

# Isometric representations of Abelian semigroups

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In the paper [2] Coburn proved that all semiunitary representations of the semigroup of nonnegative integers by isometric operators generate canonically isomorphic  $C^*$ -algebras. Later a similar result for semigroups with Archimedean order and total order have been proved by Douglas [3] and Murphy [6], respectively. A simple example of a semigroup with non-total order provides the semigroup  $Z_+ \setminus \{1\}$ , which originally was discussed by Murphy [6]. Later, Jang [5] pointed out two representations of this semigroup that generate canonically non-isomorphic  $C^*$ -algebras. Vittadello [8] studied all isometric representations of the numerical semigroups under certain condition.

Here we investigate isometric representations of Abelian semigroups and  $C^*$ -algebras generated by them. We show that this representations can be divided into two classes: inverse representations (the isometric representations that can be extended up to representations of an inverse semigroup) and non-inverse representations.

We show that all isometric representations of Abelian cancellative semigroup are inverse if and only if the natural order on semigroup is a total order. A description of all irreducible inverse representations of a semigroup without total order ( $Z_+ \setminus \{1\}$ ) is given. Also, we introduce the class of non-inverse representations ( $\beta$ -representations) of this semigroup and study his properties.

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