(t) = S^+(z)|_{z=t}$ and $(S^+(t))^*$ is its Hermitian conjugate. Such factorization plays an important role in the solution of various practical problems in Control Theory and Communications. Consequently, a number of different methods have been developed in order to actually compute $S^+(t)$ for a given matrix function $S(t)$. Recently, a new algorithm of matrix spectral factorization has been developed in [1]. In the presented talk, some components of the algorithm are analyzed which have theoretical as well as practical importance.

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Construction of Multiple Knot B-Spline Wavelet with Ideal Knots

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Multiple knot B-spline wavelets with uniform knots and multiple only in the ends have been constructed by Chui and Quak in [1]. This work deals with construction of a multiple knot B-spline wavelets (MKBSW) on the interval $[0, 1]$. They are non-uniform and multiple not only on the ends but also on the middle knots. Here, a big family of MKBSWs and their dual is presented that give a variety of basis functions with explicit formulas and locally compact supports that they have made with non-uniform knots ([2-4]). Moreover, the structure of this wavelet is conceptually simple and easy to implement. Finally, some examples of MKBSWs and their dual are also presented.

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A Note on General Kaluza-Klein Cosmology

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In 1919, Kaluza had the brilliant idea to use a 5-dimensional manifold to unify the Einstein theory of general relativity with the Maxwell theory of electromagnetism. In

1938, Einstein and Bergmann presented the first generalization of the Kaluza-Klein theory. A generalization was intensively studied in the last two decades under the name of space-time-matter theory(STM).

The lack of models for STM theory with non-zero cosmological constant we will try to presente a modle for cosmological constant.

Let M and K be two manifolds of dimensions four and one, respectively. Consider $\overline{M} = M \times K$ as a trivial bundle over M and $\overline{g}$ be a pseudo-Riemannian metric on $\overline{M}$ subject to some conditions. we denote by $V\overline{M}$ the vertical bundle on $\overline{M}$, which is tangent to the foliation whose leaves are $\{x\} \times K, x \in M$. The gauge transformations on $\overline{M}$ are determined by the horizontal distribution $H\overline{M}$, which is supposed to be a Lorentz vector bundle that is complementry orthogonal to $V\overline{M}$ in $T\overline{M}$ with resprct to $\overline{g}$. $(\overline{M}, \overline{g})$ is called general Kaliza-Klein space.

Theorem. Let $(\overline{M}, \overline{g})$ be a general Kaliza-Klein space. Suppose that the Einstein gravitational tensor field $\overline{G}$ of $(\overline{M}, \overline{g})$ satisfies the Einstein equations with cosmological $\overline{\Lambda}$ that is, we have $\overline{G} = -\overline{\Lambda}\overline{g}$, then

$$\overline{\Lambda} = -\Big(\frac{3}{8\varepsilon}\, F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\, S + \frac{1}{2\varepsilon}\, ({D_\mu}^{\mu})^2 - \frac{1}{\varepsilon}\, {D_\mu}^{\nu} {D_\nu}^{\mu}\Big),$$

where D, F are horizontal tensor fields on $\overline{M}$ and S is scaler curvature of $\overline{M}$.

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A Theoretical Measure Technique for Determining 3-D Symmetric Optimal Shapes with a Given Center of Mass

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Optimal shape design (OSD) is an application-oriented subject whose applications can be found in many engineering branches. For instance, we can address some of them in mechanical engineering, civil engineering, marine industry and chemical engineering. We remind that symmetry is an important cue for many applications in the real world, including object alignment, recognition, segmentation, computer graphics and geometric processing. Moreover, the concept of "center of mass" has a wide range of applications in science and technology. In this paper, a new approach is proposed for designing the optimal three dimensional symmetric shapes with desired physical center of mass. Herein, our purpose is to find such a shape (say domain) whose image in (r,θ)-plane is a divided region into a fixed and variable part. The optimal shape is characterized in two stages. Firstly, for each given domain, the optimal surface is determined by changing the problem into a measure-theoretical one, replacing this with an equivalent infinite dimensional linear programming problem and approximating schemes. Then, a suitable function that offers the optimal value of the objective function for any admissible given domain is defined. In the second stage, by applying a standard optimization method, the global minimizer surface and its related domain will be obtained whose smoothness is considered by applying outlier detection and smooth fitting methods. Here, the unknown bounded shape C is symmetric in cylindrical coordinate with respect to (r,z)-plane and has a specified center of mass placed on the top of the (r,θ)-plane. The boundary of the shape includes the unknown surface S with equation $z=f(r,\theta)$ and its unknown image, in (r,θ) plane is the region D; that is, a bounded region with a piecewise-smooth, closed and simple boundary ∂D which consists of a fixed and a variable part. We intend to find the optimal unknown surface S and the optimal unknown region D simultaneously, so that a given performance criteria is minimized on C. Furthermore, in general, a curve can be approximated by broken lines so that ∂D can be approximated with a number M of its points (corners of broken lines belonging to ∂D) which will be called the M-representation of D. For a fixed number M, without losing generality, the points in the M-representation set can have the fixed θ-components like $\theta_i=\theta'_i$, $i=1,2,\dots,M$. Hence, each admissible M-representation set called D_M can be characterized by M variables $r_1,r_2,\dots,r_M$. Therefore, the variable part of ∂D is defined by a finite set of M real variables$(r_1,r_2,\dots,r_M)$. By redefining the problem into a control problem one, replacing this into a measure theoretical one, using approximating schemes and outlier detection, we explain that how the optimal variables $(r_1,r_2,\dots,r_M)$ and optimal surface would be characterized.

Lower Bounds on the Multiplicative Zagreb Indices of Graph Operations

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In theoretical chemistry, the physico-chemical properties of chemical compounds are often modeled by means of molecular-graph-based structure-descriptors, which are also referred to as topological indices [1]. The multiplicative Zagreb indices were introduced by Todeschini et al. in 2010 [2]. The first and second multiplicative Zagreb indices of a simple graph G are denoted by $\Pi_1(G)$ and $\Pi_2(G)$, respectively, and defined as:

$$\Pi_1(G) = \prod_{u \in V(G)} d_G(u)^2 \text{ and } \Pi_2(G) = \prod_{uv \in E(G)} d_G(u) d_G(v),$$

where $d_G(u)$ denotes the degree of the vertex u of G.

The second multiplicative Zagreb index can also be expressed as a product over vertices of G [3]:

$$\Pi_2(G) = \prod_{u \in V(G)} d_G(u)^{d_G(u)}.$$

In this paper, we present some lower bounds for the first and second multiplicative Zagreb indices of several graph operations in terms of the multiplicative Zagreb indices of their components.

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Modeling of a Fractional-Order Economic System

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Nonlinear differential equation with fractional derivatives give general representations of real life phenomena. In this paper, reduced differential tranform method (RDTM) for solving the nonlinear fractional differential equation. A fractional Chen system is considered to demonstrate the efficiency of the algorithm. The numerical example shows that method is effective.

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A New Sequential Approach for Solving the Integro-Differential Equation Via Haar Wavelet Bases

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In this work, we present a method for numerical approximation of fixed point operator, particularly for the following mixed Volterra-Fredholm integro-differential equations:

$$\lambda u'(x) = f(x) + \alpha \int\limits_0^1 K_1(x,t,u(t))\,dt + \beta \int\limits_0^x K_2(x,t,u(t))\,dt,$$

where $x,t \in [0,1]$, $u \in C([0,1],R)$, $|\alpha|+|\beta| \neq 0$, $u(0)=u_0$, $\lambda \in R-\{0\}$ and $f:[0,1]\to R$. Also, we assume that $K_1, K_2 : [0,1]^2 \times R \to R$ are known continuous functions which satisfying the Lipschitz condition, that is, there exist $M_1, M_2 > 0$ such that:

$$\big|K_i(x,t,y) - K_i(x,t,z)\big| \leq M_i|y-z|, \quad i=1,2,$$

where $y,z \in R$, and the unknown function to be determined is $u:[0,1]\to R$.

The main tool for error analysis is the Banach fixed point theorem. The advantage of this method is that it does not use numerical integration, we use the properties of rationalized Haar wavelets for approximate of integral. The cost of our algorithm increases accuracy and reduces the calculation, considerably. Some examples are provided to illustrate its high accuracy and numerical results are compared with other methods in the other papers.

Blow-up Solutions Some Classes of the Nonlinear Parabolic Equations

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In this paper the unbounded increasing solutions of the nonlinear parabolic equations for the finite time is investigated.

The sufficient condition for nonlinearity is established. Under this condition every solution of the investigated problem is blown-up. I.e., there is number $T > 0$ such that

$$\|u(x;t)\|_{L_2(R^n)} \to \infty, \quad t \to T < \infty$$

The existence of the solution is proved by smallness of the initial function.

These type of nonlinear equations describe the processes of electron and ionic heat conductivity in plasma, fusion of neutrons and etc. One of the essential ideas in theory of evolutional equations is known as method of eigenfunctions. In this paper we apply such method. We different boundary problem is considered.

In [1] the existence of unbounded solution for finite time with a simple nonlinearity have been proved. In [2] has been shown that, under the critical exponent any nonnegative solution is unbounded increasing for the finite time. Similar results were obtained in [3] and corresponding theorems are called Fujita-Hayakawa's theorems. More detailed reviews can be found in [4-6].

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The Solutions Degenerate Nonlinear Elliptic-Parabolic Equations

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Let R^{n+1} be an $(n+1)$-dimensional Euclidian space of the points $(x,t)=(x_1,\dots,x_n,t)$ $Q_T=\Omega\times(0,T)$ be a cylindrical domain in R^{n+1}, where Ω is a bounded n-dimensional domain with the boundary $\partial\Omega$ and $T\in(0,\infty)$. Let $Q_0=\{(x,t):\ x\in\Omega,\ t=0\}$ and $\Gamma(Q_T)=Q_0\cup(\partial\Omega\times[0,T])$ be a parabolic boundary of Q_T. Consider the following second order degenerated elliptic-parabolic equation in Q_T

$$Lu=\sum_{i,j=1}^{n}a_{ij}(x,t,u)u_{x_ix_j}+\sum_{i=1}^{n}b_i(x,t)u_{x_i}+\psi(x,t)u_{tt}+C(x,t)u_t=f(x,t), \tag{1}$$

$$u\big|_{\Gamma(Q_T)}=0 \tag{2}$$

in assumption that $(a_{ij}(x,t))$ is a real symmetric matrix where for all $(x,t)\in Q_T$ and any n-dimensional vector ξ the following conditions are fulfilled

$$|\xi|^2\le\sum_{i,j=1}^{n}a_{ij}(x,t,u)\xi_i\xi_j\le\gamma^{-1}|\xi|^2 \tag{3}$$

where $\gamma\in(0,1]$-const

$$c(x,t)\le 0,\ \ c(x,t)\in L_{n+1}(Q_T),\quad \Big(\sum_{i=1}^{n}b_i^2(x,t)^{\frac{1}{2}}\in L_{2(n+1)}\Big) \tag{4}$$

We determine the function $\psi(x,t)=\omega(x)\omega(t)\varphi(T-t)$, where $\omega(x)$ nonnegative function satisfy Makenxoupt condition, ω, φ are continuous, non-negative and non-decreasing functions of their arguments. In the above mentioned conditions the coercive estimate is proved for strong solutions of (1), (2). Further the weighted space $W^{2,2}_{2,\psi}(Q_T)$ is introduced and strong solvability is proved in the following form.

Theorem. *Let the conditions* (3), (4) *be fulfilled. Then at $T\le T_0$ the problem* (1), (2) *has a unique strong solution in the space $\overset{\circ}{W}{}^{2,2}_{2,\psi}(Q_T)$ for any $f(x,t)\in L_2(Q_T)$.*

Removable Singularities of Solutions of Degenerate Nonlinear Elliptic Equations

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The goal of this work is to study the removable singularities of solutions for the Dirichlet problem for degenerate non-linear elliptic equations on the boundary of domain. For that, the method a priori energetic estimates of solutions to elliptic boundary value problems is used.We studying the growing in the vicinity of a boundary point (finite or at the infinite) for generalized solutions. The applied method differs from the way for obtaining appropriate results in linear situation. The corresponding results for linear equations were obtained in the papers of L. Carleson [1], V. A. Kondratyev, O. A. Oleynik [2], O. A. Oleynik, G. A. Iosifyan [3], V. A. Kondratyev, E. M. Landis [4], D. Gilbarg, N. Trudinger [5], T. Gadjiev, V. Mamedova [6], J. Diederich [7], R. Harvey, J. Polking [8], for non-linear equations in the papers of T. Kilpelainen, X. Zhong [9] and others.

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On Frames of Double and Unary Systems in Lebesgue Spaces

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Let X be B-space, $\mathscr{K}$ be K-space and $(\vec{x}^{+};\vec{x}^{-}) \subset X$ be some double system, where $\vec{x}^{\pm} \equiv \{x_n^{\pm}\}_{n\in N}$. Let $(\vec{\vartheta}^{+};\vec{\vartheta}^{-}) \subset X^{*}$.

Under the atomic decomposition $\{(\vec{\vartheta}^{+};\vec{\vartheta}^{-});(\vec{x}^{+};\vec{x}^{-})\}$ *of* X *with respect to* $\mathscr{K}$ *we will mean the following:*

(i) *The decomposition*

$$x=\sum_{n=1}^{\infty}\left(\vartheta_n^{+}(x)x_n^{+}+\vartheta_n^{-}(x)x_n^{-}\right), \ \ \forall\, x\in X,$$

is true;

(ii) $\exists\, A;B>0: A\|x\|_X \le \|\{\vartheta_n^{+}(x)\}_{n\in N}\|_K+\|\{\vartheta_n^{-}(x)\}_{n\in N}\|_K \le \|x\|_X$.

Consider the unary system of the form

$$x_n^{\pm}(t)\equiv \varphi_n(t)\pm\psi_n(t), \ \ n\in N,$$

where $\varphi_n; \psi_n : [0, a] \to C$ are complex-valued functions. Let us form the new system

$$\Phi_n(t) \equiv \begin{cases} \varphi_n(t), & t \in [0, a], \\ \psi_n(-t), & t \in [-a, 0) \end{cases}$$

and put

$$\Psi_n(t) = \Phi_n(-t), \ \ \forall\, t \in [-a, a].$$

Let $\{\vartheta_n^{\pm}\} \subset L_q(0, a)$ be some systems. Similarly we define

$$\Omega_k^{\pm}(t) \equiv \begin{cases} \vartheta_k^{\pm}(t), & t \in (0, a), \\ \pm\vartheta_k^{\pm}(-t), & t \in (-a, 0) \end{cases}$$

and assume

$$h_k^{\pm}(t) \equiv \frac{1}{2}\left[\Omega_k^{+}(t) \pm \Omega_k^{-}(t)\right], \ \ \forall\, t \in (-a, a). \tag{1}$$

The following theorem is true.

Theorem. *Let $(\vec{\vartheta}^{+}; \vec{x}^{+})$ and $(\vec{\vartheta}^{-}; \vec{x}^{-})$ be an atomic decomposition of $L_p(0, a)$ with respect to K-space $\mathscr{K}$. Then $((\vec{h}^{+}; \vec{h}^{-}); (\vec{\Phi}; \vec{\Psi}))$ is an atomic decomposition of $L_p(-a, a)$ with respect to $\mathscr{K}$, where $\vec{h}^{\pm}$ is determined by* (1).

On Self-Commutators of Operators

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Let A be a linear bounded operator, acting in a Hilbert space $(\mathscr{H}, \langle \bullet, \bullet \rangle)$. The difference $A^*A - AA^* = C(A)$ is said to be the self-commutator of the operator A. The norm of $C(A)$ indicates how "far" is the operator from being normal. For some classes of operators different estimates from above (Putnam's inequality, Berger-Shaw theorem) and from below (Khavinson, Ferguson) are known. In this report different operators in the finite dimensional space (tensor product of two elements, 2×2, tridiagonal, SOR matrices), as well in infinite dimensional space (composition operators in the Dirichlet space, the Volterra integration operator) are considered.

The norm of the self-commutator is connected with the numerical range of the operator. Solving an isoperimetric type problem for compact subsets of the complex plane it is proved that $C(A)$ is bounded from above by the twice of the area of the numerical range of $W(A)$ of A. For operators with elliptical numerical range the factor 2 may be replaced by $4/\pi$.

A New Approximation Algorithm for Minimum Connected Dominating set in Wireless Sensor Networks

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Mobile ad hoc networks are frequently modeled by unit disk graphs. There are several classical graph theoretic problems on unit disk graphs. We consider minimum connected dominating set, which are relevant to such networks. We use maximal independent set from [3] then connect this set. The result is minimal connected dominating set. For a unit disk graph we propose decomposition into 1-by-1 boxes and two new matrixes corresponding to graph, Packing matrix and Independent vertex matrix . If the graph is bounded, then the considered problem can be solved in polynomial time. Finally is proved this indirectly by presenting dynamic programming algorithm and we show that this result are optimal.

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Edge Detection Based on Wavelet and Direction of Gradient

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This paper propose an edge detection scheme based on wavelet transform and its angle information. The main idea of the paper is that, the method uses the gradient angle. Many edge detectors use the wavelet transform of an image in x and y directions to approximate the gradient vector at each pixel and then detect edges by thresholding modulus of the gradient vector. In present scheme we use the angle information of gradient vector which obtained by wavelet transform to increase or decrease the effect of wavelet transform in x and y directions by the fact that, every unit vector in $\mathbb{R}^2$ can be written by cosine and sine of its angle.

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Approximation in Banach Space by Linear Positive Operators

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This contribution is based on recent joint work with Dr. Yoshihiro Sawano.

In 1953, P. P. Korovkin proposed a strong and simple criteria for the convergence of positive linear operators. He proved that, for a given sequence $\{L_n\}_{n\in\mathbb{N}}$ of positive linear operators, the convergence on three test functions $\{1, x, x^2\}$ is sufficient to ensure the convergence of sequence on the space $C[a, b]$, which consists of continuous functions on a finite interval $[a, b]$ (see [1]). In fact, a classical theorem of Korovkin on approximation of continuous functions on compact intervals gives conditions with which to decide whether a sequence of positive linear operators converges to the identity operator.

Our main result is that this theorem can be extended to a large extent:

Theorem. *Let X be a Banach function space on $\mathbb{R}^n$ such that the set of all compactly supported $C^\infty(\mathbb{R}^n)$-functions is dense, and that $1 + |x|^2 \in X$. Suppose that we are given a bounded sequence $\{L_n\}_{n=1}^\infty$ of $B(X)$ such that it satisfies the following conditions:*

1. $\lim\limits_{k\to\infty} L_k(1) = 1$,
2. $\lim\limits_{k\to\infty} L_k(t_i) = x_i$ *for* $i = 1, 2, \dots, n$,
3. $\lim\limits_{k\to\infty} L_k(|t|^2) = |x|^2$,
4. *If $f \in X$ is a positive function, then so is $L_k(f)$ for each k.*

Then, $\{L_k\}_{k=1}^\infty$ converges strongly to id_X.

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Some Methods of Integral Representation of Nonsmooth Brownian Functionals of Integral Type

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We have developed a method of obtaining the stochastic integral representation of nonsmooth (in Malliavin sense) Brownian functionals based on the notion of local time of stochastic processes and on the well-known theorem of Trotter-Mayer (see [1, Theorem IV.45.1]).

For example, for the following functional

$$F = \int_0^T I_{\{a \le w_t \le b\}}$$

(where w_t, $t \in [0, T]$ is a Wiener process, $a < b$) we give the stochastic integral representation with explicit integrand. The same representation we have obtained also using the martingale representation formula which was represented by Cont and Fournie (see [2]) within of the non anticipative functional Ito Calculus (where the Malliavin derivative and the Skorokhod integral are replaced, respectively, by the vertical derivative and the Ito stochastic integral).

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New Results on Estimation of Singular Series Corresponding to Positive Quaternary Quadratic Forms

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Let $f = \sum\limits_{\alpha,\beta=1}^{4} a_{\alpha\beta}x_\alpha x_\beta$ be any positive definite, integral, primitive, quaternary quadratic form of the determinant $d = d(f)$, so the gcd $(a_{11}, a_{22}, a_{33}, a_{44}, 2a_{12}, \dots, 2a_{34}) = 1$.

We considered the main term of formulas for the number of representations $r(f, m)$ of $m \in \mathbb{N}$ by f. The main term expressed by the so-called singular series $\rho(f, m)$ can be represented as an infinite product over all primes p

$$\rho(f,m) = \frac{\pi^2 m}{d^{\frac{1}{2}}} \prod_{p\geqslant 2} \chi(p).$$

The formulas for the $\chi(p)$ (even under more general assumptions) are obtained by Malysev [3]. These formulas are simplified in some cases and represented in the convenient form in [1]. The estimates of $\rho(f, m)$ with respect to d and m are important for the investigation of the asymptotic behavior of $r(f, m)$, determination of one-class genera of the forms, the existence of the so-called Gauss type formulas for $r(f, m)$ ($r(f, m) = \rho(f, m)$) and in other applications. In the paper [3] some estimates of $\chi(p)$, $p \geqslant 2$, are given. They yield $\rho(f, m) = O(d^{\frac{1}{2}} m^{1+\varepsilon})$ for any $\varepsilon > 0$. Some analogous results are obtained by various authors too. In the paper [2] we essentially improved the existing results and proved

$$\rho(f,m) = O\big(d_0^{-\frac{1}{3}} d_1^{-\frac{1}{2}} m \ln b(d_1) \ln\ln b(m)\big),$$

where $d_0d_1 = d$, $d_0 = \prod\limits_{\substack{p|2^5d\\ p|2m}} p^{h(p)}$, $d_1 = \prod\limits_{\substack{p|2^4d\\ p\nmid 2m,\, p>2}} p^{h(p)}$, $h(p) \geqslant 0$ if $p > 2$ and $h(2) \geqslant -4$, $b(k)$ is the product of distinct prime factors of the number $16k$ if $k \neq 1$, and $b(k) = 3$ if $k = 1$.

In the present talk we consider all possible types of the quaternary forms and give the corresponding estimates for $\rho(f, m)$, which sharpens all of the previous results.

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Integration Over Open Sets Boundary

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The surface integral and the Gauss-Green formula are used in many areas of mathematics, physics, and engineering. They are well known for smooth manifolds and boundaries and last few decades they have been actively studied in generalization of Lebesgue integration process and rough boundaries for differential formω when $d\omega$ is summable in the region. For whole class of continuous on boundary forms the last assumption is not true. In the paper "Integrable Boundaries and Fractals for Hölder Classes. Gauss-Green Theorem" author introduced a surface integral without assumption that $d\omega$ is summable. In this presentation we use this definition and show condition when boundary integral exists for any continuous differential form.

Boundary Stratification

Let Q be an arbitrary closed cube in R^d and a measurable set $E \subset Q$. Partition Q into 2^d equal closed cubes by dividing its edges into halves, and repeat this procedure with each new cube. As a result we arrive at a set of dyadic subcubes of Q which we denote by 2^Q. For $P \in 2^Q$ we put $rank(P) = d^{-1} \log_2 \frac{|Q|}{|P|}$ the rank of the subcube P ($|E|$ denotes the d-dimensional Lebesgue measure of the measurable set E).

For $k = 0, 1, 2, \dots$ consider sets

$$G_k^+(Q,E) = \bigcup_{P\in 2^Q,\, rank(P)=k,\, |P\cap E|>\frac{|E|}{2}} P\,, \quad G_k^-(Q,E) = \bigcup_{P\in 2^Q,\, rank(P)=k,\, |P\cap E|\le\frac{|E|}{2}} P,$$

$$A_k(Q,E) = \bigcup_{P\in 2^Q,\, rank(P)=k,\, |P\cap G_{k+1}^+|>0,\, |P\cap G_{k+1}^-|>0} P.$$

Thus for each cube Q and set $E \subset Q$ we associate the systems of sets $G_k^\pm(Q,E)_{k=0}^\infty$ and stratification $A_k(Q,E)_{k=0}^\infty$.

Boundary Integral

Let E be an open bounded set in $R^d, d \ge 2$ and a cube $Q \subset E$. For a continuous on $\partial E(d-1)$-form ω we define the boundary integral

$$\int\limits_{\partial E} \omega = \lim_{k\to\infty} \int\limits_{\partial G_k^+(Q,E)} \omega, \tag{1}$$

where a continuous extension of form ω from ∂E into R^d we denote again as ω.

Existence

Let E be an open bounded set in R^d, $d \ge 2$ and a cube $Q \subset E$. If

$$\exists\, L > 0,\ \ \forall\, k\ \ \big(2^{-k}|Q|^{\frac{1}{d}}\big)^{d-1} N_{\partial E}\big(2^{-k}|Q|^{\frac{1}{d}}\big) \le L, \tag{2}$$

where $N_{\partial E}(2^{-k}|Q|^{\frac{1}{d}})$ is the number of cubes from 2^Q, rank k needed to cover ∂E, then boundary integral (1) exists for every $(d-1)$-form ω continuous on ∂E.

Boundary integral (1) is **properly defined**: it does not depend on an initial cube Q in boundary stratification and continuous extension of form ω from ∂E to R^d.

Gauss-Green Theorem. *Let E be an open bounded set in R^d, $d \ge 2$ for which condition* (2) *is valid. If a $(d-1)$-form ω is Absolutely Continuous on Line for each subcube from E, $\omega \in C(\partial E \cup E)$, and $d\omega$ is summable in E then*

$$\int\limits_{\partial E} \omega = \int\limits_{E} d\omega.$$

Similar results and general Stokes' theorem **take place in the case of open sets on** Lipschitz orientable n-dimensional manifolds in R^d.

Three-Dimensional Homogeneous Contact Non-Sasakian Lorentz Manifold

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A 3-dimensional strongly locally φ-symmetric space is either K-contact with constant scalar curvature or is a (k, μ)-manifold with $k < 1$. and a 3-dimensional contact metric manifold is a strongly locally φ-symmetric space if and only if it is locally contact homogeneous. Examples of strongly locally φ-symmetric spaces include the non- Sasakian (k, μ)-manifolds. Special cases of these are the non-abelian 3-dimensional unimodular Lie groups with left-invariant contact metric structures. which has been shown in [1] for Reimannian manifold. A pseudo-Riemannian manifold (M, g) is homogeneous provided that, for any points $p, q \in M$, there exists an isometry ϕ such that $\phi(p) = q$; it is locally homogeneous if there is a local isometry mapping a neighborhood of p into a neighborhood of q.

Homogeneous contact 3-manifold with (k, μ)-nulity is not yet well-understood in Lorentzian case, although some progresses have been made in the Riemannian setting [1]. Here we will introduce non-Sasakian homogeneous contact Lorentz 3-manifold. we essentially show the following theorem:

Theorem. *There is a one-to-one correspondence between homogeneous contact Riemannian three-manifolds and homogeneous contact Lorentzian three-manifolds. A simply connected homogeneous contact Lorentzian three-manifold is a Lie group G equipped with a left-invariant contact Lorentzian structure $(\varphi, \xi, \eta, g_L)$. More precisely, we have the following result for non-Sasakian case ($\|\tau\| \neq 0$ is a constant)*

(1) *If G is unimodular, then it is*

  (i) $SU(2)$ *when* $r_L > 2(1 + \frac{\|\tau\|}{2\sqrt{2}})^2$;

  (ii) $\widetilde{E}(2)$ *when* $r_L = 2(1 + \frac{\|\tau\|}{2\sqrt{2}})^2$;

  (iii) $\widetilde{SL}(2, R)$ *when* $2(1 - \frac{\|\tau\|}{2\sqrt{2}})^2 \neq r_L < 2(1 + \frac{\|\tau\|}{2\sqrt{2}})^2$;

  (iv) $E(1, 1)$ *when* $r_L = 2(1 - \frac{\|\tau\|}{2\sqrt{2}})^2$.

(2) *If G is non-unimodular, then its Lie algebra is given by $[e_1, e_2] = \alpha e_2 + 2\xi$, $[e_1, \xi] = \gamma e_2$, $[e_2, \xi] = 0$, where $\alpha \neq 0$. In this case, $r < -2(1 - \frac{\|\tau\|}{2\sqrt{2}})^2$.*

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Some Inverse Problems for Dirac Operators

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Let us denote by $L(p,q,\alpha)$ the boundary value problem

$$\left\{\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\frac{d}{dx}+\begin{pmatrix}p(x) & q(x)\\ q(x) & -p(x)\end{pmatrix}\right\}y=\lambda y,\ \ x\in(0,\pi),\ \ y=\begin{pmatrix}y_1\\ y_2\end{pmatrix},\ \ \lambda\in C, \tag{1}$$

$$y_1(0)\cos\alpha+y_2(0)\sin\alpha=0,\ \alpha\in\Big(-\frac{\pi}{2},\frac{\pi}{2}\Big],$$

$$y_1(\pi)=0,$$

where $p,q\in L^1_R[0,\pi]$, i.e. p, q are real-valued summable function on $[0,\pi]$. It is well known, that the problem $L(p,q,\alpha)$ has a purely discrete spectra of simple eigenvalues $\lambda_n=\lambda_n(p,q,\alpha)$, $n=0,\pm1,\pm2,\dots$, $\dots<\lambda_{-n}<\lambda_{-n+1}<\dots<\lambda_{-1}<\lambda_0\le 0<\lambda_1<\dots<\lambda_n<\cdots$, forming a sequence unbounded both from below and above.

By $\varphi(x,\lambda)$ and $u(x,\lambda)$ we denote the solutions of system (1), satisfying correspondingly the initial conditions $\varphi(0,\lambda)=\begin{pmatrix}\sin\alpha\\ -\cos\alpha\end{pmatrix})$, $u(\pi,\lambda)=\begin{pmatrix}0\\ -1\end{pmatrix}$.

It is easy to see that $\varphi_n(x)\overset{def}{=}\varphi(x,\lambda_n)$ and $u_n(x)\overset{def}{=}u(x,\lambda_n)$, $n\in Z$, are the eigenfunctions of $L(p,q,\alpha)$ corresponding to eigenvalues λ_n. Since all eigenvalues are simple, there exist the constants $c_n=c_n(p,q,\alpha)$, such that $u_n(x)=c_n\varphi_n(x)$, $n\in Z$.

Theorem. *If for all $n\in Z$ $\lambda_n(p,q,\alpha)=\lambda_n(\widetilde{p},\widetilde{q},\widetilde{\alpha})$, $c_n(p,q,\alpha)=c_n(\widetilde{p},\widetilde{q},\widetilde{\alpha})$, then $p(x)=\widetilde{p}(x)$, $q(x)=\widetilde{q}(x)$ almost everywhere and $\alpha=\widetilde{\alpha}$.*

As the corollaries of this theorem we have proved theorems, which are analogous to well-known theorems of Marchenko, Borg, McLaughlin-Rundell and some others in the inverse Sturm-Liouville problems.

Also, we consider the Ambarzumian type theorems for Dirac systems and compare with the results of [1, 2] and some others.

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Triangular Operational Matrix Method for Solving the Fractional Order Differential Equation

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This article proposes a simple efficient method for solving the fractional order differential equations. We derive the triangular operational matrix of the fractional order integration and use it to solve the fractional order differential equations. The method is described and illustrated with numerical example. The results show that the method is accurate and easy to apply.

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Decomposition of Euclidean Nearly Kähler Submanifolds

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Maybe one of the important questions in geometry of nearly Kähler manifolds is Butrulle conjecture [2]: *Every complete (compact) nearly Kähler manifolds is 3-symmetric or equivalently is homogeneous.* According to the Nagy decomposition [1], this conjecture can be separated two restricted questions:

Problem 1. *Every complete* (*compact*) 6*-dimensional nearly Kähler manifolds is homogenous?*

Problem 2. *Every positive quaternion-Kähler manifolds is Wolf space?*

In [3], we partially answered this conjecture by studing isometric immersions $f : M^{2n} \longrightarrow \mathbb{Q}^{2n+p}$ from a nearly Kähler manifold into a space form (especially Euclidean space). We introduced in [3] an umbilic distribution which is complex and invariant by the torsion of intrinsic Hermitian connection, we showed that, this distribution is integrable and each leaf of the generated foliation is a 6-dimensional homogeneous nearly kähler submanifold. Using this foliation we can parametrize isometric immersions from a nearly Kähler manifold into the Euclidean space and constract some examples of such submanifolds [4]. In this article, we focus on the foliation space of the above integrable distribution.

Definition. Let $f : M^{2n}(\langle\rangle, J) \longrightarrow \mathbb{Q}^{2n+p}$ be an isometric immersion from a nearly Kähler manifold into a space form with second fundamental form α and $0 \neq \eta \in T_f^{\perp}M$ is a non-zero normal vector field on M. Umblic distribitaion of f defined by $\forall\, x \in M, x \mapsto \Delta_x$ where

$$\Delta_x = \Big\{ X \in T_xM \mid \quad \alpha(X,Y) = \langle X,Y\rangle\eta \ \ \forall\, Y \in T_xM \Big\}$$

comlexification of this distribution described by $\Delta' = \Delta_x \cap J\Delta_x$ where Now we put

$$\Delta'_x = \Big\{ X \in T_xM \mid \quad \alpha(T(X,Y),Z) + \alpha(X,T(Y,Z)) = 0 \ \ \forall\, Y,Z \in T_xM \Big\}$$

where T is the torsion of intrinsic Hermitian connection and specified by $T(X,Y) = (\nabla_x J)Y$ (∇ is Levi-Civita connection on M). We define by $D_x = \Delta_x \cap \Delta'_x \cap \Delta''_x$ umblic distribution which is complex and invariant by the torsion of intrinsic Hermitian connection.

Corollary 1. *Let $f : M^{2n} \longrightarrow \mathbb{R}^{2n+p}$ be an isometric immersion from a complete and simply connected nearly Kähler manifold into Euclidean space on manifold M there exist* *two integrable distributions D, $D^\perp$ such that each leaf of generated foliations by D and $D^\perp$ is minimal and totally geodesic in M, respectively. Moreover, if $\forall\, X \in D_x^\perp$, $U \in D_x$ we have $R(X, JX, U, JU) = 0$ then M is product space of the foliation space of complex and invariant umbilic foliation and a 6-dimensional homogeneous nearly Kähler manifold which is isometric with the corresponding factor in Nagy decomposition.*

Remark. Note that 6-dimensional factor in Nagy decomposition is not necessarily homogeneous therefore this is a positive answer to the Problem 1 under assumption of Corollary 1.

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Class United Grouplikes

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Recently, we have introduced and studied a new algebraic structure, namely *"Grouplike"*. A Grouplike is something between semigroup and group and its axioms are generalizations of the four group axioms. There is an important class of grouplikes with an additional axiom. Because of the related fundamental structure theorem, we call them *Class United Grouplikes*. In this talk, I discuss the topic and introduce some related unsolved problems and show some of future directions for the researches in grouplikes (and semigroup theory). The following is a short summary about them.

A semigroup Γ is called *grouplike* if it satisfies the following axioms:

(i) There exists $\varepsilon \in \Gamma$ such that $\varepsilon x = \varepsilon^2 x = x\varepsilon^2 = x\varepsilon :\ \forall\, x \in \Gamma$.

(ii) For every ε satisfying (i) and every $x \in \Gamma$, there exists $y \in \Gamma$ such that $xy = yx = \varepsilon^2$.

We call every $\varepsilon \in \Gamma$ satisfying the axioms (i) and (ii) an *identity-like* and denote by $Iz(\Gamma)$ the set of all identity-likes.

Theorem A. *Every grouplike contains a unique idempotent identity-like.*

Let Γ be a grouplike and e the unique idempotent identity-like of Γ. Every y that is corresponded to x in axiom (ii) is called *inverse-like of* x and is denoted by x'_e or x'. By $Inv(e)$ we denote the set of all inverse-likes of e. Since $Iz(\Gamma) \subseteq Inv(e)$ we have two hypothesis for grouplikes:

($\mathbf{H_1}$) (The identity-like hypothesis) $exy = xy$, for every $x, y \in \Gamma$.

($\mathbf{H_2}$) (The inverse-like hypothesis) $Inv(e) = Iz(\Gamma)$.

We prove that (H_1) implies (H_2), but the converse is an open problem.

Theorem B (General form of grouplikes satisfying the identity-like hypothesis: Class United Grouplikes). *A binary system* $(\Gamma, \cdot)$ *is a grouplike with the identity-like hypothesis if and only if there exists a class group* $\mathscr{G}$ *and a choice function* $\varphi : \mathscr{G} \to \cup\mathscr{G}$ *such that* $\Gamma = \cup\mathscr{G}$ *and* $\cdot = \cdot^{\varphi}$, *where* $x \cdot^{\varphi} y := \varphi(\Psi(x) \circ \Psi(y))$ *and* $\Psi : \cup\mathscr{G} \to \mathscr{G}$ *is the unique class function.*

Example. Consider the additive group of real numbers R and fix $b \neq 0$. For each real number a, denote by $[a]$ the largest integer not exceeding a and put $(a) = a - [a]$. Set $[a]_b = b[\frac{a}{b}]$, $(a)_b = b(\frac{a}{b})$. We call $[a]_b$ *b-integer part of* a and $(a)_b$ *b-decimal part of* a. Now, for every real numbers x, y, we put $x +_b y = (x + y)_b$ and call $+_b$ *b-addition.*

Then, $(R, +_b)$ is a class united grouplike and $Iz(R, +_b) = bZ = \langle b \rangle = Inv(0)$.

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Relation Between Cohomology of Topological Local Groups and Reduced Cohomology Topological Group

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The cohomology group on Topological groups is introduced by Hu in the paper [4] and said relationships between cohomology groups and cohomology local groups.

Let X be a Hausdorff topological local group[1] and φ open continuous map. and X extend[3] to topological group H and let $\widetilde{\varphi} : H \to H$ be the unique extension of φ to and endomorphism of H then is a contractive near-automorphism and quotient H on union of $\ker \widetilde{\varphi}^n$ is locally isomorphic [2] with topological local group X.

Now, we prove that cohomology topological local group X is isomorphic with reduce cohomology of quotient topological group H.

Lemma ([2]). *Suppose X be a Hausdorff topological local group and φ open continuous map. Let H be enlargement of X and let $\widetilde{\varphi} : H \to H$ be the unique extension of φ to and endomorphism of H. Then the map $\widetilde{\varphi}$ is open and for $F := \bigcup_n \ker(\widetilde{\varphi}^n)$ we have*

1. *F is a discrete normal subgroup of H and $\widetilde{\varphi}^{-1}(F) = F$;*
2. *$\widetilde{\varphi}$ descends to a contractive near-automorphism*

$$\varphi_F : H/F \to H/F, \quad \varphi_F(xF) := \widetilde{\varphi}(x)F;$$

3. *for any symmetric open neighborhood $U \subseteq X$ of 1 with $U \times U \subseteq D$, the image $\pi(U)$ of U in H/F is open, and map $x \mapsto xF : U \to H/F$is an isomorphism $X|_U \to (H/F)|_{\pi(U)}$ of topological local groups.*

Main Theorem. To consider presupposition lemma. Then $H^p_L(X, C)$ cohomology of local group X on abelian group C is isomorphism to cohomology of topological group $H^p_*(H/F, C)$ for all $p > 1$.

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On Basis Properties of Root Functions of a Nonselfadjoint Boundary Value Problem

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The boundary value problem is considered which occurs in the theory of small transversal vibrations of a inhomogeneous string. The ends of the string assumed to be fixed and the midpoint of the string is damped by a pointwise force. The problem is reduced to a spectral problem for a linear operator pencil on a direct sum of two Banach spaces. The spectrum of the pencil can be presented as a union of two subsequences. The asymptotic behavior of eigenvalues and eigenfunctions are obtained, as well as of its Green function. These leads us to study the completeness of root functions in Lebesgue spaces. The uniform boundedness of Riesz projections of the problem also investigated and as a consequence the basisness of root functions of the problem is studied.

On the Local ε-Factors of Weil Representations

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Let K be a local field with finite residue-class field $\kappa_K = O_K/\mathfrak{p}_K$ of $q = p^f$ elements, where O_K denotes the ring of integers in K with the unique prime ideal $\mathfrak{p}_K$. As usual, denote the unit group of K by U_K. Fix a separable closure K^{sep} of K once and for all. The absolute Galois group of K and the absolute Weil group of K are denoted by G_K and W_K respectively. We shall denote the local Artin reciprocity map of K by $\mathsf{Art}_K : W_K^{ab} \xrightarrow{\sim} K^\times$.

If $\chi : W_K \to \mathbb{C}^\times$ is a 1-dimensional complex representation of the absolute Weil group W_K of the local field K and $\psi : K^+ \to \mathbb{C}^\times$ is a non-trivial additive character of K, then the corresponding local ε-factor (=local factor) $\varepsilon_K(\chi, \psi, d\mu) \in \mathbb{C}^\times$ of the triple $(\chi, \psi, d\mu)$ is defined (look at Tate [5]) by:

$$\varepsilon_K(\chi,\psi,d\mu) = \chi_*(\mathsf{Art}_K^{-1}(c)) \frac{\int\limits_{U_K} \chi_*(\mathsf{Art}_K^{-1}(y))^{-1}\psi(y/c)\, d\mu(y)}{\Big|\int\limits_{U_K} \chi_*(\mathsf{Art}_K^{-1}(y))^{-1}\psi(y/c)\, d\mu(y)\Big|},$$

where $\chi_* : W_K^{ab} \to \mathbb{C}^\times$ is the canonical quasi-character of W_K^{ab} attached to the 1-dimensional complex representation $\chi : W_K \to \mathbb{C}^\times$ of W_K, $d\mu$ is the additive Haar measure on K which is normalized by $\mathrm{vol}(O_K) = 1$, and $c \in K^\times$ satisfies $\nu_K(c) = a(\chi) + n(\psi)$. Here, $a(\chi)$ and $n(\psi)$ denote the conductors of χ and of ψ respectively. Therefore, in case χ is a 1-dimensional complex representation of W_K, there is an *explicit expression* of the local ε-factor $\varepsilon_K(\chi, \psi, d\mu)$ of the triple $(\chi, \psi, d\mu)$ in terms of the local Artin reciprocity map of K.

The aim of this work, which has been proposed in [1], is to give an explicit formula for the ε-factor $\varepsilon_K(\rho, \psi, d\mu)$ of the triple $(\rho, \psi, d\mu)$, where $\rho : W_K \to \mathrm{GL}(V)$ is any representation of W_K on an n-dimensional $\mathbb{C}$-linear space V, in terms of the non-abelian local class field theory developed in [2] and [3]; or equivalently, in terms of Laubie theory [4].

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Hypersurfaces of Kenmotsu Space Form

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In this paper we study hypersurfaces of Kenmotsu space form and structure of this hypersurfaces. First, we study structure of hypersurfaces of Kenmotsu space form with tangent to the structure vector field and then introduce the special of this hypersurfaces. Then introduce this hypersurfaces with special conditions.

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Similarity Conditions of Planar Bezier Curves

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In this study the G-equivalence conditions of control points in E^2 for all similarity transformations group $G = GO(E)$ as the generator invariants of the field of invariant rational functions $R(x_1, x_2, \dots, x_k)^{GO(E)}$ are investigated. As a result of this the similarity conditions of planar Bezier curves in E^2 are examined.

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The Relative Error Bounds of Fuzzy Linear Systems

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In this paper, we present a general model for solving a fuzzy linear system whose coefficient matrix is fuzzy matrix and the right hand side column is an arbitrary fuzzy vector. Numerical examples for the relative error bounds of this system for three types of perturbation, i.e. only right hand side perturbed, only coefficient matrix perturbed, and both right hand and coefficient matrix perturbed, are derived.

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Newtons Method for Finding Best Trapezoidal Solution of Fuzzy Nonlinear Equations

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Fuzzy logic is powerful tool for modeling uncertainties associated with human cognition, thinking and perception, which has been successfully applied in various fields such as neural network and financial time series. The models are leads to a nonlinear system. In this paper, we propose a method for finding trapezoidal approximation of solution of fuzzy nonlinear equations using the distance between two fuzzy numbers. Numerical test is given to state efficiency of the proposed method. Moreover, The proposed method is compared to the Newtons method for a fuzzy nonlinear equation.

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Boundary Value Problems for Real Order Differential Equations

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This paper considers differential equations with rational and irrational orders. At first some preliminary definitions of fractional derivative is introduced. Then by making use of Mitag-lefler functions and Euler exponential function, a suitable function is introduced in which this function is invariant with respect to the fractional and irrational order derivatives. similar to Euler function e^{λ} which is invariant with respect to order derivative. In order to provide the existence of solutions a suitable functional space is constructed.

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Existence and uniqueness of Solution of a Non-Self-Adjoint Initial-Boundary Value Problem for Partial Differential Equation with Non-Local Boundary Conditions

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Boundary value problems (BVP) play an important role to construct the mathematical models in many areas of physics and engineering problems. In the classic cases, these problems are considered in self-adjoint case and with local classic boundary conditions as well as Dirichlet and Neumann boundary conditions. As we know in these cases the eigenvalues of related operator are real and distinct. Consequently the eigenfunctions are orthogonal and form a complete basis system. Authors in [4] have been considered some complex constants partial differential equations in non-classic cases as well as Non-self-adjoint and non-local with non-periodic conditions. In this paper we consider an initial-boundary value problem that its spectral problem is an two dimensional elliptic equation and this problem is not a self-adjoint problem. Hence its eigenvalues are not real and the eigenfunctions do not form an orthogonal basis system. To construct the solution as infinite series, we will use the eigenfunctions of related adjoint problem. For this problem, we construct a formal and analytic solution by using eigenfunctions of related spectral problem. Finally the convergence of series solution, the existence and uniqueness of solution will be proved.

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Semi-Discrete Scheme for One Averaged System of Nonlinear Integro-Differential Equations

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It is well-known that for mathematical modeling of the process of penetrating of electromagnetic field in the substance the system of Maxwell's equations is used. In a quasistationary case the system of Maxwell's equations as it was done in the work by D. Gordeziani, T. Jangveladze and T. Korshia (1983) can be reduced in the following integro-differential form:

$$\frac{\partial H}{\partial t} = -rot\left[a\Big(\int_0^t |rot\,H|^2\,d\tau\Big) rot\,H\right], \tag{1}$$

where $H = (H_1, H_2, H_3)$ is a vector of the magnetic field and function $a = a(S)$ is defined for $S \in [0, \infty)$. By assuming the temperature of the considered body to be constant throughout the material, i.e., depending on time, but independent of the space coordinates, G. Laptev (1990) proposed some generalization of the system (1) which has the following form:

$$\frac{\partial H}{\partial t} = -rot\left[a\Big(\int_0^t \int_0^1 |rot\,H|^2\,dx\,d\tau\Big) rot\,H\right]. \tag{2}$$

System (2) with source term is studied. In particular, assuming that the magnetic field has the form $H = (0, U, V)$ and $U = U(x,t)$, $V = V(x,t)$ the following systems of nonlinear integro-differential equations is considered:

$$\begin{aligned} \frac{\partial U}{\partial t} &= \frac{\partial}{\partial x}\left[a\Big(\int_0^t \int_0^1 \Big[\Big(\frac{\partial U}{\partial x}\Big)^2 + \Big(\frac{\partial V}{\partial x}\Big)^2\Big]\,dx\,d\tau\Big)\frac{\partial U}{\partial x}\right] + f(U), \\ \frac{\partial V}{\partial t} &= \frac{\partial}{\partial x}\left[a\Big(\int_0^t \int_0^1 \Big[\Big(\frac{\partial U}{\partial x}\Big)^2 + \Big(\frac{\partial V}{\partial x}\Big)^2\Big]\,dx\,d\tau\Big)\frac{\partial V}{\partial x}\right] + g(V). \end{aligned} \tag{3}$$

The asymptotic behavior as $t \to \infty$ of solution of initial-boundary value problems with different kind boundary conditions for the systems (3) as well as numerical solution of these problems are considered. Wider classes of nonlinearity is studied than already has been investigated. Theoretical results to numerical ones are compared.

On p-Typical Curves in Algebraic K-Theory

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The title of the talk is the title of a classic paper by S. Bloch from 1970s. In that pioneering paper, Bloch considered a certain deformation of the algebraic K-theory of an algebraic variety that on one hand, looked computable, and on the other hand, was related to the usual K-theory, or at least to its $pro - p$ completion. This later gave rise to alot of beautiful mathematics, including the de Rham-Witt complex of Deligne and Illusie that computes cristalline cohomology. What I want todo in the talk is to revisit the subject and look at it from the modern point of view - so that things become simpler and more natural, and the results are stronger.

Introducing Methods of Analysis Into Fuzzy Systems

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This work introduces methods of analysis into logics via the complexification of the system. Let X be a universe, F be a family of fuzzy sets in X. It is shown that X and F generate topological groups G and Γ, respectively, that are dual to each other. These structures then provide us with powerful mathematical tools of harmonic analysis to extend the fuzzy system to a wider system.

The new system enables us to prove the assumptions $0 \Rightarrow 0$, $0 \Rightarrow 1$, $1 \Rightarrow 1$ of *boolean* logic as well as the introduction of the notions *logical bases* and *logical dependencies*, and the *comparisons* of propositions.

Certain Classes of Meromorhic Functions with Respect to (j, k) Symmetric Points

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We introduce $\mathscr{M}\mathscr{S}_p^{j,k}(\alpha, \beta)$ and $\mathscr{M}\mathscr{C}_p^{j,k}(\alpha, \beta)$ the classes of meromorphic analytic functions subject to satisfying the respective analytic criterion

$$\alpha < \mathrm{Re}\left\{ -\frac{1}{p}\,\frac{zf'(z)}{f_{j,k}(z)} \right\} < \beta \quad \big(0 \le \alpha < 1 < \beta, \;\; f_{j,k}(z) \ne 0 \;\text{ in }\; \mathscr{U}^*\big)$$

and

$$\alpha < \mathrm{Re}\left\{ -\frac{1}{p}\,\frac{(zf'(z))'}{f'_{j,k}(z)} \right\} < \beta \quad \big(0 \le \alpha < 1 < \beta, \;\; f'_{j,k}(z) \ne 0 \;\text{ in }\; \mathscr{U}^*\big).$$

Relationship with other well known classes such as meromorphic convex and starlike functions have been established. Further, very interesting integral representations have also been obtained.

On the Full Transitivity of the Cotorsion Hull of a Separable p-Group

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The question of full transitivity of a cotorsion full of the abelian separable p-group T is studied. In particular, the cases are considered, where T is a direct sum of cyclic p-groups, a torsion complete group or a direct sum of torsion complete groups. Also, the case is discussed, where the cotorsion hull is not fully transitive and then the lower semilattices used to describe lattices of fully invariant subgroups of cotorsion hulls are considered.

About Existence of the Solution of the Problem of the First and Second Type of Ratios Information Technologies of the Parties in the Mathematical Model of Information Warfare

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In the proposed paper conditions for the existence of solutions of the first and second type of ratios of information technologies of the parties in the mathematical model of information warfare. The Mathematical model of information warfare in view of information technology the parties proposed in [1] and has the following form:

$$\begin{aligned} \frac{dx(t)}{dt} &= \alpha_1 x(t)\Big(1-\frac{x(t)}{I_1}\Big) - \beta_1 z(t),\\ \frac{dy(t)}{dt} &= \alpha_2 y(t)\Big(1-\frac{y(t)}{I_2}\Big) - \beta_2 z(t), \qquad (1)\\ \frac{dz(t)}{dt} &= \gamma(x(t)+y(t))\Big(1-\frac{z(t)}{I_3}\Big), \end{aligned}$$

$$x(0)=x_0, \quad y(0)=y_0, \quad z(0)=z_0. \qquad (2)$$

Where (2) the initial conditions. In (1), (2) $x(t)$, $y(t)$ is the amount of information at a time t, disseminate relevant antagonistic parties to achieve information superiority in information warfare under consideration, $z(t)$ the amount of information disseminated third-party in peace time t to call upon the antagonistic parties, complete misinformation, i.e. stop information warfare. In the model $x(t)$, $y(t)$, $z(t)$ function defined on the interval $[0,T]$; α_1, α_2 is a indexes of aggressiveness appropriate antagonistic parties, β_1, β_2 is the index of the peacekeeping readiness appropriate antagonistic parties, γ is the indexes of the peacekeeping activities of third parties. I_1, I_2, I_3 is a kind of "equilibrium" amount of information relevant part, determined by the level of development of their own IT, financial, or other unauthorized use of IT capabilities.

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The Martingale Method in the Theory of Random Fields

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The martingale method which is very efficient in the theory of random processes in multidimensional case (random fields) is not yet so productive. The standard explanation is: the notion of martingale is based essentially on the complete ordering property of the real line, while spatial structures do not possess this property. However recently published works shown that using the notion of martingale-difference random field (see [1]) one can significantly develop the martingale method in multidimensional case.

The martingale-difference random fields are interesting from the various points of view. For instance, basic limit theorems for sums of independent random variables are valid for such fields (see, for example, [1-5]). Moreover, since under some conditions on the potential the corresponding Gibbs random field is martingale-difference, one can apply the martingale method in the problems of mathematical statistical physics (see, for example, [6, 7]).

In our talk we present a new approach for construction of martingale-difference random fields which can be applied to the Gibbs random fields. Also we present a new approach for proving limit theorems for random fields by developing the martingale method. With help of this method we extend the range of validity of various limit theorems for Gibbs random fields. Particulary one can apply the martingale method to prove the central limit theorem for the Ising model out of the critical parameters.

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Solving a Model for Recovery of Myocardial Infarction Due Diabetes

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The numbers of diabetic patients are increased very fast in over the world. Diabetes causes myocardial infarction which it is very important inadequacy of heart. This model has been successfully used to explain for the recovery of myocardial infarction due diabetes. In the present paper, solving an ordinary differential equations (ODE) are used to find the number of individuals with myocardial infarction and recovery of that. We present and discuss graphical results for our solutions.

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Dual Multi 2-Normed Space

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The notion of multi-normed space was introduced by H. G. Dales and M. E. Polyakov in [1]. This concept is somewhat similar to operator sequence space and has some connections with operator spaces and Banach latices. Motivations for the study of multi-normed spaces and many examples are given in [1]. Let $(E, \|\cdot\|)$ be a complex normed space. We denote by E^k the linear space $E \oplus \cdots \oplus E$ consisting of k-tuples $(x_1, \ldots, x_k)$, where $x_1, \ldots, x_k \in E$. The linear operations on E^k are defined coordinatewise. We denote by $\mathfrak{S}_k$ the group of permutations on k symbols. A dual multi norm on $\{E^k\}_{k\in\mathbb{N}}$ is a sequence $(\|\cdot\|_k)$ such that $\|\cdot\|_k$ is a norm on E^k for each $k \in \mathbb{N}$, $\|x\|_1 = \|x\|$ for each $x \in E$ and the following conditions are satisfied:

(A_1) for each $\sigma \in \mathfrak{S}_k$ and $x \in E^k$, $\|A_\sigma(x)\|_k = \|x\|_k$;

(A_2) for each $\alpha_1, \ldots, \alpha_k \in \mathbb{C}$ and $x \in E^k$, $\|M_\alpha(x)\|_k \leq \max\limits_{1\leq i\leq k}|\alpha_i|\|x\|_k$;

(A_3) for each $x_1, \ldots, x_k \in E$, $\|(x_1, \ldots, x_k, 0)\|_{k+1} = \|(x_1, \ldots, x_k)\|_k$;

(A_4) for each $x_1, \ldots, x_k \in E$, we have $\|(x_1, \ldots, x_k, x_k)\|_{k+1} = \|(x_1, \ldots, 2x_k)\|_k$.

The concept of linear 2-normed spaces has been introduced by S. Gähler [2] in 1965 and has been developed extensively in different subjects by many mathematicians.

Let X be a linear space of dimension greater than 1 over the field $\Bbbk (= \mathbb{R}$ or $\mathbb{C})$. Suppose $\|\cdot,\cdot\|$ is a real-valued function on $X \times X$ satisfying the following conditions:

(a) $\|x, y\| \geq 0$ and $\|x, y\| = 0$ if and only if x and y are linearly dependent,

(b) $\|x, y\| = \|y, x\|$ for all $x, y \in X$,

(c) $\|\lambda x, y\| = |\lambda|\|x, y\|$ for all $\lambda \in \Bbbk$ and all $x, y \in X$,

(d) $\|x + y, z\| \leq \|x, z\| + \|y, z\|$ for all $x, y, z \in X$.

Then $\|\cdot,\cdot\|$ is called a 2-norm on X and $(X, \|\cdot,\cdot\|)$ is called a linear 2-normed space. Taking idea from these we are motivated to define concept of dual multi 2-normed space and describe some properties of these new ones.

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b-H^*-Algebras

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The concept of linear 2-normed spaces has been introduced by S. Gähler [3] in 1965 and has been developed extensively in different subjects by many mathematicians. A concept which is related to a 2-normed space is 2-inner product space which have been intensively studied by many mathematicians in the last three decades. A systematic study of this theory can be found in the book [2].

It is shown in [4] that for each $x, y, b \in X$, $\|x, y\| = \langle x, x|y\rangle^{\frac{1}{2}}$ is a 2-norm on X and the Cauchy-Schwarz inequality $|\langle x, y|b\rangle| \leq \|x, b\|\|y, b\|$ holds. A sequence $\{x_n\}_n$ in 2-inner product space X is b-Cauchy if

$$\forall \epsilon > 0, \ \ \exists N > 0, \ \ \forall m, n \geq N, \ \ 0 < \|x_m - x_n, b\| < \epsilon.$$

We call X is b-Hilbert if every b-Cauchy sequence, b-converges in the seminormed space $(X, \|\cdot, b\|)$.

An H^*-algebra, introduced by Ambrose [1] in the associative case, is a Banach algebra A, satisfying the following conditions:

(i) A is itself a Hilbert space under an inner product $\langle \cdot, \cdot \rangle$;

(ii) For each x in A there is an element x^* in A, the so-called adjoint of x, such that we have both $\langle xy, z\rangle = \langle y, x^*z\rangle$ and $\langle xy, z\rangle = \langle x, zy^*\rangle$ for all $y, z \in A$.

Since Ambrose [1] up to now, there are many mathematicians worked on H^*-algebras and developed it in several directions, see ([5]) and references cited therein.

Let $(A, \|\cdot, \cdot\|)$ be a 2-normed space and $b \in A$. We say that A is a 2-b-normed algebra, if there is a map $\cdot : A \times A \to A$ such that $a_1(a_2+a_3) = a_1a_2+a_1a_3$, $(a_1+a_2)a_3 = a_1a_3+a_2a_3$, $a_1(a_2a_3) = (a_1a_2)a_3$ and $\|(a_1, b)(a_2, b)\| \leq \|a_1, b\| \, \|a_2, b\|$ for all $a_1, a_2, a_3 \in A$. We say A is a b-Banach algebra if it is complete with respect to seminorm $\|\cdot, b\|$.

Taking idea from these we are motivated to define b-H^*-algebras and investigate some properties of these spaces. Furthermore we describe the notion of b-ideal and obtain some important results concerned this notion.

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About MLE on Infinite Dimensional Hilbert Space

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Let $\{\mathbf{E}, \Re, P_\theta, \theta \in \Theta\}$ be the statistical structure corresponding to a random element $X = X_\theta$. Here, $\mathbf{B}$ is a separable, real, reflexive Banach space, $\Re$ is the σ-algebra of Borel sets. $\Theta \subset \mathbf{B}$ is a compact subset os the separable real Banach space $\mathbf{B}$. We assume, that the regularity conditions 1-5 are fulfilled (see [1]).

Let $g(\theta) = E_\theta T(X)$, where $T : \mathbf{B} \to R$ is a measurable mapping (statistics).

Theorem (Cramer-Rao Inequality). $Var(T(X)) \geq \frac{(g'_v(\theta))^2}{E_\theta \rho_\theta^2(X;v)}$, *where* g'_v *is the derivative along of* v *and* $\rho_\theta(X;v)$ *is the logarithmical derivative by parameter of the family probability measures* P_θ*,* $\theta \in \Theta$ *along the vector* v*.*

Consider the equation

$$\sum_{k=1}^{n} \rho_\theta(x_k; v) = 0.$$

If exists the solution $\theta = \hat{\theta}_n$ of this equation, when $v \in \Theta$, than $\hat{\theta}_n$ we call MLE for θ.

The problems of consistency and asymptotical normality of the MLE are investigated.

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Modeling the within-host Dynamics of Fractional-Order HIV Infection

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In this paper, a class of mathematical HIV infection model with fractional order is proposed. First, we prove the proposed model possesses non-negative solutions. Then, we carry out a detailed analysis to study the stability of equilibrium points.

Model description

Motivated by Yongsheng and Haiping in [1], we consider the following fractional-order HIV model:

$$\begin{gathered} D^\alpha T = f(T) - k_1(1-n_c)VT - k_2TT^*, \quad D^\alpha T^* = k_1(1-n_c)VT + k_2TT^* - \delta T^*, \\ D^\alpha V = N\delta T^* - cV - k_3VT - k_4VT^*, \end{gathered} \tag{1}$$

where $T(t)$, $T^*(t)$ and $V(t)$ respectively represent the concentration of susceptible T-cells, productively infected T-cells and free viruses at time t. Also, s, d_T, p and $T_{\max}$ are respectively, T-cells source term, death rate of healthy T-cells, growth rate of T-cells and carrying capacity of T-cells. k_1, k_2, k_3 and k_4 are viral infecting rate, contact rate between uninfected T-cells and infected T-cells, rate of absorption of free virions into healthy cells and rate of absorption of free virions into infected cells, respectively. δ, c and N are death rate of infected T-cells, clearance rate of virus and virus particles released by a productively infected T-cell over its life time, respectively. Besides, the function $f(T)$ denotes the growth rate of uninfected T cells.

Theorem 1. *For each initial value greater than, there exists a unique solution to system* (1) *on* $t \geq 0$. *Furthermore, the solution will remain in* R^3_+ *and both* $T(t)$ *and* $T^*(t)$ *are bounded by* $T_{\max}$.

Theorem 2. *The infection-free equilibrium,* E_0, *is locally asymptotically stable if* $0 < \mathscr{R}_0 < 1$ *and unstable if* $\mathscr{R}_0 > 1$.

Proposition. *The infected steady state, E_1, is locally asymptotically stable if all the eigenvalues, λ, of $J(E_1)$ satisfy $|arg(\lambda)| > \frac{\alpha\pi}{2}$.*

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Invariant Subspaces of the Hardy Space Over the Polydisc

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We describe invariant subspaces of the Hardy space over the polydisc under the multiplication operators by the independent variables that are generated by an inner function in view of classical Lax-Helson-Halmos theorem.

Boundedness Criteria for Harmonic Analysis Basic Integral Operators in Weighted Iwaniec-Sbordone Spaces

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The goal of our talk is to discuss the mapping properties of integral operators in some non-standard Banach function spaces, in particular, in weighted grand Lebesgue spaces. These spaces were introduced by T. Iwaniec and C. Sbordone in 1992. These spaces are non-reflexive, non-separable and non-rearrangement invariant. On the other hand, they are well fit to the study of a wide range of various problems of non-linear partial differential equations related to existence, uniqueness and regularity problems. Along of generalized grand Lebesgue spaces we introduce also their weak variants. We give the weak and strong estimates criteria for conjugate functions,Cauchy singular integrals defined on rectifiable curves, fractional maximal functions, potentials (including similar integral transforms with product kernels). We treat also the better bounds for the norms of the above-mentioned operators.

The talk is based on the paper [1].

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Values of Cantor-Like Analytic Functions at Rational Points

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Consider a Cantor-like analytic function of the form

$$f(z) = \sum_{h\geq 0} \frac{a_h z^h}{b_0 b_1 \cdots b_h}$$

with rational coefficients. The nature of such a function evaluated at rational points is investigated. First, using the Thue-Siegel-Roth theorem ([3, Chapter III]), the transcendence of its values evaluated at rational points is derived subject to certain conditions on the growth of the coefficients. Second, using a technique of Mahler ([2]), it is shown that the values of a lacunary Cantor-like analytic function evaluated at rational points are algebraic if and only if its corresponding partial sums vanish. These results complement those in [1].

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Recovering a Field from a One-Dimensional Structure

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Zilber conjectured in 1980's [4] that each combinatorial geometry coming from the "algebraic closure operator in a strongly minimal structure" should (more or less) coincide with one of the following:

- inequality in a pure set,
- linear dependence in a vector field,
- algebraic independence in an algebraically closed field.

This conjecture was refuted by Hrushovski [2], however it holds for many interesting classes of structures. A general feeling is that Zilber's conjecture should hold in a "geometric context". I will give a brief historical account on Zilber's conjecture and its applications. Then I will focus on my joint work with Assaf Hasson [1] and Serge Randriambololona [3] where the "geometric context" is provided either by a real closed field [1] or a valued field [3].

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Key Exchange Protocols Over Group Rings

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Units in a group ring RG are known as invertible elements with respect to the multiplication. Determination of units in a given group ring is a hard problem in the theory. In this paper, we first give some constructions of key exchange protocols over the unit groups of group rings. Then, we give some concrete examples.

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Automorphisms of Direct Limits of Symmetric Groups

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The structure and the cardinality of the automorphism group of algebraic structures are of interest for algebraists. In particular given a group G of cardinality κ. What can we say about the structure and the cardinality of $Aut(G)$? For locally finite simple groups; is it true that $|Aut(G)| = 2^{|G|}$? This question is stated for countably infinite simple locally finite groups by Kegel and Wehfritz in [1, Question VI.6].

Homogenous symmetric groups are typical examples of simple, countably infinite locally finite groups. These groups are studied by Kurosko-Sushchansky in [3] and the automorphism groups are studied by Lavrenyuk-Sushchansky in [4]. Among other things Lavrenyuk and Sushchansky proved that the automorphism group of countable homogenous symmetric group is uncountable.

By using the notion of strictly diagonal embedding, for any infinite cardinality κ one can construct uncountably many simple locally finite groups of cardinality κ as a direct limit of finitary symmetric groups. Such groups are called homogenous finitary symmetric groups and basic properties of homogenous finitary symmetric groups are studied in [2].

We will use similar technique as in [4] to show that the cardinality of the automorphism groups of homogenous finitary symmetric groups of cardinality κ has the automorphism group of cardinality 2^{κ}.

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Unconditional Convergence of Random Series

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We plan to discuss the different notions of almost sure unconditional convergence of random series.

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Discriminative Properties of Generalized Möbius Functions

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The distributive property of completely multiplicative function discovered by Lambek ([2]) asserts that an arithmetic function f is completely multiplicative if and only if $f(g*h) = fg*fh$ for every pair of arithmetic functions g, h. For arithmetic functions g, h, their product $k := g*h$ is said to be

- discriminative if the relation $k(n) = g(1)h(n) + g(n)h(1)$ holds only when n is prime;

- partially discriminative if for every prime power p^j, the equation $k(p^j) = g(1)h(p^j) + g(p^j)h(1)$ implies that $j = 1$;
- semi-discriminative if the relation $k(n) = g(1)h(n)+g(n)h(1)$ holds only when $n = 1$ or n is a prime.

Langford ([3]) showed that instead of verifying all products of two arithmetic functions, one only needs to check those products that are discriminative, or partially discriminative, while Haukkanen ([1]) showed this for semi-discriminative products. Restricting one of the two arithmetic functions to be a generalized Möbius function ([5]), conditions ensuring their product to be discrimative or partially discriminative or semi-discriminative have been derived in [4]. Complementing the work in [4], here both of the two arithmetic functions are taken to be generalized Möbius functions with real parameters. Conditions on the parameters guaranteeing their product to be discrimative or partially discriminative or semi-discriminative are derived.

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On the Chromatic Number of Replacement Product Graphs

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The edge chromatic number, $\chi'(G)$ of G is the minimum number of colors required to color the edges of G in such a way that no two adjacent edges have the same color.

Let G be an (n,k)-graph (n vertices and k-regular) and let H be a (k,k')-graph with $V(H)=[k]=\{1,\dots,k\}$ and fix a random numbering φ_G of G. The replacement product $G\circledR_{\varphi_G}H$ is the graph whose vertex set is $V(G)\times V(H)$ and there is an edge between vertices (v,k) and (w,l) whenever $v=w$ and $kl\in E(H)$ or $vw\in E(G)$, $\varphi_G^v(w)=k$ and $\varphi_G^w(v)=l$ or kth edge incident on vertex v in G is connected to the vertex w and this edge is the lth edge incident on w in G, where the numberings k and l refers to the random numberings of edges adjacent to any vertex of G.

$G\circledR_{\varphi_G}H$ is a regular graph and in fact it is a $(nk,k'+1)$-graph. In this work, we describe necessary conditions on graphs G and H, that $\chi'(G\circledR H)=k'+1$.

On Some Properties of Complete Semigroups of Binary Relations Defined by Semilattices of the Class $\Sigma_1(X,10)$

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Let X be an arbitrary nonempty set, D be an X-semilattice of unions, i.e. such a nonempty set of subsets of the set X that is closed with respect to the set-theoretic operations of unification of elements from D, f be an arbitrary mapping of the set X in the set D. To each such a mapping of f we put into correspondence a binary relation a_f on the set X that satisfies the condition $a_f=\bigcup\limits_{x\varepsilon X}(\{x\}\times f(x))$. The set of all such a_f $(f:X\to D)$ is denoted by $B_x(D)$.

In this work we have identified all XI-subsemilattices of D and seen that the maximal subgroup $G_X(D,\varepsilon)$ of $B_X(D)$ has order one or two and that the set of all regular elements of $B_X(D)$ is a subgroup. Here D is shown in Figure.

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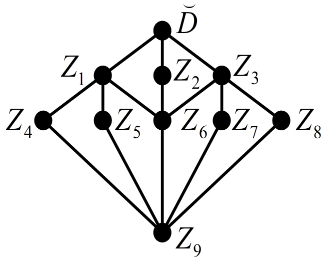

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Necessary and Sufficient Conditions for the Solvability of Inverse Problem for a Class of Dirac Operators

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In this work, inverse problem of recovering the coefficient of Dirac operator is studied from the sequences of eigenvalues and normalizing numbers. The theorem on necessary and sufficient conditions for the solvability of this inverse problem is proved and solution algorithm of inverse problem is given.

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Stochastic Differential Equations in Banach Space, Generalized Solutions end the Problem of Decomposability

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Stochastic differential equations in Banach space is very interesting field of mathematics. Special interest to this direction is observed after the eighties of the last century. The monograph of K. Ito (see [1]) is devoted to the theory of stochastic differential equations in infinite dimensional spaces. However after, the activity was relatively decreased by the reason that the traditional methods of developing the theory does not work in arbitrary Banach space. Only in the special class of Banach spaces, so called in UMD Banach spaces, some results were obtained (see [2]).

In developing of stochastic differential equations in Banach space the main problems are the construction of the stochastic integral in an arbitrary Banach space and estimation of the sequence of the successive approximation. Using traditional methods, only in UMD Banach spaces it is possible to solve partially these problems.

According to our approach we introduce generalized stochastic integral for a wide class of predictable random functions as a generalized random element (linear random function) and if this generalized random function is decomposable by the Banach space valued random element, then we say that the stochastic integral exists.

For the main stochastic differential equation we introduce the corresponding generalized stochastic differential equation. It is possible to solve this equation by traditional methods end we receive the generalized stochastic process as a solution. If there exists the Banach space valued random process corresponding to this generalized random process, it will be solution of the main stochastic differential equation. Using this approach we consider the question of the existence and uniqueness of the solution [3] and receive the solutions of the linear stochastic differential equations in Banach space.

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A Modification of Differential Transform Method for Solving System of Fractional Partial Differential Equations

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Differential equations of arbitrary order are capable of describing memory and hereditary properties of certain important materials and processes. So it is important to find efficient methods for solving FDEs. To illustrate the basic idea of the method of this paper, we consider the system of fractional partial differential equations with the initial conditions given by

$$D_t^{\alpha_i} u_i(\overline{x},t) = N_i\big(u_1(\overline{x},t),\dots,u_n(\overline{x},t)\big), \quad m_i-1<\alpha_i\le m_i, \quad i=1,2,\dots,n, \tag{1}$$

$$\frac{\partial^{k_i}}{\partial t^{k_i}} u_i(\overline{x},0) = g_{ik_i}(\overline{x}), \quad k_i=0,1,\dots,m_i-1, \quad m_i\in N, \tag{2}$$

where N_i are nonlinear operators and $u_i(\overline{x},t)$ are unknown functions. The analytic function $u_i(\overline{x},t)$ is expanded in terms of a power series in the form

$$u_i(\overline{x},t) = \sum_{k=0}^{\infty} U_i(k) t^{\frac{k}{\theta_i}}, \tag{3}$$

where θ_i should be chosen such that $\alpha_i\theta_i$ is a positive integer. Also, the standard ADM yields the solution $u_i(\overline{x},t)$ by the series

$$u_i(\overline{x},t) = \sum_{k=0}^{\infty} u_{ik}(\overline{x},t),$$

and the nonlinear term $N_i(u_1(\overline{x},t),\dots,u_n(\overline{x},t))$ is approximated by

$$N_i\big(u_1(\overline{x},t),\dots,u_n(\overline{x},t)\big) = \sum_{k=0}^{\infty} A_{ik},$$

where the $A_{ik}(u_{10}, u_{11}, \dots, u_{1k}, u_{20}, u_{21}, \dots, u_{2k}, \dots, u_{n0}, u_{n1}, \dots, u_{nk})$ are the Adomian polynomials, can be constructed using the general formula

$$A_{ik} = \frac{1}{k!}\,\frac{d^k}{d\lambda^k}\left[N_i\Big(\sum_{k=0}^{\infty} u_{1k}\lambda^k, \dots, \sum_{k=0}^{\infty} u_{nk}\lambda^k\Big)\right]_{\lambda=0} = \\ = \left[\frac{1}{k!}\,\frac{d^k}{d\lambda^k}\,N_i\Big(\sum_{k=0}^{\infty} u_{1k}\lambda^k, \dots, \sum_{k=0}^{\infty} u_{nk}\lambda^k\Big)\right]_{\lambda=0}. \tag{4}$$

Now, considering (4), (3) and definition of $U_i(k)$ we deduce that kth differential transform component of $N_i(u_1(\overline{x}, t), \dots, u_n(\overline{x}, t))$, $\widetilde{A}_{ik}$, can be obtained from the corresponding Adomian polynomial, A_{ik}, by replacing each u_{ik} with $U_i(k)$. Therefore, taking the differential transform from both side of (1), we have the following system of algebraic equations

$$\frac{\Gamma(\alpha_i + 1 + \frac{k}{\theta_i})}{\Gamma(1 + \frac{k}{\theta_i})}\,U_i(k + \alpha_i\theta_i) = \widetilde{A}_{ik}, \quad i = 1, 2, \dots, n.$$

On a Problem of Optimal Control Described by a System of Hyperbolic Integro-Differential Equations

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Let's find a minimum of the functional

$$S(u) = \varphi(z(t_1, x_1)) \tag{1}$$

under the constraints

$$u(t,x)) \in U \subset R^r, \quad (t,x) \in D = T \times X = [t_0, t_1] \times [x_0, x_1], \tag{2}$$

$$z_{tx}(t,x) = f(t, x, z(t,x), z_\tau(t,x), z_s(t,x), u(t,x)) + \\ + \int\limits_{t_0}^{t} \int\limits_{x=x_0}^{x} K(t, x, \tau, s, z(\tau, s), z_\tau(\tau, s), z_s(\tau, s), u(\tau, s))\,ds\,d\tau, \quad (t,x) \in D, \tag{3}$$

$$z(t_0, x) = a(x), \quad x \in X, \quad z(t, x_0) = b(t), \quad t \in T, \\ a(x_0) = b(t_0). \tag{4}$$

It is assumed that the data of the problem (1)-(4) satisfy usual smoothness requirements as in [1, 2]).

By applying the approach developed in [1, 2] under the condition that the domain of control is open, necessary conditions are found for the optimality of first and second order of the problem under consideration.

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Necessary Conditions for Optimality of First and Second Order in a Problem of Optimal Control by Integro-Differential Equations Under Functional Constraints

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In the report we consider a problem of minimization of the functional

$$S_0(u) = \varphi_0(x(T_1), \dots, x(T_k)) \tag{1}$$

under the following constraints

$$\begin{gathered} u(t,x)) \in U \subset R^r, \quad t \in [t_0, t_1], \qquad (2) \\ S_i(u) = \varphi_i(x(T_1), \dots, x(T_k)) \le 0, \quad i = \overline{1,p}\,, \\ S_i(u) = \varphi_i(x(T_1), \dots, x(T_k)) = 0, \quad i = \overline{p+1,q}\,, \\ \dot{x}(t) = f(t, x(t), u(t)) + \int\limits_{t_0}^{t} K(t, \tau, x(\tau), u(\tau)) d\tau, \quad t \in [t_0, t_1], \end{gathered}$$

$$x(t_0) = x_0.$$

Here $T_i \in [t_0, t_1]$ are given points, $u(t)$ is an r-dimensional vector of control impact, $x(t)$ is an n-vector of phase variables nad U is a given non-empty bounded and open set.

Some necessary conditions of first and second order optimality are found without so called normality constraints [1-3].

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Structure of Diophantine Graphs

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In this paper, we want to construct a graph for distributing n identical objects in k distinct boxes. First, arrange the k boxes in a row so that the numbers on the boxes are increasing order. Now, each distributions of n identical objects in k distinct boxes can be show by a n-sequence $a_1a_2, \dots, a_n$ such that, $1 \le a_1 \le a_2 \le \dots \le a_n \le k$. Conversely, each arrangment of n-sequence $a_1a_2, \dots, a_n$ in nondecreasing order so that they satisfy the chain of inequalities $1 \le a_1 \le a_2 \le \dots \le a_n \le k$ corresponds a distribution of n-identical objects in the k distinct boxes. For examples, let $n = 5$ and $k = 3$. Then 11223 is a 5-sequence corresponding to a distributions in which, 2 objects is in the box1,2 objects in the box2 and one object in the box3. Now, we construct a graph whose vertex set is the set of such n-sequences and two vertices are adgacent if they are different in exactly one component. We call this graph by Diophantine Graph D_k^n.

Since, there is a 1-1 correspondence between each nonnegative integer solutions of the linear (Diophantine) equation $x_1 + x_2 + \dots + x_k = n$ and different ways of distributions of n identical objects into k distinct boxes [1]. Thus, we treat by the solutions of this equation. Hence, the number of vertices of diophantine graph $D_k^n = (V_k^n, E_k^n)$ is equale to $|V_k^n| = \binom{n+k-1}{k-1}$. We show that degree of vertex $A = a_1 a_2 \cdots a_n$ depends only the first component a_1 and the last component a_n and is equale to

$$deg_{D_k^n}(A) = (a_n - a_1) + (k - 1).$$

Also, we show that the number of edges of Diophantine graphs are equale to

$$|E_k^n| = \begin{cases} \binom{k}{2}, & \text{if } n = 1, \\ \dfrac{(k-1)k(k+1)}{3}, & \text{if } n = 2, \\ \dfrac{1}{2}\sum\limits_{i=1}^{k-1}\sum\limits_{j=1}^{k-i}\binom{n+j-2}{j} j + \dfrac{1}{2}\binom{n+k-1}{k-1}(k-1), & \text{if } n \geq 3. \end{cases}$$

At last we show that The Diophantine graphs D_k^n are connected graphs and has Hamming distance properties and are k-colorable for $n \geq 1$ and $k \geq 2$.

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Two-Weight Criteria for Riesz Potentials on Cones of Radially Decreasing Functions

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We establish necessary and sufficient conditions on a weight pair (v, w) governing the boundedness of the Riesz potential operator

$$(I_\alpha f)(x) = \int\limits_G f(y)\big(r(xy^{-1})\big)^{\alpha-Q}\, dy, \;\; 0 < \alpha < Q, \;\; x \in G,$$

defined on a homogeneous group G from $L^p_{dec,r}(w,G)$ to $L^q(v,G)$, where r is homogeneous norm on G, Q is homogeneous dimension of G and $L^p_{dec,r}(w,G)$ denotes the Lebesgue space defined for non-negative radially decreasing functions on G. The same problem is also studied for the potential operator with product kernels

$$(I_{\alpha,\beta}f)(x,y)=\int\limits_{G_1}\int\limits_{G_2} f(t,\tau)\big(r_1(xt^{-1})\big)^{\alpha-Q_1}\big(r_2(y\tau^{-1})\big)^{\beta-Q_2}\,dt\,d\tau,$$

where $(x,y)\in G_1\times G_2$, $0<\alpha<Q_1$, $0<\beta<Q_2$, defined on a product of two homogeneous groups $G_1\times G_2$. In the latter case weights, in general, are not of product type.

The derived results are new even for Euclidean spaces.

Generalized Biharmonic Hypersurfaces

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A submanifold M of a Euclidean m-space E^m is said to be biharmonic if $\Delta H=0$ holds identically, where H is the mean curvature and Δ is the Laplacian on M. In 1991, B. Y. Chen conjectured in [2] that every biharmonic submanifold of a Euclidean space is minimal. The study of biharmonic submanifolds is nowadays a very active research subject. In particular, since 2000 biharmonic submanifolds have been receiving a growing attention and have become a popular subject of study with many progresses (see, for example, [1] and [3]).

The Laplace operator Δ can be seen as the first one of a sequence of n operators $L_0=\Delta, L_1,\dots,L_{n-1}$, where L_r stands for the linearized operator of the first variation of the $(r+1)$th mean curvature arising from normal variations of the hypersurface (see [4]).

Therefore, from this point of view, it seems natural and interesting to generalize the definition of biharmonic hypersurface by replacing L_r instead of Δ and study the properties of such hypersurfaces. We call these hypersurfeces L_r-biharmonic hypersurfaces. Here we present some results concerning L_r-biharmonicity implies r-minimality.

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On Some Categorical Aspects of Zero-Divisor Graphs

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A category $\mathscr{C}$ is abelian if satisfies in the following four axioms.

A1. $\mathscr{C}$ is preadditive.

A2. Every finite family of objects has a product (and coproduct).

A3. Every morphism has a kernel and a cokernel.

A4. $\overline{\alpha} : Coker(ker\alpha) \to Ker(coker\alpha)$ is an isomorphism for every morphism α.

The axiom A4 can be replaced by the following axiom:

A4′. Every morphism α has a factorization $\alpha = \gamma \circ \beta$, where β is a cokernel and is a kernel.

In this talk we will consider some categorical aspects of zero-divisor graphs in abelian categories.

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Fixed Point Theory for Cyclic Mappings

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In this paper fixed point results are presented for cyclic generalized weak φ-contraction mappings on complete metric spaces (X,d). Our results extend previous results given by Moradi and Khojasteh [1] and Karapınar [2].

Throughout the manuscript, (X,d) denotes a complete metric space. We introduce the notation Φ for the set of all mappings $\varphi:[0,+\infty)\to[0,+\infty)$ with $\varphi^{-1}(0)=\{0\}$, $\varphi(t)<t$ for all $t>0$ that satisfy the condition: $\varphi(t_n)\to 0$ implies $t_n\to 0$. For example, every nondecreasing mapping $\varphi:[0,+\infty)\to[0,+\infty)$ with $\varphi^{-1}(0)=\{0\}$, $\varphi(t)<t$ for all $t>0$ belong to Φ. Also, every lover semi-continuous (l.s.c.) mapping $\varphi:[0,+\infty)\to[0,+\infty)$ with $\varphi^{-1}(0)=\{0\}$, $\varphi(t)<t$ for all $t>0$ and $\liminf\limits_{t\to\infty}\varphi(t)>0$ belong to Φ.

Definition. Let (X,d) be a metric space, m a positive integer, $A_1,A_2,\dots,A_m$ closed non-empty subsets of X and $Y=\bigcup\limits_{i=1}^{m}A_i$. An operator $T:Y\to Y$ is called a cyclic generalized weak φ-contraction if

(i) $\bigcup\limits_{i=1}^{m}A_i$ is a cyclic representation of Y with respect to T,

(ii) there exist a function $\varphi\in\Phi$ such that

$$d(Tx,Ty)\le N(x,y)-\varphi(N(x,y))$$

for any $x\in A_i, y\in A_{i+1}, i=1,2,\dots,m$, where $A_{m+1}=A_1$, and where

$$N(x,y)=\max\left\{d(x,y),d(x,Tx),d(y,Ty),\frac{d(x,Ty)+d(y,Tx)}{2}\right\}.$$

The following theorem is the main result of this paper.

Theorem. *Let (X,d) be a complete metric space, $m\in N$, $A_1,A_2,\dots,A_m$ closed non-empty subsets of X and $Y=\bigcup\limits_{i=1}^{m}A_i$. Let $\varphi\in\Phi$ and $T:Y\to Y$ be a cyclic generalized weak ϕ-contractive mapping. Then, T has a unique fixed point $z\in\bigcap\limits_{i=1}^{m}A_i$.*

Corollary. *Let (X,d) be a complete metric space, and let $T:X\to X$ be a mapping such that for all $x,y\in X$,*

$$d(Tx,Ty)\le\psi(N(x,y)),$$

where $\psi:[0,+\infty)\to[0,+\infty)$ is u.s.c. with $\psi(t)<t$ for all $t>0$ and $\liminf\limits_{t\to\infty}(t-\psi(t))>0$. Then T has a unique fixed point.

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Generalized Weakly Contraction and its Applications

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Fixed point theory has a wide potential application not only in the branches of mathematics, but also in the several disciplines, such as, economics, computer science, biology. This paper extends the results in [1] and [2].

Let Φ denote the set of all mappings $\varphi : R^+ \to R^+$ verifying $\varphi(t_n) \to 0$ implies $t_n \to 0$.

In this paper we consider a special case of the following value problem

$$\begin{cases} u'(t) = f(t, u(t)) & \text{if } t \in [0, T], \\ u(0) = u(T) + \zeta_0, \end{cases} \tag{1}$$

where $T > 0$, $f : [0, T] \times R \longrightarrow R$ is a continuous map and ζ_0 is constant.

A lower solution for (1) is a function $\alpha \in C^1([0, T])$ such that $\alpha'(t) \le f(t, \alpha(t))$ for $t \in [0, T]$ and $\alpha(0) \le \alpha(T) + \zeta_0$.

Theorem (main theorem). *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a metric d in X such that (X, d) is a complete metric space. Let $f : X \to X$ be a non-decreasing mapping such that there exists an element $x_0 \in X$ with $x_0 \preceq f(x_0)$. Suppose that there exists $\varphi \in \Phi$ such that*

$$d(f(x), f(y)) \le d(x, y) - \varphi(d(x, y)),$$

for each $x, y \in X$ with $x \preceq y$ (i.e., weak-φ-contraction). Suppose also that either f is continuous, or, for every non-decreasing sequence $\{x_n\}$ if $x_n \to x$ then $x_n \preceq x$ for all $n \in N$. Then f has a fixed point. Moreover, if for each $x, y \in F(f)$ there exists $z \in X$ which is comparable to x and y, then the fixed point of f is unique.

Corollary 1. *Consider problem* (1) *with* f *is continuous. Suppose that there exists* $\lambda > 0$ *such that for* $x, y \in R$ *with* $y \geq x$

$$0 \leq f(t,y) + \lambda y - [f(t,x) + \lambda x] \leq \lambda[(y-x) - \varphi(y-x)],$$

where $\varphi \in \Phi$ *and* $t \mapsto t - \varphi(t)$ *is non-decreasing. Then the existence of a lower solution for* (1) *provides the existence of a unique solution of* (1).

Corollary 2. *Let* $a_0, a_1, \dots, a_{k-1} \in [0, +\infty)$ *be such that* $a_1 + a_2 + \cdots + a_{k-1} < 1$ *and* $a_0 \geq 1$. *Then, the following equation has a unique solution on* $[\sqrt[k]{a_0}, +\infty)$

$$y^k = a_{k-1}y^{k-1} + a_{k-2}y^{k-2} + \cdots + a_1 y + a_0.$$

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Stability and Hopf Bifurcation Analysis of a Discontinuous HTLV-1 Model

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Developing accurate mathematical models for host immune response in immunosuppressive diseases such as HIV and HTLV-1 are essential to achieve an optimal drug therapy regime. Since for HTLV-1 specific CTL response typically occurs after a time lag, we consider a discontinuous response function to better describe this lagged response during the early stage of the infection. Thus the system of HTLV-1 model will be a discontinuous system. For analyzing the dynamic of the system we use Filippov theory and find conditions in which the Filippov system undergoes Hopf bifurcation. Hopf bifurcation help us to find stable and unstable periodic oscillations. Also Hopf bifurcation can be use to predict whether the CTL response can return to a steady state condition. Finally we use numerical simulations to demonstrate the results by an example.

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Hadamard Majorization and its Linear Preservers on $\mathbf{M}_n$

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Let $A, B \in \mathbf{M}_n$, it is said that B is Hadamard majorized by A (written $B \prec_h A$) if there exists an $n \times n$ doubly stochastic matrix D such that $B = D \circ A$ where $\circ$ is the Hadamard (entry wise) product. In this paper we investigate some properties of Hadamard majorization on $\mathbf{M}_n$ and also we find some possible structures of linear functions preserving $\prec_h$ on $\mathbf{M}_n$.

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Free Algebras with Hyperidentities of Lattice Varieties

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A Boolean quasilattice is an algebra with hyperidentities of the variety of Boolean algebras (lattices). A De Morgan quasilattice is an algebra satisfying hyperidentities of the variety of De Morgan algebras (lattices). In this talk we give:

1) a functional representation of the free n-generated Boolean quasilattice with two binary, one unary and two nullary operations;
2) a functional representation of the free n-generated De Morgan quasilattice with two binary and one unary operations.

On Estimation of Integral Functionals of Density

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Estimation of a nonlinear integral functional of probability distribution density and its derivatives is studied. The truncated plug-in-estimator is taken for the estimation. The integrand function can be unlimited, but It cannot exceed polynomial growth. Consistency of the estimator is proved and the convergence order is established. A version of the central limit theorem is proved. As an example an extended Fisher information integral and generalized Shannon's entropy functional are considered.

Higher Order Derivative on Meromorphic Functions in Terms of Subordination

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This paper investigates sharp coefficient bounds, integral representation, extreme point, and operator properties of a certain class associated with functions which are meromorphic in the punctured unit disk.

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A Multicenter and Bidirectional Nearest Neighbour Algorithm for Traveling Salesman Problem

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The traveling salesman problem (TSP) is a well-known NP-hard problem which aims to find the shortest (least expensive) tour that visits each city (node) in a given list exactly once and then returns to the starting city. In this paper, we propose a new heuristic algorithm using the algorithms given in [2, 3]. Computational experiments show that the proposed algorithm is efficient.

A well known algorithm, developed to solve TSP, is the Nearest Neighbour (NN) algorithm. The main idea of this algorithm is to always visit the nearest city [1]. But the Nearest Neighbour Algorithm (NND) from Both End-Points we add a vertex into the tour such that this vertex has not been visited before and it is the nearest vertex to the chosen two end vertices. We update the end vertices. In both algorithms (NN and NND algorithm) mentioned above, the goal is to construct a tour using the least-weighted edges starting from any vertex. The disadvantage of these algorithms is that the largest-weighted edges remain to choose at the end. Considering this disadvantage, NND algorithm begins with a vertex whose distance from other vertices is the largest and the least-weighted edges incident to this vertex are used. Then the number of vertices with the largest distance which will be used is increased. The number of vertices with the largest distance is n/k, where $k > 0$. By changing parameter k, one can improve the computational results. The steps of the proposed algorithm are as follows:

Step 1. Find the sum of each row in the adjacency matrix and then calculate the average of the sums.

Step 2. If the number of the vertices which is larger than the average is less than n/k then, add these vertices into S, otherwise add n/k vertices which have the largest row sum into S.

Step 3. Find the least-weighted edge incident to vertices in S which does not create a subtour, and add it into the solution.

Step 4. Update S according to the edge which is added, Step 5. Repeat Step 3 and Step 4 until a tour is constructed.

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A Mathematical Model of the Multicriteria Limited Bin Packing Problem with Fuzzy Qualities

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This paper introduces a mathematical model of “The Limited Bin Packing Problem with Fuzzy Qualities” which is a new packing problem, where given a set of objects (x) which joined into different groups ($l \in L$) according to particular features, characterized by volume (w) and profit (p) and a set of bins (y) with given volume (W) which create constraits based on groups of objects and has its own constraint of upper bound of total volume of bins (V), and cost (c). The bins are seperated into types with different upper availability limits (U_t). Part of the objects, which denoted compulsory (C), must be loaded, while a selection has to be made among the non-compulsory (NC) objects. The objective is determination to compute as difference between the total cost of the used bins and the total profit of loaded objects which are non-compulsory.

The quality of assignment is calcualted by degrees. Here, $K_1(S_j)$ is the degree of consistency of separation of elements of the bin S_j from the elements that are outside the bin S_j with regard for the information on their mutual attachment, $K_2(S_j)$ is the degree of mutual compatibility of elements contained in one bin S_j, $K_3(S_j)$ is the degree of consistency of separation of elements from the bin S_j, and $K_4(S_j)$ is the degree of mutual compatibility of elements in the bin S_j and this bin. These degrees are calculated from the fuzzy relations between the objects and the bins.

The mathematical model of the proposed problem is formulated as follows.

$$\max\Big\{A = \min_{j|y_j>0,\, j\in J}\Big\{\min\{K_1(S_j), K_2(S_j), K_3(S_j), K_4(S_j)\}\Big\}\Big\},$$

$$\min\Big\{\sum_{j\in J} c_j y_j - \sum_{j\in J}\sum_{l\in L}\sum_{i\in I^{NC}} p_i x_{ijl}\Big\},$$

$$\sum_{i\in I} w_i x_{ijl} \le W_{lj},\ \ j\in J,\ \ l\in L;$$

$$\sum_{j\in J}\sum_{l\in L} y_j w_{jl} \le V, \quad \sum_{j\in J} x_{ijl} = 1,\ \ i\in I^C,\ \ l\in L;$$

$$\sum_{j\in J} x_{ijl} \le 1,\ \ i\in I^{NC},\ \ l\in L,$$

$$\sum_{j\in J:\,\sigma(j)} y_j \le U_t,\ \ t\in T,$$

$$y_j\in\{0,1\},\ \ j\in J, \quad x_{ijl}\in\{0,1\},\ \ i\in I,\ \ j\in J,\ \ l\in L.$$

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On the Location of Roots of Domination Polynomials

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Let G be a graph of order n. A dominating set of G is a subset of vertices of G, say S, such that every vertex in $V(G)\setminus S$ is adjacent to at least one vertex of S. The domination polynomial of G is the polynomial $D(G,x)=\sum\limits_{i=1}^{n} d(G,i)x^i$, where $d(G,i)$ is the number of dominating sets of G of size i. A root of $D(G,x)$ is called a domination root of G. In this paper we investigate about location of the domination roots of graphs. Let $\delta=\delta(G)$, the minimum degree of vertices of G. We prove that every root of $D(G,x)$ lies in the set $\{z:\ |z+1|\leq \sqrt[\delta+1]{2^n-1}\}$. We show that $D(G,x)$ has at least $\delta-1$ non-real roots. In particular, we prove that if all roots of $D(G,x)$ are real, then $\delta=1$. We construct an infinite family of graphs such that all roots of their polynomials are real. There is a conjecture states that every integer root of $D(G,x)$ is -2 or 0. We obtain some results about this conjecture. We prove that if $\delta\geq\frac{2n}{3}-1$, then every integer root of $D(G,x)$ is -2 or 0. Also we prove that the conjecture is valid for trees and unicyclic graphs. We characterize all graphs that their domination roots are integer. Finally we obtain a recursive formula for computing the domination polynomials of trees. As a consequence we find a infinite families of trees T so that $D(T,x)$ has no root in the interval $(-2,-1)$.

A Mixed Problem of Plane Elasticity Theory for Multi-Connected Domain with Partially Unknown Boundary

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The paper addresses a problem of plane elasticity theory for a multi-connected domain, which external boundary is rhombus boundary; the internal boundary is required two holes that are symmetric with respect to its diagonals. Absolutely rigid punches with rectilinear bases are applied to each segment of outer boundary of given body and they are under the action of the forces P that applies to their middle points. There is no friction between the surface of given elastic body and punches. Uniformly distributed normal stress Q be applied to the holes boundary. Tangential stresses are equal to zero along the entire boundary of the domain and the normal displacements on the linear parts of the boundary are constant. The shape of the holes contour and the stress state of the given body are determined, provided that the tangential normal stress arising at the holes contour takes a constant value. Such holes are called full-strength holes. Full-strength contours and stress state are found by means of complex analysis. The solution is written in quadratures. The solvability of these problems provides controlling stress optimal distribution selecting the appropriate hole boundary.

The Riemann-Hilbert Problem in Smirnov Class with a Variable Exponent and an Arbitrary Power Weight

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The report considers the Riemann-Hilbert problem $\operatorname{Re}\left[(a(t)+ib(t))\phi^{+}(t)\right]=c(t)$ in the weighted Smirnov class $E^{p(t)}(D;\omega)$ with a variable exponent. The domain D is bounded by a piecewise smooth curve possessing one angular point A, at which the size of interior with respect to D angle is equal to $\nu\pi$, $0 < \nu \leq 2$ and the weight ω is an arbitrary power function of type $\omega(z) = (z - A)^\beta$, $\beta \in \mathbf{R}$.

Depending on values of numbers ν, β, $p(A)$ and functions $a(t)$, $b(t)$, there arise different situations for solvability of the problem. All possible cases are investigated. The necessary and sufficient conditions of solvability are pointed out and all solutions (when they exist) are constructed.

Normal Edge-Transitive Cayley Graphs on Non-Abelian Groups of Order $4p^2$, where p is a Prime Number

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Let G be a finite group and S be an inverse closed subset of G, i.e., $S = S^{-1}$, such that $1 \notin S$, the Cayley graph $\Gamma = Cay(G, S)$ on G with respect to S is a graph with vertex set G and edge set $\{\{g, sg\} \mid \ g \in G,\ s \in S\}$. Γ is connected if and only if $G = \langle S \rangle$. For $g \in G$, define the mapping $\rho_g : G \to G$ by $\rho_g(x) = xg$, $x \in G$. $\rho_g \in Aut(\Gamma)$ for every $g \in G$, thus $R(G) = \{\rho_g \mid \ g \in G\}$ is a regular subgroup of $Aut(\Gamma)$ isomorphic to G, forcing Γ to be a vertex-transitive graph. The Cayley graph Γ is said to be normal edge transitive, if $N_{Aut(\Gamma)}(R(G))$ is transitive on edges. In this paper, we determine all connected normal edge-transitive Cayley graphs on non-abelian groups with order $4p^2$, where p is a prime number.

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On r-Stablility Index of Spacelike Hypersurfaces in the De Sitter Space

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The minimal hypersurfaces of Riemannian manifolds are the critical points of a well known variational problem and then the notion of stability is arisen from this subject. Many people have studied the stability of hypersurfaces with constant mean curvature and the r-stability of hypersurfaces with null r-mean curvature in Riemannian space forms. Recently, the concept of index of stability as a criterion to show the distance from stability has been considered by Barros and Sousa who gave an estimation for the index of stability of minimal hypersurfaces and the r-stability of hypersurfaces with null r-mean curvature in Riemannian sphere. They proved that closed oriented non-totally geodesic minimal hypersurfaces of Euclidean sphere have index of stability greater than or equal to $n+3$ with equality in only Clifford tori. Also, they extended such result to closed oriented hypersurfaces with null $(r+1)$-mean curvature by estimating the index of r-stability. Up to Clifford tori, for closed oriented hypersurfaces in $\S^{n+1}$ with conditions $H_{r+1}=0$ and $H_{r+2}<0$ we have $Ind^r(M^n)\geq 2n+5$. We extend the notion of index of r-stability to spacelike hypersurfaces of de Sitter spaces with emphasizing on Clifford tori provided their higher order mean curvatures satisfy certain conditions.

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On the Convergence of an Iteration Method for a Nonlinear Static Beam Equation

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We consider a Kirchhoff type beam equation

$$u''''(x)-m\bigg(\int\limits_0^l u'^2(x)\,dx\bigg)u''(x)=f(x,u),\ \ 0<x<l, \tag{1}$$

with the conditions

$$u(0)=u(l)=0,\quad u''(0)=u''(l)=0. \tag{2}$$

Here $u=u(x)$ is the displacement function of the length l of the beam subjected to a force given by the function $f(x,u)$, the function $m(z)$, $m(z)\geq\alpha>0$, $0\leq z<\infty$, describes the type of relationship between stress and strain.

From problem (1), (2) we come to the equivalent nonlinear integral equation

$$u(x)=\int\limits_0^l G(x,\xi)f(\xi,u(\xi))\,d\xi+\frac{1}{\tau}\,\varphi(x), \tag{3}$$

where

$$G(x,\xi)=\frac{1}{\tau\sqrt{\tau}\,\sinh(\sqrt{\tau}\,l)}\begin{cases}\sinh(\sqrt{\tau}\,(x-l))\sinh(\sqrt{\tau}\,\xi), & 0<\xi\leq x<l,\\ \sinh(\sqrt{\tau}\,(\xi-l))\sinh(\sqrt{\tau}\,x), & 0<x\leq\xi<l,\end{cases}$$

$$\tau=m\bigg(\int\limits_0^l u'^2(x)\,dx\bigg),$$

$$\varphi(x)=\frac{1}{l}\Bigg((l-x)\int\limits_0^x \xi f(\xi,u(\xi))\,d\xi+x\int\limits_x^l (l-\xi)f(\xi,u(\xi))\,d\xi\Bigg).$$

Equation (3) is solved by the method of ordinary iterations

$$u_{k+1}(x)=\int\limits_0^l G_k(x,\xi)f(\xi,u_k(\xi))\,d\xi+\frac{1}{\tau_k}\,\varphi(x),\ \ 0<x<l,\quad k=0,1,\dots,$$

where

$$G_k(x,\xi) = \frac{1}{\tau_k\sqrt{\tau_k}\,\sinh(\sqrt{\tau_k}\,l)} \begin{cases} \sinh(\sqrt{\tau_k}\,(x-l))\sinh(\sqrt{\tau_k}\,\xi), & 0<\xi\le x<l, \\ \sinh(\sqrt{\tau_k}\,(\xi-l))\sinh(\sqrt{\tau_k}\,x)), & 0<x\le\xi<l, \end{cases}$$

$$\tau_k = m\left(\int_0^l {u'_k}^2(x)\,dx\right), \quad k=0,1,\dots,$$

The convergence of the iteration method is established.

Compactness

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This is a general talk on the interplay of logic, algebra, and topology. The starting point is an occasional confusion in model theory about the Compactness Theorem. The name of this theorem is sometimes explained as deriving from the compactness of a certain Stone space. This explanation is misleading, since all Stone spaces are compact. In the most general sense, a *logic* can be understood as a relation of *truth,* connecting certain *structures* to certain *sentences.* The truth relation establishes a Galois correspondence between classes of structures and sets of sentences. The set of all sentences can be replaced by a quotient, which is usually a Boolean algebra. Then a topology is induced on the class of structures, and the Kolmogorov quotient of this class embeds in the Stone space of the Boolean algebra of sentences. In general, the embedding might be proper. In *first-order logic* (and, under a further restriction, in no other logic, by Lindstrom's Theorem), this embedding is a bijection: this is the Compactness Theorem. Equivalently, the image of the embedding is dense and closed; and then Łoś's Theorem on ultraproducts is just an explicit way of establishing this closedness-albeit a way that relies on the full Axiom of Choice, or equivalently the Maximal Ideal Theorem, rather than the weaker Prime Ideal Theorem, which is equivalent to the Compactness Theorem as such.

Existence, Asymptotic Behavior and Blow Up of Solutions for the Timoshenko Equation

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We consider the existence, both locally and globally in time, the asymptotic behavior and the blow up of the solution for the Timoshenko equation with nonlinear damping and source terms. We prove the existence of the solution by Banach contraction mapping principle. The asymptotic behavior of the solution are proved by using Nakao's inequality. Moreover, under suitable conditions on the initial datum, we prove that the solution blow up in finite time.

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Existence of Solutions to Cauchy Problem for Boussinesq Type Equations

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This work is concerned with the study of Cauchy problem for the Boussinesq type equations. Assuming that the nonlinear function admits an exponential growth at infinity we prove global existence and nonexistence of solutions.

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On the Distribution of Prime Numbers

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The report is devoted to a consideration of following problems:

1) Relation of prime and odd numbers on the number axis.
2) List (sieve) of prime numbers.
3) Process of cancellation of prime numbers.
4) Distribution of prime numbers.

Comparison of Operator Jensen Functionals of Two Convex Functions

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Let f, g be two convex functions on an interval J of the real line. We say f is $\alpha-$lower convex dominant by g if there exists $\alpha > 0$ such that $f - \alpha g$ is convex on J. Similarly, f is $\beta-$upper convex dominant by g if there exists $\beta > 0$ such that $\beta g - f$ is convex on J. Moreover, f is (α, β)-convex dominant by g if f is $\alpha-$lower and β-upper convex dominant by g.

Let H be a Hilbert space, $x \in H$ be a unit vector and A be a self-adjoint operator with spectrum contained in J. We introduce the operator Jensen functional of f as

$$(0 \leq)\langle f(A)x, x\rangle - f\big(\langle Ax, x\rangle\big).$$

In this talk, we compare operator Jensen functionals of two convex functions f, g when f is (α, β)-convex dominant by g. Then as applications we consider a special class of convex functions and get the best α, β.

Oscillation Theorems for Second-Order Nonlinear Differential Equations of Generalized Euler Type

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The oscillation problem for second order nonlinear differential equations has been studied in many papers. In this work the second order nonlinear differential equation of generalized Euler type

$$t^2\ddot{u} + f(u)t\dot{u} + g(u) = 0, \quad t > 0, \tag{1}$$

is considered and some implicit necessary and sufficient conditions and some explicit sufficient conditions are given for all nontrivial solutions of this equation to be oscillatory. Here, $f(u)$ and $g(u)$ satisfy smoothness conditions which guarantee the uniqueness of solutions of initial value problems and

$$ug(u) > 0 \text{ if } u \neq 0.$$

It is supposed that all solutions of (1) are continuable in the future. A nontrivial solution of (1) is said to be *oscillatory* if it has arbitrarily large zeros. Otherwise, the solution is said to be *nonoscillatory*. For brevity, equation (1) is called oscillatory (respectively nonoscillatory) if all nontrivial solutions are oscillatory (respectively nonoscillatory). Because of Sturm's separation theorem, the solutions of second order linear differential equations are either all oscillatory or all nonoscillatory, but not both. Thus, the second order linear differential equations can be classified into two types. However, the oscillation problem for (1) is not easy, because $g(u)$ and $f(u)$ are nonlinear.

Also, especial case of the equation (1) is considered as follows.

$$t^2\ddot{u} + g(u) = 0, \quad t > 0. \tag{2}$$

In this case the oscillation problem has been solved when

$$\limsup_{|u|\to\infty} \frac{g(u)}{u} < \frac{1}{4} \quad \text{or} \quad \liminf_{|u|\to\infty} \frac{g(u)}{u} > \frac{1}{4}.$$

In the continuation sufficient conditions are presented for all nontrivial solutions of (2) to be oscillatory which can be applied in the case:

$$\liminf_{|u|\to\infty} \frac{g(u)}{u} \leq \frac{1}{4} \leq \limsup_{|u|\ \to\infty} \frac{g(u)}{u}.$$

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An Operational Approach for Solving Fractional-Order Integro-Differential Equations

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Fractional calculus is an emerging field both in the theoretical and applied aspects. In this approach, a truncated Legendre series together with the Legendre operational matrix of fractional derivatives are used for numerical solution of integro-differential equations. The fractional derivative is described in the Caputo sense. An Illustrative example is included to demonstrate the validity and applicability of the technique.

Canonical Functions of Admissible Measures in Half-Plane

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For an admissible measure in upper half-plane the concept of canonical function is entered. This concept is generalization of Nevanlinna canonical product for analytical in half-plane functions of a finite order. It is shown that for function which growth is defined by a proximate order in the sense of Butru, entered definition and Nevanlinna canonical product coincide.

Mathematical Modeling of Biofilter Used for Air Ow Through Lung Tissues

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Toxicity of nanoparticles is of rising concern due to fast development and growth of nanotechnology [1, 2]. Nanoparticles are potentially hazardous to human health; thus, it is necessary to control their motion in air. We need a biofilter to protect from nanoparticles which enter human lung and damage it. Biofiltration refers to the biological transformation or treatment of contaminants present in the gas phase, usually air. Biofilter is typically a porous media. Supported on the ideal biofilter model, numerical simulation is carried out to examine the effect of Darcy number and porosity on removal efficiency of low headloss biofilter. The generalized Navier-Stokes equations are applied making various hypotheses. The mathematical model for analysis of mass transport phenomena in the biofilter was solved using a two-step, explicit finite difference approximation technique and computer simulation was carried out. It is found that the Darcy number has determinant influence on the removal efficiency, and the effect of porosity on removal efficiency is very weak at lower Darcy numbers but very strong at higher Darcy numbers. The aim of this paper is to study the effect of Darcy number and porosity on air removal efficiency of biofilter.

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Numerical Solution of Reaction-Diffusion Equations in the Reproducing Kernel Spaces

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In this study we discuss the numerical solution of Euler-Tricomi equation in the reproducing kernel Hilbert space. The representation of solution is given by the form of convergent series and the n-term approximation solution is obtained by truncating the series.

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Extension of the Best Polynomial Approximation Operator in Variable Exponent Lebesgue Spaces

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Let Ω be a bounded measurable set in $\mathbf{R}^n$ and Π^m be the space of all polynomials (algebraic or trigonometric), defined on $\mathbf{R}^n$, of degree at most m. The best polynomial approximation operator was recently extended by Cuenya [1] from L^p to L^{p-1}. Guenya proved that, for each $f \in L^{p-1}(\Omega)$, $p > 1$, there exists a polynomial $Q \in \Pi^m$ satisfying following condition

$$\int\limits_{\Omega} |f(x) - Q(x)|^{p-1} \operatorname{sign}(f(x) - Q(x)) S(x)\, dx = 0, \tag{1}$$

for every $S \in \Pi^m$. Such a polynomial $Q := T(f)$ there always exists and it is unique. This polynomial will be called an extended polynomial approximant. Guenya proved that the extended polynomial approximant is unique. Denote by $\overline{T}(f)$ solution of (1) for $f \in L^{p-1}(\Omega)$ where $1 < p < \infty$. The operator $\overline{T} : L^{p-1} \to \Pi^m$ is continuous and consequence $\overline{T} : L^{p-1} \to \Pi^m$ is the unique extension of T preserving the property of continuity.

In case of Π^0 (class of constant functions) the operator $\overline{T}$ for $1 \leq p < \infty$ was studied in [5]. In that paper the authors extended the best constant approximation operator to $L^{p-1}(\Omega)$ if $p > 1$, and to the space of finite a.e., measurable functions if $p = 1$. Lather in [2] and [3] the authors considered the operator $\overline{T}$ defined in Orlicz spaces and in [4] the operator $\overline{T}$ was studied in Orlicz–Lorentz spaces.

In this report we will talk about extension of the operator of the best polynomial approximation from $L^{p(\cdot)}(\Omega)$ to the space $L^{p(\cdot)-1}(\Omega)$, $1 < p_- \leq p_+ < \infty$.

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Multistep Collocation Methods for Solving Fractional Differential Equations

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In this work we present multistep collocation methods for the numerical solution of fractional differential equations. Our aim is obtaining convergence order and investigating superconvergence of the method. Further, linear stability of the proposed method is analyzed. Numerical experiments are given confirming and demonstrating the theoretical results.

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The Boundary Value Contact Problems of Electroelasticity for Piezo-Electric Half Space with Cut or Elastic Inclusion

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The boundary value contact problems of the theory of electroelasticity for piezo-elastic half space with an cut or with an elastic inclusion of variable rigidity are considered. The problems are reduced to the singular integral or singular integro-differential equations with fixed singularity. Using the methods of analytic functions and integral transformation we obtain the Riemann boundary value problem. We can manage to investigate the obtained problem, the solution of which is represented in explicit form.

Various Chaotic Generalized Shift Dynamical Systems

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Considering dynamical properties of well-known one sided and two sided shifts are common in topological dynamical systems' approach. For nonempty set Γ, arbitrary self-map $\varphi : \Gamma \to \Gamma$ and discrete topological space X with at least two elements consider generalized shift $\sigma_\varphi : X^\Gamma \to X^\Gamma$ with $\sigma_\varphi((x_\alpha)_{\alpha\in\Gamma}) = (x_{\varphi(\alpha)})_{\alpha\in\Gamma}$ for $(x_\alpha)_{\alpha\in\Gamma} \in X^\Gamma$, where X^Γ is equipped with pointwise convergence topology (product topology). Hence one may consider generalized shift dynamical system $(X^\Gamma, \sigma_\varphi)$. The following facts has been stablished on generalized shift dynamical system $(X^\Gamma, \sigma_\varphi)$ for infinite countable Γ and finite (discrete) X with at least two elements, We have, the dynamical system $(X^\Gamma, \sigma_\varphi)$ is Devaney chaotic if and only if $\varphi : \Gamma \to \Gamma$ is one to one without periodic points; the dynamical system $(X^\Gamma, \sigma_\varphi)$ is exact Devaney chaotic if and only if $\varphi : \Gamma \to \Gamma$ is one to one and $\varphi : \Gamma \to \Gamma$ has neither periodic points nor φ-backwarding infinite sequences; the dynamical system $(X^\Gamma, \sigma_\varphi)$ is Li–Yorke chaotic if and only if $\varphi : \Gamma \to \Gamma$ has at least one non-quasi periodic point (i.e., there exists $\alpha \in \Gamma$ such that $\{\varphi^n(\alpha) : n \geq 1\}$ is infinite), for bijection $\varphi : \Gamma \to \Gamma$, the dynamical system $(X^\Gamma, \sigma_\varphi)$ is e-chaotic if and only if $\{\{\varphi^n(\alpha) :\ n \in \mathbb{Z}\} :\ \alpha \in \Gamma\}$ is a finite partition of Γ; also for topological chaotic (positive topological entropy) generalized shift dynamical system $(X^\Gamma, \sigma_\varphi)$. Here we consider and compare the above cases with more details motivated with counterexamples and more results.

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Sign of Eigenvalues for Toeplitz Matrices

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In this paper, we try to get, sign of eigenvalues (Inertia) for symmetric Toeplitz matrices. In the first step we factor symmetric Toeplitz matrix A by $A = LPL^T$ for finding inertia. In [3] is shown A and P have the same inertia $In(P) = \{n, z, p\} = In(A)$, $P(i,i) \in \{0, 1, -1\}$, $P(i,j) = P(j,i) \in \{0, 1\}$. We use of some properties for orthogonal and invertible matrix and some changes in factor $A = LPL^T$. These changes make this process easier and faster than basic factor. We put O_n for orthogonal, S_n for symmetric, T_n for Toeplitz matrices, $ST_n = S_n \cap T_n$, $OST_n = O_n \cap S_n \cap T_n$ and $A_n = A$, that

$A \in \mathbb{R}^{n\times n}$,

$$A = \begin{pmatrix} a & b_1 & b_2 & & \dots & & b_{n-1} \\ b_1 & a & b_1 & b_2 & \dots & & b_{n-2} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ b_{n-2} & \dots & & b_2 & b_1 & a & b_1 \\ b_{n-1} & \dots & & & b_2 & b_1 & a \end{pmatrix}.$$

Four possible changes are shown by $e_k = 0$ or $e_k \neq 0$ in the following factorization:

$$\begin{pmatrix} A_{k-1} & \mathbf{b_k} \\ \mathbf{b_k}^T & a \end{pmatrix} = \begin{pmatrix} L_{k-1} & \\ l_k^T & 1 \end{pmatrix} \begin{pmatrix} P_{k-1} & e_k \\ e_k^T & f_k \end{pmatrix} \begin{pmatrix} L_{k-1}^T & l_k \\ & 1 \end{pmatrix}, \quad f_k = a - \mathbf{b_k^T} P_{k-1} \mathbf{b_k},$$

that $\mathbf{b_k} = (b_{k-1}, b_{k-2}, \dots, b_1)^T$. We can show that the inertia of all the upper left square submatrices P_k and A_k agree in $A_k = L_k P_k L_k^T$, $In(P_k) = \{n_k, z_k, p_k\} = In(A_k)$. Changes from the sign of eigenvalues $In(A_{k-1}) = (n_{k-1}, z_{k-1}, p_{k-1})$ to $In(A_k)$ are:

1. $\{n_k, z_k, p_k\} = \{n_{k-1}, z_{k-1} + 1, p_{k-1}\}$,
2. $\{n_k, z_k, p_k\} = \{n_{k-1}, z_{k-1}, p_{k-1} + 1\}$,
3. $\{n_k, z_k, p_k\} = \{n_{k-1} + 1, z_{k-1}, p_{k-1}\}$,
4. $\{n_k, z_k, p_k\} = \{n_{k-1} + 1, z_{k-1} - 1, p_{k-1} + 1\}$.

We can transfer $\mathbf{b_k}^T (L_{k-1} P_{k-1} L_{k-1}^T)^{-1} \mathbf{b_k}$ to $\mathbf{b_k}^T A_{k-1}^{-1} \mathbf{b_k}$. We check matrix $A_k \in ST_n$ or $A_k \in OST_n$, even in every step, we check A_{k-1} is invertible matrix or no. Then we try to find inertia for matrix A_k. It does n't need lower triangular matrix L_k and inertia matrix P_k. It can be done by invertible matrix A_{k-1},

$$\begin{pmatrix} A_{k-1} & \mathbf{b_k} \\ \mathbf{b_k}^T & a \end{pmatrix} = \begin{pmatrix} I & \\ \mathbf{b_k}^T A_{k-1}^{-1} & 1 \end{pmatrix} \begin{pmatrix} A_{k-1} & \\ & s_{k-1} \end{pmatrix} \begin{pmatrix} I & A_{k-1}^{-1} \mathbf{b_k} \\ & 1 \end{pmatrix}$$

that $s_{k-1} = a - \mathbf{b_k}^T A_{k-1}^{-1} \mathbf{b_k}$ (see [1, 2]). Then $s_{k-1} = 0$ or $s_{k-1} < 0$ or $s_{k-1} > 0$,

$$\begin{aligned} a = \mathbf{b_k}^T A_{k-1}^{-1} \mathbf{b_k} &\longrightarrow In(A_k) = \{n_{k-1}, z_{k-1} + 1, p_{k-1}\}, \\ a < \mathbf{b_k}^T A_{k-1}^{-1} \mathbf{b_k} &\longrightarrow In(A_k) = \{n_{k-1} + 1, z_{k-1}, p_{k-1}\}, \\ a > \mathbf{b_k}^T A_{k-1}^{-1} \mathbf{b_k} &\longrightarrow In(A_k) = \{n_{k-1}, z_{k-1}, p_{k-1} + 1\}. \end{aligned}$$

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Existence of Global Solutions of an Initial-Boundary Value Problem for a Nonlinear Timoshenko Equation

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We study global weak solutions for initial-boundary value problem of the Timoshenko equation in case of high energy initial data by using the potential well method. We define a new functional and prove the existence of global solutions by use of sign invariance of this functional.

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On New Summability Methods with a Variable Order

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We introduce summability methods with a variable order and prove theorems connected with these methods. Namely, one of our theorems shows close connection between summabilities of an orthogonal series by these methods and the convergence of this series. Using above-mentioned theorem and such well-known results as Menshov-Rademacher (see [1] and [2]), Menshov (see [3, 4] and [5]) and Tandori (see [6]) ones we prove theorems on divergence of an orthogonal series almost everywhere by the introduced methods.

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Uniqueness Theorems for Rademacher Series

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We prove uniqueness theorems for Rademacher series. Namely, we obtain the formulae for reconstruction of coefficients of the Rademacher single rearranged series by the sum of the series. As a result we strengthen such well-known theorems as Stechkin-Ul'yanov (see [1]) and Bakhshetsyan (see [2]) ones. It should be noted that mentioned formulae generalize well-known Fourier-Rademacher formulae. Indeed, using our formulae one can reconstruct the coefficients of the series for such case too if the values of the series sum are known only at the countable union of the appropriate pairs of points. Of course, Fourier-Rademacher formulae cannot be used for such case. We also prove the uniqueness of a Rademacher d-multiple series ($d \geq 2$) if the values of the series sum are known only at the countable union of sets of the appropriate 2^d points. As a result we strengthen known theorem on the uniqueness of multiple series in the Rademacher system obtained by Mushegyan (see [3]) and Tetunashvili (see [4]).

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On Equivariant Fiber Shape Theory

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The main aim of report is to give the shape classification of equivariant maps of spaces with an action of a given compact group G [7].

From the point of view of retracts and shape theory ([1, 6]), the categories $\mathbf{Mor_{Top^G}}$ and $\mathbf{Mor_{M^G}}$ [3] of morphisms of categories $\mathbf{Top^G}$ and $\mathbf{M^G}$ [1, 2] of G-spaces and metrizable G-spaces, are studied.

Here we prove equivariant versions of main theorems of retracts theory for maps [3, 4]. This theorems play a very important role in constructions of equivariant fiber shape theories for the categories $\mathbf{Mor_{Top^G}}$ and $\mathbf{Mor_{M^G}}$.

Let $\mathbf{H}(\mathbf{ANR}(\mathbf{Mor_{M^G}}))$ be the G-homotopy category of the category $\mathbf{ANR}(\mathbf{M^G})$ of absolute neighbourhood retracts for the category $\mathbf{M^G}$.

Theorem 1. *The G-homotopy category $\mathbf{H}(\mathbf{ANR}(\mathbf{Mor_{M^G}}))$ is dense subcategory of G-homotopy categories $\mathbf{H}(\mathbf{M^G})$ and $\mathbf{H}(\mathbf{Top^G})$.*

Using the results of [5] we define the equivariant fiber shape invariants of G-maps and study the problem of extension of functors defined on the category $\mathbf{ANR}(\mathbf{M^G})$.

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On Čech's Type Continuous Functors

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The uniform shape theory ([1, 4, 5]) is closely connected with the problem of extension of functors from the category of uniform spaces with the uniform homotopy type of uniform polyhedras to the category of uniform topological spaces.

According to [6] we give formulation of the uniform continuity axiom. Let $T : \mathbf{Unif}^2 \to \mathbf{Ab}$ be a covariant (contravariant) functor. The functor T is continuous at pair (X, X_0) of uniform spaces if it satisfies the condition:

(UCA) If $\mathbf{p} = (p_\alpha) : (X, X_0) \to (\mathbf{X}, \mathbf{X_0}) = (X_\alpha, p_{\alpha\alpha'}, A)$ is an uniform resolution [4] of $(X, X_0) \in \mathbf{Unif}$, than

$$T(\mathbf{p}) = (T(p_\alpha)) : T(X, X_0) \to T(\mathbf{X}, \mathbf{X_0}) = (T(X_\alpha, X_{\alpha'}), T(P_{\alpha\alpha'}), A)$$

is an inverse limit. We say that T is uniform continuous functor if T is continuous at any pair (X, X_0) of uniform spaces.

Let T : $\mathbf{ANRU}^2 \to \mathbf{Ab}$ be a covariant (contravariant) functor. We say that T satisfies the uniform homotopy axiom, if f : $(X, X_0) \to (Y, Y_0)$ is uniform homotopy to g :$(X, X_0) \to (Y, Y_0)$, then $T(\mathrm{f}) = T(\mathrm{g})$.

Using the methods of proof of Theorem 13 in [2] and Theorem 2 in [3, Ch. II, § 3.1] we can prove the following theorems.

Theorem 1. *Let* $T : \mathbf{ANRU}^2 \to \mathbf{Ab}$ *be a covariant (contravariant) functor satisfying the uniform homotopy axiom. Then there exists the covariant (contravariant) uniform continuous functor* $\check{T} : \mathbf{fUnif}^2 \to \mathbf{Ab}$ ($\widehat{T} : \mathbf{fUnif}^2 \to \mathbf{Ab}$)*, which is an extension of* T *and satisfies the uniform homotopy axiom.*

Theorem 2. *Let* $T : \mathbf{fUnif}^2 \to \mathbf{Ab}$ *be a uniform continuous covariant (contravariant) functor satisfying the uniform homotopy axiom. Then* T *and* $(\check{T}_{|\mathbf{ANRU}^2})(\widehat{T}_{|\mathbf{ANRU}^2})$ *are equivalent functors.*

Besides,it is shown that these Čech's type extensions of functors have equal values on pairs of uniform spaces and their completions.

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Numerical Solution of Euler-Tricomi Equation in the Reproducing Kernel Hilbert Spaces

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This work is concerned with the study of Cauchy problem for the Boussinesq type equations. Assuming that the nonlinear function admits an exponential growth at infinity we prove global existence and nonexistence of solutions.

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Some Results of One-Alpha Descriptor of Graphs

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Recently, one-two descriptor has been defined and it has been shown that it is a good predictor of the heat capacity at P constant (CP) and of the total surface area (TSA). In this paper, we analyze its generalizations by replacing the value 2 by arbitrary positive value α. One-alpha descriptor can be expected that this more general descriptor can find a wider range of application than the original one. The extremal values of trees have been found for all values of α. One-alpha descriptor is defined as the sum of the vertex contributions is such a way that each pendant vertex contributes 1, each vertex of degree two adjacent to pendant vertex contributes α, and also each vertex of degree higher than two also contributes α. If we take $\alpha = 2$, we get the previously defined [1] one-two descriptor. All graphs considered in this paper will be finite and simple. The notation we use is mostly standard and taken from standard graph theory textbooks, such as [2].

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Submanifolds of a Riemannian Product Manifold Admitting a Quarter-Symmetric Metric Connection

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We study the relations between the geometry of the semi-invariant submanifolds of Riemannian product manifold with Levi-Civita connection and the geometry of the semi-invariant submanifolds of Riemannian product manifold with quarter-symmetric metric connection. We obtain fundamental properties of the semi-invariant submanifolds of Riemannian product manifold with quarter-symmetric metric connection such as the integrarability of the distribution D and $D^{\perp}$ and mixed-geodesic property for a quarter symmetric metric connection.

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Optimal Vibration Control of a Smart Beam with Time Delay

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Dynamic response control of a smart beam, with Kelvin-Voigt damping and time delay which occurs in voltage applied to the piezoelectric patch actuator, subject to the displacement and moment boundary conditions is considered. The optimal control law is derived using a maximum principle which involves a Hamiltonian expressed in terms of an adjoint variable with the state and adjoint variables linked by terminal conditions leading to a boundary-initial-terminal value problem. The explicit solution of the control problem is obtained by eigenfunction expansions of the state and adjoint variables. To show the effectiveness and applicability of the piezoelectric actuator to suppress the undesirable vibrations in the control system, numerical results are given in the tables and graphical forms.

A Positive Cone in p-Operator Projective Tensor Product of Herz Algebras

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Throughout this paper G is a locally compact group, p is a real number in $(1,\infty)$ and $q \in (1,\infty)$ is the conjugate of p, that is $\frac{1}{p}+\frac{1}{q}=1$. The Fourier algebra $A(G)$ consists of all coefficient functions of the left regular representation λ of G

$$A(G)=\big\{w=(\lambda\xi,\eta):\ \xi,\ \eta\in L_2(G)\big\}.$$

This is a Banach algebra with the norm $\|w\|_{A(G)}=\inf\{\|\xi\|_2\|\eta\|_2:\ w=(\lambda\xi,\eta)\}$. When G is abelian, the Fourier transform yields an isometric isomorphism from $A(G)$ onto $L_1(\widehat{G})$, where $\widehat{G}$ is the Pontryagin dual of G. In general, $A(G)$ is a two-sided closed ideal of the Fourier-Stieltjes algebra $B(G)$. This is the linear span of the set $P(G)$ of all positive definite continuous functions on G [2]. In an earlier paper, the author et al. studied the order structure of the Fourier algebra $A(G)$ [5].

In [3], Figà-Talamanca introduced a natural generalization of the Fourier algebra, for a compact abelian group G, by replacing $L_2(G)$ by $L_p(G)$. In [4], Herz extended the notion to an arbitrary group, leading to the commutative Banach algebra $A_p(G)$, called the Figà-Talamanca-Herz algebra.

In this talk we introduce a positive cone on $A_p(G)$, firstly. Then using the p-operator space structure on $A_p(G)\widehat{\otimes}^p A_p(H)$, for locally compact groups G, H, we introduce an order structure on this space. Then we show that the isometrically isomorphism between $A_p(G\times H)$ and $A_p(G)\widehat{\otimes}^p A_p(H)$, studied in [1], is order isomorphism on amenable groups G and H.

We shall give some complementary results in this area.

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Minimal Free Resolution of Monomial Ideals

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In this talk, we study square-free monomial ideals generated in degree d having linear resolution. We define some operations on the simplicial complexes associated to these ideals and prove that linearity of the resolution is conserved under these operations. We apply the operations to construct classes of simplicial complex with and without linear resolution. This id a joint work with A. Nasrollah Nejad, M. Morales and A. Yazdanpour.

On Approximate Solution Some Dirichlet Generalized Problems for Cylindrical Shells of Revolution

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In this report we propose an algorithm of approximate solution of the Dirichlet generalized problem for a harmonic function in the case of a finite cylindrical shell of revolution. Under the generalized problem the case when a boundary function has a finite number of first kind curves of discontinuity is meant.

The problem of the indicated type is considered for the case when the curves of discontinuity are circles whose centres are situated on the axis of the cylindrical shell and the boundary function is independent on an angle of rotation with respect to the axis.

An example of application of the proposed algorithm is considered.

A New Practical Approach for Generalized High Order Derivatives of Nonsmooth Functions

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In this paper, We proposed a Extension Definition to derive, simultaneously, the first, second and high order generalized derivative for non-smooth functions which is based on optimization, in which the involved functions are Riemann integrable but not necessarily locally Lipschitz or continuous. Indeed, we define a functional optimization problem corresponding to smooth functions where its optimal solutions are the first and second derivatives of these functions in a domain. Then by applied these functional optimization problems for non-smooth functions and using this method we obtain generalized first derivative (GFD) and generalized second derivative (GSD). Here, the optimization problem is approximated with a linear programming problem that by solving of which, we can obtain these derivatives, as simple as possible. We extend this approach for obtaining generalized high order derivatives (GHODs) of non-smooth functions, simultineously. Finally, for efficiency of our approach some numerical examples have been presented.

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Numerical Solution of Fuzzy Nonlinear Fredholm Integral Equation

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In this paper, we will study numerical solution of the nonlinear fuzzy Hammerstein integral equations of the form

$$u(x) = f(x) + \lambda(K\Phi u)(x), \quad x \in [0,1],$$

where

$$(K\Phi u)(x) = \int_0^1 k(x,t)\phi(t,u(t))\,dt,$$

and $f \in C([0,1],\mathbb{R}_{\mathscr{F}})$, $k \in C([0,1]\times[0,1],\mathbb{R})$ and ϕ is uniformly continuous. For this purpose, we will use fuzzy B-spline series. We will use B-spline functions of order 4 to find approximate solution of this equation. B-spline functions have compact support which makes them helpful in numerical points of view. Let $t_0 \le t_1 \le \cdots \le t_r$ be points in R, with $t_0 \neq t_r$. The B-spline B is given by $B(x) = B(x,t_0,\ldots,t_r) = r[t_0,\ldots,t_r](\cdots - x)_+^{r-1}$, where $[t_0,\ldots,t_r]f$ denotes the divided difference of f ([1]). In [1], approximation of fuzzy-number-valued function $f : [0,1] \to R_{\mathscr{F}}$ introduced by fuzzy B-spline series $S : [0,1] \to R_{\mathscr{F}}$ as follows:

Let $0 = t_0 \le t_1 \le \cdots \le t_n = 1$ be a partition of the interval $[0,1]$. Choose auxiliary knots $t_{-t+1} \le \cdots \le t_0 = 0, 1 = t_{n+1} \le \cdots \le t_{n+r}$, where r is the order of B-spline. Let $\xi_j \in [0,1] \cap \operatorname{supp} B_j$, $j = -r+1,\ldots,n$. Then the fuzzy B-spline series will be

$$S(f,x) = \sum_{-r+1}^{n} B_j f(\xi_j).$$

We will use fuzzy B-spline series of ϕ and approximate the unknown function u.

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