LATE ABSTRACTS:

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Robustness and Rate of Optimality in Linear Programming

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This paper deals with robustness and rate of optimality in linear programming. This study is based on a new concept namely, the *angle deviation* between the objective vector c and the *binding cone* (the cone generated by the gradients of binding constraints at the given point). Using this new concept, some criteria for determining the robustness and sensitivity analysis are defined. In the sequel, a signed distance (oriented distance) is considered. It can be shown that the angle deviation between the objective vector c and the binding cone can be computed using this signed distance.

Consider the following LP Problem:

$$\begin{array}{lll} LP(c) & \max & \langle c, x\rangle \\ & s.t. & Ax \leq b, \end{array}$$

where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$. The feasible set of this problem is denoted by X. By LP(c) we denote an LP with objective coefficient vector c.

Let $x^* \in X$ be a feasible solution. $I(x^*)$ denotes the index set of binding constraints of Problem (LP) at x^* and $A_{I(x^*)}$ is the sub-matrix of A containing rows of A whose indices are in $I(x^*)$. The polyhedral cone generated by the rows of $A_{I(x^*)}$, denoted by $\mathcal{A}(x^*)$, is an important cone. Since given feasible solution x^* remains optimal if and only if $c \in \mathcal{A}(x^*)$, thus the angle deviation between c and $\mathcal{A}(x^*)$ or the distance between them can be a suitable criterion for determining the rate of optimality for x^*. For optimal feasible solution x^* (where, $c \in \mathcal{A}(x^*)$), the position of c in $\mathcal{A}(x^*)$ can be considered as a meaningful criterion for determining the robustness of x^* as a factor denoting the quality of optimality for x^*. Since $\mathcal{A}(x^*)$ has a conic structure and optimality of Problem LP(c) dose not depend on the positive scalar multiplications of c, the best way for comparing c and $\mathcal{A}(x^*)$ is computing the angle between c and $\mathcal{A}(x^*)$. Therefore, the rate of optimality of a given feasible solution $\bar{x}$ is defined as

$$opt(\bar{x}) = \cos(\theta(\bar{x})),$$

where $\theta(\bar{x}) := \angle(c, \mathcal{A}(\bar{x}))$ is the angle between c and $\mathcal{A}(\bar{x})$. So, we investigate this angle from both theoretical and computational points of view.

[1] P. G. Georgiev, D. T. Luc, and P. M. Pardalos: Robust aspects of solutions in deterministic multiple objective linear programming, *European Journal of Operational Research*, 1 (2013), 29-36.

[2] A. Ben-Tal, E. L. Ghaoui, and A. Nemirovski: *Robust optimization.* Princeton University Press, 2009.

Overridden boundary value problems for ODE with continuous solutions

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In the work of S.V. Israilov, I.T Kiguradze and others, in the case when the systems

$$y_i^{/} = f_i(x, y_1, y_2, ..., y_n), \qquad i = \overline{1, n} \tag{1}$$

is singular for $x = x_i$, $i = \overline{1, n}$, i.e. the right hand side of (1) or the functions $f_i(i = \overline{1, n})$ have unbounded discontinuous in these points and on the phase coordinates continuous in a certain domain, selected cases of existence of a continuum of solutions satisfying the Nicolettis conditions

$$y_i(x_i) = 0, \qquad i = \overline{1, n}. \tag{2}$$

In presented report reduced the results, besides the condition (2) satisfy yet additional boundary conditions

$$y_i(a) = 0, y_i(b) = 0, \qquad i = \overline{1, n,} \tag{3}$$

at the ends of the segment $[a, b], x_i \in (a, b)$, $i = \overline{1, n}$, and with additional restrictions continuous of function $f_i(i = \overline{1, n})$ for $x = a$ and $x = b$.

Naturally, overridden boundary value problems of the type (1), (2) and (3) having precedence applied problems.

It is noted that instead of condition (3) can be taking the functional conditions

$$y_i(a) = F_i(x, y_1, y_2, ..., y_n), \qquad y_i(b) = F_i^*(x, y_1, y_2, ..., y_n), \qquad i = \overline{1, n}, \tag{4}$$

where $F_i,\ F_i^*$ $(i = \overline{1, n})$ are some functionals.

Overridden boundary value problems for ODE with discontinuous solutions

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In some works S.V. Israilov and A. A. Sagitov we studied boundary value problem for a system

$$y_i^{/} = f_i(x, y_1, y_2, ..., y_n), \qquad i = \overline{1, n}, \tag{1}$$

when marginal parts f_i, $i = \overline{1,n}$, are defined and continuous in the domain $y_i^{/} = f_i(x, y_1, y_2, ..., y_n)$, $i = \overline{1,n}$, $D_i : \{\ x \in [a,b] - \sigma_i, |y_i| \le d_i, \sigma_i = \{x_i\ \}$, $i = \overline{1,n}$, but for $x = x_i$ the functions f_i having a stronger singularity than in the works of V. A. Chechika, S. V. Israilov, I. T. Kiguradze. So this singularity reduced well known Nicolettis conditions

$$y(x_i) = 0, i = \overline{1,n},$$

in the boundary conditions of the form

$$y = (x_i - 0) = d_i^-, y_i(x_i + 0) = d_i^+, d_i^- \ne d_i^+, y_i(x_i) = \frac{y_i(x_i - 0) + y_i(x_i + 0)}{2}, \qquad i = \overline{1,n}. \tag{2}$$

In the proposed report are proved theorems, satisfying the existence of a solution of this system (1) satisfying not only the conditions (2) and the additional boundary conditions

$$y_i(a) = 0, y_i(b) = 0, \qquad i = \overline{1,n}, \tag{3}$$

or more general functional conditions

$$y_i(a) = \Phi_i y, \qquad y_i(b) = \Phi_i^*, \qquad i = \overline{1,n},$$

where Φ_i, Φ_i^*, $(i = \overline{1,n})$ are some functionals.

The type of such a predetermined boundary value problems very less studied and is of interest in applied problems.

Bohr's phenomenon for analytic functions mapping into hyperbolic domains and the hyperbolic metric

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A link is established between Bohr's inequality for classes of analytic functions mapping into a hyperbolic domain and the hyperbolic metric.

Development of fractional differential equation on variable coefficients and its applications on PDE

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It is commonly accepted that fractional differential equations play an important role in the explanation of many physical phenomena. For this reason we need a reliable and efficient technique for the solution of fractional differential equations. This paper deals with the numerical solution of fractional partial differential equation with variable coefficient of fractional differential equation in various continues functions of spatial and time orders. In the examples, we describe new numerical solution and this efficiency on FPDE,s.

[1] Chang-Ming Chen, F. Liu, V. Anh, I. Turner, Numerical schemes with high spatial accuracy for a variable-order anomalous sub-diffusion equation, *SIAM J. Sci. Comput.* 32 (4) (2010) 1740-1760.

[2] S. Irandoust-pakchin, M. Javidi, H. Kheiri, Analytical solutions for the fractional nonlinear cable equation using a modified homotopy perturbation and separation of variables methods *Computational Mathematics and Mathematical Physics* Springer Verlag, 2014, (Accepted)

Characterization of best proximity pairs

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In this talk we give some necessary conditions for existence and uniqueness of proximity pair. Also we shall characterize best proximity pairs by linear functional. Moreover, if the mapping under consideration is a self-mapping, it may be noted that under suitable conditions, this best proximity theorem boils down to a fixed-point theorem. Thus, best proximity pair theorems also serve as a generalization of fixed-point theorems. Best proximity theorems have variety applied in other branches of mathematics for instance game theory.

Polynomial optimization with application to solving optimal control problems

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In this talk, a convexification method for finding the global minimums of polynomial optimization problems is presented. In this method, using moments theory, sightly problems are converted to a sequence of linear convex optimization problems with linear matrix inequality constraints. In the sequel, some optimal control problems are changed to polynomial optimization problems by using state parameterization approach and solved by the mentioned method.

Minimal free resolution of monomial ideals

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In this talk, we study square-free monomial ideals generated in degree d having linear resolution. We define some operations on the simplicial complexes associated to these ideals and prove that linearity of the resolution is conserved under these operations. We apply the operations to construct classes of simplicial complex with and without linear resolution. This id a joint work with A. Nasrollah Nejad, M. Morales and A. Yazdanpour.