საქართველოს
მათემატიკოსთა კავშირი

Georgian
Mathematical Union

ბათუმის შოთა რუსთაველის
სახელმწიფო უნივერსიტეტი

Batumi Shota Rustaveli
State University

საქართველოს მათემატიკოსთა კავშირის XIII საერთაშორისო კონფერენცია

XIII International Conference of the Georgian Mathematical Union

თეზისების კრებული
BOOK OF ABSTRACTS

ბათუმი, 4 – 9 სექტემბერი

Batumi, September 4 – 9

2023

საორგანიზაციო კომიტეტი:

ალექსანდრე მესხი (თავმჯდომარე), ვლადიმერ ბალაძე, მარიამ ბერიაშვილი, ანზორ ბერიძე, თენგიზ ბოკელავაძე, მიხეილ გაგოშიძე, გურამ გოგიშვილი, თინათინ დავითაშვილი (კონფერენციის მდივანი), ზურაბ ვაშაკიძე, გიორგი იმერლიშვილი, ნუგზარ კერესელიძე, ლელა თურმანიძე (თავმჯდომარის მოადგილე), დალი მახარაძე, ლაზარე ნათელაშვილი, მიხეილ რუხაია, თემურ ჯანგველაძე (თავმჯდომარის მოადგილე).

Organizing Committee:

Alexander Meskhi (Chairman), Vladimer Baladze, Mariam Beriashvili, Anzor Beridze, Tengiz Bokelavadze, Tinatin Davitashvili (Conference Secretary), Mikheil Gagoshidze, Lazare Natelashvili, Guram Gogishvili, Temur Jangveladze (Deputy Chairman), George Imerlishvili, Nugzar Kereselidze, Dali Makharadze, Mikheil Rukhaia, Lela Turmanidze (Deputy Chairman), Zurab Vashakidze.

სამეცნიერო კომიტეტი:

მიხეილ ამაღლობელი, მალხაზ ბაკურაძე, ვლადიმერ ბალაძე, ანზორ ბერიძე (თანათავმჯდომარე), როსტომ გეწაძე, გურამ გოგიშვილი, ლუკას გრაფაკოსი, თეიმურაზ დავითაშვილი, მასსიმო ლანცა დე კრისტოფერის, როლანდ დუდუჩავა, ბესიკ დუნდუა, ლაშა ეფრემიძე, თეიმურაზ ვეფხვაძე, დავით კაპანაძე, ვახტანგ კვარაცხელია, ალექსანდრე კვინიხიძე, ალექს კირთაძე, ტარიელ კიღურაძე, ჰამლეტ მელაძე, სერგეი მიხაილოვი, მარიუს მიტრეა, ელიზბარ ნადარაია, დავით ნატროშვილი (თანათავმჯდომარე), როლანდ ომანაძე, გიორგი ონიანი, მიხეილ რუხაია, ილია სპიტკოვსკი, ვაჟა ტარიელაძე, გიორგი ტეფნაძე, შაქრო ტეტუნაშვილი, თეიმურაზ ფირაშვილი, კონსტანტინე ფხაკაძე, ომარ ფურთუხია, თორნიკე ქადეიშვილი, რამაზ ქვათაძე, ევგენი შარგოროდსკი, ნუგზარ შავლაყაძე, თემურ ჩილაჩავა, ოთარ ჭკადუა, ალექსანდრე ხარაზიშვილი, მანანა ხაჩიძე, გიორგი ხიმშიაშვილი, ემზარ ხმალაძე, გიორგი ჯაიანი, თემურ ჯანგველაძე.

Scientific Committee:

Mikheil Amaglobeli, Malkhaz Bakuradze, Vladimer Baladze, Anzor Beridze (co-Chair), Temur Chilachava, Otar Chkadua, Teimuraz Davitashvili, Roland Duduchava, Besik Dundua, Lasha Ephremidze, Rostom Getsadze, Guram Gogishvili, Loukas Grafakos, George Jaiani, Temur Jangveladze, Tornike Kadeishvili, David Kapanadze, Manana Khachidze, Alexander Kharazishvili, George Khimshiashvili, Emzar Khmaladze, Tariel Kiguradze, Aleks Kirtadze, Vakhtang Kvaratskhelia, Ramaz Kvatadze, Aleksander Kvinikhidze, Massimo Lanza de Cristoforis, Hamlet Meladze, Sergey Mikhailov, Marius Mitrea, Elizbar Nadaraia, David Natroshvili (co-Chair), Roland Omanadze, George Oniani, Teimuraz Pirashvili, Konstantine Pkhakadze, Omar Purtukhia, Mikheil Rukhaia, Eugene Shargorodsky, Nugzar Shavlakadze, Ilia Spitkovsky, Vaja Tarieladze, George Tephnadze, Shakro Tetunashvili, Teimuraz Vepkhvadze.

რედაქტორები: გურამ გოგიშვილი, მაია ჯაფოშვილი

Editors: Guram Gogishvili, Maia Japoshvili

Dedicated to the 120-th anniversary of Academician Victor Kupradze

(02.11.1903 – 25.04.1985)

Contents

Abstracts of Sectional Talks 33

Abstracts of Plenary Talks

Exact Continuum Representation of Long-range Interacting Systems and Emerging Exotic Phases in Unconventional Superconductors

Andreas A. Buchheit[1], **Torsten Keßler**[2], **Peter K. Schuhmacher**[3], **Benedikt Fauseweh**[3]

[1] *Applied Mathematics, Saarland University, 66123 Saarbrücken, Germany*
E-mail: buchheit@num.uni-sb.de

[2] *Eindhoven University of Technology, 5600 MB Eindhoven, Netherlands*

[3] *German Aerospace Center (DLR), 51147 Cologne, Germany*

Continuum limits are a powerful tool in the study of many-body systems in condensed matter physics, yet their validity is often unclear when long-range interactions are present. In this work, we rigorously address this issue and put forth an exact representation of long-range interacting lattices that separates the model into a term describing its continuous analog, the integral contribution, and a term that fully resolves the microstructure, the lattice contribution. Here, we use the recently developed Singular Euler–Maclaurin expansion [2–4], a generalization of the 300-year old Euler–Maclaurin summation formula to multidimensional sums that involve functions with algebraic singularities. For any system dimension, any lattice, any power-law interaction, and for linear, nonlinear, and multi-atomic lattices, we show that the lattice contribution can be described by a differential operator based on the multidimensional generalization of the Riemann zeta function, namely the Epstein zeta function. We employ our representation in Fourier space to solve the long-standing problem of long-range interacting unconventional superconductors. We derive a generalized Bardeen–Cooper–Schrieffer gap equation, solve it numerically, and find emerging exotic phases in two-dimensional superconductors with topological phase transitions. Finally, we determine the quantum time evolution and utilize non-equilibrium Higgs spectroscopy to analyze the impact of long-range interactions on the collective excitations of the condensate.

Acknowledgments

The authors gratefully acknowledge the scientific support and HPC resources provided by the Erlangen National High Performance Computing Center (NHR@FAU) of the Friedrich–Alexander-Universität Erlangen–Nürnberg (FAU) under the NHR project # n101af. In particular, we thank Thomas Gruber for providing excellent code review. NHR funding is provided by federal and Bavarian state authorities. NHR@FAU hardware is partially funded by the German Research Foundation (DFG) – 440719683. TK acknowledges funding received from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska–Curie grant agreement # 899987.

References

[1] A. A. Buchheit, T. Keßler, P. K. Schuhmacher and B. Fauseweh, Exact continuum representation of long-range interacting systems and emerging exotic phases in unconventional superconductors. *Preprint* arXiv:2201.11101; `https://arxiv.org/abs/2201.11101`

[2] A. A. Buchheit and T. Keßler, Singular Euler–Maclaurin expansion on multidimensional lattices. *Nonlinearity* **35** (2022), no. 7, 3706–3754.

[3] A. A. Buchheit and T. Keßler, On the efficient computation of large scale singular sums with applications to long-range forces in crystal lattices. *J. Sci. Comput.* **90** (2022), no. 1, Paper no. 53, 20 pp.

[4] A. A. Buchheit, On the efficient computation of multidimensional singular sums. *Dissertation*, 2021.

The Torsion of Stellar Streams Due to a Nonspherical Dark Matter Halo

Felipe José Llanes Estrada[1], **Adriana Bariego Quintana**[2]

[1] *Universidad Complutense de Madrid, Madrid, Spain*
E-mail: fllanes@fis.ucm.es

[2] *IFC – University of Valencia, Valencia, Spain*

We have recently pointed out that flattening rotation curves v(r) are naturally explained by elongated (prolate) Dark Matter (DM) distributions, and provided competitive fits to the SPARC database. To further probe the geometry of the halo one needs out-of-plane observables.

Stellar streams in the Milky Way, poetically analogous to airplane contrails, but caused by tidal dispersion of massive substructures such as satellite dwarf galaxies, would lie on a plane (consistently with angular momentum conservation) should the gravitational field of the DM halo be spherically symmetric.

Entire orbits are seldom available because their periods are commensurable with Hubble time, with streams often presenting themselves as short segments.

Therefore, the systematic study of the stellar stream torsion, a local observable that measures the deviation from planarity in differential curve geometry, provides sensitivity to aspherical DM distributions and ensures the use of even short streams.

Knowledge Acquisition in Multi-Agent Systems: A Formalisation of the Eleusis Card Game

Natalia Garanina[1], **Sergei Gorlatch**[2]

[1] *Institute of Informatics Systems, Novosibirsk, Russia*
E-mail: natta.garanina@gmail.com

[2] *University of Müenster, Münster, Germany*
E-mail: gorlatch@uni-muenster.de

We deal with logical approaches to knowledge acquisition in multi-agent systems. We enhance previous work by considering the inductive card game Eleusis [2] where dynamic knowledge about the behaviour of other agents or environment is acquired, rather than traditional static knowledge about system states. We also extend related work where an agent acquires knowledge about the system state or takes knowledge from other agents: we rather aim at acquiring knowledge about the *behavior* of other agents or environment. In addition to the theoretical interest, our work is also motivated by potential application fields, including cryptography where an intruder tries to guess the behavior of an environment in order to pretend to be a legitimate agent. Using card games in cryptography is a currently popular research topic.

In Eleusis game, two or more players try to guess the secret rule for a card sequence made by the dealer who starts a pile and then tells 'yes' if a player puts a right card on top of the pile in its turn or tells 'no' if the card is wrong. Our main contribution is two-fold: 1) we formalize the infinite version of the Eleusis game, i.e., its rules, strategies and the knowledge acquisition process, using the Propositional Logic of Knowledge and Branching Time *Act*-CTL-K_n [5] and a notion of an interpreted system [3] enriched by perfect recall [4]; and 2) we formally prove that the Eleusis system with perfect recall is well-structured [1].

Theorem 1 *A binary relation $\leq$ is a partial order on states of Eleusis formal model El, such that El provided with this partial order is an ideal-based interpreted system and its perfect recall interpreted system $prs(El)$ with order $\preceq$ generated by $\leq$ is an ideal-based interpreted system also.*

Acknowledgments

The work was supported by the scholarship of DAAD (German Academic Exchange Service).

References

[1] N. O. Garanina, Common knowledge in well-structured perfect recall systems. *Aut. Control Comp. Sci.* **48** (2014), 381–388.

[2] M. Gardner, *The Second Scientific American Book of Mathematical Puzzles and Diversions.* University of Chicago Press Edition, 1987.

[3] M. Kacprzak, A. Lomuscio and W. Penczek, Verification of Multiagent Systems via Unbounded Model Checking. *Formal Approaches to Agent-Based Systems (FAABS 2004)*, 638-645, Lecture Notes in Comp. Sci., 3228, Springer, Berlin, 2004.

[4] A. R. Lomuscio, R. Van Der Meyden and M. Ryan, Knowledge in Multiagent Systems: Initial Configurations and Broadcast. *ACM Trans. Comput. Log.* **1** (2000), no. 2, 247–284.

[5] N.V. Shilov, N.O. Garanina and K.-M. Choe, Update and abstraction in model checking of knowledge and branching time. *Fund. Inform.* **72** (2006), no. 1-3, 347–361.

Frictional Unilateral Contact Problems in Continuum Mechanics – Analytical and Numerical Treatment

Joachim Gwinner

Institute of Applied Mathematics, Department of Aerospace Engineering, Universität der Bundeswehr München, Neubiberg/Munich, Germany

E-mail: joachim.gwinner@unibw-muenchen.de

This talk surveys recent progress in the analytical and numerical treatment of unilateral contact problems with friction in continuum mechanics. The variational formulation of such problems within the range of small strain linearized elasticity leads to variational inequalities (VIs), respectively to hemivariational inequalities (HVIs) when nonmonotone friction conditions are present. A broad existence theory for such VI-HVIs can be based on the Fan-KKM principle of nonlinear analysis [2]. For the numerical solution of nonmonotone frictional contact problems we combine regularization technics of nonsmooth optimization with finite element methods [1, 4]. In these papers we consider the delicate situation, where the elastic body is not fixed along some boundary part, but is only subjected to surface tractions and body forces. Thus there is a loss of coercivity leading to so-called semicoercive/noncoercive problems. For the solution of nonlinear interface problems for nonlinear material in the interior and linear elasticity in the exterior with nonmonotone set-valued transmission conditions we couple finite element and boundary element methods with regularization [5]. Finally in the talk we address unilateral contact with nonsmooth Tresca friction within finite strain elasticity and present existence and convergence results for a smoothing procedure [3].

References

[1] O. Chadli, J. Gwinner and M. Z. Nashed, Noncoercive variational-hemivariational inequalities: existence, approximation by double regularization, and application to nonmonotone contact problems. *J. Optim. Theory Appl.* **193** (2022), no. 1-3, 42–65.

[2] J. Gwinner, From the Fan-KKM principle to extended real-valued equilibria and to variational-hemivariational inequalities with application to nonmonotone contact problems. *Fixed Point Theory Algorithms Sci. Eng.* **2022**, Paper no. 4, 28 pp.

[3] J. Gwinner, On a frictional unilateral contact problem in nonlinear elasticity—existence and smoothing procedure. *Philos. Trans. Roy. Soc. A* **380** (2022), no. 2236, Paper no. 20210356, 12 pp.

[4] J. Gwinner and N. Ovcharova, A Gårding inequality based unified approach to various classes of semi-coercive variational inequalities applied to non-monotone contact problems with a nested max-min superpotential. *Minimax Theory Appl.* **5** (2020), no. 1, 103–128.

[5] J. Gwinner and N. Ovcharova, Coupling of finite element and boundary element methods with regularization for a nonlinear interface problem with nonmonotone set-valued transmission conditions. *Comput. Math. Appl.* **134** (2023), 45–54.

Duality and Extrapolation in Function Spaces of Lebesgue Type

Pankaj Jain

Department of Mathematics, South Asian University, New Delhi, India
E-mail: pankaj.jain@sau.ac.in

In this talk, we shall focus on two function spaces, namely, Lebesgue space and grand Lebesgue space. Along with Hardy type inequalities in these spaces, the aim would be to discuss the duality and extrapolation properties in these spaces. Of particular interest, in duality, is the Sawyer's duality principle which deals with function spaces consisting of monotone functions. As for the extrapolation is concerned, we would talk about Rubio de Francia type extrapolation, both in Lebesgue as well as in grand Lebesgue spaces. As applications to both duality and extrapolation, the boundedness of several well known integral operators will be deduced.

The Mystery of Binary Matrix Properties of Categories

Zurab Janelidze

Department of Mathematical Sciences, Stellenbosch University
Stellenbosch, South Africa
E-mail: zurab@sun.ac.za

Matrix properties of categories were introduced in [1]. They generalize a special type of Mal'tsev conditions on varieties of universal algebras. The name refers to the fact that these properties can be encoded as matrices (with integer entries). In [2], a computer-implementable algorithm was formulated for deciding when does one matrix property imply another. This allowed to visualise some fragments of the (infinite) poset of matrix properties, ordered by implication. In these talk we discuss the case of binary matrices: the entries of the matrix are 0's and 1's. The poset of binary matrix properties is already a mystery. In this talk, based on [3], we present results that determine some characteristics of this poset.

References

[1] Z. Janelidze, Matrices of terms and relations in categories. *MSc Thesis, Ivane Javakhishvili Tbilisi State University*, 2004.

[2] M. Hoefnagel, P.-A. Jacqmin and Z. Janelidze, The matrix taxonomy of finitely complete categories. *Theory Appl. Categ.* **38** (2022), Paper no. 19, 737–790.

[3] M. Hoefnagel, P.-A. Jacqmin, Z. Janelidze and E. van der Walt, On binary matrix properties. (*in press*).

Nonlinear Composition Operators in Grand Lebesgue Spaces

Alexey Karapetyants[1], **Massimo Lanza de Cristoforis**[2]

[1] *Institute of Mathematics, Mechanic and Computer Sciences & Regional Mathematical Center, Southern Federal University, Rostov-on-Don, Russia*
E-mail: karapetyants@gmail.com

[2] *Dipartimento di Matematica 'Tullio Levi-Civita', Università degli Studi di Padova Padova, Italia*
E-mail: mldc@math.unipd.it

Let Ω be an open subset of $\mathbb{R}^n$ of finite measure. Let f be a Borel measurable function from $\mathbb{R}$ to $\mathbb{R}$. We prove necessary and sufficient conditions on f in order that the composite function $T_f[g] = f \circ g$ belongs to the Grand Lebesgue space $L_{p),\theta}(\Omega)$ whenever g belongs to $L_{p),\theta}(\Omega)$.

We also study continuity, uniform continuity, Hölder and Lipschitz continuity of the composition operator T_f in $L_{p),\theta}(\Omega)$.

References

[1] A. Karapetyants and M. Lanza de Cristoforis, Composition operators in Grand Lebesgue spaces. *Analysis Math.*, 2023 (to appear).

Poroelastic Problem of a Non-Penetrating Crack with Cohesive Contact for Fluid-Driven Fracture

Victor Kovtunenko

Institut für Mathematik und Wissenschaftliches Rechnen KFU-Graz
Graz, Österreich, Austria
E-mail: victor.kovtunenko@uni-graz.at

This talk surveys recent progress in the analytical and numerical treatment. We introduce a new class of unilaterally constrained problems for fully coupled poroelastic models stemming from hydraulic fracturing and study its well-posedness. The poroelastic medium contains a fluid-driven crack, which is subjected to non-penetrating conditions and cohesion forces between the crack faces. Compared to the classical model of a hydraulically open fracture, non-penetration allows compression at which the fracture can be mechanically close. Solvability of the governing elliptic-parabolic variational inequality under the unilateral constraint with a small cohesion is established using the incremental approximation based on Rothés semi-discretization in time.

For the poroelastic system with cohesionless non-penetrating crack, the incremental model is expressed by a saddle-point problem with respect to the unknown solid phase displacement, pore pressure, and contact force. Applying the Lagrange multiplier approach and Delfour–Zolesio theorem, formula of the shape derivative under crack perturbation is derived. In the plane isotropic setting, a Fourier series solution is obtained in the sector of angle 2π with respect to distance to the crack-tip. A square-root singularity takes place, and no logarithmic terms occur in the asymptotic expansion. Integral formulas calculating stress intensity factors are rigorously calculated.

High-Dimensional Approximation in Periodic Function Spaces

Thomas Kühn

Faculty of Mathematics and Computer Science, Institute of Mathematics,
Leipzig University, Leipzig, Saxony, Germany
E-mail: Thomas.Kuehn@math.uni-leipzig.de

This talk is a survey of recent results on approximation in periodic function spaces, mainly in Sobolev spaces of finite smoothness, but also in Gevrey spaces, which consist of C^∞-functions. The approximation error can be expressed in terms of approximation numbers a_n of the embedding of the corresponding function space into L_2 or L_∞.

For classical isotropic, anisotropic or mixed-order Sobolev spaces the asymptotic rate of a_n – up to unspecified multiplicative constants – is well known since long ago. From a theoretical point of view this is fully satisfactory. For computational aspects of high-dimensional problems, however, it is rather useless to know only the asymptotic rate; then one needs additional information on the hidden constants, in particular how they depend on the dimension d of the underlying domain and the chosen norm. Even more important is the preasymptotic behaviour of a_n for 'small' n, say $n \leq 2^d$.

I will discuss recent progress in this direction for periodic Sobolev and Gevrey embeddings. The proofs, a combination of functional-analytic arguments and combinatorial estimates, provide optimal algorithms. The results are closely related to tractability issues in the sense of information-based complexity.

Verification of Neural Networks?

Martin Leucker
Institute for Software Engineering and Programming Languages, University of Lübeck
Lübeck, Schleswig-Holstein, Germany
E-mail: leucker@isp.uni-luebeck.de

Machine learning is a popular tool for building state of the art software systems. It is more and more used also in safety critical areas. This demands for verification techniques ensuring the safety and security of machine learning based solutions. However, in this presentation, we argue that the popularity of machine learning comes from the fact that no formal specification exists which renders traditional verification inappropriate. Instead, validation is typically demanded and we present a recent technique that validates certain correctness properties for an underlying recurrent neural network.

Weil Conjectures, Moduli of Bundles and Homotopy Types

Frank Neumann

Dipartimento di Matematica 'Felice Casorati', Università di Pavia, Pavia, Italy
E-mail: frank.neumann@unipv.it

After a gentle introduction into the classical Weil conjectures on how to count rational points on an algebraic variety over a finite field, I will give an overview on related recent results concerning the geometry, topology and arithmetic of moduli of vector bundles and principal bundles over an algebraic curve and other algebraic varieties. A main ingredient will be the understanding and calculation of cohomology algebras and homotopy types of algebraic stacks.

References

[1] F. Neumann, *Geometry of Algebraic Stacks and Moduli of Vector Bundles.* IMPA Research Monographs, Springer Verlag, Berlin–New York, 300 pp. (to appear).

[2] A. Castorena and F. Neumann, On actions of Frobenius morphisms for moduli stacks of principal bundles over algebraic curves. (in preparation).

[3] P. L. del Ángel Rodríguez and F. Neumann, Mixed Hodge theory of algebraic stacks. (in preparation).

[4] F. Neumann and U. Stuhler, Moduli stacks of vector bundles and Frobenius morphisms. *Algebra and number theory*, 126–146, *Hindustan Book Agency, Delhi*, 2005.

Rewriting Logic and Some of Its Applications to Distributed and Real-Time Systems

Peter Ölveczky
Department of Informatics, University of Oslo, Oslo, Norway
E-mail: peterol@ifi.uio.no

Rewriting logic is a simple and intuitive, yet expressive and general, computation logic developed by José Meseguer in the 1990s. Rewriting logic has a model theory with initial models, and a sound and complete proof system. Because of its simplicity and generality, rewriting logic is a convenient logical and semantic framework, in which many logics, modeling and programming languages, and computer systems in general, can be naturally represented.

Maude is a programming/modeling language and high-performance analysis tool for rewriting logic. Data types are defined by equational specifications, while dynamic behaviors are specified by rewrite rules. Maude models can then be subjected to: simulation by rewriting; reachability analysis; temporal logic model checking; and various forms of theorem proving using associated tools.

Rewriting logic and Maude have been applied to a wide range of problems and systems, including: transforming large libraries between different higher-order logics; providing formal semantics and analysis methods to programming and industrial modeling languages; commercially analyzing Ethereum smart contracts; analyzing distributed systems such as web browsers, security protocols, distributed algorithms, DDoS defense mechanisms, airplane controllers and turning algorithms; human cognition and thermoregulation; biochemical reactions; and so on.

In this talk I give a gentle introduction to rewriting logic and Maude and their extensions to real-time and probabilistic systems. I then give a sample of some Maude applications, focusing on real-time systems and industrial cloud-based transaction systems.

Morava K-Theory of Infinite Groups and Euler Characteristic

Irakli Patchkoria

Fraser Noble Building (Department of Mathematics, University of Aberdeen, Aberdeen, UK)
E-mail: irakli.patchkoria@abdn.ac.uk

Given a discrete group G with a finite model for $\underline{E}G$, we study $K(n)^*(BG)$ and $E^*(BG)$, where $K(n)$ is the n-th Morava K-theory on a given prime and E is the height n Morava E-theory. In particular, we generalize the character theory of Hopkins, Kuhn and Ravenel [1] who studied these objects for G finite. We give a formula for a localization of $E^*(BG)$ and the $K(n)$-theoretic Euler characteristic of BG in terms of centralizers. In certain cases these calculations lead to a full computation of $E^*(BG)$, for example when G is a right angled Coxeter group and $SL_3(\mathbb{Z})$. We also compute localized $E^*(BG)$ and the $K(n)$-theoretic Euler characteristic for certain special linear groups, symplectic groups and mapping class groups in terms of class numbers and special values of zeta functions. Finally, we introduce the asymptotic Euler characteristic at the infinite height and compute it for the latter groups. The answers are related to the p-fractional parts of the special values of zeta functions using the classical class number formula and Von Staudt–Clausen theorem. These results are partially joint with W. Lück and S. Schwede.

Acknowledgments

The work is supported by the EPSRC grant # EP/X038424/1 *"Classifying spaces, proper actions and stable homotopy theory"*.

References

[1] M. J. Hopkins, N. J. Kuhn and D. Ravenel, Generalized group characters and complex oriented cohomology theories. *J. Amer. Math. Soc.* **13** (2000), no. 3, 553–594.

Grand Lebesgue Spaces: Old and New

Humberto Rafeiro

Department of Mathematical Sciences, United Arab Emirates University
Al Ain, United Arab Emirates
E-mail: rafeiro@uaeu.ac.ae

In this talk, we will briefly discuss the notion of *old* grand Lebesgue spaces, as defined by Iwaniec–Sbordone, and present some recent progress in this topic, specifically the introduction of the concept of *local grand Lebesgue spaces*. Within the framework of these newly introduced spaces, we will explore their properties and demonstrate the boundedness of the maximal operator, singular operators with standard kernel, and potential operators.

This work is based on joint research with S. Samko and S. Umarkhadzhiev [1].

References

[1] H. Rafeiro, S. Samko and S. Umarkhadzhiev, Local grand Lebesgue spaces on quasi-metric measure spaces and some applications. *Positivity* **26** (2022), no. 3, Paper no. 53, 16 pp.

Investigation of Local and Nonlocal Problems

Maria Alessandra Ragusa

University of Catania, Catania CT, Italy
E-mail: mariaalessandra.ragusa@unict.it

I show the advances on the regularity problem and present recent results related to minimizers

$$u(x) : \Omega \to R^n$$

of quadratic and non quadratic growth functionals of the following type

$$\int_\Omega A(x, u, Du)\, dx,$$

where $\Omega \subset R^m$ is a bounded domain. About the dependence on the variable x is supposed that $A(\,\cdot\,, u, p)$ is in the vanishing mean oscillation class, as a function of x. Then, is pointed out that the continuity of $A(x, u, p)$, with respect to x, is not assumed.

Differentiation of Integrals and Multiple Fourier Series

Alex Stokolos

Department of Mathematics, Georgia Southern University, Statesboro Georgia, USA

E-mail: astokolos@georgiasouthern.edu

The subject of differentiation of integrals originated with problems discussed in the theory of the multiple Fourier series. Many well known mathematicians made contributions in the development of this area of research, Antoni Zygmund, Charles Fefferman, Levan Vladimirovich Zhizhiashvili, and their students and collaborators being among them. In the talk, I will present recent results related to the subject which were obtained in collaboration with Dmitriy Dmitrishin, Paul Hagelstein, Satbir Malhi and Giorgi Oniani.

o-Minimality and Arithmetic Geometry

Andrei Yafaev

Department of Mathematics, University College London (UCL), London, England
E-mail: yafaev@math.ucl.ac.uk

The theory of *o*-minimality stems from Model Theory (Mathematical Logic). Recently it proved to be extremely useful in Arithmetic Geometry and Number theory, in particular in questions about the distribution of certain types of points on certain types of algebraic varieties such as abelian or Shimura varieties. I has lead to a complete resolution of a 30 year old problem – the André–Oort conjecture. We will give an overview of the theory and its applications.

Abstracts of Sectional Talks

The Role of the Electronic Library in the Post-Pandemic Period

Inga Abuladze, Saba Vachnadze, Nana Maglakelidze

Georgian Technical University, Tbilisi, Georgia

E-mails: i.abuladze@gtu.ge; s.vachnadze@gtu.ge; n.maglakelidze@gtu.ge

The 21st century is considered the century of the digital world. The purpose of my research is the advantages of using cloud services in *e*-libraries around the world. The *e*-library system played a particularly big role during the pandemic, when people's movement was restricted. Using the electronic library saves people's time, makes the work process more efficient and interesting. In the post-pandemic period, *e*-library use has become even more practical and accessible from geographically distant locations. In the paper, cloud service models were developed for effective management of the electronic library system, according to which books of the same category will be placed in one section. These models will help the reader to find this or that book much easier. With the help of the cloud service, registration in the system will become flexible. At the same time, the cloud system can record how many readers visited this section every day. Allows readers to leave comments. The work is done in the metaprogramming Python language and SQLite is used as a database.

Some Results for Fixed Point Theory on Ultrametric Space

Özlem Acar, Aybala Sevde Özkapu

Mathematics, Selçuk University, Konya, Turkey

E-mail: acarozlem@ymail.com; aybalasevdeozkapu@yahoo.com

Let (X,d) be a metric space. If the metric d satisfies strong triangle inequality: for all $x,y,z\in X$

$$d(x,y)\leq \max\big\{d(x,z),d(y,z)\big\}$$

it is called ultrametric on X. Pair (X,d) now is called ultrametric space [2]. In 2001, Gajic [1] studied some fixed point results on ultrametric space. Gajic said that; Let (X,d) be a spherically complete ultrametric space. If $T:X\to X$ is a mapping such that for every $x,y\in X$, $x\neq y$,

$$d(Tx,Ty)\leq \max\big\{d(x,Tx),d(x,y),d(y,Ty)\big\}$$

then T has a unique fixed point.

In this talk, I will tell you about some fixed point results in ultrametric space.

References

[1] L. Gajić, On ultrametric space. *Novi Sad J. Math.* **31** (2001), no. 2, 69–71.

[2] C. Petalas and T. Vidalis, A fixed point theorem in non-Archimedean vector spaces. *Proc. Amer. Math. Soc.* **118** (1993), no. 3, 819–821.

Some Recent Results on Orthogonal Metric Space

Özlem Acar

Mathematics, Selçuk University, Konya, Turkey
E-mail: acarozlem@ymail.com

Fixed point problems are part of the famous and traditional theories in mathematics and have a wide range of applications and are studied in three different theories; topological, discrete and metric fixed point theory. The beginning of the metric fixed point problem is the famous Banach Contraction Principle given by Banach [1] in 1922. The crucial role of the principle in existence and uniqueness problems arising in mathematics has been realized which fact directed the researchers to extend and generalize the principle in many ways. In 2017, Gordji et. al. [2] defined the concept of an orthogonal set and gave an extension of the Banach Contraction Principle in orthogonal metric spaces and also they gave the application of this results.

My purpose in this talk is to interpret these studies by examining the fixed point studies given in orthogonal metric spaces.

References

[1] S. Banach, Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. (French) [On the operations in abstract sets and their application to integral equations] *Fund. Math.* **3** (1922), 133–181.

[2] M. E. Gordji, M. Ramezani, M. de la Sen and Y. J. Cho, On orthogonal sets and Banach fixed point theorem. *Fixed Point Theory* **18** (2017), no. 2, 569–578.

Approximation by Generalized Sampling Type Series: Recent Results

Tuncer Acar

Department of Mathematics, Selcuk University, Konya, Türkiye
E-mail: tunceracar@ymail.com

In this work, we present recent results on approximation by sampling type operators. We start with approximation properties of generalized sampling series and generalized sampling Kantorovich series in weighted spaces of functions. Then, in the second part, we study strong converse inequalities for generalized sampling series. At the end, we show rate of the simultaneous approximation by generalized sampling series and their Kantorovich modifications. This part will include the following topics:

1. Convergence of generalized sampling series in weighted spaces;
2. Approximation by sampling Kantorovich series in weighted spaces of functions;
3. A strong converse inequality for generalized sampling operators;
4. A characterization of the rate of the simultaneous approximation by generalized sampling operators and their Kantorovich modification.

References

[1] T. Acar, O. Alagöz, A. Aral, D. Costarelli, M. Turgay and G. Vinti, Convergence of generalized sampling series in weighted spaces. *Demonstr. Math.* **55** (2022), no. 1, 153–162.

[2] T. Acar, O. Alagöz, A. Aral, D. Costarelli and M. Turgay, Approximation by sampling Kantorovich series in weighted space of functions. *Turkish J. Math.* **46** (2022), no. 7, 2663–2676.

[3] T. Acar and B. R. Draganov, A strong converse inequality for generalized sampling operators. *Ann. Funct. Anal.* **13** (2022), no. 3, Paper no. 36, 16 pp.

[4] T. Acar and B. R. Draganov, A characterization of the rate of the simultaneous approximation by generalized sampling operators and their Kantorovich modification. *J. Math. Anal. Appl.* (accepted).

[5] C. Bardaro, G. Vinti, P. L. Butzer and R. L. Stens, Kantorovich-type generalized sampling series in the setting of Orlicz spaces. *Sampl. Theory Signal Image Process.* **6** (2007), no. 1, 29–52.

[6] P. L. Butzer, W. Splettstösser, A sampling theorem for duration-limited functions with error estimates. *Information and Control* **34** (1977), no. 1, 55–65.

[7] P. L. Butzer, W. Engels, S. Ries and R. L. Stens. The Shannon sampling series and the reconstruction of signals in terms of linear, quadratic and cubic splines. *SIAM J. Appl. Math.* **46** (1986), no. 2, 299–323.

[8] D. Costarelli and G. Vinti, Rate of approximation for multivariate sampling Kantorovich operators on some functions spaces. *J. Integral Equations Appl.* **26** (2014), no. 4, 455–481.

[9] A. D. Gadjiev, A problem on the convergence of a sequence of positive linear operators on unbounded sets, and theorems that are analogous to P. P. Korovkin's theorem. (Russian) *Dokl. Akad. Nauk SSSR* **218** (1974), 1001–1004.

Approximation Properties of Sampling Type Operators in Orlicz Spaces

Tuncer Acar, Dilek Özer, Metin Turgay

Mathematics, Selcuk University, Konya, Turkey

E-mails: tunceracar@ymail.com; dilekozer@yahoo.com; metinturgay@yahoo.com

In this talk, we construct a new form of sampling type operators. After, we focus on approximation properties in Orlicz spaces. We also furnish a quantitative estimate for the order of approximation, using the suitable modulus of continuity of the target functions. Then, we obtain a modular convergence theorem in the general setting of Orlicz spaces, $L^\eta(\mathbb{R})$.

References

[1] T. Acar, O. Alagöz, A. Aral, D. Costarelli and M. Turgay, Approximation by sampling Kantorovich series in weighted space of functions. *Turkish J. Math.* **46** (2022), no. 7, 2663–2676.

[2] T. Acar, O. Alagöz, A. Aral, D. Costarelli, M. Turgay and G. Vinti, Convergence of generalized sampling series in weighted spaces. *Demonstr. Math.* **55** (2022), no. 1, 153–162.

[3] O. Alagöz, M. Turgay, T. Acar, M. Parlak, Approximation by sampling Durrmeyer operators in weighted space of functions. *Numer. Funct. Anal. Optim.* **43** (2022), no. 10, 1223–1239.

[4] C. Bardaro, G. Vinti, P. L. Butzer and R. L. Stens, Kantorovich-type generalized sampling series in the setting of Orlicz spaces. *Sampl. Theory Signal Image Process.* **6** (2007), no. 1, 29–52.

[5] B. R. Draganov, A fast converging sampling operator. *Constr. Math. Anal.* **5** (2022), no. 4, 190–201.

Jensen's inequality in mathematics Olympiad problems

Vladimer Adeishvili[1], **Ivane Gokadze**[2]

[1]*Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: vladimer.adeishvili@atsu.edu.ge

[2]*Ltd School# 1, Kutaisi, Georgia*
E-mail: ivane.gokadze@atsu.edu.ge

In mathematical Olympiads of various ranks, it is not uncommon to find inequalities to be proven, in the process of proving the truth of which Jensen's inequality is used with considerable success. In order to be able to apply Jensen's inequality appropriately, it would be good if we have a solid knowledge and a well-thought-out understanding of the increasing and decreasing functions of some well-known functions, because often it is involving these functions and relying on their properties that the desired result is obtained. It is also worth noting that the question of the truth of many well-known inequalities is proved quite easily by using Jensen's inequality. Several tasks are discussed in the paper, in the solution of which we need Jensen's inequality. Also, tasks for independent work are offered to interested readers.

Development of a Block Method for Solving Multiple Order Odes

Emmanuel O. Adeyefa

Mathematics Department, Federal University Oye-Ekiti, Nigeria
E-mail: emmanuel.adeyefa@fuoye.edu.ng

In this work, a convergent hybrid block method (CHBM) with two off-grid points for direct integration of first, second, and third-order initial value problems (IVPs) is proposed. The development of a block method for the solution of IVPs has been considered overwhelmingly in the literature. However, using a block method to directly solve multi-order IVPs has not been so common. Thus, the formulation of a single numerical algorithm for the direct numerical integration of first, second and third-order IVPs is our focus. The method is formulated from a continuous scheme derived using collocation and interpolation techniques and implemented in a block-by-block manner as a numerical integrator for IVPs. To assess the method's applicability, efficiency, and accuracy, the convergence analysis has been investigated, and six test problems are considered

References

[1] E. O. Adeyefa and J. O. Kuboye, Derivation of new numerical model capable of solving second and third order ordinary differential equations directly. *IAENG Inter. J. Appl. Math.* **50** (2020), no. 2, 233–241.

Applying Percolation and Q Analysis Methods to Design Sustainable Urban Systems

Merab Akhobadze, Elguja Kurtskhalia

Department of Interdisciplinary Informatics, Georgian Technical University
Tbilisi, Georgia

E-mails: m.akhobadze@gtu.ge; e.kurtskhalia@gtu.ge

The theory of percolation explores the features and properties of both isolated and interconnected territories and landscapes. Therefore, it is a very valuable and applicable theory for the planning and management of urban systems, as well as for determining the placement of buildings, blocks, green areas and their optimal density. To create sustainable systems, it is very important to achieve a certain compromise between urbanization, nature conservation, agricultural expansion and other areas, because buildings and road infrastructure are the anthropogenic concerns of the landscape. Indeed, among them, distances, topology determine the environmental, social, economic and other problems of the urban system.

The paper presents algorithms created using the theories of percolation, q-analysis and Voronoi diagrams that allow you to systematically study the morphology of buildings and detect those buildings whose "presence" creates a continuous connection during the spread of pandemics, fires and other adverse events

References

[1] R. H. Atkin, From cohomology in physics to q-connectivity in social science. *Inter. J. Man-Machine Stud.* **4** (1972), no. 2, 139–167.

[2] I. Zakharkin, Voronoi diagram and its applications. (Russian) `https://habr.com/ru/post/309252/`, 2016.

[3] M. Akhobadze and E. Kurtskhalia, Mathematical model of urban planning for sustainable development and reconstruction of the city. *Bull. Georgian Natl. Acad. Sci. (N.S.)* **16** (2022), no. 1, 7–10.

[4] E. Kurtskhalia, N. Imnadze and O. Mchedlishvili, For optimal planning and management of the urban system. *Collection of Scientific Works of Stu*, 2022, no. 4(526).

[5] M. Ahobadze, *Issues of Mathematical Modeling of Macrosystems (Based on the Principle of Entropy Maximization).* (Georgian) Monograph, 2021.

Mathematical Model and Algorithm for Technical Diagnostics and Rehabilitation of Building Structures

Merab Akhobadze, Elguja Kurtskhalia

Department of Interdisciplinary Informatics, Technical University of Georgia
Tbilisi, Georgia
E-mails: m.akhobadze@gtu.ge; *E-mail:* e.kurtskhalia@gtu.ge

There are many methods for diagnosing building structures of a particular class and purpose, most of which are based on the use of mathematical statistics methods, the use of which is often incorrect, since mathematical statistics methods can only be used when the object under study is described by a random process model.

In practice, the measurements of the parameters of building structures are carried out inaccurately, the data are fuzzy, the object is affected by hard-to-observe external influences. Operational control of a number of parameters is impossible. In many cases, the relationships between the structural elements of buildings are very difficult to formalize, and the measurement procedures are carried out by specialists with low qualifications. Obviously, in such a situation, it is necessary to create such mathematical models and diagnostic algorithms that will be less sensitive to measurement errors, qualifications of maintenance personnel, and in the presence of other uncertainties.

In work using fuzzy set theories and Q analysis, a technique for mathematical modeling of building structures and algorithms for assessing the state of building structures were created, which are implemented by combining expert knowledge and current measurements carried out at the diagnostic object.

References

[1] M. Akhobadze, *Issues of Mathematical Modeling of Macrosystems (Based on the Principle of Entropy Maximization).* (Georgian) Monograph, ISBN 978-9941-8-3483-7, 2021.

[2] M. Akhobadze and E. Kurtskhalia, Mathematical model of urban planning for sustainable development and reconstruction of the city. *Bull. Georgian Natl. Acad. Sci.* **16** (2022), no. 1, 7–10.

[3] R. H. Atkin, From cohomology in physics to q-connectivity in social science. *Internat. J. Man-Mach. Stud.* **4** (1972), 139–167.

[4] D. Gurgenidze, M. Akhobadze, M. Tsikarishvili and T. Bulia, New optimal planning of restoration and technical diagnostics of sea berths on the basis of Q analysis and fuzzy set theory. *Bull. Georgian Natl. Acad. Sci.* **17** (2023), no. 1, 38–42.

[5] V. N. Tutubalin, *Theory of Probability. A Short Course, and Scientific and Methodological Remarks.* (Russian) Izdat. Moskov. Univ., Moscow, 1972.

[6] L. A. Zadeh, Fuzzy sets. *Information and Control* **8** (1965), 338–353.

On the Countable Spectrum of Weakly o-Minimal Theories

Gulnaz Akimbekova[1], **Beibut Kulpeshov**[2]

[1]*Department of Science and Innovation, Kazakh-British Technical University Almaty, Kazakhstan*
E-mail: akguka@gmail.com

[2]*School of Applied Mathematics, Kazakh-British Technical University Almaty, Kazakhstan*
E-mail: kulpesh@mail.ru

The lecture concerns the notion of *weak o-minimality* originally deeply studied in joint work by D. Macpherson, D. Marker and C. Steinhorn [5]. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ such that any definable (with parameters) subset of the structure M is a union of finitely many convex sets in M. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures. All the necessary definitions and statements can be found in [1, 2, 5]. In [3, 4, 6] the countable spectrum of variants of o-minimality was studied. As usual, we denote by $I(T,\omega)$ the countable spectrum of a complete theory T, i.e. the number of pairwise non-isomorphic countable models of T.

Theorem 1 *Let T be a weakly o-minimal theory of finite convexity rank having fewer than 2^ω countable models. Then there exist $\Gamma_1 = \{p_1, p_2, \dots, p_m\}$, $\Gamma_2 = \{q_1, q_2, \dots, q_l\}$ – maximal pairwise weakly orthogonal families of quasirational and irrational 1-types over $\varnothing$ respectively for some $m, l < \omega$ such that*

$$I(T,\omega) = \prod_{i=1}^{m} (\kappa_i + 3) * \prod_{j=1}^{l} (\lambda_j^2 + 5\lambda_j + 6),$$

where κ_i (λ_j) is maximal number of non-algebraic pairwise almost quite orthogonal 1-types over $\varnothing$ that are non-weakly orthogonal, but almost quite orthogonal to p_i (q_j) for each $1 \le i \le m$ ($1 \le j \le l$).

Acknowledgments

The work was supported by Science Committee of Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant # AP19674850).

References

[1] B. S. Baizhanov, Expansion of a model of a weakly o-minimal theory by a family of unary predicates. *J. Symbolic Logic* **66** (2001), no. 3, 1382–1414.

[2] B. Sh. Kulpeshov, Weakly o-minimal structures and some of their properties. *J. Symbolic Logic* **63** (1998), no. 4, 1511–1528.

[3] B. Sh. Kulpeshov, Vaught's conjecture for weakly o-minimal theories of finite convexity rank. (Russian) *Izv. Ross. Akad. Nauk Ser. Mat.* **84** (2020), no. 2, 126–151; translation in *Izv. Math.* **84** (2020), no. 2, 324–347.

[4] B. Sh. Kulpeshov and S. V. Sudoplatov, Vaught's conjecture for quite o-minimal theories. *Ann. Pure Appl. Logic* **168** (2017), no. 1, 129–149.

[5] D. Macpherson, D. Marker and Ch. Steinhorn, Weakly o-minimal structures and real closed fields. *Trans. Amer. Math. Soc.* **352** (2000), no. 12, 5435–5483.

[6] L. L. Mayer, Vaught's conjecture for o-minimal theories. *J. Symbolic Logic* **53** (1988), no. 1, 146–159.

Varieties of Exponential R-Groups

Mikheil Amaglobeli[1], **Alexei Myasnikov**[2]

[1] *Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: mikheil.amaglobeli@tsu.ge

[2] *Schaefer School of Engineering & Science, Department of Mathematic Sciences, Stevens Institute of Technology, Castle Point on Hudson, Hoboken NJ, USA*
E-mail: amiasnikov@gmail.com

The concept of a exponential R-group, where R is an arbitrary associative ring with identity, was introduced by R. Lindon in [1]. A. G. Myasnikov and V. N. Remeslennikov refined the concept of an exponential R-group by introducing additional axiom [2]. The new notion of an R-group is a straightforward generalization of the notion of an R-module to the case of non-commutative groups. In the article by M. G. Amaglobeli and V. N. Remeslennikov [3] R-groups with this additional axiom are called MR-groups. It turned out that all previously studied Lyndon R-groups are actually MR-groups (including the free Lyndon $\mathbb{Z}[t]$-group $F^{\mathbb{Z}[t]}$). In this talk we discuss MR-groups. Tensor extensions of the scalar rings for modules play an important part in the theory if modules. The authors of [2] defined an exact analogue of this construction for an arbitrary R-group, called a tensor completion.

The talk introduces the concepts of a variety of exponential R-groups and a tensor completion of groups in a variety. We study connections between the free groups of a given variety for different scalar rings. Abelian varieties of R-groups are described. In addition, in the category of R-groups, various analogues of the concept of an n-nilpotent R-group are introduced and their comparison in this category is given. It is shown that the completion of a 2-nilpotent R-group is 2-nilpotent [4].

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation (SRNSF grant # FR 21-4713).

References

[1] R. C. Lyndon, Groups with parametric exponents. *Trans. Amer. Math. Soc.* **96** (1960), 518–533.

[2] A. G. Myasnikov and V. N. Remeslennikov, Groups with exponents I. Fundamentals of the theory and tensor completions, *Sib. Math. J.* **35** (1994), no. 5, 986–996.

[3] M. G. Amaglobeli and V. N. Remeslennikov, Free nilpotent R-groups of class 2. (Russian) *Dokl. Akad. Nauk* **443** (2012), no. 4, 410–413; translation in *Dokl. Math.* **85** (2012), no. 2, 236–239.

[4] M. G. Amaglobeli, Varieties of exponential *MR*-groups. (Russian) *Dokl. Akad. Nauk* **490** (2020), no. 1, 5–8; translation in *Dokl. Math.* **101** (2020), no. 1, 1–4.

Yolov8 Platform-Based OCR Tool for Georgian Handwritten Text Recognition

Maia Archuadze, Ana Chikashua, Magda Tsintsadze

[1]*Department of Computer Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: ana.chikashua5118@ens.tsu.edu.ge; maia.archuadze@tsu.ge; magda.tsintsadze@tsu.ge

Optical character recognition (OCR) is a widely used field of research in artificial intelligence, pattern recognition and computer vision. The field is broad and has use cases across various industries like financial and banking systems, healthcare, logistics, government services etc. [3]. In the work we concentrate on handwriting recognition (HWR) that is a technique of recognizing and interpreting handwritten data into machine-readable output, which is still considered a challenging problem, especially for not widely recognized and use languages like Georgian Language.

The paper is focused on the development of an optical character recognition system specially designed for the Georgian language. The main goal is to create an accurate and efficient OCR model using the YOLOv8 architecture [2] based on the Georgian-language database (with 48625 unit data).

The solution was carried out in several stages: collection and augmentation [1], model selection, testing and comparative analysis. Based on the results of the performance of three models: CNN, ResNet50 and YOLOv8. last one was the best fit for our data.

After training and evaluating the aforementioned models on the training dataset, it was determined that the YOLOv8 model yielded the best results. The CNN model, incorporating convolutional, pooling, and dropout layers, achieved an accuracy of 0.9230, whereas the ResNet model achieved an accuracy of 0.9302. In contrast, YOLOv8 achieved an accuracy of 0.98848, with a loss value of 0.04479. These results indicate that YOLOv8 performed exceptionally well during live testing. Furthermore, it is noteworthy that YOLOv8 demonstrated remarkable speed in generating results during the testing phase.

References

[1] R. Atienza, Data augmentation for scene text recognition. *IEEE/CVF International Conference on Computer Vision Workshops (ICCVW)*, pp. 1561–1570, Montreal, BC, Canada, 2021.

[2] M. Krishnakumar, A gentle introduction to YOLOv8. https://wandb.ai/mukilan/wildlife-yolov8/reports/A-Gentle-Introduction-to-YOLOv8--Vmlldzo0MDU5NDA2, 2023.

[3] D. Mangnani, J. Sukheja, H. Mohinani and P. Kanade, Handwritten character recognition. *IJCRT* **9** (2021), no. 6, a268–a275.

REST and Event-Driven Approaches in Microservices Architecture

Maia Archuadze, Magda Tsintsadze

Computer Science Department, Iv.Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mails: maia.archuadze@tsu.ge; magda.tsintsadze@tsu.ge

Microservices offer a solution to the challenges posed by monolithic applications [1]. Monolithic applications are tightly coupled systems that perform multiple tasks as a single entity. In contrast, a microservice architecture, composed of small interconnected services, presents a more efficient solution for handling heavy loads. However, ensuring efficient communication between microservices remains a significant challenge in system design.

Microservices can communicate with each other using both event-based and RESTful APIs. While RESTful APIs still hold relevance in today's world, the nature of event-based communication and the scalability of messaging platforms like Kafka have made it a dominant force in the software industry. It is worth noting that a combination of both approaches can be advantageous in cases where synchronous communication is required [2].

In the work we present a comparative analysis of event-based and RESTful API approaches. Results demonstrate that the event-based approach outperforms RESTful APIs in several key metrics [3]. These findings confirm the advantages of employing an event-based architecture in microservices applications. However, we acknowledge that a hybrid approach combining both event-based and RESTful API communication can be beneficial in specific scenarios.

References

[1] P. Irudayaraj, P. Saravanan, Adoption advantages of micro-service architecture in software industries. *International Journal Of Scientific And Technology Research* **8** (2019), no. 09, 183–186.

[2] S. Zhelev and A. Rozeva, Using microservices and event driven architecture for big data stream processing. *AIP Conference Proceedings* **2172** (2019), no. 1, 8 pp.

[3] Z. Qin, X. Zheng and J. Xing, Evaluating Software Architecture. In: *Software Architecture. Advanced Topics in Science and Technology in China*, pp. 221–273, Springer, Berlin, Heidelberg, 2008.

Using Parallel Data in Forecasting the Currency Exchange Rate

Natela Archvadze[1], **Givi Lemonjava**[2], **Merab Pkhovelishvili**[3]

[1]*Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: natela.archvadze@tsu.ge

[2]*University of Georgia, Tbilisi, Georgia*
E-mail: givilemonjava@yahoo.com

[3]*Faculty of Informatics and Control Systems, Georgian Technical University Tbilisi, Georgia*
E-mail: m.pkhovelishvili@gtu.ge

Parallel data algorithms can be effectively used in various economic and financial tasks, especially in forecasting tasks with several models. Each of these models has its advantages, but none of these models allows determining the prognostic value with minimal possible error.

We consider the performance of the algorithm of parallel forecasting data. For this, we took the following models: Averaging Methods, Moving Averages, Simple Exponential Smoothing, Holt's Exponential Smoothing, and Winter's Exponential Smoothing. When choosing the best pair, they discussed the exchange rates of 2022 in Georgia and calculated the error of these models when calculating the values.

In the presented method, the average arithmetic forecast for each day is taken for each pair. During the next month, these average values are summed up and divided to quantity days. Next, a pair is used, the intersection of the forecast values with time gives us the smallest error compared to the actual value. Pairs should be selected monthly, as subsequent models may find the best pair based on both their value and the predictions of the other pair.

A particularly effective pair is selected when the value of the prediction determined by one model is greater, and the value of the other model is less than the actual value. Then the arithmetic mean of this pair is very close to the real value.

Forecasting exchange course next, a pair is used, the intersection of the forecast values with time gives us the smallest error compared to the actual value. Pairs should be selected monthly, as subsequent models may find the best pair based on both their value and the predictions of the other pair.

References

[1] G. Lemonjava, Time series models for forecasting exchange rates. *Globalization & Business*, 2019; `https://doi.org/10.35945/gb.2019.08.020`.

[2] A. Prangishvili, Z. Gasitashvili, M. Pkhovelishvili and N. Archvadze, Theory of universal approach to improve predictive models using parallel data and application examples. *AIP Conference Proceedings* **2757** (2023), no. 1.

Consistent Criteria for Hypothesis Testing for Haar's Statistical Structures

Lela Aleksidze, Laura Eliauri, Zurab Zerakidze

[1]*Faculty of Educations, Exact and Natural Sciences, Gori State Teaching University Gori, Georgia*

E-mails: l.aleksidze@gmail.com; l.eliauri@gmail.com; zura.zerakidze@mail.ru

Definition 1 Let E be an arbitrary locally compact and σ-compact topological group and $\mathcal{B}(E)$ is σ-algebra of subsets of E. We say that measure μ defined on $\mathcal{B}(E)$ is Haar measure if μ is regular measure and

$$\mu(sX) = \mu(X), \ \ \forall\, s \in E, \ \ \forall\, X \in \mathcal{B}(E).$$

Definition 2 An object $\{E, B(E), \mu_h, h \in H\}$ is called Haar's statistical structure, where $\{\mu_h, h \in H\}$ is a family of Haar probability measures.

For each $h \in H$ denote by $\overline{\mu}_h$ the completion of the measure μ_h and denote by $dom(\overline{\mu}_h)$ the σ-algebra of all $\overline{\mu}_h$-measurable of E. Let

$$S_1 = \bigcap_{h \in H} dom(\overline{\mu}_h).$$

Definition 3 Haar statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ is called strongly separable if there exists a family of S_1-measurable sets $\{Z_h, h \in H\}$ such that the following realations are fulfilled:

(1) $\overline{\mu}_h(Z_h) = 1, \forall\, h \in H$;

(2) $Z_{h_1} \cap Z_{h_2} = \varnothing, \forall\, h_1 \neq h_2,\ h_1, h_2 \in H$;

(3) $\bigcup_{h \in H} Z_h = E$.

Definition 4 We will say that the Haar's statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ admits a consistent criterion for hypothesis testing if there exists at least one measurable mapping $\delta : (E, S_1) \to (H, \mathcal{B}(H))$ such that

$$\overline{\mu}_h(\{x : \ \delta(x) = h\}) = 1, \ \ \forall \ h \in H.$$

Theorem *Let Haar's statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $CardH = c$ strongly separable, then Haar's statistical structure admits δ consistent criterion for hypothesis testing and $\delta^{-1}(\mathcal{B}(H))$ is a sufficient statistic.*

Counting Complex Points of Two Dimensional Surfaces

Teimuraz Aliashvili[1], **Gvantsa Kapanadze**[2]

[1] *Institute of Fundamental and Interdisciplinary Mathematics Study, Ilia State University Tbilisi, Georgia*

E-mail: teimuraz.aliashvili@iliauni.edu.ge

[2] *Ilia State University, Tbilisi, Georgia*

E-mail: gvantsa.kapanadze.3@iliauni.edu.ge

We deal with complex point of two dimensional surfaces. For algebraic surfaces, a formula is proved which expresses the number of complex points as local degree of an explicitly constructible polynomial endomorphism.

Theorem 1 *For a generic $X \subset C^2$ the number of complex points of $X = \{f = 0, g = 0\} \subset C^2$ is given by*

$$c(X) = \frac{1}{2}\,(1 - \deg_0 \bigtriangledown H),$$

where H is Bruce polynomial.

Theorem 2 *The number of complex points of a generic algebraic surface defined by two equations of degree $m \geqslant 2$ does not exceed $P_5(4m-1)$.*

References

[1] 1 T. Aliashvili, Counting real roots of polynomial endomorphisms. Complex analysis. *J. Math. Sci. (N.Y.)* **118** (2003), no. 5, 5325–5346.

[2] J. W. Bruce, Euler characteristics of real varieties. *Bull. London Math. Soc.* **22** (1990), no. 6, 547–552.

[3] G. N. Khimshiashvili, The local degree of a smooth mapping. (Russian) *Sakharth. SSR Mecn. Akad. Moambe* **85** (1977), no. 2, 309–312..

On Characterization of Two-Weight Norm Inequalities for Multidimensional Hausdorff Operators on Lebesgue Spaces

Dunya R. Aliyeva[1], **Rovshan A. Bandaliyev**[2]

[1]*Mathematical Analysis, Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan Baku, Azerbaijan*
E-mail: dunya@box.az

[2]*Higher Mathematics, Azerbaijan University of Architecture and Construction Baku, Azerbaijan*
E-mail: bandaliyevr@gmail.com

In this abstract we give necessary conditions and sufficient conditions for the boundedness of multidimensional Hausdorff operator on weighted Lebesgue spaces. In particular, we establish necessary and sufficient conditions for the boundedness of special type multidimensional Hausdorff operator on weighted Lebesgue spaces for monotone radial weight functions. Also, we get similar results for important operators of harmonic analysis which are special cases of the multidimensional Hausdorff operator.

For detail, see [1–4].

References

[1] E. Liflyand, Hardy type inequalities in the category of Hausdorff operators. *Modern methods in operator theory and harmonic analysis*, 81–91, Springer Proc. Math. Stat., 291, *Springer, Cham*, 2019.

[2] R. A. Bandaliyev and P. Górka, Hausdorff operator in Lebesgue spaces. *Math. Inequal. Appl.* **22** (2019), no. 2, 657–676.

[3] R. A. Bandaliyev and K. H. Safarova, On boundedness of multidimensional Hausdorff operator in weighted Lebesgue spaces. *Tbilisi Math. J.* **13** (2020), no. 1, 39–45.

[4] R. A. Bandaliyev and K. H. Safarova, On two-weight inequalities for Hausdorff operators of special kind in Lebesgue spaces. *Hacet. J. Math. Stat.* **50** (2021), no. 5, 1334–1346.

Generating Matches Between Georgian and English Nouns and Adjectives in a Grammatical Dictionary Software Application

Nino Amirezashvili, Liana Lortkipanidze, Liana Samsonadze

Institute of control systems of the Technical University of Georgia
Tbilisi, Georgia

E-mails: ninomaskh@yahoo.com; l_lordkipanidze@yahoo.com; liasams@yahoo.com

A grammatical dictionary is a language guide containing extensive information about the morphological and syntactic properties of words, which serves as the basis for understanding the rules for their use [1]. One of the important issues for the creation of the Georgian-English grammatical dictionary is to determine the correspondence between Georgian and English characteristics. In the article grammatical categories typical for Georgian adjectives and nouns and their characteristics have been discussed. It is shown the categories which are carried by the Georgian adjectives or nouns and, if they are not present in English, how such forms are compensated during generate of adequate words.

Acknowledgments

This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant # FR-21-3509.

References

[1] Russian Grammatical Dictionary,
https://seelrc-iis.trinity.duke.edu/russdict/index.aspx?doc=2.

About the Genetic Algorithm for Solving the Traveling Salesman Problem

Natela Ananiashvili

[1]*Georgian Aviation University, Tbilisi, Georgia*
E-mail: n.ananiashvili@ssu.edu.ge

The traveling salesman problem belongs to the class of discrete optimization NP-hard problems [3]. Its solution is very important both from a practical and a theoretical point of view. This task involves finding the minimum weight Hamiltonian cycle in a weighted complete graph. Since the exact solution can be obtained through complete selection, which is impossible in the case of large dimensions, therefore, approximate algorithms are used, which obtain solutions close to the optimal in a short time interval [1, 2]. The paper proposes a genetic algorithm for solving the salesman's problem for graphs for which the vertices are located on the plane and the distance between them is calculated according to the coordinates (in Cartesian rectangular coordinate system). With the proposed algorithm, it is possible to get a solution close to the optimal one.

References

[1] N. Ananiashvili, About one heuristic algorithm for solving the traveling salesman problem. (Georgian) *Archil Eliashvili Institute of Control Systems of the Georgian Technical University Proceedings*, no. 26, (2022), 194–198.

[2] R. Bellman, Dynamic programming treatment of the travelling salesman problem. *J. Assoc. Comput. Mach.* **9** (1962), 61–63.

[3] M. R. Garey and D. S. Johnson, *Computers and Intractability. A Guide to the Theory of NP-Completeness.* A Series of Books in the Mathematical Sciences. W. H. Freeman and Co., San Francisco, Calif., 1979.

On The Absolute Convergence of The Multiple Series of Fourier–Haar Coefficients

Aleksandre Aplakovi

Department of Mathematics, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: aleksandre.aplakovi@tsu.ge

As is well known, the Haar and Walsh systems are successfully applied in signal transmission processes. In this direction an important role is played by the study of the behavior of the signal, as a sum of the absolute values of the Fourier coefficients.

In this paper we study the problem of absolute convergence of the N-dimensional series of Fourier–Haar coefficients for the classes of functions with bounded partial p-variations.

Let

$$\chi_{\vec{m}}(\vec{x})=\prod_{i=1}^{N}\chi_{m_i}(x_i),\ \ x_i\in I=[0,1]\ \ (i=1,2,\dots,N),\ \ N\geq 2$$

is the multiple Haar system on $I^N=[0,1]^N$, where

$$\vec{x}=(x_1,\dots,x_N),\ \ \vec{m}=(m_1,\dots,m_N)\ \ (m_i=1,2,\dots;\ \ i=1,\dots,N)$$

Definition ([1]) Let f be a function defined on $[0,1]^N$ and 1-periodic with respect to each variable. f is said to be a function of bounded partial p-variation ($f\in PBV_p(I^N)$), if for any $i=1,2,\dots,N$ and $n=1,2,\dots$

$$V_i(f)=\sup_{x_j,j\in\{1,\dots,N\}\setminus\{i\}}\sup_{\Pi}\sum_{k=0}^{n-1}\Big|f\big(x_1,\dots,x_{i-1},x_i^{(2k)},x_{i+1},\dots,x_N\big)-f\big(x_1,\dots,x_{i-1},x_i^{(2k+1)},x_{i+1},\dots,x_N\big)\Big|^p<\infty,$$

where Π is an arbitrary system of disjoint intervals $(x_i^{(2k)},x_i^{(2k+1)})$ $(k=0,1,\dots,n-1)$ on $[0,1]$, i.e.

$$0\leq x_i^{(0)}<x_i^{(1)}<x_i^{(2)}<\dots<x_i^{(2n-2)}<x_i^{(2n-1)}\leq 1.$$

Theorem 1 *Let $f\in PBV_p(I^N)$, $p\geq 1$ and $\beta>0$, $\alpha+1<\beta(\frac{1}{pN}+\frac{1}{2})$. Then*

$$\sum_{n_1=0}^{\infty}\cdots\sum_{n_N=0}^{\infty}\prod_{i=1}^{N}(n_i+1)^{\alpha}|C_{n_1,\dots,n_N}(f)|^{\beta}<\infty.$$

References

[1] U. Goginava, On the uniform convergence of multiple Fourier series with respect to the trigonometric system. *Bull. Georgian Acad. Sci.* **159** (1999), no. 3, 392–395.

Approximation by Nörlund Means with Respect to Walsh System in Lebesgue Spaces

Nika Areshidze[1], **George Tephnadze**[2]

[1] *Department of Mathematics, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: nika.areshidze15@gmail.com

[2] *School of Science and Technology, The University of Georgia, Tbilisi, Georgia*
E-mail: g.tephnadze@ug.edu.ge

The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves. The development of the theory of Vilenkin-Fourier series has been strongly influenced by the classical theory of trigonometric series. Because of this it is inevitable to compare results of Vilenkin series to those on trigonometric series. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, which studies orthonormal systems from the point of view of the structure of a topological group. Some important steps in the early development can be found in the book by F. Schipp, W. R. Wade, P. Simon and J. Pál [3] from 1990. The research continued intensively also after this. Some of the most important steps in these developments are presented in the resent book by L. E. Persson, G. Tephnadze and F. Weisz [4] from 2022.

This talk is devoted to improve and complement a result by Móricz and Siddiqi [2]. In particular, we prove that their estimate of the Nörlund means with respect to the Walsh system holds also without their additional condition. Moreover, we prove a similar approximation result in Lebesgue spaces for any $1 \leq p < \infty$ (for details see [1]).

References

[1] N. Areshidze and G. Tephnadze, Approximation by Nörlund means with respect to Walsh system in Lebesgue spaces. *Math. Inequal. Appl.* (to appear).

[2] F. Móricz and A. Siddiqi, Approximation by Nörlund means of Walsh–Fourier series. *J. Approx. Theory* **70** (1992), no. 3, 375–389.

[3] L. E. Persson, G. Tephnadze and F. Weisz, *Martingale Hardy Spaces and Summability of Vilenkin–Fourier Series.* Birkhäuser Cham, 2022.

[4] F. Schipp, W. R. Wade and P. Simon, *Walsh Series. An Introduction to Dyadic Harmonic Analysis.* With the collaboration of J. Pśl. Adam Hilger, Ltd., Bristol, 1990.

Challenges of Media Digitization in Georgia

Elisabed Asabashvili, **David Demetradze**

[1]*IT Department, School of Science and Technologies, University of Georgia*
Tbilisi, Georgia

E-mails: z.asabashvili@ug.edu.ge; davidemetradze@gmail.com

New social networks and models are likely to emerge in the near future. We will witness the implications of the new wave of technological innovation. For example, in 2022, significant advancements in artificial intelligence (AI) have provided journalism with both opportunities and challenges. AI enables publishers to access more information and formats to deliver content, offering opportunities to overcome channel fragmentation and information overload. In this context, media organizations that have not fully embraced digital technologies will face a disadvantage. The coming years will be determined not only by the speed of digital technology adoption but also by how effectively we transform digital content to meet rapidly changing audience expectations.

Ruby and Python are two popular programming languages used in web development to create applications. Both languages are clean, easy to read, and open-source high-level languages used on the server-side to support program interfaces.

Python, in particular, has gained popularity in journalism. The "Pandas Library" plays an interesting role in data cleaning and analysis in journalism. It facilitates tasks such as cleaning datasets, importing CSV files into "Jupyter Notebook", finding and replacing values in columns, changing column data types, and removing or filling columns with new data. These digital tools provide journalists with affordable access to vast amounts of data and enable them to tell compelling stories through infographics.

Modern journalists need to familiarize themselves with various technical innovations, such as "Python Conda" environments, "Jupyter Notebook" usage, and data cleaning techniques. They should also possess data literacy skills, including knowledge of statistics, working with big datasets, connecting and interpreting data, and using data to write articles. Data journalism allows journalists to present complex stories in engaging ways, utilizing data both as a source and as a storytelling tool. Unfortunately, the implementation of digital technologies in Georgian media is currently limited. Journalists from institutions like the National Statistics Office of Georgia, the National Bank, and other agencies still rely on traditional methods to work with large databases. This approach requires significant effort, time, and resources. Therefore, I believe it would be beneficial for journalists to study mathematics and computer science. Offering lectures on these subjects can help reporters apply mathematical and informatics concepts, understand different data types and structures, and gain a deeper understanding of the principles and applications necessary for their work.

Boundary-Domain Integral Equations for Dirichlet BVP for Variable-Coefficient Helmholtz Equation in 2D

Tsegaye Ayele[1], **Bizuneh Demissie**[1], **Sergey Mikhailov**[2]

[1]*Department of Mathematics, Addis Ababa University, Addis Ababa, Ethiopia*
E-mails: tsegaye.ayele@aau.edu.et; bizuneh.minda@aau.edu.et

[2]*Department of Mathematics, Brunel University, London, UK*
E-mail: sergey.mikhailov@brunel.ac.uk

In this talk, we construct boundary-domain integral equations (BDIEs) for Dirichlet boundary value problem (BVP) for a two-dimensional variable coefficient Helmholtz equation. Using appropriate parametrix this problem is reduced to two systems of BDIEs. It is shown that the BVP and the formulated BDIE systems are equivalent. Unique solvability and invertibility of BDIE systems are investigated in appropriate Sobolev spaces.

Acknowledgments

The first two authors would like to thank the Department of Mathematics, Addis Ababa University, and the International Science Program (ISP) in Uppsala, Sweden.

Boundary-Domain Integral Equations to the Mixed BVP for Variable-Coefficient Helmholtz Equation in 2D

Tsegaye Ayele[1], **Bizuneh Demissie**[1], **Sergey Mikhailov**[2]

[1]*Department of Mathematics, Addis Ababa University, Addis Ababa, Ethiopia*
E-mails: tsegaye.ayele@aau.edu.et; bizuneh.minda@aau.edu.et

[2]*Department of Mathematics, Brunel University, London, UK*
E-mail: sergey.mikhailov@brunel.ac.uk

The boundary-domain integral equations (BDIEs) to the mixed boundary value problem (BVP) for variable-coefficient Helmholtz equation in 2D are considered in this paper. An appropriate parametrix (Levi function) is used to reduced this BVP to four different systems BDIEs. The BDIEs in 2D needs special consideration due to their different equivalence properties. The equivalence of the original BVP and the obtained BDIEs is analyzed. The properties of the corresponding boundary integral operators are investigated.

Acknowledgments

The first two authors would like to thank the Department of Mathematics, Addis Ababa University, and the International Science Program (ISP) in Uppsala, Sweden.

Some Structures on Lie Groupoids

Rezvaneh Ayoubi[1], **Ghorbanali Haghighatdoost**[1], **Hossein Kheiri**[2]

[1] *Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran*
E-mails: gorbanali@azaruniv.ac.ir; rezvaneha@yahoo.com

[2] *Mathematics, University of Tabriz, Tabriz, Iran*
E-mail: kheirihossein@yahoo.com

In this paper, Our aim to introduce some structures on Lie groupoids [1] analogue to Lie groups (see [3, 4]) for example we try to introduce invariant Poisson–Nijenhuis structures on Lie groupoids and their infinitesimal counterparts as called $(\wedge, \mathbf{n})$-structures.It seems there is a mutual correspondence between $(\wedge, \mathbf{n})$-structures on Lie algebroids with Poisson–Nijenhuis structures $(\Pi, \mathbf{N})$ on their Lie groupoids under some conditions. By an illustrative example we end the paper.

Definition Let Π be a Poisson structure on the Lie groupoid $G \rightrightarrows M$ we call Π right invariant if there exists a bivector $\Lambda \in \Gamma(\wedge^2 AG)$ such that $\Pi = \overrightarrow{\Lambda}$.

Theorem 1 *Let s-connected and s-simply connected Lie groupoid $G \rightrightarrows M$ with Lie algebroid AG. Consider $\Lambda \in \Gamma(\wedge^2 AG)$ be an element satisfying $[\Lambda, \Lambda] = 0$. Then $\Pi = \overrightarrow{\Lambda}$ defines a Poisson groupoid structure on G. Furthermore for the endomorphisms $n : \Gamma(AG) \to \Gamma(AG)$ and $n_M : TM \to TM$ there exists multiplicative $(1,1)$-tensors $N : TG \to TG$, $N_M : TM \to TM$ such that $\overrightarrow{n} = N$, $n_M = N_M$ and compatible with Π.*

Example *Let M be a manifold and G be a Lie group. Consider $\Upsilon := M \times G \times M$. As mentioned in, Υ has a Lie groupoid structure over M, called trivial Lie groupoidand the Lie algebroid associated to the trivial Lie algebroid is $A\Upsilon = TM \oplus (M \times \mathfrak{g})$. The dual bundle of this Lie algebroid is $A^*\Upsilon = T^*M \oplus (M \times \mathfrak{g}^*)$. Here we introduce a right-invariant Poisson–Nijenhuis Structure on the trivial Lie groupoid Υ.*

References

[1] A. Das, Poisson–Nijenhuis groupoids. *Rep. Math. Phys.* **84** (2019), no. 3, 303–331.

[2] Gh. Haghighatdoost and R. Ayoubi, Hamiltonian systems on co-adjoint Lie groupoids. *J. Lie Theory* **31** (2021), no. 2, 493–516.

[3] Gh. Haghighatdoost, Z. Ravanpak and A. Rezaei-Aghdam, Some remarks on invariant Poisson quasi-Nijenhuis structures on Lie groups. *Int. J. Geom. Methods Mod. Phys.* **16** (2019), no. 7, 1950097, 28 pp.

[4] Z. Ravanpak, A. Rezaei-Aghdam and Gh. Haghighatdoost, Invariant Poisson–Nijenhuis structures on Lie groups and classification. *Int. J. Geom. Methods Mod. Phys.* **15** (2018), no. 4, 1850059, 37 pp.

About Hypothesis Testing of Equality of Two Bernoulli Regression Functions

Petre Babilua, Elizbar Nadaraya

Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mails: petre.babilua@tsu.ge; elizbar.nadaraya@tsu.ge

We establish the limit distribution of the square-integrable deviation of two nonparametric kernel-type estimations for the Bernoulli regression functions. The criterion of testing the hypothesis of two Bernoulli regression functions. The question as to its consistency is studied. The power asymptotics of the constructed criterion is also studied for certain types of close alternatives.

On an Algorithm for Numerical Solution of Non-Linear Goursat Problem

Giorgi Baghaturia[1], **Marine Menteshashvili**[1,2]

[1] *Muskhelishvili Institute of Computational Mathematics, Georgian Technical University Tbilisi, Georgia*

[2] *Sokhumi State University; Tbilisi, Georgia*

E-mails: gi.baghaturia@gtu.ge; m.menteshashvili@gtu.ge

We consider the Goursat problem for one class of quasi-linear equations of mixed type. We have proved, that the Goursat problem is well posed [1]. The families of characteristic curves are described and the area of definition of the solution is constructed. In order to solve the problem, a difference scheme is written and the approximation and stability of the scheme are studied. Calculation results are given for some test examples.

Acknowledgments

The work was partially supported by European Commission HORIZON EUROPE WIDERA-2021-ACCESS-03, Grant Project (GAIN), grant agreement # 101078950.

References

[1] G. G. Bagaturia and M. Z. Menteshashvili, General integral of a quasilinear equation and its application for solving a nonlinear characteristic problem. (Russian) *Sibirsk. Mat. Zh.* **60** (2019), no. 6, 1209–1222; translation in *Sib. Math. J.* **60** (2019), no. 6, 940–951.

Complex Cobordism Modulo c_1-Spherical Cobordism and Related Genera

Malkhaz Bakuradze

Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: malkhaz.bakuradze@tsu.ge

We prove that the set $S = (x_1, x_k, k \geq 3)$ of polynomial generators of c_1-spherical cobordism ring W_*, treated as a set in complex cobordism ring MU_* by forgetful map is regular. For any subset Σ in S the quotient map defines a genus on MU_* with values in integral domain. This yields two new complex oriented cohomology theories with the coefficient rings isomorphic to complex cobordiams modulo flops in spherical cobordiams and modulo special unitary flops respectively.

References

[1] M. Bakuradze, Complex cobordism modulo c_1-spherical cobordism and related genera. *Preprint* arXiv:2306.02163v1; https://arxiv.org/abs/2306.02163.

On a Problem of Kolmogorov

Mzevinar Bakuridze[1], **Sergei Chobanyan**[2], **Vaja Tarieladze**[2]

[1] *Batumi Shota Rustaveli State University, Batumi, Georgia*
E-mail: bakuridzemzevinari@mail.ru

[2] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*
E-mails: s.chobanyan@gtu.ge, v.tarieladze@gtu.ge

Dedicated to Andrei Nikolaevich Kolmogorov (1903-1987)

An orthonormal sequence $\varphi_n \in H = L_2[0,1]$, $n = 1, 2, \dots$ is called a *convergence system* if for every sequence $(a_n)_{n\in\mathbb{N}} \in l_2$ the series $\sum\limits_n a_n\varphi_n(t)$ converges in $\mathbb{R}$ for almost all $t \in [0,1]$.

The first orthonormal sequence, which **is not** a convergence system was found by Menchov in 1923.

Let us call an orthonormal sequence $(\varphi_n)_{n\in\mathbb{N}}$

- a *potentially* convergence system [1], if there exists a bijection $\sigma : \mathbb{N} \to \mathbb{N}$ such that the sequence $(\varphi_{\sigma(n)})_{n\in\mathbb{N}}$ is a convergence system.

According to [2] "…the following problem which goes back to A. N. Kolmogorov remains open": *prove that every orthonormal sequence is potentially convergence system.*

The first article, which title mentions Kolmogorov's problem was [3] written by Jean Bourgain (28 February 1954 – 22 December 2018), a Belgian mathematician, who was awarded the Fields Medal in 1994 in recognition of his work on several core topics of mathematical analysis such as the geometry of Banach spaces, harmonic analysis, Ergodic theory and nonlinear partial differential equations from mathematical physics.

It is interesting to note that in [2] Olevskii did not write where Kolmogorov posed the problem, while in [3] it is written that the problem was posed in one of Kolmogorov's papers. However Bourgain's reference is not correct!

In our talk we plan to discuss some recent results about **Kolmogorov's rearrangement problem** and **Garsia's conjecture** (mentioned in [3], *which remain open so far.*

References

[1] M. Bakuridze, S. Chobanyan and V. Tarieladze, On Perfect And Potentially Convergence Systems. *XII International Conference of the Georgian Mathematical Union (Batumi, August 29 – September 3, 2022)*, Book of Abstracts, 2022, p. 63.

[2] A. M. Olevskii, *Fourier Series with Respect to General Orthonormal Systems.* Ergebnisse der Mathematik und ihrer Grenzgebiete. 2. Folge (MATHE2, vol. 86) Springer-Verlag, New York–Heidelberg, 1975.

[3] J. Bourgain, On Kolmogorov'S Rearrangement problem for orthogonal systems and Garsia'S conjecture. In *Geometric Aspects of Functional Analysis.* Lecture Notes in Mathematics, vol. 1376. Springer, Berlin, Heidelberg, 1989.

Solving Various Types of Functional Equations

Bakur Bakuradze[1], **Giorgi Bregadze**[2]

[1] *Department of Teaching Methods, Akaki Tsereli State University, Kutaisi, Georgia*
E-mail: bakurbakuradze@rambler.ru

[2] *41-st Physics and Mathematics Public School of Kutaisi, Kutaisi, Georgia*
E-mail: giorgi.bregadze1@mail.ru

In school mathematics course, such functions are studied, which are written in the analytical form, and then the properties of the function are determined through this expression.

Area of interest is the consideration of such problems, when some properties of a function are given, and with these properties we need to reconstruct the function and write it in an analytical form.

The problems of restoring functions by means of some properties of a function fall into the category of non-standard, exploratory or Olympiad problems. The general algorithm for solving such problems is unknown.

In such problems, the analytic image of $f(x)$ is generally sought. In the period when the derivative and integral of a function were studied at school, then the students were more or less certain about the essence of restoring the function with some given properties of a function. Currently, when the elements of differential accounting are no longer studied at school, the teacher has to work harder to clarify the essence of the problem of restoring the function, but it is a very good thing that the students show a special interest in solving this type of problems.

Methodical approaches to solving such type of problems are less elaborated, therefore we have considered different types of problems of this type, to which we have attached methodical recommendations and guidelines. In addition, we discussed some types of sequences and some types of mathematical operations, when correspondence between numbers is established in a certain way, we should note that such problems are also related to the functions.

Revisiting Population Based Optimization Algorithms

Jagdish Chand Bansal

Mathematics Department, South Asian University, New Delhi, India
E-mail: jcbansal@sau.ac.in

Broadly, there are three types of solution methodologies for an optimization problem, graphical, analytical and numerical. Finding the numerical solution to an optimization problem is usually referred to as Numerical Optimization. Based on the number of solutions used for numerical optimization, the optimization algorithms can be categorized in two ways, single solution based and population based optimization algorithms. Newton–Raphson, Bisection, Secant, Box's Evolutionary Optimization, Hooke–Jeeves Pattern Search and Powell's Conjugate Direction algorithm are some of the single solution based optimization algorithms. While Particle Swarm Optimization, Ant Colony Optimization, Artificial Bee Colony, Differential Evolution, Grey Wolf Optimizer, Gravitational Search Algorithm, Spider Monkey Optimization, Bio-geography Based Optimization, Salp Swarm Algorithm are some of the population based optimization algorithms or nature inspired optimization algorithms [1].

However, the population based algorithms are probabilistic in nature while most of the single solution based algorithms are deterministic, the search process share certain similarities among all these algorithms. This study is an attempt to discuss these similarities. Finding these similarities will help the researchers in the field of numerical optimization, in further development of new numerical optimization algorithms, particularly the population based or the probabilistic algorithms.

Acknowledgments

The work was supported by the South Asian University New Delhi, India.

References

[1] H. Zang, S. Zhang and K. Hapeshi, A review of nature-inspired algorithms. *J. Bionic Eng.* **7** (2010), no. 4, S232–S237.

Summability of Tkebuchava's Means of One and Two Dimensional Trigonometric Fourier Series

Lasha Baramidze, Ushangi Goginava

[1]*Department of Mathematics, Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

E-mails: lashabara@gmail.com; ushangi.goginava@tsu.ge

The talk is devoted to characterize the set of convergence of the general logarithmic means of trigonometric Fourier series and also establish a condition that guarantees convergence in the measure of logarithmic means of the two-dimensional Fourier series. We also consider the summability of two-dimensional Tkebuchava means.

References

[1] L. Baramidze, Pointwise convergence of logarithmic means of Fourier series. *Acta Math. Acad. Paedagog. Nyházi. (N.S.)* **32** (2016), no. 2, 225–232.

[2] L. Baramidze and U. Goginava, Convergence in measure of logarithmic means of double Fourier series. *Semin. I. Vekua Inst. Appl. Math. Rep.* **41** (2015), 3–11.

Maximal Operators of Partial Sums of Walsh–Fourier Seriesv in the Martingale Hardy Spaces

Davit Baramidze[1,2], **Lars-Erik Persson**[2], **George Tephnadze**[3]

[1]*School of Science and Technology, The University of Georgia, Tbilisi, Georgia*
[2]*Department of Computer Science and Computational Engineering, UiT-The Arctic University of Norway, Narvik, Norway*
E-mails: davit.baramidze@ug.edu.ge; lars.e.persson@uit.no

[3]*School of Science and Technology, The University of Georgia, Tbilisi, Georgia*
E-mail: g.tephnadze@ug.edu.ge

The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves. The development of the theory of Vilenkin-Fourier series has been strongly influenced by the classical theory of trigonometric series. Some important steps in the early development can be found in the book by F. Schipp, W. R. Wade, P. Simon and J. Pál [4] from 1990. Some of the most important steps in developments of the theory of martingale Hardy spaces are presented in the book [5] by F. Weisz from 1994. The research continued intensively also after this. Some of the most important steps in these developments are presented in the resent book [3].

This talk is devoted to introduce some new weighted maximal operators of the partial sums of the Walsh–Fourier series with some "optimal" weights and investigate $(H_p - H_p)$ and $(H_p - weak - H_p)$ type inequalities for these new operators, for $0 < p < 1$. Moreover, we also show sharpness of this result. As a consequence we obtain some new and well-known results (for details see [1] and [2]).

References

[1] D. Baramidze, L.-E. Persson and G. Tephnadze, Some new $(H_p - L_p)$ type inequalities for weighted maximal operators of partial sums of Walsh–Fourier series. *Positivity* **27** (2023), Article no. 38, 27–38.

[2] D. Baramidze, L.-E. Persson, H. Singh and G. Tephnadze, Some new weak $(H_p - L_p)$ type inequality for weighted maximal operators of partial sums of Walsh–Fourier series. *Mediterr. J. Math.*, 2023 (to appear).

[3] L. E. Persson, G. Tephnadze and F. Weisz, *Martingale Hardy Spaces and Summability of Vilenkin–Fourier Series.* Birkhäuser Cham, 2022.

[4] F. Schipp, W. R. Wade and P. Simon, *Walsh Series: an Introduction to Dyadic Harmonic Analysis.* Adam Hilger, Bristol, England, 1990.

[5] F. Weisz, *Martingale Hardy Spaces and Their Applications in Fourier Analysis.* Lecture Notes in Mathematics, 1568. Springer-Verlag, Berlin, 1994.

On the Bases of Quasivarieties Generated by Certain Finite Lattices

Ainur Basheyeva[1,2], **Svetlana Lutsak**[3]

[1]*Department of Algebra and Geometry, L. N. Gumilev Eurasian National University Astana, Kazakhstan*

[2]*Department of Computational and Data Science, Astana IT University Astana, Kazakhstan*

E-mail: basheyeva3006@gmail.com

[3]*Department of Mathematics and Computer Science, M. Kozybayev North Kazakhstan University, Petropavlovsk, Kazakhstan*

E-mail: sveta_lutsak@mail.ru

It is known that any finite lattice has a finite basis of identities (R. McKenzie 1970 [4]). But the similar result for quasi-identities is incorrect: there is a finite lattice that has no finite basis of quasi-identities (V. P. Belkin 1979 [1]). These results naturally arose the problem "Which finite lattices have finite bases of quasi-identities?" (V. A. Gorbunov and D. M. Smirnov 1979 [3]). In 1984 V. I. Tumanov [5] found sufficient condition consisting of two parts under which the locally finite quasivariety of modular lattices has no finite (independent) basis of quasi-identities. Also he assumed that a finite (modular) lattice has a finite basis of quasi-identities if and only if a quasivariety generated by this lattice is a variety. In general, the assumption is wrong. In 1989 W. Dziobiak [2] found a finite lattice that generates finitely axiomatizable proper quasivariety. However the Tumanov's conjecture for modular lattices is still open.

The main purpose of this work is to present a particular finite modular lattice such that the quasivariety generated by this lattice does not satisfy all Tumanov's conditions and is not finitely based (has no finite basis of quasi-identities). The proof of this result gives many examples of finite lattices confirming Tumanov's conjecture.

Acknowledgments

This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan ([1]Grant # AP13268735, [3]Grant # AP09058390).

References

[1] V. P. Belkin, Quasi-identities of finite rings and lattices. (Russian) *Algebra i Logika* **17** (1978), no. 3, 247–259; translation in *Algebra and Logic* **17** (1979), 171–179.

[2] W. Dziobiak, Finitely generated congruence distributive quasivarieties of algebras. *Fund. Math.* **133** (1989), no. 1, 47–57.

[3] V. A. Gorbunov and D. M. Smirnov, Finite algebras and the general theory of quasivarieties. *Finite algebra and multiple-valued logic (Szeged, 1979)*, pp. 325–332, Colloq. Math. Soc. János Bolyai, 28, *North-Holland, Amsterdam–New York*, 1981.

[4] R. McKenzie, Equational bases for lattice theories. *Math. Scand.* **27** (1970), 24–38.

[5] V. I. Tumanov, Finite lattices with no independent basis of quasi-identities. (Russian) *Mat. Zametki* **36** (1984), no. 5, 625–634; translation in *Math. Notes* **36** (1984), 811–815.

Periodic on Part of Variables Solution of a System of Equations in the Broad Sense

Altynshash Bekbauova, **Yergali Kurmangaliyev**

K. Zhubanov Aktobe Regional University, Aktobe, Kazakhstan

E-mails: altynshash.bekbauova@gmail.com; ergali715@gmail.com

The paper considers a system of equations in first-order quotients

$$Dx = P(t, \varphi, \psi)x + \mu Q(t, \varphi, \psi, x), \tag{1}$$

where

$$D = \frac{\partial}{\partial t} + \Big\langle a(t, \varphi, \psi), \frac{\partial}{\partial \varphi} \Big\rangle + \Big\langle b(t, \varphi, \psi), \frac{\partial}{\partial \psi} \Big\rangle$$

is a differentiation operator,

$$\frac{\partial}{\partial \varphi} = \Big(\frac{\partial}{\partial \varphi_1}, \dots, \frac{\partial}{\partial \varphi_m} \Big), \quad \frac{\partial}{\partial \psi} = \Big(\frac{\partial}{\partial \psi_1}, \dots, \frac{\partial}{\partial \psi_k} \Big)$$

is a differentiation vectors, $\varphi = (\varphi_1, \dots, \varphi_m)$ and $a = (a_1, \dots, a_m)$ is a m-vectors, $\psi = (\psi_1, \dots, \psi_k)$ and $b = (b_1, \dots, b_k)$ is a k-vectors, $\langle \ , \ \rangle$ is a means the scalar product of this vectors, P is a $n \times n$-matrix, x, Q is a n-vectors, μ is a parameter.

Known, that the classical solution $x(t, \varphi, \psi)$ of system (1) is continuous differentiable in all variables. If the solution $x(t, \varphi, \psi)$ has less smoothness, but in satisfies the system (1) in some sense, then it is called a generalized solution of the system (1).

In this paper, we construct a periodic in variables (t, φ) solution of system (1) in the broad Friedrichs sense [2].

Note, that the solution in the broad sense of system (1) does not require smoothness of the function a, b, Q and matrices P. If these input data have the desired smoothness, then the constructed the solution in the broad sense is also the classical solution of the system (1).

Acknowledgements

The work was carried out within the framework of the project # 19675358 on grant financing of the Ministry of Science and Higher Education of the Republic of Kazakhstan.

References

[1] R. Courant and D. Hilbert, *Methods of Mathematical Physics.* Vol. II. *Partial Differential Equations.* Reprint of the 1962 original. Wiley Classics Library. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1989.

[2] B. L. Rozhdestvenskiĭ and N. N. Janenko, *Systems of Quasilinear Equations and Their Applications to Gas Dynamics.* (Russian) Nauka, Moscow, 1978; Translated from the second Russian edition by J. R. Schulenberger. Translations of Mathematical Monographs, 55. American Mathematical Society, Providence, RI, 1983.

A Note on Controlled Degenerate Systems in Hilbert Spaces

Nor-EI-Houda Beghersa, Mehdi Benabdallah

Department of Mathematics, University of Sciences and Technology USTO-MB
Oran, Algeria

E-mail: norelhouda.beghersa@univ-usto.dz; mehdi.benabdallah@univ-usto.dz

In the paper, we show the important role of the General Lyapounov Theorem on the stability for the investigation of the controlled degenerate system in Hilbert spaces. We also obtain some conditions of stabilization of such systems using the spectral theory of the corresponding pencil of operators. Finally we give an illustrative example in the last section of this work.

Theorem 1 *A necessary condition, for the spectrum $\sigma(A,B)$ of the pencil $\lambda A - B$ to lie in the interior of the half plane $\operatorname{Re}(\lambda) < \alpha$ is that for any uniformly operator $U \gg 0$, there exists an operator $W \gg 0$, such that:*

$$A^*WB + B^*WA - 2\alpha A^*WA = -2U,$$

and a sufficient condition is that $\alpha + 1$ is regular value of the pencil $\lambda A - B$ also, there exists an operator $W \gg 0$ such that

$$A^*WB + B^*WA - 2\alpha A^*WA \ll 0.$$

References

[1] R. Bellman and K. L. Cooke, *Differential-Difference Equations.* Academic Press, New York–London, 1963.

[2] M. Benabdallah, A. G. Rutkas and A. A. Soloviev, On the stability of degenerate difference systems in Banach spaces. *J. Sov. Math.* **57** (1991), 3435–3439.

[3] L. A. Vlasenko, *Evolutionary Models with and Degenerate Differential Equations.* (Russian) System Technologies, Dniepropetrovsk, 2006.

On Some Paradoxical Point Sets in the Euclidean Plane

Lika Beraia

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: beraianiko@gmail.com

In the talk some paradoxical point sets are considered from the measure-theoretical and topological measurability (in the Lebesgue sense and the Baire property).

On Teaching the Solution of a Non-Standard Functional Equation of One Type in the Secondary School

Giorgi Berdzulishvili

Department of Teaching Methods, Akaki Tsereli State University, Kutaisi, Georgia
E-mail: giorgi.berdzulishvili@mail.ru

Theorem 1 *If the function*

$$y = f(x) \tag{1}$$

is an increasing (decreasing) function, then the equations (1) and

$$g(x) = h(x) \tag{2}$$

are equal to the set of admissible values of the functions included in the equation (1).

Result If the function $y = f(x)$ is increasing (decreasing) and in the regions of values of functions $y = g(x)$ and $y = h(x)$, then (1) and $g(x) = h(x)$ equations are equal.

It should be noted that while solving the equation (1), it is necessary to carefully consider the case when the function $y = f(x)$ is even.

Theorem 2 *If the function $y = f(x)$ is even on the section $-l \leq x \leq l$ and is increasing (decreasing) when $0 \leq x \leq l$ then on the given section, the equation (1) is equal to the set of equations*

$$\begin{cases} g(x) = h(x), \\ g(x) = -h(x), \end{cases}$$

with the requirement that $-l \leq g(x) \leq l$ and $-l \leq h(x) \leq l$.

Based on them, the solution of various types of equations and systems of equations is discussed, which gives students a perfect idea of how to solve these types of equations and excludes the logical errors in the solution process.

Acknowledgments

The research was conducted within the framework of the SCR-23-082 project *"Science Begins at School – Research with the Participation of Students"*, winner of the grant competition announced by the Shota Rustaveli National Science Foundation of Georgia.

References

[1] G. Berdzulishvili, *Methods of Solving School and Olympiad Mathematical Problems.* Akaki Tsereteli State University Publishing House, Kutaisi, 2018.

[2] G. Berdzulishvili, G. Bregadze and G. Margvelashvili, *Methodology of Teaching Solving Olympic Mathematical Problems.* Akaki Tsereteli State University Publishing House, Kutaisi, 2019.

[3] T. Moralishvili, *Searching for Solutions to Problems in High School.* "Education", Tbilisi, 1991.

On Absolutely Negligible Uniform Sets

Mariam Beriashvili[1,2], **Wieslaw Kubis**[3]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*

E-mail: mariam_beriashvili@yahoo.com

[3] *Department of Abstract Analysis, Institute of Mathematics of the Czech Academy of Sciences, Prague, Czech Republic*

E-mail: kubis@math.cas.cz

Let G be an arbitrary uncountable group and X be a subset of G. X is an G-absolutely negligible set, if for each countable family $\{g_i: i \in N\}$ of elements from G, there exists a countable family $\{h_k: k \in N\}$ of elements from G such that

$$\bigcap_{k\in N} h_k\Big(\bigcup_{i\in N} g_i(X)\Big) = \varnothing.$$

Its well known, that

- there exists a countable family of uniform sets, whose union is identical to R^2;
- there exist uniform sets which are not absolutely negligible.

The goal of the presented talk is to discuss briefly uniform subsets of the Euclidean plane in the context of absolutely negligible property and describe some conditions to be an absolutely negligible sets.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Grant # YS-21-1667.

References

[1] M. Beriashvili, M. Khachidze and A. Kirtadze, Absolutely negligible sets and their algebraic sums. *Trans. A. Razmadze Math. Inst.* **177** (2023), no. 1, 131–133.

[2] A. Kharazishvili and A. Kirtadze, On algebraic sums of measure zero sets in uncountable commutative groups. *Proc. A. Razmadze Math. Inst.* **135** (2004), 97–103.

On Some Definitions of Isosceles Simplexes

Shalva Beriashvili

Georgian National University (SEU), Assistant Professor, Tbilisi, Georgia
E-mail: s.beriashvili@seu.edu.ge

It is well known that the k-isosceles simplex plays an important role in combinatorial geometry and has many applications in various fields [1, 2].

Suppose, S is an m-dimensional simplex in $R^m (m \geq 2)$ Euclidean space. A simplex S is called a k-isosceles $(1 \leq k \leq m-1)$ simplex if given in a simplex, at least one such vertex is found that all k-dimensional faces are congruent.

In the presentation talk we will discuss k-isosceles simplex and some combinatorial properties to it.

References

[1] Sh. Beriashvili, On some properties of primitive polyhedrons. *Trans. A. Razmadze Math. Inst.* **174** (2020), no. 1, 23–28.

[2] A. Kharazishvili, *Elements of Combinatorial Geometry*, Part II. Publish. House Georgian Natl. Acad. Sci., Tbilisi, 2020.

On Axiomatic Homology Theory of General Topological Spaces

Anzor Beridze[1,2]

[1]*Batumi Shota Rustaveli State University, Batumi, Georegia*
[2]*Kutaisi International University, Kutaisi, Georgia*
E-mails: a.beridze@bsu.edu.ge, anzor.beridze@kiu.edu.ge

On the category $\mathcal{K}^2_{CM}$ of pairs of compact metric spaces the exact homology theory was defined by N. Steenrod, that is known as the classical Steenrod homology theory. J. Milnor constructed the exact homology theory on the category $\mathcal{K}^2_C$ of pairs of compact Hausdorff spaces, which is isomorphic to the Steenrod homology theory on the subcategory $\mathcal{K}^2_{CM}$ and which satisfies the so-called "modified continuity" property: if $X_1 \leftarrow X_2 \leftarrow X_3 \leftarrow \dots$ is an inverse sequence of compact metric spaces with inverse limit X, then for each integer n there is an exact sequence:

$$0 \longrightarrow \varprojlim{}^1 H_{n+1}(X_i) \xrightarrow{\beta} H_n(X) \xrightarrow{\gamma} \varprojlim H_n(X_i) \longrightarrow 0\,, \tag{1}$$

where H_* is the Steenrod (Milnor) homology theory. There are exact homology theories defined by other authors (A. N. Kolmogoroff, G. Chogoshvili, K. A. Sitnikov, A. Borel and J. C. Moore, H. N. Inasaridze, D. A. Edwards and H. M. Hastings, W. S. Massey, E. G. Sklyarenko) that are isomorphic to the Steenrod homology theory on the category $\mathcal{K}^2_{CM}$ and so, satisfy the modified continuity axiom.

On the category $\mathcal{K}^2_C$ the axiomatic characterization is obtained by N. Berikashvili, L. Mdzinarishvili and Kh. Inasaridze, L. Mdzinarishvili, Kh. Inasaridze. The connection between these axiomatic systems is studied in the paper [2].

In the paper [1] we have generalized the result for general topological spaces. In particular, we have defined the Alexander–Spanier normal cohomology theory based on all normal coverings and have shown that it is isomorphic to the Alexandroff–Čech normal cohomology [1]. Using this fact and methods developed in [3], we constructed an exact, the so-called Alexander–Spanier normal homology theory $\bar{H}^N_*(-,-;G)$ on the category $\mathcal{K}^2_{Top}$, which is isomorphic to the Steenrod homology theory on the subcategory of compact pairs $\mathcal{K}^2_C$. Moreover, we gave an axiomatic characterization of the constructed homology theory [1]. In this paper we will use the method of construction of the strong homology theory to show that the homology theory $\overline{H}^N_*(-,-;G)$ is strong shape invariant.

The talk partially is based on joint works with co-authors Vladimir Baladze (BSU) and Leonard Mdzinarishvili (GTU).

References

[1] V. Baladze, A. Beridze and L. Mdzinarishvili, On axiomatic characterization of Alexander–Spanier normal homology theory of general topological spaces. *Topology Appl.* **317** (2022), Paper no. 108166, 25 pp.

[2] A. Beridze and L. Mdzinarishvili, On the axiomatic systems of Steenrod homology theory of compact spaces. *Topology Appl.* **249** (2018), 73–82.

[3] A. Beridze and L. Mdzinarishvili, On the universal coefficient formula and derivative $\varprojlim^{(i)}$ functor. *Preprint* arXiv:2102.00468; `https://arxiv.org/abs/2102.00468`

Different Approaches in Inductive Logic Programming

Merihan Hazem Anwar Labib Bishara

International Black Sea University, Tbilisi, Georgia
E-mail: merihan.hazem20@gmail.com

Probabilistic inductive logic programming aka. statistical relational learning addresses one of the central open questions of artificial intelligence which is the integration of probabilistic reasoning with machine learning and first-order and relational logic representations [1]. Traditional machine-learning approaches are able to cope either with uncertainty or with relational representations but typically not with both, therefore it is not a surprise that there has been a significant interest in integrating statistical learning with first-order logic and relational representations [1]. From our point of view, we will start by studying inductive logic programming and how its formalisms, settings, and techniques can be extended to deal with probabilistic issues [2].

In this talk, we will introduce three probabilistic inductive logic programming settings, derived from the learning from entailment, from interpretations, and from proofs [3]. Each of these settings contribute different notions of probabilistic logic representations, examples, and probability distributions [2].

References

[1] L. De Raedt and K. Kersting, Probabilistic inductive logic programming. *Probabilistic inductive logic programming*, 1–27, Lecture Notes in Comput. Sci., 4911, Lecture Notes in Artificial Intelligence, *Springer, Berlin*, 2008.

[2] L. De Raedt and K. Kersting, Probabilistic inductive logic programming. In: S. Ben-David, J. Case, A. Maruoka (Eds.), *Algorithmic Learning Theory, ALT 2004, Lecture Notes in Computer Science*, vol. 3244. pp. 19–16. *Springer, Berlin, Heidelberg*, 2004.

[3] L. De Raedt, Logical settings for concept-learning. *Artificial Intelligence* **95** (1997), no. 1, 197–201.

Probability Quantifiers in σ-Additive Frameworks

Merium Hazem Anwar Labib Bishara

International Black Sea University, Tbilisi, Georgia
E-mail: merium.hazem20@gmail.com

The probability logic had a significant advancement that Howard Jerome Keisler made. Keisler's purpose [1] was to develop, within Robinson's nonstandard infinitesimal analysis [8], a model theory that would be appropriate for studying and classifying probability models arising in applied mathematics.

Instead of classical universal and existential quantifiers, Keisler introduced probability quantifiers, for example $Px > r$.

In this talk we describe the notion of σ-additive framework and explain why mixing ordinary (universal and existential) quantifiers and probability quantifiers in such framework is still an open problem [2–4]. We go through different examples showing the idea of the problem [5–7].

References

[1] H. J. Keisler, Hyperfinite model theory. *Logic Colloquium 76 (Oxford, 1976)*, pp. 5–110. Studies in Logic and the Foundations of Mathematics, Vol. 87, *North-Holland, Amsterdam*, 1977.

[2] H. J. Keisler, Probability quantifiers. *Model-theoretic logics*, 509–556, Perspect. Math. Logic, *Springer, New York*, 1985.

[3] Z. Ognjanović, *Probabilistic Extensions of Various Logical Systems.* Springer, Nature Switzerland AG, 2020.

[4] Z. Ognjanović, M. Rašković and Z. Marković, *Probability Logics: Probability-Based Formalization of Uncertain Reasoning.* Springer, 2016.

[5] M. D. Rašković, Weak completeness theorem for $L_{\mathcal{A}\mathrm{P}\forall}$ logic. *Zb. Rad. (Kragujevac)*, no. 8 (1987), 69–72.

[6] M. Rašković, A completeness theorem for an infinitary intuitionistic logic with both ordinary and probability quantifiers. *Publ. Inst. Math. (Beograd) (N.S.)* **50(64)** (1991), 7–13.

[7] M. Rašković and P. Tanović, Completeness theorem for a monadic logic with both first-order and probability quantifiers. *Publ. Inst. Math. (Beograd) (N.S.)* **47(61)** (1990), 1–4.

[8] A. Robinson, *Non-Standard Analysis.* North-Holland Publishing Co., Amsterdam, 1966.

Lattice Isomorphisms of 2-Nilpotent W-Power Hall Groups and Lie Algebras

Tengiz Bokelavadze

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: tengiz.bokelavadze@atsu.edu.ge

The paper deals with lattice isomorphisms of 2-nilpotent Hall W-power groups and Lie algebras. Analogues of the fundamental theorem of projective geometry are proved. A corresponding example is constructed. The main definitions and notation are standard and can be found in [2, 3, 5] for W-power groups and in [1, 4, 6] for Lie algebras.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia, Grant # FR 21-4713.

References

[1] D. W. Barnes and G. E. Wall, On normaliser preserving lattice isomorphisms between nilpotent groups. *J. Austral. Math. Soc.* **4** (1964), 454–469.

[2] T. Bokelavadze, On some properties of W-power groups. *Bull. Georgian Acad. Sci.* **172** (2005), no. 2, 202–204.

[3] Ph. Hall, *The Edmonton Notes on Nilpotent Groups*. Queen Mary College Mathematics Notes. Queen Mary College, Mathematics Department, London, 1969.

[4] A. A. Lashkhi and T. Z. Bokelavadze, A subgroup lattice and the geometry of Hall's W-power groups. (Russian) *Dokl. Akad. Nauk* **429** (2009), no. 6, 731–734; translation in *Dokl. Math.* **80** (2009), no. 3, 891–894.

[5] A. G. Myasnikov and V. N. Remeslennikov, Degree groups. I. Foundations of the theory and tensor completions. (Russian) *Sibirsk. Mat. Zh.* **35** (1994), no. 5, 1106–1118; translation in *Siberian Math. J.* **35** (1994), no. 5, 986–996.

[6] V. R. Varea, Lie algebras whose maximal subalgebras are modular. *Proc. Roy. Soc. Edinburgh Sect. A* **94** (1983), no. 1-2, 9–13.

Nuclear Evaluation Type Statistical Challenges of Probability Distribution

Tristan Buadze[1], **Vazha Giorgadze**[2], **Revaz Kakubava**[1]

[1] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mails: T.buadze@gtu.ge; r.kakubava@gmail.com

[2] *International Black Sea University, Tbilisi, Georgia*
E-mail: Vazha.Giorgadze@kiu.edu.ge

This thesis introduces a robust and versatile approach for evaluating multi-dimensional unknown probability distribution density using the Nadaraya–Rosenblatt–Parzen nuclear type. The proposed model accommodates independent observations and is specifically designed for Lebesgue square integrable evaluations in function space. Advanced statistical techniques and mathematical modeling are employed to estimate density functions, making it applicable to complex data sets. Furthermore, the thesis investigates the Integral Square Deviation Measure with the Weight of "Delta-Functions" of the Rosenblatt–Parzen Probability Density Estimator.

Algebraic and Logical Properties of Chevalley Groups

Elena Bunina

Mathematics Department, Bar Ilan University, Israel
E-mail: helenbunina@gmail.com

In our talk we consider Chevalley groups with irreducible root systems of rank > 1 over almost arbitrary commutative rings with 1 from algebraic and logical points of view:

(1) we describe their automorphisms and isomorphisms and show that they are in some sense standard;

(2) we also describe some special types of endomorphisms of Chevalley groups over local rings;

(3) we show that two Chevalley groups are elementarily equivalent if and only if the corresponding root systems and weight lattices coincide and the rings are elementarily equivalent;

(4) We show that for some very wide class of Chevalley groups these groups are regularly bi-interpretable with the corresponding rings and the class of all Chevalley groups of a given type is elementary definable.

References

[1] E. Bunina, Regular bi-interpretability of Chevalley groups over local rings. *Eur. J. Math.* **9** (2023), no. 3, Article no. 64.

[2] E. Bunina, Automorphisms of Chevalley groups of different types over commutative rings. *Preprint* arXiv:2304.13447; https://arxiv.org/abs/2304.13447.

Expectations of Large Data Power Means

Tomislav Burić, **Lenka Mihoković**

Department of Applied Mathematics, Faculty of Electrical Engineering and Computing, University of Zagreb, Zagreb, Croatia

E-mails: tomislav.buric@fer.hr; lenka.mihokovic@fer.hr

We present estimation formulas for the expectations of power means of large data and associate them with means of probability distribution and means of random sample X. Namely, we study and obtain coefficients in the following asymptotic expansion:

$$\mathbb{E}[M_r(X)] = \mu + \frac{d_2}{\mu} + \frac{d_3}{\mu^2} + \frac{d_4}{\mu^3} + \mathcal{O}(\mu^{-4}), \quad \mu \to \infty,$$

where M_r is n-variable power mean.

The proposed method follows from the asymptotic expansion of power means which is applicable for sufficiently large data and it is especially useful when value of such expectation is hard to obtain. We will show the accuracy of these approximations for random samples which have uniform and normal distribution and analyse their behaviour for large sample volume.

Special cases of power means have been studied in the last few decades within theory of financial mathematics. For example, geometric and harmonic means have been applied to establishing criteria for choosing among strategies for maximizing the income. Other applications in statistics and related areas are also investigated.

On the Histogram of Relative Frequencies

Mamuli Butchukhishvili, Teimuraz Giorgadze

Faculty of Pedagogics, Akaki Tsereteli State University, Kutaisi, Georgia

E-mails: mamuli.buchukhishvili@atsu.edu.ge; Teimuraz.giorgadze@atsu.edu.ge

When grouping data, we can choose equal length intervals, although sometimes intervals of different lengths are also considered. For example, one interval has a length of 15 and has 60 observations, while another one has a length of 5 and has 50 observations. So it's safe to say that in the first interval? there are on average 4 observations per unit of length, and in the second one – 10. That is why the notions of frequency density and relative frequency density are introduced. The frequency density is called the ratio:

$$x_k = \frac{n_k}{\Delta d_k}\,, \quad k = 1, 2, \dots, m,$$

where $\Delta d_k = d_k - d_{k-1}$ is the length of the interval. Similarly, the relative frequency density is called the ratio:

$$h_k = \frac{n_k}{n \cdots \Delta d_k}\,, \quad k = 1, 2, \dots, m.$$

If we plot the frequency densities of the intervals in the corresponding intervals with parallel sections of the abscissa axis at a height of X_k from the axis, we shall obtain the $x(i)$ frequency histogram, and with a similar representation of the relative densities with parallel sections of the abscissa axis at a height of h_k from the axis, we shall obtain a $h(i)$ histogram of the relative frequencies. On each (d_{k-1}, d_k) interval as a base, in the first case, a rectangle with the area of n_k is constructed, and in the second case – with area of $\frac{n_k}{n}$.

In this case, the sum of these areas for the $x(i)$ histogram of frequencies is equal to n, and for the $h(i)$ histogram of relative frequencies, it is equal to 1.

Mathematical Model Describing the Transformation of the Proto-Kartvelian Population

Temur Chilachava, Gia Kvashilava, George Pochkhua

Sokhumi State University, Department of Applied Mathematics, Tbilisi, Georgia
E-mails: temo_chilachava@yahoo.com; gia.kvashilava@tsu.ge; g.pochkhua@sou.edu.ge

This work discusses two periods of transformation of the Proto-Kartvelian population: the first (L–XXV centuries BC) when the entire population spoke one Proto-Kartvelian language and lived in a relatively large area; the second period – (XXV–X centuries BC), when the population divided into three parts: Proto-Svan; speaking the Colchian-Georgian language and the third part was scattered on the European continent.

The second period is described by two different mathematical models: a part of the Proto-Kartvelian speaking population went to Europe and slowly began the process of their assimilation on the European continent. The unknown function that determines the number of Proto-Kartvelian-speaking people in Europe at the time is described by a Pearl - Verhulst-type mathematical model with variable coefficients that also take the assimilation process into account. The analytical solution of the Cauchy problem is found in quadratures.

The population that remained primarily in former Asia and the Caucasus region was gradually divided into two groups: those who spoke the Proto-Svan and those who spoke Colchian-Georgian languages. To describe their interference and development, a mathematical model is used, which is described by a nonlinear dynamic system with nonlinear terms of self-limitation and takes into account the unnatural reduction of the Colchian-Georgian population as a result of hostilities with neighboring peoples. For a dynamic system without nonlinear terms of self-constraint, in the case of certain relationships between variable coefficients, the first integral was found, by means of which the Bernoulli equation with variable coefficients was obtained for one of the unknown functions. In the case of constant coefficients of the dynamic system, for certain dependences between the coefficients, the dynamic system follows the system of Lotka–Volterra equations, with corresponding periodic solutions.

For the general mathematical model (nonlinear terms of self-limitation and unnatural reduction of the Colchian-Georgian population due to hostilities with neighboring peoples) in two cases of certain interdependencies between constant coefficients, it is shown that the divergence of an unknown vector-function in the physically meaningful first quarter of the phase plane changes the sign when passing through some half-direct one. Taking into account the principle (theorem) of Bendixson, theorems have been proved, on the variability of the divergence of the vector field and the existence of closed trajectories in some singly connected domain of the point located on this half-direct (starting point of the trajectory).

Thus, for the general dynamic system, with some dependencies between constant coefficients, it is shown that there is no assimilation of the Proto-Svan population by the Colchian-Georgian population and these two populations (Colchian-Georgian and Proto-Svan) coexist peacefully in virtually the same region due to the transformation of the Proto-Kartvelian population.

Mixed Type Dynamical Transmission Problems with Interior Cracks of the Thermo-Piezo-Electricity Theory Without Energy Dissipation

Otar Chkadua[1,2], **Anika Toloraia**[3]

[1]*Department of Mathematical Physics, Andrea Razmadze Mathematical Institute of Ivane Jacakhishvili Tbilisi State University, Tbilisi, Georgia*

[2]*Sokhumi State University, Tbilisi, Georgia*

E-mail: otar.chkadua@gmail.com

[3]*Department of Applied Mathematics, Sokhumi State University Tbilisi, Georgia*

E-mail: anikatoloraia@gmail.com

In the paper, we study mixed type interaction dynamical problem with interior cracks between thermo-elastic and thermo-piezo-elastic bodies. The model under consideration is based on the Green–Naghdi theory of thermo-piezo-electricity without energy dissipation. This theory allows the thermal waves to propagate only with a finite speed. Using the Laplace transform, potential theory and the method of boundary pseudodifferential equations, we prove the existence and uniqueness of solutions and analyze their smoothness.

On One Nonlinear Parabolic Integro-Differential Model

Teimuraz Chkhikvadze[1], **Mikheil Gagoshidze**[2], **Temur Jangveladze**[1,2], **Zurab Kiguradze**[1,3]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*

[3] *Electromagnetic Compatibility Laboratory, Missouri University of Science and Technology Rolla, MO 65409, USA*

E-mails: m.zarzma@gmail.com; mishagagoshidze@gmail.com; tjangv@yahoo.com; kiguradzz@mst.edu

One type of model of nonlinear parabolic integro-differential equations is considered. The analogous models partially are derived, on one hand, from the description of real diffusion processes and on the other hand, in the generalization of well-known equations and systems of equations, the study of which devoted many scientific papers (see, for example, [1-8] and references therein). Models of such types still yield to the investigation for special cases. In this direction, the latest and rather complete bibliography can be found in the following monographs [6, 7]. In our research uniqueness, stability and asymptotic behavior of the solutions of the initial-boundary value problems are studied. The finite-difference scheme is constructed and its convergence property is established. The approximate algorithm based on this scheme is constructed. Numerical implementation with various experiments for different values of the input parameters is performed to validate the theoretical conclusions.

Acknowledgments

This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant FR-21-2101.

References

[1] M. M. Aptsiauri, T. A. Jangveladze and Z. V. Kiguradze, Asymptotic behavior of the solution of a system of nonlinear integro-differential equations. (Russian) *Differ. Uravn.* **48** (2012), no. 1, 70–78; translation in *Differ. Equ.* **48** (2012), no. 1, 72–80.

[2] T. Chkhikvadze, On one system of nonlinear partial differential equations. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **36** (2022), 19–22.

[3] T. Dzhangveladze, *An Investigation of the First Boundary-Value Problem for Some Nonlinear Parabolic Integro-differential Equations.* (Russian) Tbilisi State University, Tbilisi, 1983.

[4] D. G. Gordeziani, T. A. Dzhangveladze and T. K. Korshiya, Existence and uniqueness of the solution of a class of nonlinear parabolic problems. (Russian) *Differentsial'nye Uravneniya* **19** (1983), no. 7, 1197–1207; translation in *Differ. Equ.* **19** (1984), no. 7, 887–895.

[5] F. Hecht, T. Jangveladze, Z. Kiguradze and O. Pironneau, Finite difference scheme for one system of nonlinear partial integro-differential equations. *Appl. Math. Comput.* **328** (2018), 287–300.

[6] T. Jangveladze, Investigation and numerical solution of nonlinear partial differential and integro-differential models based on system of Maxwell equations. *Mem. Differ. Equ. Math. Phys.* **76** (2019), 1–118.

[7] T. Jangveladze, Z. Kiguradze and B. Neta, *Numerical Solutions of Three Classes of Nonlinear Parabolic Integro-Differential Equations.* Elsevier/Academic Press, Amsterdam, 2016.

[8] T. Jangveladze, Z. Kiguradze, B. Neta and S. Reich, Finite element approximations of a nonlinear diffusion model with memory. *Numer. Algorithms* **64** (2013), no. 1, 127–155.

Strongly and Weakly Separable Haar Statistical Structures

Tamar Chkonia, **Mimoza Tkebuchava**

Department of Mathematics, Faculty of Exact and Natural Sciences,
Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mails: tamar.chkonia@tsu.ge; mimoza.tkebuchava@tsu.ge

Definition 1 *Let E be an arbitrary locally compact and σ-compact topological group and $B(E)$ is σ-algebra of subsets of E. We say that measure μ defined on $B(E)$ is Haar measure if μ is regular measure and*

$$\mu(sx)=\mu(x),\ \ \forall\, s\in E,\ \ \forall\, x\in B(E).$$

Definition 2 *An object $\{E,S,\mu_i,i\in I\}$ is called Haar statistical structure, where $\{\mu_i,i\in I\}$ is a family of Haar probability measures on $(E,B(E))$.*

Theorem 1 *Let M_H be Hilbert pace of measures, then in M_H there exist a family of pairwise orthogonal Haar probability measures $\{\mu_i,i\in I\}$ such that $M_H=\bigoplus\limits_{i\in I}M_H(\mu_i)$, where $M_H(\mu_i)$ is the Hilbert space of measures of the form*

$$\nu(B)=\int\limits_B f(x)\,\mu_i(dx)\ \ \text{with}\ \ \int\limits_E |f(x)|^2\,\mu_i(dx)<\infty$$

with the norm

$$\|\mu\|_{M_H(\mu_i)}=\bigg(\int\limits_E |f(x)|^2\,\mu_i(dx)\bigg)^{1/2}.$$

Theorem 2 *Let $M_H=\bigoplus\limits_{i\in I}M_H(\mu_i)$ be Hilbert space of measures. For an orthogonal Haar statistical structure $\{E,S,\mu_i,i\in I\}$ to be weakly separable it is necessary and sufficient that the correspondence $f\leftrightarrow\psi_f$ given by the equality $\int\limits_E f(x)\,\psi(dx)=(\psi_f,\psi)$, $\forall\,\psi\in M_H$ would be one-to-one.*

Theorem 3 *Let $M_H=\bigoplus\limits_{i\in I}M_H(\mu_i)$, E be a total metric space. In the $(\mathrm{ZFC})\,\&\,(\mathrm{MA})$ theory, for on orthogonal Borel Haar statistical structure $\{E,S,\mu_i,i\in I\}$, $card\,I=2^{\chi_0}$ to be strongly separable Haar statistical structure it is necessary and sufficient that the correspondence $f\leftrightarrow\psi_f$ given by the equality $\int\limits_H f(x)\,\psi(dx)=(\psi_f,\psi)$, $\forall\,\psi\in M_H$ would be one-to-one.*

On the Differentiation of Random Measures with Respect to Homothecy Invariant Convex Bases

Kakha Chubinidze[1], **Giorgi Oniani**[2]

[1] *Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: kakha.chubinidze@atsu.edu.ge

[2] *School of Computer Science and Mathematics, Kutaisi International University Kutaisi, Georgia*
E-mail: giorgi.oniani@kiu.edu.ge

For every homothecy invariant convex density differentiation basis B in $\mathbb{R}^d$ there are characterized sequences of weights $w=(w_j)_{j\in\mathbb{N}}$ for which the random measures $\mu_{w,\theta}=\sum\limits_{j=1}^{\infty} w_j\delta_{\theta_j}$ are differentiable with respect to the basis B for almost every selection of a sequence of points $\theta_1,\theta_2,\dots$ from the unit cube $[0,1]^d$.

References

[1] K. Chubinidze and G. Oniani, On the differentiation of random measures with respect to homothecy invariant convex bases. *Colloq. Math.* **173** (2023), no. 1, 111–121.

Verb Markers for Georgian–English Automatic Dictionary

Anna Chutkerashvili, Liana Lortkipanidze, Nino Javashvili

Archil Eliashvili Institute of Control Systems of the Georgian Technical University
Tbilisi, Georgia

E-mails: annachutkerashvili@gmail.com; l_lordkipanidze@yahoo.com; ninojavashvili@yahoo.com

Electronic grammar dictionaries are rather important for annotating large text corpora. They serve to assign the correct morphological and syntactic markers to the lexical unit in order to construct grammatically correct phrases. That kind of dictionaries, in addition to annotating texts, are used in translation, language teaching and in the process of managing dialogue systems. The ways of searching as well as correspondence of appropriate English markers to the classification markers of Georgian verb forms for the Georgian-English automatic dictionary will be presented.

Acknowledgments

This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant # FR-21-3509.

The finite Hilbert Transform on $(-1, 1)$

Guillermo P. Curbera

[1]*Facultad de Matemáticas and IMUS, Universidad de Sevilla, Sevilla, Spain*
E-mail: curbera@us.es

We review recent advances [1–5] on the study of the action of the finite Hilbert transform on $(-1, 1)$ on function spaces.

The work is joint with Werner J. Ricker from Germany and Susumu Okada from Australia.

References

[1] G. P. Curbera, S. Okada and W. J. Ricker, Inversion and extension of the finite Hilbert transform on $(-1, 1)$. *Ann. Mat. Pura Appl. (4)* **198** (2019), no. 5, 1835–1860.

[2] G. P. Curbera, S. Okada and W. J. Ricker, Extension and integral representation of the finite Hilbert transform in rearrangement invariant spaces. *Quaest. Math.* **43** (2020), no. 5-6, 783–812.

[3] G. P. Curbera, S. Okada and W. J. Ricker, Non-extendability of the finite Hilbert transform. *Monatsh. Math.* **195** (2021), no. 4, 649–657.

[4] G. P. Curbera, S. Okada and W. J. Ricker, Fine spectra of the finite Hilbert transform in function spaces. *Adv. Math.* **380** (2021), Paper no. 107597, 29 pp.

[5] G. P. Curbera, S. Okada and W. J. Ricker, The finite Hilbert transform acting in the Zygmund space $L\log L$. *Ann. Sc. Norm. Super. Pisa Cl. Sci.* (to appear).

Difference Schemes of Increased Order of Accuracy for Systems of Elliptic and Parabolic Equations with Constant Coefficients

Tinatin Davitashvili[1], Hamlet Meladze[2], Sergo Tsiramua[3]

[1] *The Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: tinatin.davitashvili@tsu.ge

[2] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*

E-mail: h_meladze@hotmail.com

[3] *University of Georgia, Tbilisi, Georgia*

E-mail: s.tsiramua@ug.edu.ge

The systems of elliptic and parabolic equations in a p-dimensional parallelepiped ($p = 2, 3$) with constant coefficients without mixed derivatives are considered. For elliptic equations, the difference scheme was constructed and the uniform convergence of this scheme with a speed of $O(|h|^4)$ was proved. For parabolic equations, a three-layer economic difference scheme of increased order of accuracy was constructed. This scheme is stable in grid norms $\overset{\circ}{W}{}_2^{(1)}$ and $\overset{\circ}{W}{}_2^{(2)}$. A parallel algorithm can be used to solve the obtained difference equations. Convergence with the speed of $O(\tau^2 + |h|^4)$ in a uniform metric is proved.

Reconfigurable Systems Based on Multifunctional Elements

Tinatin Davitashvili[1], **Hamlet Meladze**[2], **Sergo Tsiramua**[3]

[1] *The Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: tinatin.davitashvili@tsu.ge

[2] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*
E-mail: h_meladze@hotmail.com

[3] *School of Science and Technologies, University of Georgia, Tbilisi, Georgia*
E-mail: s.tsiramua@ug.edu.ge

Multifunctional elements (MFE) are a special class of elements, whose reliability model differs from the classical, two-pole, "works – does not work" model. In addition to the faultless and faulty states, MFE can have partial fault states. The multifunctionality of elements determines the formation of a flexible, structurally reconfigurable, adaptive system. In such systems, in the event of partial failure of elements, it is possible to reconfigure the structure by redistributing functions between elements and continuing the successful operation of the system [1].

At the initial stage of forming a system with multifunctional elements, optimal distribution of functions between MFEs is established, and in the case of partial failure of elements during the operation process, the problem of optimal reconfiguration of systems arises. In order to solve these problems, it is necessary to transition from the logical description of the system's functional resources to a probabilistic description of functional capabilities. The logical $(0,1)$ matrix of the system's functional resources should be replaced with a probabilistic matrix $P(m \times n) = [p_i(f_j)]$, where $p_i(f_j) \in [0,1]$ is the probability of performing the j-th function by the i-th element. It should also be noted that since MFEs belong to the class of multipolar elements, it is possible to use Fuzzy Logic methods to obtain $p_i(f_j)$ estimates [2].

Accordingly, the shortest routes to successful system operation are recorded in the following format

$$P_F(S_q) = p_{i1}(f_{j1}) \times p_{i2}(f_{j2}) \times \dots \times p_{jm}(f_{jm}),$$

where $i_1 \neq i_2 \neq \dots \neq i_m$, $j_1 \neq j_2 \neq \dots \neq j_m$, $q \in [1, N_S]$.

In most cases, the MFEs vary in relation to different functions. From this it follows that $P_F(S_1) \neq P_F(S_2) \neq \dots \neq P_F(S_{Ns})$, which implies that a significant value is attributed to how the system starts functioning and how it continues to operate after reconfiguration. In this paper, we consider the issues of optimal reconfiguration of systems to ensure high reliability.

References

[1] S. Tsiramua, System reliability: Organizational design of highly reliable human-machine systems based on multi-functional operators. (Russian) *Ed. Lambert Academic Publishing*, 2017.

[2] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic. Theory and Applications. Prentice Hall PTR, Upper Saddle River, New Jersey*, 1995.

Exploring the Development of Hydrogen Energy in Georgia in the Face of Climate Change

Teimuraz Davitashvili[1], **Inga Samkharadze**[2], **Giorgi Rukhaia**[1]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University*
Tbilisi, Georgia
E-mails: tedavitashvili@gmail.com; thegr1992@gmail.com

[2] *Hydrometeorological Institute of Georgian Technical University*
Tbilisi, Georgia
E-mail: inga.samkharadze562@ens.tsu.edu.ge

Despite the fact that hydrogen in nature is not replenished naturally and is not depleted (by analogy with renewable energy sources), there is a growing interest around the world in using hydrogen for electricity generation or in industry, transport and other areas as a highly efficient energy source. Currently, Georgia uses only hydro, wind and geothermal energy from renewable energy sources and has good opportunities for producing and transporting hydrogen. Indeed, Kazakhstan, Turkmenistan and Azerbaijan are planning to produce "green" and "blue" hydrogen (having a modern production infrastructure for petrochemicals and huge resource potential) and develop the infrastructure and operational components of the "Middle Corridor" for its transportation using the TRACECA route through Georgia and Turkey to EU countries. While efforts are being made in the long term to build a dedicated hydrogen infrastructure (pipeline), blending hydrogen into the existing gas pipeline network is a more promising strategy for transporting hydrogen in the short term. Thus, studying the behavior of mixed flow in a pipeline is relevant to the analysis of several potential problems that arise when mixing hydrogen in natural gas networks. This article focuses on exploring how much hydrogen can be integrated into a gas pipeline from an operational point of view. Namely, on the basis of one mathematical model describing the flow of a mixture of natural gas and hydrogen substances in a pipeline, the distribution of pressure and gas flow through a branched gas pipeline was analytically obtained. Some aspects of the production and transportation of hydrogen as a highly efficient source of energy on the territory of Georgia under the conditions of climate change are discussed.

Acknowledgments

The research is funded by Shota Rustaveli National Scientific Foundation Grant # FR-22-18445.

Development of Critical Thinking in Mathematics Classes

Manana Deisadze, Shalva Kirtadze

Department of Teaching Methods, Akaki Tsereteli State University, Kutaisi, Georgia
E-mails: manana.deisadze@atsu.edu.ge; shalva.kirtadze@atsu.edu.ge

Learning and teaching of National Curriculum aims to provide students with the development of see-through abilities and values, such as critical thinking, which is an ability to critically discuss and analyze facts, views and concepts; to state questions and search for answers; to lead argumentative discussion, i.e. to prove their own assumptions and statements with relevant arguments and examples; to make meaningful choice and prove it. Critical thinking is an intellectually organised process, when concept thinking, application, analysis and synthesis are active and skillful. This work introduces exercises for the development of Critical thinking and problem solving abilities through math teaching in real academic practice. Topic - geometrical shapes and measures by using area calculation formulae to measure the area of square (rectangle) shaped figure.

The Role of Developmental Evaluation in Mathematics Classes on the Entrance Level

Manana Deisadze, Shalva Kirtadze

Department of Teaching Methods, Akaki Tsereteli State University, Kutaisi, Georgia
E-mails: manana.deisadze@atsu.edu.ge; shalva.kirtadze@atsu.edu.ge

Evaluation is a process of collecting, monitoring, analysing and applying information. This describes student's progress and academic advancement with respect to knowledge acquisition, skills development and attitude. In accordance with National Curriculum of the third generation, defining evaluation marks student's achievement level with regard to the Subject National Plan outcomes, while developmental evaluation defines the dynamics of every student's development and is directed towards the improvement of teaching level. It is important and noteworthy that the developmental evaluation is not only concerned with the improvement of student's learning quality, but rather, developmental evaluation evidences are used to adapt teaching to the needs of students. This work focuses on the idea that the developmental evaluation is the unity of learning activities which not only supports the development of students' learning quality but at the same time, it is a tool to help teachers develop teaching, and define the priorities in order to develop and enhance students.

Calculus with Bayesian User Model and GeoGebra in Assessment

Luis Descalço

Department of Mathematics, University of Aveiro, Aveiro, Portugal
E-mail: luisd@ua.pt

Although there are students intrinsically motivated for learning mathematics, many others see it as a hindrance. The poor success rate in calculus courses reflects this unfavorable student's attitude. We present here some tools that seem to contribute for improving the results.

For more than ten years, we have been developing, improving and using with students a computer system for aiding autonomous learning in calculus, with concept maps, including aggregation and pre-requisite relations, together with a Bayesian user model to provide feedback from collected evidence. Moreover, a methodology of teaching, allowing the use of GeoGebra in assessment, has been used for motivating students to learn mathematics together with a CAS. Finally, to help students dealing with decreasing attention span and stress, we have been improving class breaks in calculus, using simple technics from modern yoga.

We describe these tools, present the results of using them with students from several courses of sciences and engineering, and discuss the benefits of their application in similar contexts.

Acknowledgments

This work is supported by The Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT – Fundação para a Ciência e a Tecnologia), references UIDB/04106/2020 and UIDP/04106/2020.

References

[1] E. Millán, L. Descalço, G. Castillo, P. Oliveira and S. Diogo, Using Bayesian networks to improve knowledge assessment. *Computers and Education* **60** (2013), no. 1, 436–447.

[2] L. Descalço, A. do Canto Filho and J. V. de Lima, Learning Trajectories with Bayesian student model for autonomous study in flipped learning. *Proceedings of EDULEARN17 Conference, 3th - 5th July, International Academy of Technology, Education and Development (IATED)*, pp. 564–569, Barcelona, Spain, 2017.

[3] L. Descalço and P. Carvalho, Comparing Learning Objects for Effective Learning in Mathematics. *Proceedings of EDULEARN20 Conference, 6th–7th July, 2020, International Academy of Technology, Education and Development (IATED)*, pp. 1173–1179, Palma, Spain, 2020.

[4] P. Carvalho, L. Descalço and H. F. Gonçalves, Using Computer Algebra Systems in Teaching and Assessment in Calculus. *Proceedings of EDULEARN23 Conference, 3rd–4th July, International Academy of Technology, Education and Development (IATED)*, Palma, Spain, 2023 (to appear).

[5] P. Carvalho, L. Descalço, H.F. Gonçalves. Attention Span in Calculus Classes and Yoga Breaks. *Proceedings of EDULEARN23 Conference, 3rd–4th July, International Academy of Technology, Education and Development (IATED)*, Palma, Spain, 2023 (to appear).

On Bicentric Configurations of Pentagon Linkage

Ana Diakvnishvili

Ilia State University, Tbilisi, Georgia
E-mail: ana.diakvnishvili.1@iliauni.edu.ge

We investigate the existence of bicentric configurations of a given pentagon linkage. Recall that a polygon is called cyclic if its vertices lie on a circle, it is called tangential if it has an inscribed circle, and it is called bicentric if it simultaneously has a circumscribed circle and an inscribed circle. Bicentric polygons are determined up to a congruence by three positive numbers (R, r, d), where R is the radius of circumscribed circle, r is the radius of inscribed circle and d is the distance between their centers.

It will be proven that, if we have a tangential pentagon with all sides different from each other, and if its area is a root of the so-called Robbins polynomial for the same sides, then this polygon is bicentric. The existence of tangential configuration of a given pentagon linkage can be verified by solving a linear system with circulant matrix.

Detailed computations will be presented in a number of examples. We will also discuss the possibility of applying this approach to the hexagon case.

References

[1] D. P. Robbins, Areas of polygons inscribed in a circle. *Discrete Comput. Geom.* **12** (1994), no. 2, 223–236.

[2] D. P. Robbins, Areas of polygons inscribed in a circle. *Amer. Math. Monthly* **102** (1995), no. 6, 523–530.

[3] D. Svrtan, D. Veljan and V. Volenec, Geometry of pentagons: from Gauss to Robbins. *Department of Mathematics, University of Zagreb*, 2004.

[4] M. Radić, Certain relations concerning bicentric polygons and 2-parametric presentation of Fuss' relations. *Math. Pannon.* **20** (2009), no. 2, 219–248.

[5] M. Radić and A. Zatezalo, About some kinds of bicentric polygons and concerning relations. *Math. Maced.* **4** (2006), 47–73.

Generic Bessel Potential Spaces on Lie Groups and Their Applications

Roland Duduchava[1,2]

[1]*Institute of Mathematics of the University of Georgia, Tbilisi, Georgia*
[2]*Department of Mathematical Physics, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: RolDud@gmail.com

The purpose of the presentation is to discuss the role of adapted Bessel potential space to the structure of underlying Lie group, defined based on generic differential operators from the Lie algebra of the Lie group. Such generic Bessel potential spaces are well adapted to the investigation of integro-differential (of pseudo-differential) operators on Lie groups.

We will expose several examples of Lie groups and corresponding generic Bessel potenrial spaces and then concentrate on the investigation of boundary value problems (BVPs) for the Laplace-Beltrami equation on a hypersurface $\mathcal{S}\subset\mathbb{R}^3$ with the Lipschitz boundary $\Gamma=\partial\mathcal{S}$, containing a finite number of angular points (knots) c_j of magnitude α_j, $j=1,2,\dots,n$. The Dirichlet, Neumann and mixed type BVPs are considered in a non-classical setting, when solutions are sought in the generic Bessel potential spaces (GBPS) $\mathbb{GH}^s_p(\mathcal{S},\rho)$, $s>1/p$, $1<p<\infty$ with weight $\rho(t)=\prod\limits_{j=1}^{n}|t-c_j|^{\gamma_j}$. By the localization the problem is reduced to the investigation of Model Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domain $\Omega_{\alpha_j}\subset\mathbb{R}^2$ of magnitude α_j, $j=1,2\dots,n$. Further the model problem in the GBPS with weight $\mathbb{GH}^s_p(\Omega_{\alpha_j},t^{\gamma_j})$ is investigated by means of Mellin convolution operators on the semi-axes $\mathbb{R}^+=(0,\infty)$. Explicit criteria for the Fredholm property and the unique solvability of the initial BVPs are obtained and singularities of solutions at knots to the mentioned BVPs are indicated. In contrast to the same BVPs in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{S})$, the Fredholm property in the GBPS $\mathbb{GH}^s_p(\mathcal{S},\rho)$ with weight is independent of the smoothness parameter s.

Dirichlet and Neumann Boundary Value Problems for the Helmholtz Equation in a Double Angle

Roland Duduchava[1,2], **Medea Tsaava**[2], **Margarita Tutberidze**[3]

[1] *The University of Georgia, Tbilisi, Georgia*

[2] *Department of Mathematical Physics, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi Staye University, Tbilisi, Georgia*

E-mails: RolDud@gmail.com; m.caava@yahoo.com; margarita.tutberidze@ug.edu.ge

We investigate Dirichlet and Neumann boundary value problems for the anisotropic Helmholtz equation in a double angle of magnitude $\alpha > 0$ and $-\beta < 0$, having in common the positive semi axes $\mathbb{R}^+$. On the outer boundary is prescribed a Dirichlet or Neumann condition, while along the common boundary of angles (interface) $\mathbb{R}^+$ is prescribed the continuity)the transmission) conditions. We consider the non-classical $\mathbb{L}_p$-based Bessel potential space setting of the problem. for $1 < p < \infty$ We apply the potential method and reduce the boundary value problem to the system of boundary pseudodifferential equation, which is further reduced to an equivalent system of 6×6 Mellin-type convolution equations on $\mathbb{R}^+$ the Bessel potential space $\mathbb{H}^s_p(\mathbb{R}^+)$. By using the results obtained earlier by V. Didenko and R. Duduchava for such equations, we write symbol of the equation and derive the criteria for solvability (Fredholmness) of such systems of equations in the Bessel potential spaces $\mathbb{H}^s_p(\mathbb{R}^+)$. Moreover, we indicate the range of space parameters (s, p) for which the original Dirichlet and Neumann boundary value problems have unique solutions in the non-classical space settings.

References

[1] T. Buchukuri, R. Duduchava, D. Kapanadze and M. Tsaava, Localization of a Helmholtz boundary value problem in a domain with piecewise-smooth boundary. *Proc. A. Razmadze Math. Inst.* **162** (2013), 37–44.

[2] V. D. Didenko and R. Duduchava, Mellin convolution operators in Bessel potential spaces. *J. Math. Anal. Appl.* **443** (2016), no. 2, 707–731.

[3] R. Duduchava and M. Tsaava, Mixed boundary value problems for the Laplace–Beltrami equation. *Complex Var. Elliptic Equ.* **63** (2018), no. 10, 1468–1496.

[4] R. Duduchava, Mixed type boundary value problems for Laplace-Beltrami equation on a surface with the Lipschitz boundary. *Georgian Math. J.* **28** (2021), no. 2, 219–232.

CLP(MS): Programming Using Multiple Similarity Constraints

Besik Dundua[1,2]

[1] *Kutaisi International University, Kutaisi, Georgia*
[2] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: bdundua@gmail.com

We describe the semantics of CLP(MS): constraint logic programming over multiple similarity relations. Similarity relations are reflexive, symmetric, and transitive fuzzy relations. They help to make approximate inferences, replacing the notion of equality. Similarity-based unification has been quite intensively investigated, as a core computational method for approximate reasoning and declarative programming. In this talk we consider solving constraints over several similarity relations [1], instead of a single one. Multiple similarities pose challenges to constraint solving, since we can not rely on the transitivity property anymore. Existing methods for unification with fuzzy proximity relations (reflexive, symmetric, non-transitive relations) do not provide a solution that would adequately reflect particularities of dealing with multiple similarities. To address this problem, we develop a constraint solving algorithm for multiple similarity relations, prove its termination, soundness, and completeness properties. We integrate the solving algorithm into constraint logic programming schema and study semantics of obtained CLP(MS).

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia under the project # FR-21-7973.

References

[1] B. Dundua, T. Kutsia, M. Marin and C. Pau, Constraint solving over multiple similarity relations. In Zena M. Ariola (Ed.), *5th International Conference on Formal Structures for Computation and Deduction, FSCD 2020 (June 29-July 6, 2020), Paris, France (Virtual Conference)*, vol. 167 of LIPIcs, pp. 30:1–30:19, *Schloss Dagstuhl–Leibniz–Zentrum für Informatik*, 2020.

New Algorithm for Spectral Factorization of Rational Matrix Functions with Applications to Paraunitary Filter Banks

Lasha Ephremidze[1,3], **Gennady Mishuris**[2], **Ilya Spitkovsky**[3]

[1]*Department of Mathematical Analysis, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

E-mail: lasha@rmi.ge

[2]*Department of Mathematics, Aberystwyth University, Aberystwyth, UK*

E-mail: ggm@aber.ac.uk

[3]*Division of Science and Mathematics, New York University Abu Dhabi, Abu Dhabi, UAE*

E-mails: le23@nyu.edu; ims2@nyu.edu

Spectral factorization is the process by which a positive (scalar or matrix-valued) function S is expressed in the form $S(t) = S_+(t)S_+^*(t)$, $t \in \mathbb{T}$, where S_+ can be analytically extended inside the unit circle $\mathbb{T}$ and S_+^* is its Hermitian conjugate. There are multiple contexts in which this factorization naturally arises, e.g., linear prediction theory of stationary processes, optimal control, digital communications, etc. Spectral factorization is used to construct certain wavelets and multiwavelets as well. Therefore, many authors contributed to development different computational methods for spectral factorization. Unlike the scalar case, where an explicit formula exists for factorization, in general, there is no explicit expression for spectral factorization in the matrix case. The existing algorithms for approximate factorization are, therefore, more demanding in the matrix case.

The Janashia–Lagvilava algorithm [1, 2] is a relatively new method of matrix spectral factorization which proved to be effective [3, 4] and provides several generalizations. Nevertheless, the algorithm, as it was designed so far, was not able to factorize exactly even simple polynomial matrices. In the proposed work, we cast a new light on the capabilities of the method eliminating the above-mentioned flaw. In particular, we can factorize explicitly matrices whose rational entries in the lower-upper triangular factorization can be determined (indicating their poles inside $\mathbb{T}$ and the principle parts at these poles). This extension allows to construct rational paraunitary filter banks with preassigned poles and zeros which are multidimensional lossless infinite impulse response filters and play an important role in linear time invariant systems.

Acknowledgments

The work was supported by the by the EU through the H2020-MSCA-RISE-2020 project EffectFact, Grant agreement ID: 101008140. The first author thanks also to Martin Dutko for fruitful discussions and hospitality during his recent visit to software company "Rockfield" under the framework of the grant.

References

[1] G. Janashia and E. Lagvilava, A method of approximate factorization of positive definite matrix functions. *Studia Math.* **137** (1999), no. 1, 93–100.

[2] G. Janashia, E. Lagvilava and L. Ephremidze, A new method of matrix spectral factorization. *IEEE Trans. Inform. Theory* **57** (2011), no. 4, 2318–2326.

[3] L. Ephremidze, F. Saied and I. M. Spitkovsky, On the algorithmization of Janashia–Lagvilava matrix spectral factorization method. *IEEE Trans. Inform. Theory* **64** (2018), no. 2, 728–737.

[4] L. Ephremidze, A. Gamkrelidze and I. M. Spitkovsky, On the spectral factorization of singular, noisy, and large matrices by Janashia–Lagvilava method. *Trans. A. Razmadze Math. Inst.* **176** (2022), no. 3, 361–366.

Qualitative Analysis of Nonregular Differential-Algebraic Equations and Applications in the Gas Dynamics

Maria Filipkovska[1,2]

[1] *Chair for Dynamics, Control, Machine Learning and Numerics, Friedrich–Alexander-Universität Erlangen-Nürnberg, Erlangen, Germany*

[2] *Department of Mathematical Physics, B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine*

E-mails: maria.filipkovska@fau.de, filipkovskaya@ilt.kharkov.ua

In this work, the initial value problem (IVP) for implicit differential equations of the form $\frac{d}{dt}[Ax] + Bx = f(t,x)$, where $t \in [t_+,\infty)$, $t_+ \geq 0$, $x \in \mathbb{R}^n$, $A, B \in \mathrm{L}(\mathbb{R}^n,\mathbb{R}^m)$ and $f(t,x) \in C([t_+,\infty)\times\mathbb{R}^n,\mathbb{R}^m)$, is considered. When $m \neq n$ or $m = n$ and the operator A is noninvertible (degenerate) these equations are called differential-algebraic equations (DAEs) or degenerate differential equations. DAEs of the considered type are commonly referred to as semilinear. It is assumed that the characteristic pencil $\lambda A + B$ corresponding to the linear part $\frac{d}{dt}[Ax] + Bx$ of the equation is nonregular (singular) and accordingly the DAE is called nonregular or singular. In the general case, a singular pencil contains a block which is a regular pencil, and singular DAEs comprise regular ones. If $\operatorname{rank}(\lambda A + B) = m < n$, then the singular semilinear DAE corresponds to an underdetermined system of equations (the number of equations is less than the number of unknowns); if $\operatorname{rank}(\lambda A + B) = n < m$, then DAE corresponds to an overdetermined system of equations (the number of equations is greater than the number of unknowns).

We present the obtained theorems on the existence and uniqueness of global solutions, the Lagrange stability, the dissipativity (i.e., the ultimate boundedness of solutions) and the Lagrange instability of singular semilinear DAEs. To prove them, the special block form for the characteristic pencil of the operator (the operator coefficients of the DAE) was developed [1, 2]. We also use the spectral projectors of the Riesz type and differential inequalities with the functions of the Lyapunov type. We do not use the global Lipschitz condition to prove the global solvability of the DAE that allows one to solve more general classes of applied problems. In the present work, the theorems with the most general conditions were obtained. Particular cases of the theorems on the Lagrange stability and instability were proved in [1]. The application of the obtained theorems to the study of isothermal models of gas networks are discussed. The models of gas networks consisting of pipes, valves and regulating elements are similar to the ones presented in [3].

Acknowledgments

The work was partially supported by the Alexander von Humboldt Foundation.

References

[1] M. S. Filipkovska, Lagrange stability and instability of irregular semilinear differential-algebraic equations and applications. (Russian) *Ukraïn. Mat. Zh.* **70** (2018), no. 6, 823–847; translation in *Ukrainian Math. J.* **70** (2018), no. 6, 947–979.

[2] M. S. Filipkovska, A block form of a singular pencil of operators and a method of obtaining it. *Visnyk of V. N. Karazin Kharkiv National University. Ser. Mathematics, Applied Mathematics and Mechanics* **89** (2019), 33–58.

[3] T. Kreimeier, H. Sauter, T. Streubel, C. Tischendorf and A. Walther, Solving least-squares collocated differential algebraic equations by successive abs-linear minimization a case study on gas network simulation, 2022, 22 pp. [*Preprint*].

Sobolev Meets Riesz: an Alternative Characterization of Weighted Sobolev Spaces via Weighted Riesz Bounded Variation Spaces

Oscar Fonseca

Science Department, America University, Bogotá, Colombia
E-mail: oscar.guzman@profesores.uamerica.edu.co

We introduce weighted Riesz bounded variation spaces defined in an open subset of the n-dimensional Euclidean space. Using the newly introduced space we give a characterization of weighted Sobolev spaces when the weight belongs to the Muckenhoupt class. We also provide, as an application of the main result, a characterization of variable exponent Sobolev spaces via variable exponent Riesz bounded variation spaces. This is a join work with Prof. David Cruz Uribe from University of Alabama and Prof. Humberto Rafeiro from United Araba Emirates University.

Acknowledgments

We thank America University for make possible the deliver of this talk by providing the expenses of my trip.

Boundary-Contact Problems with Regard to Friction of Couple-Stress Viscoelasticity for Inhomogeneous Anisotropic Bodies (Quasi-Static Cases)

Avtandil Gachechiladze[1,2], **Roland Gachechiladze**[1]

[1]*Department of Mathematical Physics, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

[2]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*

E-mails: avtogach@yahoo.com; r.gachechiladze@yahoo.com

The paper deals with three-dimensional boundary-contact problems of couple-stress viscoelasticity for inhomogeneous anisotropic bodies with regard to friction. We prove the uniqueness theorem using the corresponding Green formulas and positive definiteness of the potential energy. To analyze the existence of solutions we reduce equivalently the problem under consideration to a spatial variational inequality. We consider a special parameter-dependent regularization of this variational inequality which is equivalent to the relevant regularized variational equation depending on a real parameter and study its solvability by the Faedo–Galerkin method. Some a priori estimates for solutions of the regularized variational equation are established and with the help of an appropriate limiting procedure the existence theorem for the original contact problem with friction is proved.

Some Properties and Numerical Solution of Initial-Boundary Value Problem for One System of Nonlinear Partial Differential Equations

Mikheil Gagoshidze[1], **Temur Jangveladze**[1,2], **Zurab Kiguradze**[1,3]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*

[3] *Electromagnetic Compatibility Laboratory, Missouri University of Science and Technology Rolla, MO 65409, USA*

E-mails: mishagagoshidze@gmail.com; tjangv@yahoo.com; kiguradzz@mst.edu

Investigated model is based on the well-known system of Maxwell's equations and represents some of its generalizations. Such type models are studied in many works (see, for example, [1–6] and references therein). The one-dimensional case with a three-component magnetic field is considered. The asymptotic behavior of solution for initial-boundary value problem as time variable tends to infinity is studied. The question of linear stability of the stationary solution of the system and the possibility of the Hopf-type bifurcation is investigated. A finite-difference scheme is constructed. The convergence of this scheme is studied and an estimate of the error of the approximate solution is obtained. Corresponding numerical experiments are carried out.

Acknowledgments

This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant # FR-21-2101.

References

[1] T. A. Dzhangveladze, Stability of the stationary solution of a system of nonlinear partial differential equations. (Russian) *Current problems in mathematical physics, Vol. I (Russian) (Tbilisi, 1987)*, 214–221, 481–482, *Tbilis. Gos. Univ., Tbilisi*, 1987.

[2] T. Jangveladze and M. Gagoshidze, Hopf bifurcation and its computer simulation for one-dimensional Maxwell's model. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **30** (2016), 27–30.

[3] T. Jangveladze, Investigation and numerical solution of nonlinear partial differential and integro-differential models based on system of Maxwell equations. *Mem. Differ. Equ. Math. Phys.* **76** (2019), 1–118.

[4] T. Jangveladze, Some properties of the initial-boundary value problem for one system of nonlinear partial differential equations. *Bull. TICMI* **25** (2021), no. 2, 137–143.

[5] T. Jangveladze, Finite difference scheme for one system of nonlinear partial differential equations. *Bull. Georgian Natl. Acad. Sci. (N.S.)* **16** (2022), no. 2, 7–13.

[6] Z. V. Kiguradze, On the stationary solution for one diffusion model. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **16** (2001), no. 1-3, 17–20.

Numerical Solution of One Two-Dimensional System of Nonlinear Partial Differential Equations

Mikheil Gagoshidze[1], **Temur Jangveladze**[1,2], **Zurab Kiguradze**[1,3], **Besiki Tabatadze**[4]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*

[3] *Electromagnetic Compatibility Laboratory, Missouri University of Science and Technology Rolla, MO 65409, USA*

[4] *European University, Tbilisi, Georgia*

E-mails: mishagagoshidze@gmail.com; tjangv@yahoo.com; kiguradzz@mst.edu; tabatadze.besik@eu.edu.ge

The two-dimensional system of nonlinear partial differential equations is considered. This system arises in the process of vein formation of young leaves [7]. There are many works where this and many models describing similar processes are also presented and discussed (see, for example, [1, 2, 8, 9] and references therein). Investigation and numerical solution of such type systems are discussed in many papers (see, for example, [1, 3–6] and references therein). In our note, the averaged model of sum approximation is used [3] and the variable directions difference scheme is also considered [4]. Comparison of numerical experiments of the proposed methods is done.

Acknowledgments

This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant # FR-21-2101.

References

[1] J. Bell, C. Cosner and W. Bertiger, Solutions for a flux-dependent diffusion model. *SIAM J. Math. Anal.* **13** (1982), no. 5, 758–769.

[2] H. Candela, A. Martı́nez-Laborda and J. L. Micol, Venation Pattern Formation in*Arabidopsis thaliana*Vegetative Leaves. *Developmental Biology* **205** (1999), no. 1, 205–216.

[3] T. A. Dzhangveladze, An averaged model of summary approximation for a system of nonlinear partial differential equations. (Russian) *Tbiliss. Gos. Univ. Inst. Prikl. Mat. Trudy* **19** (1987), 60–73.

[4] T. A. Jangveladze, The difference scheme of the type of variable directions for one system of nonlinear partial differential equations. *Tbiliss. Gos. Univ. Inst. Prikl. Mat. Trudy* **47** (1992), 45–66.

[5] T. Jangveladze, Z. Kiguradze and M. Gagoshidze, Economical difference scheme for one multi-dimensional nonlinear system. *Acta Math. Sci. Ser. B (Engl. Ed.)* **39** (2019), no. 4, 971–988.

[6] T. Jangveladze, M. Nikolishvili and B. Tabatadze, On one nonlinear two-dimensional diffusion system. *Proc. 15th WSEAS Int. Conf. Applied Math. (MATH 10)*, (2010), 105–108.

[7] G. J. Mitchison, A model for vein formation in higher plants. *Proc. R. Soc. Lond. B.* **207** (1980), no. 1166, 79–109.

[8] P. Prusinkiewicz, S. Crawford, R. S. Smith, K. Ljung, T. Bennett, V. Ongaro and O. Leyser, Control of Bud Activation by an Auxin Transport Switch. *Proc. Nat. Acad. Sci.* **106(41)** (2009), 17431–17436.

[9] C. J. Roussel and M. R. Roussel, Reaction–diffusion models of development with state-dependent chemical diffusion coefficients, *Progress Biophys. Molecular Biology* **86** (2004), no. 1, 113–160.

On Subsequences of Lebesgue Functions of General Uniformly Bounded ONS

Rostom Getsadze

Department of Mathematics, Uppsala University, Uppsala, Sweden
E-mails: rostom.getsadze@math.uu.se, rostom@kth.se, rostom.getsadze@telia.com

In the present paper we study growth of subsequences of Lebesgue functions of general uniformly bounded ONS for wide class of subsequences of indices.

Subgaussian Random Elements in Infinite Dimensional Spaces

George Giorgobiani, Vakhtang Kvaratskhelia, Vaja Tarieladze

Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University
Tbilisi, Georgia

E-mails: giorgobiani.g@gtu.edu.ge; v.kvaratskhelia@gtu.ge; v.tarieladze@gtu.ge

The concept of Subgaussian random variable first appeared in a paper [1] of well-known French mathematician J.P. Kahane. As a motivation for introducing this concept Kahane cited the cycle of works of Paley and Zygmund in the early 1930s. Later, Subgaussian random variables and processes were discussed and studied by many authors. In our presentation, different definitions of Subgaussian random elements (weakly, T- and F-Subgaussian) in infinite-dimensional spaces are discussed and compared with each other.

One of the problems considered will be the problem of characterization of T-Subgaussian random elements in an infinite-dimensional Banach space X. A solution of this problem in the case (which includes the case of an infinite dimensional Hilbert space) when X is a reflexive Banach space of type 2 will be discussed too.

Acknowledgments

The work was partially supported by European Commission HORIZON EUROPE WIDERA-2021-ACCESS-03, Grant Project (GAIN), grant agreement #101078950.

References

[1] J.-P. Kahane, Propriétés locales des fonctions à séries de Fourier aléatoires. (French) *Studia Math.* **19** (1960), 1–25.

On One Notable Method of Developmental Teaching in Mathematics

Guram Gogishvili

St. Andrew the First Called Georgian University, Faculty of Business, Computing and Social Sciences, Tbilisi, Georgia

E-mail: guramgog@gmail.com

At the primary level of school education, it is very important to focus on such developmental and multifaceted interesting mathematical tasks, the further research, generalization, and obtaining noteworthy results of which can be continued throughout the school period and, moreover, even at the higher level of education. Besides, it is clear that presenting tasks in a fun and enjoyable way adds charm to such tasks and increases student engagement. In this way – repeatedly returning to and exploring problems begun as "harmless fun" – can make a significant contribution to a young person's mathematical education.

An example of such a task is the problem proposed by the 1st century historian Josephus Flavius about the escape from the tragic decision of 41 captive Jew fighters. Obviously, at the primary school level, you can think of a peaceful analogue of the content of the task and reduce the number of characters to 6, 10. It is a feasible, fun, but at the same time non-trivial task for them.

When considering the task in these simple particular cases, the idea of a further generalization of the task naturally arises, which is connected with a substantial increase of the number of persons. Students should express a hypothesis about the representation of the values of search numbers in both recursive and explicit (non-recursive) forms and prove these hypotheses. The inclusion of the binary number system in the research, the use of programming and the visualization of the solution will also deserve attention, which will make a deep impression on the students and undoubtedly push them to further similar multifaceted research.

General approaches to similar issues on the example of the task set and research methods for its generalization, their presentation are discussed in the report.

About Isospectrality

Vladimir Gol'dshtein

Department of Mathematics, Ben Gurion University of the Negev
Beer Sheva, Israel
E-mail: vladimirshtein@gmail.com

In this talk we discuss isospectrality property of the Dirichlet boundary value problem for elliptic operators in divergence form in bounded planar simply connected domains. It is based on a version of the Rayleigh–Faber–Krahn inequality for a special case of these elliptic operators. As a preliminary result we obtained estimates for first eigenvalues Dirichlet boundary value problem for elliptic operators in divergence form (i.e. for the principal frequen cy of non-homogeneous membranes) in bounded domains satisfying quasihyperbolic boundary conditions. The suggested method is based on the quasiconformal composition operators on Sobolev spaces and their applications to estimates of constantsin the corresponding Sobolev inequalities. Some examples of isocpectral operators will be discussed.

References

[1] V. Gol'dshtein and V. Pchelintsev, On the Principal Frequency of Nonhomogeneous membranes. *Preprint* arXiv:2301.03197; `https://arxiv.org/abs/2301.03197`.

Embeddings of Smoothness Morrey Spaces on Domains

Helena F. Gonçalves

CIDMA, University of Aveiro, Aveiro, Portugal
E-mail: helenag@ua.pt

Smoothness Morrey spaces are built upon Morrey spaces $\mathcal{M}_{u,p}(\mathbb{R}^d)$, $0 < p \le u < \infty$ and its study has been motivated by several applications. This class of function spaces includes not only Besov–Morrey spaces $\mathcal{N}^s_{u,p,q}(\mathbb{R}^d)$ and Triebel–Lizorkin–Morrey spaces $\mathcal{E}^s_{u,p,q}(\mathbb{R}^d)$ with $0 < p \le u < \infty$, $0 < q \le \infty$, $s \in \mathbb{R}$, but also Besov-type spaces $B^{s,\tau}_{p,q}(\mathbb{R}^d)$ and Triebel–Lizorkin-type spaces $F^{s,\tau}_{p,q}(\mathbb{R}^d)$, with $0 < p < \infty$, $0 < q \le \infty$, $\tau \ge 0$, $s \in \mathbb{R}$. Although these scales are defined in different ways, they share some properties and are related to each other by a number of embeddings and coincidences. For instance, they both include the classical spaces of type $B^s_{p,q}(\mathbb{R}^d)$ and $F^s_{p,q}(\mathbb{R}^d)$ as special cases.

In this talk, embeddings of Besov-type and Triebel–Lizorkin-type spaces,

$$\mathrm{id}_\tau\colon B^{s_1,\tau_1}_{p_1,q_1}(\Omega) \hookrightarrow B^{s_2,\tau_2}_{p_2,q_2}(\Omega) \ \text{ and } \ \mathrm{id}_\tau\colon F^{s_1,\tau_1}_{p_1,q_1}(\Omega) \hookrightarrow F^{s_2,\tau_2}_{p_2,q_2}(\Omega),$$

where $\Omega \subset \mathbb{R}^d$ is a bounded domain, are studied. Namely, we present necessary and sufficient conditions for the continuity and compactness of id_τ.

This talk is based on a joint work with D. D. Haroske and L. Skrzypczak.

Acknowledgments

This work is supported by The Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT – Fundação para a Ciência e a Tecnologia), references UIDB/04106/2020 and UIDP/04106/2020.

References

[1] H. F. Gonçalves, D. D. Haroske and L. Skrzypczak, Compact embeddings in Besov-type and Triebel–Lizorkin-type spaces on bounded domains. *Rev. Mat. Complut.* **34** (2021), no. 3, 761–795.

[2] H. F. Gonçalves, D. D. Haroske and L. Skrzypczak, Limiting embeddings of Besov-type and Triebel–Lizorkin-type spaces on domains and an extension operator. *Annali di Matematica* (2023); `https://doi.org/10.1007/s10231-023-01327-w`.

On New Fibonacci Identities Involving Multinomial Coefficients

Taras Goy

Faculty of Mathematics and Computer Science,
Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine
E-mail: taras.goy@pnu.edu.ua

In this note, we evaluate determinants of several families of Hessenberg matrices having various translates of the Fibonacci numbers as their nonzero entries. By the generalized Trudi formula, these determinant identities we write equivalently as identities with multinomial coefficients (see [1, 2] for more details).

Let $(F_n)_{n\ge 0}$ be the Fibonacci sequence satisfying the recurrence $F_0=0$, $F_1=1$, $F_n=F_{n-1}+F_{n-2}$, $n\ge 2$.

Theorem 1 *Let $n\ge 1$, except when noted otherwise. Then*

$$\sum_{2t_1+3t_2+\cdots+nt_{n-1}=n}(-1)^{T_{n-1}}s_{n-1}(t)(F_2-1)^{t_1}\cdots(F_n-1)^{t_{n-1}}=2^{n-3},\ \ n\ge 3,$$

$$\sum_{2t_1+3t_2+\cdots+nt_{n-1}=n}(-1)^{T_{n-1}}s_{n-1}(t)(F_3-1)^{t_1}\cdots(F_{n+1}-1)^{t_{n-1}}=\sum_{k=1}^{\lfloor\frac{n+1}{3}\rfloor}(-1)^k\binom{n-k}{2k-1},$$

$$\sum_{t_1+2t_2+\cdots+nt_n=n}(-1)^{T_n}s_n(t)(F_3-1)^{t_1}\cdots(F_{n+2}-1)^{t_n}=\sum_{k=0}^{\lfloor\frac{n-1}{3}\rfloor}(-1)^{k-1}\binom{n-1-2k}{k},$$

$$\sum_{t_1+2t_2+\cdots+nt_n=n}s_n(t)(F_3-1)^{t_1}\cdots(F_{n+2}-1)^{t_n}=3^{n-1}\sum_{k=0}^{\lfloor\frac{n-1}{3}\rfloor}\Big(-\frac{1}{27}\Big)^k\binom{n-1-2k}{k},$$

$$\sum_{t_1+2t_2+\cdots+nt_n=n}(-1)^{T_n}s_n(t)(F_4-1)^{t_1}\cdots(F_{n+3}-1)^{t_n}=0,\ \ n\ge 4,$$

$$\sum_{2t_1+3t_2+\cdots+nt_{n-1}=n}s_n(t)(F_3-1)^{t_1}\cdots(F_{2n-1}-1)^{t_{n-1}}=\sum_{k=0}^{n-1}\binom{n+1+2k}{3k+2},$$

$$\sum_{t_1+2t_2+\cdots+nt_n=n}(-1)^{T_n}s_n(t)(F_5-1)^{t_1}\cdots(F_{2n+3}-1)^{t_n}=0,\ \ n\ge 4,$$

$$\sum_{2t_1+3t_2+\cdots+nt_{n-1}=n}(-1)^{T_{n-1}}s_{n-1}(t)(F_4-1)^{t_1}\cdots(F_{2n}-1)^{t_{n-1}}=\frac{(2+\sqrt{2})^n-(2-\sqrt{2})^n}{4\sqrt{2}},\ \ n\ge 2,$$

$$\sum_{t_1+2t_2+\cdots+nt_n=n}(-1)^{T_n}s_n(t)(F_4-1)^{t_1}\cdots(F_{2n+2}-1)^{t_n}=\sum_{k=0}^{n}(-1)^{k-1}\binom{n+2k}{3k},\ \ n\ge 2,$$

where $s_n(t)=\binom{|t|}{t_1,\dots,t_n}=\frac{|t|!}{t_1!\cdots t_n!}$, $T_n=t_1+\cdots+t_n$ *with* $t_i\ge 0$.

References

[1] T. Goy and M. Shattuck, Fibonacci and Lucas identities from Toeplitz–Hessenberg matrices. *Appl. Appl. Math.* **14** (2019), no. 2, 699–715.

[2] T. Goy and M. Shattuck, Fibonacci–Lucas identities and the generalized Trudi formula. *Notes Number Theory Discrete Math.* **26** (2020), no. 3, 203–217.

Perfect Generalized 3-Valued Post Algebras

Revaz Grigolia[1,2], **Ramaz Liparteliani**[1,2]

[1] *Ivane Javakhishvili Tbilisi State University, Tbilisi Georgia*

[2] *Institute of Cybernetics of Georgian Technical University, Tbilisi, Georgia*

E-mails: revaz.grigolia@tsu.ge; r.liparteliani@yahoo.com

Generalized 3-valued Post algebra (GP_3-algebra) is a system $(A, \vee, \wedge, \oplus, \odot, \neg, 0, 1/2, 1)$, where A is a nonempty set of elements, 0, 1/2 and 1 are distinct constant elements of A, $\vee$, $\wedge$, $\oplus$, $\odot$ are binary operations on elements of A, and $\neg$ is a unary operation on elements of A, obeying finite set of axioms (identities).

$([0, 1], \vee, \wedge, \oplus, \odot, \neg, 0, 1/2, 1)$ is an example of generalized 3-valued Post algebra with the following operations:

$$x \vee y = \max(x, y), \quad x \wedge y = \min(x, y),$$
$$x \oplus y = \min(1, x + y), \quad x \odot y = \max(0, x + y - 1), \quad \neg x = 1 - x,$$

becomes an GP_3-algebra. Notice, $(\{0, 1/2, 1\}, \vee, \wedge, \oplus, \odot, \neg, 0, 1/2, 1)$ is a subalgebra of the algebra $([0, 1], \vee, \wedge, \oplus, \odot, \neg, 0, 1/2, 1)$, which is functionally equivalent to the 3-element Post algebra P_3. Indeed, it is enough to express the cyclic negation $x = (1/2 \odot x) \vee (\neg x \odot \neg x)$.

Generalized 3-valued Post algebra is said to be perfect generalized 3-valued Post algebra if it satisfies the following identity

$$\big((x \oplus x) \odot (x \oplus x)\big) \oplus \big((x \oplus x) \odot (x \oplus x)\big) = (x \oplus x \oplus x \oplus x) \odot (x \oplus x \oplus x \oplus x).$$

The theory of Generalized 3-valued Post algebras are developed.

Lifting of Endomorphism Fields

Narmina Gurbanova, Arif Salimov

Baku State University, Baku, Azerbaijan

E-mails: qurbanova.nermine.97@mail.ru; asalimov@hotmail.com

Let $T^2(M_r)$ be the bundle of 2-jets, i.e. the tangent bundle of order 2 over C^∞-manifold M_r, $\dim T^2(M_r)=3r$ and let

$$(x^i,x^{\bar{i}},x^{\bar{\bar{i}}})=(x^i,x^{r+i},x^{2r+i}),\ \ x^i=x^i(t),\ \ x^{\bar{i}}=\frac{dx^i}{dt}\,,\ \ x^{\bar{\bar{i}}}=\frac{1}{2}\,\frac{d^2x^i}{dt^2}\,,\ \ t\in\mathbb{R},\ \ i=1,\dots,r$$

be an induced local coordinates in $T^2(M_r)$. Let $\tilde{t}$ be a pure (1,1) -tensor field on $T^2(M_r)$ with respect to Π, where Π is the regular structure naturally existing on $T^2(M_r)$. Firstly, we prove that the bundle $T^2(M_r)$ is a real modeling of $R(\varepsilon^2)$-holomorphic manifold $X_r(R(\varepsilon^2))$ (see [1]), where $R(\varepsilon^2)$ is the algebra of order 3 with a canonical basis $\{e_1,e_2,e_3\}=\{1,\varepsilon,\varepsilon^2\}$, $\varepsilon^3=0$. We would like to find a local expression of pure tensor field $\tilde{t}=(\tilde{t}^I_J)$ on $T^2(M_r)$ which is corresponding to the $R(\varepsilon^2)$-holomorphic $(1,1)$-tensor field $\overset{*}{t}$ on $X_r(R(\varepsilon^2))$.

The pure $(1,1)$-tensor (endomorphism) field $\tilde{t}=(\tilde{t}^I_J)$ on $T^2(M_r)$ which is corresponding to the $R(\varepsilon^2)$-holomorphic $(1,1)$-tensor field $\overset{*}{t}$ on $X_r(R(\varepsilon^2))$ has components

$$\tilde{t}=\begin{pmatrix} t^i_j & 0 & 0\\ x^{r+k}\partial_k t^i_j+H^i_j & t^i_j & 0\\ x^{2r+s}\partial_s t^i_j+\dfrac{1}{2}\,x^{r+k}x^{r+s}\partial_k\partial_s t^i_j+x^{r+k}\partial_k H^i_j+K^i_j & x^{r+k}\partial_k t^i_j+H^i_j & t^i_j\end{pmatrix}$$

with respect to the induced coordinates $(x^r,x^{\bar{r}},x^{\bar{\bar{r}}})$ in $T^2(M_r)$, where $H=(H^i_j(x^1,\dots,x^r))$ and $K=(K^i_j(x^1,\dots,x^r))$ are arbitrary (1,1)-tensor fields on M_r. If $H=K=0$, then we have $\tilde{t}=t^{II}$, where t^{II} is the 2-nd lift of t to $T^2(M_r)$ [2, p. 331]. The $(1,1)$-tensor field $\tilde{t}$ on $T^2(M_r)$ is called *the deformed 2-nd lift* of t to $T^2(M_r)$.

References

[1] T. Sultanova and A. Salimov, On holomorphic metrics of 2-jet bundles. *Mediterr. J. Math.* **19** (2022), no. 1, Paper no. 29, 12 pp.

[2] K. Yano and S. Ishihara, *Tangent and Cotangent Bundles: Differential Geometry.* Pure and Applied Mathematics, No. 16. Marcel Dekker, Inc., New York, 1973.

A New Mathematical Model for HIV Infectious

Ghorbanali Haghighatdoost[1], **Hossein Kheiri**[2]

[1] *Department of Mathematical Sciences, Azarbaijan Shahid Madani University Tabriz, Iran*
E-mail: gorbanali@azaruniv.ac.ir

[2] *Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran*
E-mail: h-kheiri@tabrizu.ac.ir

In this work, a new mathematical model for HIV with logistic growth for target cells is considered. Quality behavior of model is studied. The reproduction number is obtained. It is shown that for reproduction number less than of one, the free equilibrium point is asymptotically stable. The dynamic behavior of endemic equilibrium points is investigated for reproduction number greater than one. Also, numerical simulations are given to confirm the obtained results.

Introduction

Human immunodeficiency virus (HIV) which targets the $CD4^+T$-cells is now a great epidemic worldwide. It causes the destruction and decline of $CD4^+T$-cells which results in decreasing the body's ability to fight infection. Many studies have been derived using mathematical models for the interaction between HIV and $CD4^+T$-cells [1, 2].

Discription of Model

To description of dynamical behavior HIV, we consider following model

$$\begin{aligned}
\dot{T} &= \Lambda - \mu_T T(t) + rT(t)\Big(1 - \frac{T(t)}{K}\Big) - \beta_1 V(t)T(t) - \beta_2 I(t)T(t),\\
\dot{L} &= \beta_1\gamma V(t)T(t) + \beta_2\gamma I(t)T(t) - \mu_L L(t) - a_L L(t),\\
\dot{I} &= \beta_1(1-\gamma)V(t)T(t) + \beta_2(1-\gamma)I(t)T(t) - \mu_I I(t) + a_L L(t),\\
\dot{V} &= pI(t) - \mu_V V(t) - \beta_1 V(t)T(t),
\end{aligned} \tag{1}$$

where T, L, I, and V are population number of $CD4^+T$ cells, latent cells, infectious cells and free virus particles, respectively.

Theorem 1 *The region* $\Omega_+ = \{(T,L,I,V);\ T>0,\ L\ge 0,\ I\ge 0,\ V\ge 0\}$ *is a positive invariant set for system* (1)*.*

Now, we show the boundedness of the solution of the model.

Theorem 2 *The region* $\Omega = \big\{(T,L,I,V);\ T>0,\ L\ge 0,\ I\ge 0,\ V\ge 0,\ A(t)=(T+I+L)(t)\le M/\mu_{min} + ce^{-\mu_{min}t},\ \text{where } \mu_{\min} = \min\{\mu_T,\mu_I,\mu_L\}\big\}$ *is a positive invariant set for system* (1)*.*

Acknowledgments

The work was supported by the University of Tabriz and Azarbaijan Shahid Madani University.

References

[1] A. Mojaver and H. Kheiri, Mathematical analysis of a class of HIV infection models of $CD4^+$ T-cells with combined antiretroviral therapy. *Appl. Math. Comput.* **259** (2015), 258–270.

[2] H. Kheiri and M. Jafari, Stability analysis of a fractional order model for the HIV/AIDS epidemic in a patchy environment. *J. Comput. Appl. Math.* **346** (2019), 323–339.

One-Sided Potentials in Weighted Central Morrey Spaces

Giorgi Imerlishvili[1], **Alexander Meskhi**[1,2]

[1]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2]*Kutaisi International University, School of Mathematics, Kutaisi, Georgia*

E-mails: imerlishvili18@gmail.com; alexander.meskhi@kiu.edu.ge

The boundedness of one-sided potential operators defined, generally speaking, with respect to a Borel measure μ, in the classical and central Morrey spaces is established. Weighted estimates for these operators in the case of power-type weights are also derived in central Morrey spaces and in complementary central Morrey spaces. Similar problems are studied for vanishing Morrey spaces.

The Boundary-Contact Problem of the Dynamical Viscoelasticity

Tsiala Jamaspishvili[1], **Bachuki Pachulia**[1], **Nugzar Shavlakadze**[1,2]

[1] *Georgian Technical University, Tbilisi, Georgia*

[2] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: nusha1961@yahoo.com

It is considered the dynamical contact problem for a viscoelastic half-plane which is reinforced along its boundary by an elastic thin finite cover plate (inclusion, patch) and excited by harmonic forces. It is solved the auxiliary dynamical problem of the viscoelastic half-plane whose boundary is under the action of harmonic horizontal and vertical forces with oscillation frequency ω.

The problem is formulated in the form of the Lame's differential equation

$$(\mu+\mu_0\partial_t)\Delta u_i+(\lambda+\mu+(\lambda_0+\mu_0)\partial_t)\theta_{,i}=\rho\partial_{tt}u_i,\ \ i=1,2$$

with the boundary conditions

$$\big((\lambda+\lambda_0\partial_t)\theta+2(\mu+\mu_0\partial_t)u_{2,2}\big)_{x_2=0}=p(x_1)e^{-i\omega t},\quad (\mu+\mu_0\partial_t)(u_{1,2}+u_{2,1})_{x_2=0}=-\tau(x_1)e^{-i\omega t},$$

where $u_i(x_1,x_2,t)$, $|x_1|<\infty$, $x_2\le 0$, $i=1,2$ are the components of the displacement vector, λ, μ and λ_0, μ_0 are the elastic and viscoelastic Lame's parameters, respectively, ρ is density of the plate material. $\tau(x_1)$ and $p(x_1)$ are the unknown tangential and normal contact stresses, respectively, $\theta(x_1,x_2,t)=u_{1,1}+u_{2,2}$.

In case when the viscoelastic half-plane is under the action of harmonic tangential forces $-\tau_0\delta(x+1)e^{-i\omega t}$ ($\delta(x)$ is the Dirac function, $x=x_1$), the boundary-contact problem is reduced to the following integro-differential equation

$$\Big(\frac{d^2}{dx^2}+k^2\Big)\bigg[\frac{1}{2\pi q}\int_{-1}^{1}\ln\frac{1}{|p_2|\,|x-s|}\,\tau(s)\,ds+\int_{-1}^{1}R(|p_2|\,|x-s|)\tau(s)\,ds\bigg]$$
$$=-\frac{1}{hE_0}\,\tau(x)-\frac{1}{hE_0}\,\tau_0\delta(x+1),\ \ -1<x<1,$$

with condition

$$\int_{-1}^{1}\tau(s)\,ds=-\tau_0,$$

where $\tau(x)=\tau(x_1)$, $|x_1|\le 1$; $\tau(x)=0$, $|x|>1$, h, E_0, ρ_0 are the thickness, modulus of elasticity and density of the inclusion material, respectively, $\lambda^*=\lambda-i\lambda_0\omega$, $\mu^*=\mu-i\mu_0\omega$,

$$q=\mu*\Big(1-\frac{p_1^2}{p_2^2}\Big),\quad p_1^2=\frac{\omega^2\rho}{(\lambda^*+2\mu^*)}\,,\quad p_2^2=\frac{\omega^2\rho}{\mu^*}\,,\quad k^2=\frac{\rho_0\omega^2}{E_0}\,,$$

$R(|p_2||x-s|)$ is the regular kernel.

Using the method of orthogonal polynomials the integro-differential equation is reduced to an infinite system of linear algebraic equations. The quasi-completely regularity of the obtained system is proved.

Acknowledgement

This work is supported by the Shota Rustaveli National science foundation of Georgia (Project # FR-21-7307).

On the Integrability of Multi-Dimensional Rare Maximal Functions

Irakli Japaridze[1], **Giorgi Oniani**[2]

[1]*Department of Mathematics, Akaki Tsereteli State University*
Kutaisi, Georgia
E-mail: irakli.japaridze@atsu.edu.ge

[2] *School of Computer Science and Mathematics, Kutaisi International University*
Kutaisi, Georgia
E-mail: giorgi.oniani@kiu.edu.ge

There are characterized the translation invariant monotone collections of multi-dimensional intervals for maximal functions associated to which (known in the literature as rare maximal functions), the analogue of Stein's criterion for the integrability of the Hardy–Littlewood maximal function is true. Namely, there are characterized the collections B of the mentioned type for which the conditions $\int\limits_{[0,1]^d} M_B(f) < \infty$ and $\int\limits_{[0,1]^d} |f| \log^+ |f| < \infty$ are equivalent for functions f supported on the unit cube $[0,1]^d$. Here M_B denotes the maximal operator associated to a collection B. The talk is based on the article [1].

References

[1] I. Japaridze and G. Oniani, On the Integrability of Multi-Dimensional Rare Maximal Functions. *Acta Math. Hungar.* (to appear)

Multiquadric RBFs Combined with Compact Discretization for Non-Linear Elliptic PDEs

Navnit Jha, Shikha Verma

[1]*Department of Mathematics, South Asian University, New Delhi, India*
E-mails: navnitjha@sau.ac.in; shikhavns4@gmail.com

An approach combining multiquadric radial basis functions and compact discretization is proposed for estimating solutions of two-dimensional nonlinear elliptic partial differential equations. By using a scattered grid network with variable step sizes, the accuracy of the solutions can be adjusted according to the presence of high oscillations in different regions. The implementation of radial basis functions on a nine-point grid network enhances the efficiency of functional evaluations through a compact formulation, resulting in savings in memory space and computation time. The proposed strategy significantly improves the accuracy of approximate solutions for elliptic equations with sharp variations occurring in narrow zones. The convergence theory is thoroughly described and also various numerical simulations are conducted to demonstrate the effectiveness of the novel algorithm.

Theorem 1 *The mesh-step sequence* $\{h_l\}_{l=1}^{N+1}$ *is convergent in* $\mathbb{R}$ *as* $N \to \infty$.

Theorem 2 *The MQ-RBF approximations of First-order partial derivatives are* $O(h_l^2)$ *accurate on a scattered grid network.*

Acknowledgments

S. Verma acknowledges South Asian University, New Delhi for providing research facilities and Council of Scientific and Industrial Research Grant-in-aid (# 09/1112(0009)/2020-EMR-I) in the form of research fellowship.

References

[1] V. Bayona, M. Moscoso, M. Carretero and M. Kindelan, RBF-FD formulas and convergence properties. *J. Comput. Phys.* **229** (2010), no. 22, 8281–8295.

[2] L. Jianyu, L. Siwei, Q. Yingjian and H. Yaping, Numerical solution of elliptic partial differential equation using radial basis function neural networks. *Neural Netw.* **16** (2003), no. 5–6, 729–734.

[3] C. M. T. Tien, N. Mai-Duy, C. D. Tran and T. Ttran-Cong, A numerical study of compact approximations based on flat integrated radial basis functions for second-order differential equations. *Comput. Math. Appl.* **72** (2016), no. 9, 2364–2387.

[4] G. B. Wright and B. Fornberg, Scattered node compact finite difference-type formulas generated from radial basis functions. *J. Comput. Phys.* **212** (2006), no. 1, 99–123.

[5] N. Jha and B. Singh, Fourth-order compact scheme based on quasi-variable mesh for three-dimensional mildly nonlinear stationary convection-diffusion equations. *Numer. Methods Partial Differential Equations* **38** (2022), no. 4, 803–829.

[6] R. S. Varga, *Matrix Iterative Analysis.* Second revised and expanded edition. Springer Series in Computational Mathematics, 27. Springer-Verlag, Berlin, 2000.

Realization of Hybrid Cryptosystem Based on AES and Vigenere Encryption Algorithms

Elza Jintcharadze, **Tornike Kartsivadze**

Department of Computer Sciences, Batumi Shota Rustaveli State University
Batumi, Georgia
E-mail: elza.jintcharadze@bsu.edu.ge; tokaqarco2001@gmail.com

Protecting digital data from unauthorized access is an important issue today. Cryptographic algorithms are used in many systems such as banking transactions, computer passwords, *e*-commerce, secure communications, and more.

The paper discusses new hybrid cryptosystem developed with classical and modern encryption algorithms. Based on this cryptosystem, using modern technologies, was developed online communication system, in which the transmitted information is encrypted.

Based on the study and research of classical and modern encryption algorithms, in particular Vigenere cipher and AES encryption algorithms, in the paper is presented a new hybrid algorithm. Using React.JS has been created software product. The paper describes the implementation methods of the new algorithm.

New hybrid cryptosystem, can be easily integrated into any system of secure data exchange. In this case, the paper discusses an example of an online chat. Comparison results of the presented cryptosystems are following: The cryptosystem discussed in the paper allows encryption and decryption of any symbol; The developed software allows users to exchange data in the most secure environment; The size of the data encrypted by the hybrid algorithm increases by an average of 4.64 times; Implementation of the presented hybrid cryptographic algorithm for data security is easy in different systems.

References

[1] D. Boneh and V. Shoup, *A Graduate Course in Applied Cryptography*, 2017; `https://crypto.stanford.edu/ dabo/cryptobook/BonehShoup_0_4.pdf`.

[2] J. von zur Gathen, *Classical Cryptography*, 2008; `https://cosec.bit.uni-bonn.de/fileadmin/user_upload/teaching/08ss/08ss-classical-crypto/notes.pdf`.

[3] D. R. Stinson and M. B. Peterson, *Cryptography Theory and Practice*. CRC Press, Taylor & Francis Group, London–New-York, 2019.

[4] W. Stallings, *Cryptography and Network Security. Principles and Practices*, Pearson Education, Inc., Prentice Hall, Boston–New York, 2011.;

[5] M. Barakat, C. Eder and T. Hanke, *An Introduction to Cryptography*, 2018; `https://www.cs.stonybrook.edu/sites/default/files/PGP70IntroToCrypto.pdf`.

On a Periodic Problem in an Infinite Strip for Second-Order Hyperbolic Equations

Otar Jokhadze, Sergo Kharibegashvili

[1] *Department of Differential Equations, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

E-mails: ojokhadze@yahoo.com; kharibegashvili@yahoo.com

The periodic problem for second-order hyperbolic equations in an infinite strip is studied. Taking into account the behavior of the solution at infinity, the questions of its uniqueness and non-uniqueness, existence and non-existence are established.

Acknowledgments

The work was supported by the Shota Rustavely National Science Foundation, Grant # FR-21-7307.

On Martingale Representations of Non-Smooth Brownian Functionals

Valeriane Jokhadze[1], **Ekaterine Namgalauri**[2], **Omari Purtukhia**[2,3]

[1]*Faculty of Economics and Business, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: jokhadze.valeriane@gmail.com

[2]*Department of Mathematics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

[3]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: E-mail: ekanamgalauri96@gmail.com; o.purtukhia@gmail.com

In the theory of random processes, a special place take a martingale representation theorems, which (along with Girsanov's measure change theorem) play an essential role as in the modern stochastic financial mathematics, so in the problem of nonlinear filtering. The martingale representation theorem states that any square integrable Brownian functional is represented as a stochastic integral with respect to a Brownian Motion.The first proof of the martingale representation theorem was implicitly provided by Ito (1951) himself. Many years later, Dellacherie (1974) gave a simple new proof of Ito's theorem using Hilbert space techniques. Many other articles were written afterward on this problem and its applications but one of the pioneer work on explicit descriptions of the integrand is certainly the one by Clark (1970). But, taking into account the needs of inancial mathematics, it is not enough to know only the existence of an integral representation, it is necessary to be able to find the explicit form of the integrand of the integral representation. It is known that for stochastically smooth functionals, the integrand is calculated by Ocone's formula ([2]), which was later generalized by Glonti and Purtukhia ([1]), when only the filter of the functional is stochastically smooth. Here we study functionals whose filter is no longer smooth and propose a method for finding the integrand.

Theorem 1 *For any real numbers $a<b$, the following stochastic integral representation is fulfilled*

$$\int\limits_0^T I_{\{a\le B_t\le b\}}\,\mathrm{d}t=\int\limits_0^T\Big[\Phi\Big(\frac{x}{\sqrt{t}}\Big)\Big]\Big|_{x=a}^{x=b}\,\mathrm{d}t-\int\limits_0^T\bigg(\int\limits_t^T\frac{1}{\sqrt{u-t}}\,\varphi\Big(\frac{x-B_t}{\sqrt{u-t}}\Big)\Big|_{x=a}^{x=b}\,\mathrm{d}u\bigg)\,\mathrm{d}B_t,$$

where Φ is the standard normal distribution function and φ is its density.

Theorem 2 *The following stochastic integral representation is valid*

$$I_{\{B_T^*\le a\}}=P(B_T^*\le a)-2\int\limits_0^T I_{\{B_t^*\le a\}}\frac{1}{\sqrt{T-t}}\,\varphi\Big(\frac{a-B_t}{\sqrt{T-t}}\Big)\,dB_t\ \ (P\text{-a.s}).$$

Acknowledgments

The work was partially supported by the grant STEM-22-226.

References

[1] O. A. Glonti and O. G. ; Purtukhia, On an integral representation of a Brownian functional. (Russian) *Teor. Veroyatn. Primen.* **61** (2016), no. 1, 158–164; translation in *Theory Probab. Appl.* **61** (2017), no. 1, 133–139.

[2] D. Ocone, Malliavin's calculus and stochastic integral representations of functionals of diffusion processes. *Stochastics* **12** (1984), no. 3-4, 161–185.

Solution of a Family of Boundary Value Problems for Nonlinear Loaded Hyperbolic Equations

Symbat Kabdrakhova[1,2], **Zhazira Kadirbayeva**[1,2]

[2] *Al-Farabi Kazakh National University, Almaty, Kazakhstan*

[2] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*

E-mails: symbat2909.sks@gmail.com; apelman86pm@mail.ru

We consider the following boundary value problem for loaded hyperbolic equations with mixed derivatives:

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\,\frac{\partial u}{\partial x} + A_0(x,t)\,\frac{\partial u}{\partial x}\Big|_{x=x_0} + f\Big(x, x_0, t, u, \frac{\partial u}{\partial t}\Big), \tag{1}$$

$$u(x,0) = u(x,T), \ \ x \in [0,\omega], \tag{2}$$

$$u(0,t) = \psi(t), \ \ t \in [0,T], \tag{3}$$

Here $f : \overline{\Omega} \times R^2 \to R$ continuous on $\bar{\Omega}$, $\psi(t)$ is continuously differentiable on $[0,T]$ and satisfies the condition $\psi(0) = \psi(T)$. Functions $A(x,t)$, $A_0(x,t)$ are continuous on $\bar{\Omega}$, x_0 is the load point. Nonlinear hyperbolic equations with loading arise in various fields such as hydromechanics, acoustics and geophysics. Boundary value problems for loaded differential equations have been studied by many authors Nakhushev A. M., Dzhumabaev D. S., Ladyzhenskay O. A., Genaliev M. T., Ramazanov M. I., Assanova A. T. and another.

To find a solution to problem (1)–(3), we used a modification of the Euler polygonal method [2]. Let us divide the interval $[0,\omega]$ with a step $h > o$ into N parts: $Nh = \omega$, and at each step we obtain families of periodic boundary value problems for nonlinear loaded hyperbolic equations. To find a solution of a family of boundary value problems we used the parametrization method proposed by D. S. Dzhumabaev [1]. Based on two methods, conditions for the solvability of a family of boundary value problems and problems (1)–(3) are obtained.

References

[1] D. S. Dzhumabaev and S. M. Temesheva, A parametrization method for solving nonlinear two-point boundary value problems. (Russian) *Zh. Vychisl. Mat. Mat. Fiz.* **47** (2007), no. 1, 39–63; translation in *Comput. Math. Math. Phys.* **47** (2007), no. 1, 37–61.

[2] S. S. Kabdrakhova, A modification of Euler's broken line method to solve a semiperiodic boundary value problem for a nonlinear hyperbolic equation. (Russian) *Mat. Zh.* **8** (2008), no. 2(28), 55–62.

On the Use of One Numerical Method for Solving a Nonlinear Beam Equation

Nikoloz Kachakhidze

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: n.kachakhidze@gtu.ge

The boundary value problem

$$w''''(x) - a\bigg(\int\limits_0^L u'^2(x)\,dx\bigg)w''(x) = f(x), \;\; 0 < x < L,$$
$$u(0) = u(L) = 0, \quad u''(0) = u''(L) = 0,$$

$a(\lambda) \geq const > 0$, $0 \leq \lambda < \infty$, describing the behavior of a static beam of the Kirchhoff type [2] is considered. To solve it, the method from [1] is used, which is a combination of the Galerkin method and Newton's iterative process. Two test examples have been solved. The results of the calculations are given by means of tables and graphs.

References

[1] N. Kachakhidze, J. Peradze and Z. Tsiklauri, A Galerkin–Newton algorithm for solution of a Kirchhoff-type static equation. *Int. J. Comput. Methods* **19** (2022), no. 1, Paper no. 2150057, 22 pp.

[2] S. Woinowsky-Krieger, The effect of an axial force on the vibration of hinged bars. *J. Appl. Mech.* **17** (1950), 35–36.

Automatic Diagnosis of Lung Disease on the Basis of an X-Ray Images of a Patient with Given Reliability

J. Kachiashvili[1,2], **Kartlos Kachiashvili**[1,2,3], **R. Kalandadze**[3], **Vakhtang Kvaratskhelia**[1,4]

[1]*Faculty of Informatics and Control Systems, Georgian Technical University Tbilisi, Georgia*
E-mail: kkachiashvili@gmail.com; k.kachiashvili@gtu.edu.ge

[2]*Shalva Mikeladze Computing Center of the Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia*

[3]*Department of Probability Theory and Mathematical Statistics, Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[4]*Department of Probabilistic and Statistical Methods, Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia*
E-mail: v.kvaratskhelia@gtu.ge

The article proposes algorithms for the automatic diagnosis of the facts of human lung diseases with pneumonia and cancer based on images obtained by radiation irradiation, which allow making decisions with the necessary reliability, that is, by limiting the probabilities of making possible errors to a pre-planned level. The proposed algorithms have been tested using statistical simulation and real data, which fully confirmed the correctness of theoretical reasoning and the ability to make decisions with the required reliability using artificial intelligence.

Acknowledgments

The work was partially supported by European Commission HORIZON EUROPE WIDERA-2021-ACCESS-03, Grant Project (GAIN), grant agreement # 101078950.

Propagation of Waves in a Triangular Grid with Discrete Sources Positioned Along Line Segments

David Kapanadze[1], **Zurab Vashakidze**[2,3]

[1]*Department of Mathematical Physics, Aandrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: david.kapanadze@gmail.com

[2]*Institute of Mathematics, The University of Georgia (UG), Tbilisi, Georgia*
[3]*Department of Numerical Mathematics and Modelling, Ilia Vekua Institute of Applied Mathematics (VIAM), Tbilisi, Georgia*
E-mails: z.vashakidze@ug.edu.ge, zurab.vashakidze@gmail.com

Our research delves into the behaviour of time-harmonic waves as they traverse a triangular lattice containing sources positioned along line segments. Specifically, the study is centred around the discrete Helmholtz equation. In this scenario, the wave number k is confined to the interval $(0, 2\sqrt{2})$, and the input data is assigned to finite rows and columns of lattice sites without the need for complex wave numbers. Analogous to the concepts in continuum theory, we introduce the concept of a radiating solution. This serves to establish a unique solvability result and employs difference (discrete) potentials to formulate Green's representation formula. Additionally, we employ a numerical computation approach that effectively showcases its efficiency in tackling challenges associated with the propagation of left-handed 2D inductor-capacitor metamaterials.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant # FR-21-301, project title: *"Metamaterials with Cracks and Wave Diffraction Problems"*].

Wave Diffraction by a Crack in Triangular and Hexagonal Lattices

David Kapanadze, Ekaterina Pesetskaya

[1]*Andrea Razmadze Mathematical Institute of Ivane Jacakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: david.kapanadze@gmail.com; kate.pesetskaya@gmail.com

Motivated by applications of recent interest related to analog circuits, crystalline materials, and metamaterials, we investigate problems of wave diffraction by a crack in infinite triangular and hexagonal lattices. Namely, we study Dirichlet problems for the discrete Helmholtz equation in a plane with a hole. Using the notion of the radiating solution, we prove the existence and uniqueness of a solution for triangular and hexagonal lattices for the real wave number $k \in (0, 2\sqrt{2})$ (cf. [1]) and $k \in (0, \sqrt{2}) \cup (2, \sqrt{6})$, respectively. Green's representation formula for the solution is derived with the help of difference potentials. We also developed a numerical calculation method and demonstrate by examples the effectiveness of our approach related to the propagation of wavefronts in metamaterials with small defects such as cracks and rigid constraints.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Grant # FR-21-301.

References

[1] D. Kapanadze and E. Pesetskaya, Exterior diffraction problems for a triangular lattice. *Math. Mech. Solids.*, 2023; DOI:`https://doi.org/10.1002/mma.7575`.

On One Example of the Existence of a Non-Measurable Set on the Real Line R

Tamar Kasrashvili[1,2]

[1]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
[2]*Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: kasrashvilitamar08@gtu.ge

It is well known the existence of non-measurable sets on the real line R (Vital's set, Bernstein's set, etc., see, for example, [2]). This report will present relationship between countable equid composability of sets and existence a nonmeasurability set with respect to Lebesgue measure on real line R. Since any two (bounded or unbounded) point sets of R with nonempty interiors are countably equid composable (see, [1]), we get that there exists a Lebesgue non-measurable set on R.

References

[1] H. Hadwiger, *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie.* (German) Springer-Verlag, Berlin–Göttingen–Heidelberg, 1957.

[2] A. B. Kharazishvili, *Nonmeasurable Sets and Functions.* North-Holland Mathematics Studies, 195. Elsevier Science B.V., Amsterdam, 2004.

Riemann Boundary Value Problem on Spirals and Generalized Cauchy-Type Integral

David Katz

Chair of Mathematics, Moscow Polytechnic University, Moscow, Russia
E-mail: katzdavid89@gmail.com

The Riemann boundary value problem is one of the most famous boundary value problems of complex analysis, the solution and applications of which are the subject of a large number of monographs and scientific articles (see, for example, [3–5] and references in these monographs). Publications devoted to new applications of this problem appear systematically (see, for example, [1, 2]).

Let Γ be a simple non-closed piecewise smooth Jordan curve on the complex plane $\mathbf{C}$ with endpoints a_1 and a_2, on which the functions $G(t)$ and $g(t)$, and the first of them does not vanish. We need to find all functions $\Phi(z)$ that are holomorphic in $\overline{\mathbf{C}} \setminus \Gamma$ and vanish at a point at infinity and having at each point $\Gamma \setminus \{a_1, a_2\}$ limit values on the left and on the right $\Phi^{\pm}(t)$, respectively, connected by the boundary condition $\Phi^+(t) = G(t)\Phi^-(t) + g(t)$, $t \in \Gamma' := \Gamma \setminus \{a_1, a_2\}$.

We study the jump problem on spiral arcs. We introduce the concept of a generalized curvilinear integral, prove its existence, a generalized Cauchy-type integral over a spiral is constructed and an existence theorem is proved for the jump problem on a spiral arc.

We obtain the Cauchy-type integral in the following form

$$\Phi(z) = \frac{1}{2\pi i} \int\limits_{\Gamma} \frac{f(z)}{t-z}\, dt = K(z)F(z) - \frac{1}{2\pi i} \iint\limits_{\mathbb{C}} \frac{\partial F}{\partial \overline{\zeta}} \frac{K(\zeta)}{\zeta - z}\, d\zeta\, d\overline{\zeta}.$$

It allows us to formulate the main result in the form of

Theorem *Let the spiral Γ satisfy the condition $|\theta(r)| \le Cr^{-q}$, $q < 2$, the jump $g(t)$ given on it satisfy the Lipschitz condition and vanish at the origin. Then the jump problem on it has a solution in the class $|\Phi(z)| \le C|z - a_j|^{-\gamma}$, $j = 1, 2$, defined by a generalized Cauchy-type integral, and such a solution is unique.*

Acknowledgments

The work was supported by the grant of Russian Science Foundation # № 22-71-10094, `https://rscf.ru/project/22-71-10094/`.

References

[1] S. I. Bezrodnykh, V. I. Vlasov and B. V. Somov, Analytical models of generalized Syrovatskii's current layer with MHD shock waves. *Astronomic and Space Science (Berlin, Heidelberg, 2012)*, 133–144, Astronomic and Space Science Proc., 30, Springer-Verlag, Berlin, 2012.

[2] P. A. Deift, *Orthogonal Polynomials and Random Matrices: a Riemann–Hilbert Approach.* Courant Lecture Notes in Mathematics, 3. New York University, Courant Institute of Mathematical Sciences, New York, American Mathematical Society, Providence, RI, 1999.

[3] F. D. Gakhov, *Boundary Value Problems.* Pergamon Press, Oxford–New York–Paris; Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1966.

[4] J. K. Lu, *Boundary Value Problems for Analytic Functions.* Series in Pure Mathematics, 16. World Scientific Publishing Co., Inc., River Edge, NJ, 1993.

[5] N. I. Muskhelishvili, *Singular Integral Equations.* (Russian) Nauka, Moscow, 1962.

To the Problem of Full Transitivity of a Group

Tariel Kemoklidze

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mails: tariel.kemoklidze@atsu.edu.ge, kemoklidze@gmail.com

The problem of full transitivity of the cotorsion hull of the primary abelian group without non-zero elements of infinite higth is considered. Using the finite topology of the ring endomorphisms $E(T)$ of mentioned group T it is shown that if $B \subseteq T \subseteq \overline{B}$ where B is a basic subgroup of T and $\overline{B}$ is a torsion complete group $a, c \in \overline{B}$, a does not belong to T, $O(a) = p = O(c)$, height $h_{\overline{B}}(a) \leq h_{\overline{B}}(b)$ and T is not a direct sum of cyclic p groups or a torsion complete group, then there is no endomorphism $a \in E(T)$ for which $\overline{\alpha} a = c$, where $\overline{\alpha}$ is extension of endomorphism α to an endomorphisms of group $\overline{B}$. Our claim gives a key for clarifying the problem of full transitivity of a cotorsion T^* hull of the group T.

The Issue of Manageability of the Task of Optimal Fight Against Disinformation

Nugzar Kereselidze

Faculty of Natural Sciences, Mathematics, Technology and Pharmacy, Sokhumi State University, Tbilisi, Georgia

E-mail: nkereselidze@sou.edu.ge

Mathematical and computer models of combating disinformation are considered. The distribution of false and its opposite – objective information is described by a dynamic system with variable coefficients. From the point of view of the information security of society, the permissible number of persons (adepts) under the influence of misinformation (desempidara) is determined. The goal of the anti-disinformation task is to control the number of adherents. Bringing the number of adepts with the help of special measures to a value of a smaller desempidara at the end of the considered period of time is a process of combating disinformation. Special events are evaluated according to a certain criterion, say, financial value. The problem of optimal control of the fight against disinformation is set. Computer simulation studies the controllability of the problem of combating disinformation. A numerical experiment establishes an expanded set of new control parameters, and identifies important factors in the fight against disinformation.

References

[1] EUvsDisinfo, Disinformation Review. https://euvsdisinfo.eu/disinfo-review/; *Accessed* Oct. 01, **1** (2019).

[2] N. Kereselidze, Misinformation models. *Proceedings of XXVII International Conference "Problems of Security Management of Complex Systems" (Moscow, 18 December, 2021)*, 167–172.

[3] N. Kereselidze, Mathematical and computer model of effective fight against disinformation. *XXXVII International Enlarged Sessions of the Seminar of Ilia Vekua Institute of Applied Mathematics of Ivane Javakhisvili Tbilisi State University (April 19-22, 2023)*, Book of Abstracts, 2023, p. 55; http://www.viam.science.tsu.ge/enlarged/2023/abstracts_eng.pdf.

[4] W. O. Kermack and A. G. McKendrick, Contributions to the mathematical theory of epidemics. II. – The problem of endemicity. *Proc. Royal Soc. A, Math., Phys, Eng. Sci.* **115**, (1927), 700–721.

On an Existence of an Almost Non-Invariant Set

Marika Khachidze

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: m.khachidze1995@gmail.com

In the presented talk we introduce the notion of the almost non-invariant sets and discuss several properties of them.

Let E be a nonempty set, G be a group of transformations of E and X be a subset of E. We shall say that X is under the group G if the following assertion is true

$$card\big\{g: \ \ g(X)\cap X\neq\varnothing\big\}<card(G).$$

Theorem *There exists an almost non-invariant set $X\subset R$ such that its measurable hull is an almost invariant set.*

References

[1] M. Khachidze and A. Kirtadze, One example of application of almost invariant sets. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **32** (2018), 31–34.

[2] A. B. Kharazishvili, *Nonmeasurable Sets and Functions.* North-Holland Mathematics Studies, 195. Elsevier Science B.V., Amsterdam, 2004.

[3] K. Kunen, Inaccessibility properties of cardinals. *Ph.D. Thesis, Department of Mathematics, Stanford University*, 1968.

The Task of Summarization in Georgian Language Documents

Manana Khachidze, **Ana Shvelidze**

Computer Science Department, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: manana.khachidze@tsu.ge

With the rapid development of technology, the volume of electronic resources is growing exponentially. Consequently, the need for efficient processing of these resources has become increasingly urgent. One major challenge in the field of natural language processing is text summarization. The objective of this task is to reduce the document's size while preserving its content. This involves extracting only the key information from the text, allowing readers to comprehend the document without reading it entirety.

Different approaches can be employed to accomplish this, such as extractive summarization, which generates summaries using sentences from the original text, and abstractive summarization, which creates summaries based on semantic understanding. The choice and utilization of these methods depend significantly on the language's unique characteristics, as accurate semantic and lexical analysis is essential [1].

Key considerations include maintaining information consistency, avoiding loss of relevant information, and minimizing redundancy. We'll introduce a model that employs deep learning methods to summarize extensive documents written in the Georgian language. Specifically, the model utilizes the pre-trained BERT language model, known as Bert LM Head Model, which has been fine-tuned using Georgian language texts. The model was trained on a dataset comprising up to 28,000 records.

To evaluate the quality of the model's summaries, a set of widely-used ROUGE metrics was employed [2]. The following results were obtained: ROUGE-1: Recall 0.218, Precision 0.854, F1 Score 0.347. ROUGE-2: Recall -0.159, Precision -0.681, F1 Score 0.257.

References

[1] N. Andhale and L. A. Bewoor, An overview of Text Summarization techniques. *Proceedings 2016 International Conference on Computing Communication Control and automation (ICCUBEA)*, 2016.

[2] J. Steinberger and K. Jezek, Evaluation Measures for Textsummarization. *Computing and Informatics* **28** (2009), 251–275.

A New Generalization of (m, n)-Closed Ideals

Hani A. Khashan

Department of Mathematice, Al al-Bayt University, Al-Mafraq, Jordan
E-mail: hakhashan@aabu.edu.jo

Let R be a commutative ring with identity. For positive integers m and n, Anderson and Badawi [2] defined an ideal I of a ring R to be an (m, n)-closed if whenever $x^m \in I$, then $x^n \in I$. In this paper we define and study a new generalization of the class of (m, n)-closed ideals which is the class of quasi (m, n)-closed ideals. A proper ideals I is called a quasi (m, n)-closed in R if for $x \in R$, $x^m \in I$ implies either $x^n \in I$ or $x^{m-n} \in I$. That is, I is quasi (m, n)-closed in R if and only if I is either (m, n)-closed or (m, n)-closed in R. We justify several properties and characterizations of quasi (m, n)-closed ideals with many supporting examples. Furthermore, we investigate quasi (m, n)-closed ideals under various contexts of constructions such as direct products, localizations and homomorphic images. Finally, we discuss the behaviour of this class of ideals in idealization rings.

References

[1] D. F. Anderson and A. Badawi, On n-absorbing ideals of commutative rings. *Comm. Algebra* **39** (2011), no. 5, 1646–1672.

[2] D. F. Anderson and A. Badawi, On (m, n)-closed ideals of commutative rings. *J. Algebra Appl.* **16** (2017), no. 1, 1750013, 21 pp.

[3] M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra.* Addison–Wesley Publishing Co., Reading, Mass.–London–Don Mills, Ont. 1969.

[4] A. Badawi, On 2-absorbing ideals of commutative rings. *Bull. Austral. Math. Soc.* **75** (2007), no. 3, 417–429.

[5] A. Badawi, Ü. Tekir and E. Yetkin, On 2-absorbing primary ideals in commutative rings. *Bull. Korean Math. Soc.* **51** (2014), no. 4, 1163–1173.

[6] A. Badawi, n-absorbing ideals of commutative rings and recent progress on three conjectures: a survey. *Rings, polynomials, and modules*, 33–52, *Springer, Cham*, 2017.

[7] A. Badawi, M. Issoual and N. Mahdou, On n-absorbing ideals and (m, n)-closed ideals in trivial ring extensions of commutative rings. *J. Algebra Appl.* **18** (2019), no. 7, 1950123, 19 pp.

[9] A. Badawi and E. Y. Celikel, On 1-absorbing primary ideals of commutative rings. *J. Algebra Appl.* **19** (2020), no. 6, 2050111, 12 pp.

On the Stokes Flows

Nino Khatiashvili

Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: ninakhatia@gmail.com

We consider incompressible viscous fluid flow for the small Reynolds number in the infinite domains. The velocity components of the flow satisfy the Stokes linear system with the equation of continuity and suitable initial-boundary conditions [1, 8–11]. The steady and unsteady cases are considered. The novel exact solutions for the axial fluid flow over the ellipsoid and the countable number of discs are obtained. The non-axial flows were studied in the works [2, 3–7].

References

[1] G. K. Batchelor, *An Introduction to Fluid Dynamics.* Cambridge Univ. Press, UK, 1967.

[2] N. Khatiashvili, On the 2D free boundary problem for the Stokes flow. *Proc. I. Vekua Inst. Appl. Math.* **70** (2020), 32–40.

[3] N. Khatiashvili, On the free boundary problem for the creeping flows. *Proceedings of the World Congress on Engineering (WCE 2021, July 7-9, 2021)*, pp. 60–65, London, U.K., 2021.

[4] N. Khatiashvili, On the Stokes flow in pipes with the polygonal cross-section. *Proceedings of the World Congress on Engineering 2022 (WCE 2022, July 6-8, 2022)*, London, U.K., 2022, 6 p.

[5] N. Khatiashvili, On the creeping flow in prismatic pipes. *IAENG Inter. J. Appl. Math.* **52** (2022), no. 4, 1144–1149.

[6] N. Khatiashvili, On the Free Boundary Problem for the Low Reynolds Number. In: SI. Ao, L. Gelman, (Eds.) *Transactions on Engineering Technologies*, pp. 17–31. Lecture Notes in Electrical Engineering, vol. 919. *Springer, Singapore*, 2023.

[7] N. Khatiashvili and D. Janjgava, On the Stokes flow in the infinite domains. *Proc. I. Vekua Inst. Appl. Math.* **71** (2021), 60–70.

[8] N. Khatiashvili, K. Pirumova and D. Janjgava, On the Stokes flow over ellipsoidal type bodies. *Proceedings of the World Congress on Engineering (WCE 2013, July 3-5, 2013)*, Vol. I, pp. 148–151, London, U.K., 2013.

[9] N. Khatiashvili, K. Pirumova and D. Janjgava, On some effective solutions of Stokes axisymmetric equation for a viscous fluid. *Proceedings of World Academy of Science, Engineering and Technology* **79** (2013), 690–694.

[10] N. Khatiashvili, K. Pirumova, I. Khatiashvili and V. Akhobadze, On the influence of the cancer proteins on the blood flow. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **29** (2015), 60–63.

[11] G. G. Stokes, On the steady motion of incompressible fluids. *Trans. Cambridge Philos. Soc.* **7** (1880), 75–129.

Cohomology and Crossed Extensions of Algebras with Brackets

Emzar Khmaladze[1,2]

[1] *The University of Georgia, Tbilisi, Georgia*

[2] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: e.khmal@gmail.com, e.khmaladze@ug.edu.ge

In this talk, we present the investigation of the extensibility problem of a pair of derivations associated with an abelian extension of algebras with bracket using the cohomology of algebras with bracket developed in [1], and derive an exact sequence of the Wells type connecting various vector spaces of derivations. We introduce crossed modules for algebras with bracket and prove their equivalence with internal categories in the category of algebras with bracket. Then we interpret the set of equivalence classes of crossed extensions as the second cohomology. The results of this research are presented in [2].

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia, grant # FR-22-199.

References

[1] J. M. Casas and T. Pirashvili, Algebras with bracket. *Manuscripta Math.* **119** (2006), no. 1, 1–15.

[2] J. M. Casas, E. Khmaladze and M. Ladra, Wells type exact sequence and crossed extensions of algebras with bracket. (submitted for publication in 2023).

Independent Family of Sets and Their Applications

Aleks Kirtadze

Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: kirtadze2@yahoo.com

Recall that a family $\{E_i : \ i \in I\}$ of subsets of an infinite set E is independent if the relation

$$\cap\{\overline{E_i} : \ i \in J\} = card(E)$$

holds, whenever J is a countable subset of I and

$$(\forall\, i)\ \big(i \in J \Longrightarrow (\overline{E_i} = E_i) \vee (\overline{E_i} = E \setminus E_i)\big).$$

Theorem 1 *Let $(G,+)$ be an uncountable commutative group with $card(G) = \alpha$ and let β be an infinite cardinal such that $\beta^\omega \leq \alpha$. Then there exists a stochastically independent family of subsets of G having cardinality $(\beta^\omega)^+$.*

Theorem 2 *Let $(G,+)$ be an arbitrary uncountable commutative group with $card(G) = \alpha$ and let β be an infinite cardinal such that $\beta^\omega \leq \alpha$. Then there exists an invariant probability measure on G, whose weight is $(\beta^\omega)^+$.*

Logarithmically Superquadratic Functions

Mario Krnić[1], **Hamid Reza Moradi**[2], **Mohammad Sababheh**[3]

[1] *Faculty of Electrical Engineering and Computing, University of Zagreb*
Zagreb, Croatia
E-mail: mario.krnic@fer.hr

[2] *Department of Mathematics, Mashhad Branch, Islamic Azad University*
Mashhad, Iran
E-mail: hrmoradi@mshdiau.ac.ir

[3] *Department of Basic Sciences, Princess Sumaya University for Technology*
Al Jubaiha, Amman, Jordan
E-mail: sababheh@psut.edu.jo

In this talk, we develop a concept of a logarithmically superquadratic function. Such a class of functions is defined via superquadratic functions. These functions possess some superior properties, especially if they take values greater than or equal to one. We prove that they are convex and superadditive in the latter case. In particular, we also obtain the corresponding refinement of the Jensen inequality in a product form. Furthermore, we derive an external form of the Jensen inequality and the corresponding reverse. Finally, we give a variant of the Jensen operator inequality for logarithmically superquadratic functions. All established results are derived via the corresponding relations for superquadratic functions.

References

[1] S. Abramovich, S. Banić and M. Matić, Superquadratic functions in several variables. *J. Math. Anal. Appl.* **327** (2007), no. 2, 1444–1460.

[2] S. Abramovich, G. Jameson and G. Sinnamon, Inequalities for averages of convex and superquadratic functions. *JIPAM. J. Inequal. Pure Appl. Math.* **5** (2004), no. 4, Article 91, 14 pp.

[3] S. Abramovich, G. Jameson and G. Sinnamon, Refining Jensen's inequality. *Bull. Math. Soc. Sci. Math. Roumanie (N.S.)* **47(95)** (2004), no. 1-2, 3–14.

[4] M. Kian, Operator Jensen inequality for superquadratic functions. *Linear Algebra Appl.* **456** (2014), 82–87.

[5] M. Kian and S. S. Dragomir, Inequalities involving superquadratic functions and operators. *Mediterr. J. Math.* **11** (2014), no. 4, 1205–1214.

On Numerical Solutions of Crack Problems

Mirian Kublashvili, Murman Kublashvili

Muskhelishvili Institute of Computational Mathematics, Georgian Technical University
Tbilisi, Georgia

E-mails: m_kublashvili@gtu.ge; murman.kublashvili@gtu.ge

Curved cracks on a plane are considered. Simplified algorithms for numerical solution of high order accuracy are built for them. Specific tasks are presented, computer programs are compiled in the symbolic language "Wolfram Mathematica".

Logical Interpretation of Probability

Lia Kurtanidze

Faculty of Business and Technology, Georgian National University SEU
Tbilisi, Georgia
E-mail: l.kurtanidze@seu.edu.ge

The history of relationship between mathematical logic and probability theory started centuries ago [1]. Actually, these two branches of science were strongly connected since the appearance of the first concept of probability in the second half of the seventeenth century [2]. The probability theory was often considered as an extension of logic. Nowadays, they are developing independently and we have the basic notions of probability theory, like probability itself, conditional probability, etc [3]. Beside the classical definition where the probability of an event is the quotient of the numbers of the favorable and all possible outcomes, some alternative views were proposed: frequency, logical, and subjective interpretations, etc.

Probability logic can be viewed as a generalization of deductive logic which formalizes a broader notion of inference based on the notion of confirmation which is one set of propositions [4].

In this talk we present the logical interpretation of probability and we will talk about relationship history between mathematical logic and probability.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under the project # FR-22-4254

References

[1] Z. Ognjanović, M. Rašković and Z. Marković, *Probability Logics: Probability-Based Formalization of Uncertain Reasoning*. Springer, 2016.

[2] J. Barone and A. Novikoff, A history of the axiomatic formulation of probability from Borel to Kolmogorov, Part I. *Archive for History of Exact Sciences* **18** (1978), no. 2, 123–190.

[3] E. Adams, The logic of Conditional. Inquiry: An Interdisciplinary Journal of Philosophy **8** (1965), no. 1-4, 166–197 .

[4] Z. Ognjanović, *Probabilistic Extensions of Various Logical Systems*. Springer, Nature Switzerland AG, 2020.

On a Mathematical Model of a Single Dynamic Problem of Discrete Optimization

Ketevan Kutkhashvili

School of Science and Technology, The University of Georgia, Tbilisi, Georgia
E-mail: kkutkhashvili@yahoo.com

The task of optimal planning of project financing in a certain long-term period by this or that company is considered at the expense of managing the distribution of existing resources. System dynamism will be taken into account in the task. The state of the system changes at certain points in time, and at each new step the state of the system at the previous step is taken into account. In addition, the expected disturbances of the state of the system are probabilistic quantities, which are also considered in the task. Minimization of total costs is chosen as the optimality criterion.

References

[1] R. Bellman, *Dynamic Programming.* Princeton University Press, 2010.

[2] T. Hürlimann, *Mathematical Modeling and Optimization. An Essay for the Design of Computer-Based Modelling Tools.* Springer Science & Business Media, 2013.

[3] K. Kutkhashvili and V. Gabisonia, A mathematical model of a single discrete optimization problem under uncertainty. *Archil Eliashvili Institute of Control Systems of the Georgian Technical University Proceedings*, no. 21, 2017.

[4] D. P. Loucks, Discrete Optimization Modeling. In *Public Systems Modeling*, International Series in Operations Research & Management Science, vol. 318. *Springer, Cham*, 2022.

Variants of Fuzzy Anti-Unification and Generalization

Temur Kutsia

RISC, Johannes Kepler University, Linz, Austria
E-mail: kutsia@risc.jku.at

Anti-unification is a fundamental operation for computing common generalizations of expressions in some logic language. Such generalizations are supposed to maximally retain commonalities between the given expressions, while the differences are abstracted uniformly by fresh variables.

Anti-unification was introduced in the 1970s [5,6], but currently it is experiencing a renewed interest due to novel applications in various problems of artificial intelligence, knowledge representation, inductive reasoning, natural language processing, program synthesis and analysis, software code clone detection, automatic program repair, etc. In a recent survey [2], the authors provided a general framework for the generalization problem, an overview of the corresponding solving algorithms, existing theoretical results and application domains, and some future directions of research.

In this talk, we review some of the recent generalization algorithms developed for quantitative theories [1, 3, 4]. In them, the exact equality is replaced by its approximation, expressed by fuzzy similarity or proximity relations. These extensions affect the notion of generalization, which also becomes approximate. We formulate such approximate fuzzy generalization problems, show how they fit into the generalization framework described in [2], and illustrate the properties of the corresponding anti-unification algorithms.

Acknowledgments

The work was supported by the Austrian Science Fund (FWF), Project P 35530.

References

[1] H. Aït-Kaci and G. Pasi. Fuzzy lattice operations on first-order terms over signatures with similar constructors: a constraint-based approach. *Fuzzy Sets and Systems* **391** (2020), 1–46.

[2] D. Cerna and T. Kutsia, Anti-unification and generalization: a survey. In: Edith Elkind (Ed.), *Proceedings of IJCAI 2023 – 32nd International Joint Conference on Artifical Intelligence.* ijcai.org, 2023.

[3] T. Kutsia and C. Pau, A Framework for Approximate Generalization in Quantitative Theories. In: J. Blanchette, L. Kovács, D. Pattinson (Eds.), *Automated Reasoning. IJCAR 2022*, pp. 578–596. Lecture Notes in Computer Science, vol. 13385. Springer, Cham, 2022.

[4] T. Kutsia and C. Pau, Matching and Generalization Modulo Proximity and Tolerance Relations. In: A. Özgün and Y. Zinova (Eds.), *Language, Logic, and Computation. TbiLLC 2019*, pp. 323–342. Lecture Notes in Computer Science, vol 13206. *Springer, Cham*, 2019.

[5] G. D. Plotkin, A note on inductive generalization. *1970 Machine Intelligence*, 5 pp. 153–163, *American Elsevier, New York.*

[6] J. C. Reynolds. Transformational systems and the algebraic structure of atomic formulas. *1970 Machine Intelligence*, 5 pp. 135–151 American Elsevier, New York.

Similarity-Based Set Matching

Temur Kutsia[1], **Mircea Marin**[2], **Mikheil Rukhaia**[3]

[1]*RISC, Johannes Kepler University Linz, Austria*
E-mail: kutsia@risc.jku.at

[2]*West University of Timişoara, Romania*
E-mail: mircea.marin@e-uvt.ro

[3]*Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: mrukhaia@logic.at

Similarity relations are fuzzy counterparts of equivalence relations. A binary fuzzy relation $\mathcal{R}$ on a set S (a mapping from S to the real interval $[0,1]$) is a similarity relation if it satisfies

Reflexivity: $\mathcal{R}(s,s)=1$ for all $s \in S$,

Symmetry: $\mathcal{R}(s_1,s_2)=\mathcal{R}(s_2,s_1)$ for all $s_1,s_2 \in S$,

Transitivity: $\mathcal{R}(s_1,s_2) \geq \mathcal{R}(s_1,s) \wedge \mathcal{R}(s,s_2)$,

where $\wedge$ is a T-norm: an associative, commutative, non-decreasing binary operation on $[0,1]$ with 1 as the unit element. We assume that the T-norm is minimum (Gödel T-norm). A fuzzy relation is a proximity relation if it is reflexive and symmetric but not necessarily transitive.

Basic operations for many deduction and computational formalisms are matching and unification. These are methods for solving systems of equations. In unification, variables can be replaced in both sides of equations. In matching, it is allowed only in one side. These techniques have been intensively investigated for the crisp (two-valued) case. Some work is done about equation solving (including also matching as a special case) in the presence of similarity relations. Equational matching and unification are important problems in this area, where equality is considered modulo background theories. However, unlike the crisp case, they have not attracted much attention in the fuzzy setting. One such background theory is the theory of sets. In this context, a set is represented by a first-order term, called a set-term, using a special function symbol as its constructor. Set unification and set matching problems have been studied in the crisp case. It can be also formulated as unification/matching modulo associativity, commutativity, and idempotence of the set constructor, together with its unit element (ACIU-unification/matching). These algorithms have found applications in e.g., deductive databases, theorem proving, static analysis, and rapid software prototyping, just to name a few.

In this talk, we propose extending set matching to similarity relations. In this way, we incorporate some background knowledge into solving techniques with similarity relations. Although our set terms are interpreted as (finite) classical sets, their elements (arguments of set terms) might be related to each other by a similarity relation, which induces also a notion of similarity between set terms. We design a matching algorithm and study its properties. It can be useful in applications where the exact set matching techniques need to be relaxed to deal with quantitative extensions of equality such as similarity relations.

This work can be further extended to several directions. A natural next step would be to allow approximate background knowledge expressed by, e.g., fuzzy sets or rough sets. Another direction would be to generalize the problem from matching to unification. Bringing in multisets together with sets in the theory, generalizing similarity to proximity relations would be also some other interesting extensions to investigate.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia under the project # FR-21-16725.

Esakia Duality for Étale Heyting Algebras

Evgeny Kuznetsov

Department of Mathematical Logic, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: e.kuznetsov@freeuni.edu.ge

The objective of the present study is an attempt to investigate a class of morphisms of Esakia spaces, called *Esakia local homeomorphisms*, hoping of some evidence that for an Esakia space X the corresponding category LH_{ES}/X of Esakia local homeomorphisms over X enjoys many properties of elementary topoi.

Motivation to do so is as follows. Andrew M. Pitts, in his paper [4] asked, whether for arbitrary Heyting algebra H there exists an elementary topos with the lattice of all sub-objects of its terminal object isomorphic to H. For Boolean algebra B, the category LH_{Stone} of local homeomorphisms over the Stone dual space X_B is equivalent to the category obtained using construction by Peter Freyd [3, Exercise 11 Ch. 9], which answers positively a question for Boolean algebras. Even though we believe the approach considered in this talk will not solve the problem of Pitts, it seems that it would be useful to investigate more accessible closely related questions like ours.

We try to generalize the Freyd construction from Boolean algebras to Heyting algebras. To do so we employ Esakia ordered-topological duality for Heyting algebras [2], [1]. For Heyting algebras, we consider continuous strict p-morphisms as the analog of local homeomorphisms of Stone spaces. By SE/X we denotes the category of strict p-morphisms over X. We establish the connection of the category SE/X to algebraic varieties $\acute{E}t(H)$ called *étale Heyting H-algebras* (relative to H) and $\mathcal{V}_{(\acute{E}t)_H}$, the variety of algebras validating axiom $(\acute{E}t)_H$ $\bigvee_{h\in H}(x \iff c_h) = \top$.

Because of its transparency, we consider a finite case separately. The first part of the talk will be devoted to investigation of the relationship between $\acute{E}t(H)$, $Stone^{X_H}$ of Stone-space-valued spresheaves on the dual X_H of H, and the variety of $\mathcal{V}_{(\acute{E}t)_H}$. We summarize the first part of the talk by the following statement:

Theorem 1 *For finite Heyting algebra H, the categories $\acute{E}t(H)$, $\mathcal{V}_{(\acute{E}t)_H}$, SE/X, $Stone^X$ are (dually) equivalent.*

In the second part of the talk we attempt to establish the following:

Theorem 2 *For arbitrary Heyting algebra H the categories $\acute{E}t(H)$ and ES^X are dually equivalent.*

Acknowledgments

The work has been partially supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) grant # FR-22-6700.

References

[1] N. Bezhanishvili, Lattices of intermediate and cylindric modal logics. *ILLC Dissertation (DS) Series*, Amsterdam, 2006; *Institute for Logic, Language and Computation. Thesis, fully internal, Universiteit van Amsterdam.*

[2] L. L. Esakia, Topological Kripke models. (Russian) *Dokl. Akad. Nauk SSSR* **214** (1974), 298–301; translation in *Soviet Math Dokladi* **15** (1974), 147–151.

[3] P. T. Johnstone, *Topos Theory.* London Mathematical Society Monographs, Vol. 10. Academic Press [Harcourt Brace Jovanovich, Publishers], London–New York, 1977.

[4] A. M. Pitts. On an interpretation of second-order quantification in first-order intuitionistic propositional logic. *J. Symbolic Logic* **57** (1992), no. 1, 33–52.

e-Infrastructure and Services for Research and Education in Eastern Partnership Countries EaPConnec

Ramaz Kvatadze

Georgian Research and Educational Networking Association GRENA
Tbilisi, Georgia
E-mail: ramaz@grena.ge

EaPConnect project `https://eapconnect.eu/` is funded by the European Union and contributes to the development of Information Technologies (IT) in education and research in Eastern Partnership (EaP) countries (Armenia, Azerbaijan, Georgia, Moldova and Ukraine) by:

- establishing a high-capacity reliable, long term connections to internet and European Research and Education Network GÉANT for universities and research institutes;
- developing data centers (Cloud, HPC) with computing and storage resources essential for education and research;
- implementing IT services and tools that meet the needs of education and research in the EaP region;
- supporting development of Open Science in the region by integrating countries' digital resources in European Open Science Cloud EOSC;
- enabling and fostering engagement with European e-infrastructures and collaborations on an international scale.

The following topics are discussed:

- Eastern Partnership region and EaPConnect projec;
- *e*-Infrastructure and services;
- Benefits for the research and education communities;
- Situation in Ukraine.

Developed with the support of European Union high capacity network, computing and storage resources, as well as services for research and education community are presented. More than 300 institutions with over 700,000 users are benefiting from the created infrastructure. Several flagship scientific investigations and projects performed by research teams of the Eastern Partnership countries using above mentioned facilities are also presented

Weather Forecast Model Using Markov Chain

Tsiala Kvatadze[1], **Zurab Kvatadze**[2], **Beqnu Parjiani**[2]

[1]*International Black Sea University, Tbilisi, Georgia*
E-mail: ttkvatadze@gmail.com

[2]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mails: zurakvatadze@yahoo.com; beqnufarjiani@yahoo.com

A weather forecast model for the Telavi region has been created. The Markov chain is built on daily weather data for the period [2015–2021]. The initial distribution π and the matrix of transition probabilities P are determined. The limit distribution vector γ, the fundamental matrix Z, and the matrix of first-reach moments M are calculated. The conclusions about the weather forecast are checked.

Analog of Perturbation Theory j_n Quantum Mechanics for the General Case of the Homogeneous Equations

Alexander Kvinikhidze

Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia

E-mail: sasha_kvinikhidze@hotmail.com

We propose a new derivation of Time-Independent Perturbation Theory (PT) that has a fundamental advantage over the usual derivations presented in textbooks on Quantum Mechanics (QM): it is simpler and much shorter. As such, it can provide an easier and quicker way for students to learn PT, than afforded by current methods. In spite of that, our approach does not requirethe potentials to be energy independent or the inverse free Green function to be a linear function of energy E, as is the case in QM, and can be applied directly to various extensions of QM including Relativistic QM, the Bethe–Salpeter equation, and all kinds of quasipotential approaches n Quantum Field Theory .

Three-Statement Financial Models

Givi Lemonjava

University of Georgia, Tbilisi, Georgia
E-mail: givi_lemonjava@yahoo.com

Financial modeling is one of the most highly valued tools in financial analysis. The objective of financial modeling is to combine accounting, finance, and business metrics to create a forecast of a company's future results. A three-statement financial model is an integrated model that forecasts an organization's income statements, balance sheets and cash flow statements. It is the foundation on which we can build additional (and more advanced) models. These include merger models, DCF models, leveraged buyout (LBO) models and various other financial model types. This paper presents an overview of the three core elements (income statements, balance sheets and cash flow statements) model, which is built into Excel by Visual Basic for Applications (VBA) programming language. This VBA Software, named – FSAM.xlsm, we have developed for building the three-statement financial model and financial ratio analysis tools for management. These integrated models' software is powerful tools which allow us to modify assumptions in one part of the model to see how it accurately and consistently influences the other areas of the model. A three-statement financial model is an integrated model that forecasts an organization's income statements, balance sheets and cash flow statements. This three-statement model includes various outputs and schedules. These three key elements accurately capture the association of the multiple line items across the financial statements. To build this three-statement financial model we have taken following steps:

(1) enter historical five years financial data into an Excel-formatted platform – data input sheet;

(2) define the prediction's assumptions that drive forecasting;

(3) predict the income statement;

(4) predict capital investments and assets;

(5) predict financing activity;

(6) predict the balance sheet;

(7) make a cash flow statement.

Overall, this ensured the integrity of the model, its adequacy and accuracy.

On the Topology of Linear Differential Systems

Vakhtang Lomadze

Mathematics Department, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: vakhtang.lomadze@tsu.ge

Let $\mathbb{F}$ be one of the fields $\mathbb{R}$ or $\mathbb{C}$, $\mathcal{U} = C^\infty(\mathbb{R}^n, \mathbb{F})$, $s = (s_1, \dots, s_n)$ the sequence of indeterminates, $\partial = (\partial_1, \dots, \partial_n)$ the sequence of partial differentiation operators, and q a fixed positive integer.

A polynomial matrix $R \in \mathbb{F}[s]^{\bullet \times q}$ translates into a partial differential equation with constant coefficients

$$R(\partial)w = 0 \ \ (w \in \mathcal{U}^q),$$

and one defines a linear differential system (LDS) with signal number q to be a subset of $\mathcal{U}^q$ that is representable as the solution set of such an equation. Crucial for applications in systems and control theory is the availability of a "good topology" on the set of all LDSs with a given signal number. One such a topology has already been defined in Nieuwenhuis and Willems [2, 3]. In this talk we aim to propose a different approach to this concept.

It was observed by Willems [4] that LDSs possess with a remarkable property, the so-called, *jet-completeness* property. And it turned out (see Lomadze [1] that this property (together with the evident properties of linearity and shift-invariance) completely characterizes LDSs among all other dynamical systems. It should be natural therefore to describe a topology of LDSs in terms of jets. This will be done in the talk. It will be shown also that the space of all LDSs with a given complexity polynomial (which is the most important numerical invariant of a LDS) is homeomorphic to a subspace of a certain Grassmanian.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), grant # STEM-22-1601.

References

[1] V. Lomadze, Characterization of linear differential systems (in several variables). *Systems Control Lett.* **68** (2014), 20–24.

[2] J. W. Nieuwenhuis and J. C. Willems, Continuity of dynamical systems: a system theoretic approach. *Math. Control Signals Systems* **1** (1988), no. 2, 147–165.

[3] J. W. Nieuwenhuis and J. C. Willems, Continuity of dynamical systems: the continuous-time case. *Math. Control Signals Systems* **5** (1992), no. 4, 391–400.

[4] J. C. Willems, From time series to linear system, I. Finite-dimensional linear time invariant systems. *Automatica J. IFAC* **22** (1986), no. 5, 561–580.

Embeddings and Regularity of Potentials in Grand Variable Exponent Function Space

Dali Makharadze[1], **Alexander Meskhi**[2,3]

[1]*Department of Mathematics, Batumi Shota Rustaveli State University Batumi, Georgia*
E-mail: dali.makharadze@bsu.edu.ge

[2]*Kutaisi International University, Kutaisi, Georgia*
[3]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: alexander.meskhi@kiu.edu.ge

Our aim is to introduce grand variable exponent Hajłasz–Morrey spaces, and to establish embeddings from these spaces to variable parameter Hölder spaces under the log-Hölder continuity condition on exponents of spaces. The boundedness of the fractional integral operator from grand variable exponent Morrey space to grand variable parameter Hölder space is also established. Similar problems in the frame of grand variable exponent Lebesgue spaces were studied in [1] and [4], respectively. Grand variable exponent Morrey spaces were introduced and studied in [3] (see [5] and [6] for the constant exponent grand Morrey spaces and related topics). In general, the spaces are defined on quasi-metric measure spaces, however, the results are new even for Euclidean spaces. Our investigation was carried out jointly with D. Edmunds, and appeared as a short communication in [2]. It will be published with proofs later.

Acknowledgments

The work was supported by Batumi Shota Rustaveli State University Grant (Resolution # 06-01/04, 31.01.2023).

References

[1] D. E. Edmunds, V. Kokilashvili and A. Meskhi, Embeddings in grand variable exponent function spaces. *Results Math.* **76** (2021), no. 3, Paper no. 137, 27 pp.

[2] D. E. Edmunds, D. Makharadze and A. Meskhi, Embeddings and regularity of potentials in grand variable exponent function spaces. *Trans. A. Razmadze Math. Inst.* **177** (2023), no. 2, 309–314.

[3] V. Kokilashvili and A. Meskhi, Boundedness of operators of harmonic analysis in grand variable exponent Morrey spaces. *Mediterr. J. Math.* **20** (2023), no. 2, Paper no. 71, 25 pp.

[4] V. Kokilashvili, A. Meskhi, H. Rafeiro and S. Samko, *Integral Operators in Non-Standard Function Spaces.* Vol. 1. *Variable Exponent Lebesgue and Amalgam Spaces.* Operator Theory: Advances and Applications, 248. Birkhäuser/Springer, [Cham], 2016; Vol. 2. *Variable Exponent Hölder, Morrey–Campanato and Grand Spaces.* Operator Theory: Advances and Applications, 249. Birkhäuser/Springer, [Cham], 2016.

[5] A. Meskhi, Maximal functions, potentials and singular integrals in grand Morrey spaces. *Complex Var. Elliptic Equ.* **56** (2011), no. 10-11, 1003–1019.

[6] H. Rafeiro, A note on boundedness of operators in grand grand Morrey spaces. *Advances in harmonic analysis and operator theory*, 349–356, Oper. Theory Adv. Appl., 229, *Birkhäuser/Springer Basel AG, Basel*, 2013.

On Popularization of Some Questions of Continuum Mechanics

Vardan Manukyan[1,2], **Gagik Nikoghosyan**[2]

[1]*Reasarch Institute of Physics, Yerevan State University, Yerevan, Armenia*
E-mail: v.f.manukyan@gmail.com

[2]*Faculty of Natural Science and Mathematics, Shirak State University*
Gyumri, Armenia
E-mail: gnikoghosyan@gmail.com

The work is devoted to an accessible presentation of some ideas of solid mechanics. At present, one of the important issues of education is accessible coverage and presentation of the achievements of science to school students. Popularization of scientific ideas motivates. This can increase interest in learning and serve as a basis for project research by schoolchildren. From this point of view, it is important and useful to present in a popular way some of the basic ideas of continuum mechanics, problems and approaches to their solution at school, which can be done within the framework of mathematics and physics courses. Familiarization with scientific tasks at school is associated with certain difficulties, the main of which is their complexity. However, there is always a way to explain complex things in simple terms. As noted by R. Feynman, "If you cannot explain something in simple terms, you don't understand it".

In this paper, an attempt is made to give an accessible and simple introduction to high school students in some topics studied in high school courses in mechanics of a deformable solid body and mathematical physics. For the analysis of static problems of the theory of elasticity, along with simple mathematical and physical representations, elements of the method of dimensions were used. When considering dynamic problems, including the propagation of waves, the ideas of superposition and symmetry were used, as well as the method of changing the reference frame. We have shown that these fundamental principles and approaches, along with other graphical and visualization tools, can often help avoid complex and large-scale mathematical calculations. We have presented a way of determining the propagation speed of mechanical waves within the framework of school knowledge. We also covered the issues of reflection and superposition of waves, as well as the introduction of the idea of standing waves in school. Such questions have both theoretical and practical significance, which can be useful for the development of knowledge and skills of high school students.

Acknowledgments

The study was carried out with the financial support of ShSU, within the framework of scientific project # ShSU 02-SCI-2022.

A Mathematician Visits the Alhambra – a Universal Dialogue

José Martínez-Aroza

Department of Applied Mathematics, University of Granada, Granada, Spain
E-mail: jmaroza@ugr.es

The Alhambra is a palace and fortress complex located in Granada, Andalusia, Spain. The complex was built in 1238 and continuously modified by the successive Nasrid rulers, until the conclusion of the Christian Reconquest in 1492.

The palace complex is designed in the Nasrid style, the last blooming of Islamic art in the Iberian Peninsula, that had a great influence on the Maghreb to the present day, and on contemporary Mudejar art, which is characteristic of western elements reinterpreted into Islamic forms and widely popular during the Reconquest in Spain.

In this presentation we will see some mathematical aspects of the complex, paying special attention to the richness of proportions and the mathematical structure of the surprisingly wide variety and complexity of tile mosaics that can be found in this monument.

References

[1] C. Alsina Català, J.M. Fortuny Aymemí and R. Pérez Gómez, *Por qué Geometría?: propuestas didácticas para la ESO*. (Spanish) Sintesis, Madrid, 1997.

[2] R. Pérez-Gómez, The four regular mosaics missing in the Alhambra. *Comput. Math. Appl.* **14** (1987), no. 2, 133–137.

On the Classification of Schreier Extensions of Monoids with Non-Abelian Kernel

Nelson Martins-Ferreira[1], **Andrea Montoli**[2], **Alex Patchkoria**[3], **Manuela Sobral**[4]

[1] *Polytechnic Institute of Leiria, Leiria, Portugal*
E-mail: martins.ferreira@ipleiria.pt

[2] *University of Milan, Milan, Italy*
E-mail: andrea.montoli@unimi.it

[3] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: alex.patchkoria@tsu.ge

[4] *University of Coimbra, Coimbra, Portugal*
E-mail: sobral@mat.uc.pt

We show that any regular (right) Schreier extension of a monoid M by a monoid A induces an abstract kernel $\Phi\colon M \to \frac{End(A)}{Inn(A)}$. If an abstract kernel factors through $\frac{SEnd(A)}{Inn(A)}$, where $SEnd(A)$ is the monoid of surjective endomorphisms of A, then we associate to it an obstruction, which is an element of the third cohomology group of M with coefficients in the abelian group $U(Z(A))$ of invertible elements of the center $Z(A)$ of A, on which M acts via Φ. An abstract kernel $\Phi\colon M \to \frac{SEnd(A)}{Inn(A)}$ (resp. $\Phi\colon M \to \frac{Aut(A)}{Inn(A)}$) is induced by a regular weakly homogeneous (resp. homogeneous) Schreier extension of M by A if and only if its obstruction is zero. We also show that the set of isomorphism classes of regular weakly homogeneous (resp. homogeneous) Schreier extensions inducing a given abstract kernel $\Phi\colon M \to \frac{SEnd(A)}{Inn(A)}$ (resp. $\Phi\colon M \to \frac{Aut(A)}{Inn(A)}$), when it is not empty, is in bijection with the second cohomology group of M with coefficients in $U(Z(A))$.

Acknowledgments

The third author was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG), grant # FR-18-10849, *"Stable Structures in Homological Algebra"*.

Additive Functors and Control Theory

Alex Martsinkovsky

Mathematics Department, Northeastern University, Boston, MA 02115, USA
E-mail: a.martsinkovsky@northeastern.edu

The goal of this talk is to propose a functorial algebraic framework for the duality between observability and controllability of linear control systems.

Acknowledgments

The work was partially supported by the Shota Rustaveli National Science Foundation of Georgia Grant # NFR-18-10849.

References

[1] A. Martsinkovsky and J. Russell, Injective stabilization of additive functors, I. Preliminaries. *J. Algebra* **530** (2019), 429–469.

[2] A. Martsinkovsky and J. Russell, Injective stabilization of additive functors, II. (Co)torsion and the Auslander–Gruson–Jensen functor. *J. Algebra* **548** (2020), 53–95.

[3] A. Martsinkovsky and J. Russell, Injective stabilization of additive functors, III. Asymptotic stabilization of the tensor product. *Algebra Discrete Math.* **31** (2021), no. 1, 120–151.

Hilbert's Theorem 90 in Monoidal Categories

Bachuki Mesablishvili[1,2]

[1]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2]*Department of Mathematics, Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

E-mail: bachuki.mesablishvili@tsu.ge

We present the following categorified version of Hilbert's Theorem 90 that includes and extends known forms of Hilbert's Theorem 90.

Theorem 1. *Let $\mathscr{V}$ be a symmetric monoidal category in which equalizers and coequalizers exist and the latter are preserved by taking the monoidal product with any object, $\mathbf{H}$ be a $\mathscr{V}$-bialgebra and A be a commutative right $\mathbf{H}$-comodule $\mathscr{V}$-algebra. Suppose that either*

(i) $\mathrm{A}^{\mathbf{H}} \rightarrowtail \mathrm{A}$ *is a split monomorphism of* $\mathrm{A}^{\mathbf{H}}$*-modules,*

or

(ii) *the functor* $- \otimes_{\mathrm{A}^{\mathbf{H}}} A : \mathscr{V}_{\mathrm{A}^{\mathbf{H}}} \to \mathscr{V}_{\mathrm{A}}$ *is comonadic and* $\mathrm{A}^{\mathbf{H}} \rightarrowtail \mathrm{A}$ *descents invertibility of modules.*

Then there are isomorphisms of abelian groups

$$\mathscr{D}^1(\mathrm{A}\natural\boldsymbol{H}, A) \simeq \mathbf{Ker}(\mathbf{Pic}(\mathrm{A}^{\mathbf{H}} \rightarrowtail \mathrm{A})) \simeq \mathcal{H}^1(\mathrm{A}^{\mathbf{H}} \rightarrowtail \mathrm{A}\,, \underline{\mathbf{Aut}}_{\mathrm{A}^{\mathbf{H}}}).$$

Each of these three groups is trivial, provided that $\mathbf{Pic}(\mathrm{A}^{\mathbf{H}}) = 0$.

Here $A\natural\mathbf{H}$ is an A-coring induced by entwining A and $\mathbf{H}$ (see, [1]) and $\mathscr{D}^1(A\natural\mathbf{H}, A)$ is the 1-descent cohomology set of $A\natural\mathbf{H}$ with coefficients in A (see [2]).

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation Grant # FR-22-4923.

References

[1] A. Al-Rawashdeh and B. Mesablishvili, Hilbert's theorem 90 in monoidal categories. *J. Algebra* **602** (2022), 1–36.

[2] B. Mesablishvili, On descent cohomology. *Trans. A. Razmadze Math. Inst.* **173** (2019), no. 2, 137–155.

Two-Weight Criteria for Multiple Fractional Integrals in Mixed-Normed Lebesgue Spaces

Alexander Meskhi[1,2], Lazare Natelashvili[3]

[1]*Kutaisi International University, Kutaisi, Georgia*

[2]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: alexander.meskhi@kiu.edu.ge

[3]*Georgian Technical University, Tbilisi, Georgia*

E-mail: lazarenatelashvili@gmail.com

Two-weight norm estimates for strong fractional maximal operator

$$(M_{\vec{\alpha}}f)(x_1,\dots,x_n)=\sup\frac{1}{\prod\limits_{j=1}^{n}|Q_j|^{1-\frac{\alpha_j}{d}}}\int\limits_{Q_1\times\cdots\times Q_n}|f(y_1,\cdots,y_n)|\,dy_1\cdots dy_n,\ \ \vec{\alpha}:=(\alpha_1,\dots,\alpha_n),$$

where the supremum is taken over all cubes $Q_j\subset\mathbb{R}^d$ with sides parallel to the coordinate axis such that $x_j\in Q_j$, $j=1,\dots,n$, are established in mixed-normed Lebesgue spaces. In particular, we study the two-weight inequality

$$\|V(M_{\vec{\alpha}}f)\Big\|_{L^{\vec{q}}(\mathbb{R}^{d\times n})}\le C\|Wf\|_{L^{\vec{p}}(\mathbb{R}^{d\times n})},\ \ \vec{q}:=(q_1,\dots,q_n),\ \ \vec{p}:=(p_1,\dots,p_n),$$

where $L^{\vec{p}}$ and $L^{\vec{q}}$ are mixed-normed Lebesgue spaces, and V and W are weight functions on $\mathbb{R}^{d\times n}$. Here under the symbol $\mathbb{R}^{d\times n}$ we mean n-fold Cartesian product of $\mathbb{R}^d$, i.e., $\mathbb{R}^{d\times n}:=\mathbb{R}^d\times\cdots\times\mathbb{R}^d$.

As a consequence, we have complete characterizations of the trace inequality

$$\|V(M_{\vec{\alpha}}f)\|_{L^{\vec{q}}(\mathbb{R}^{d\times n})}\le C\|f\|_{L^{\vec{p}}(\mathbb{R}^{d\times n})}. \tag{1}$$

Fefferman–Stein type two-weight inequality for strong fractional maximal operator $M_{\vec{\alpha}}$ is also derived.

A complete characterization of the one-weight Sobolev inequality

$$\|W(I_{\vec{\alpha}}f)\|_{L^{\vec{q}}(\mathbb{R}^{d\times n})}\le C\|Wf\|_{L^{\vec{p}}(\mathbb{R}^{d\times n})},\ \ \frac{1}{p_j}-\frac{1}{q_j}=\frac{\alpha_j}{d}\,,\ \ j=1,\dots,n, \tag{2}$$

for the multiple fractional integral operator

$$(I_{\vec{\alpha}}f)(x_1,\dots,x_n)=\int\limits_{\mathbb{R}^d}\cdots\int\limits_{\mathbb{R}^d}\frac{f(y_1,\dots,y_n)}{\prod\limits_{j=1}^{n}|x_j-y_j|^{d-\alpha_j}}\,dy_1\cdots dy_n,$$

when $W(x_1,\dots,x_n)=W_1(x_1)\cdots W_n(x_n)$ is also derived.

On Teaching Problems of Constructing the Graph of the Function

Rusudan Meskhia

Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: rusudan.meskhia@tsu.ge

The question of constructing the graph of the function by the transformation of the graphs of main Elementary functions is considered. It is important, that in order to construct the graph of the function in this way it is not necessary to study the properties of the function.

On the contrary, from the formed graph we can establish the properties of the function. This fact is more interesting, as the derivative of the function, in the school course of mathematics is not taught.

Besides, we consider some examples of constructing the graph for some composition of functions.

Asymptotic Expansions of Stable, Stabilizable and Stabilized Means

Lenka Mihoković

University of Zagreb, Faculty of Electrical Engineering and Computing, Zagreb, Croatia
E-mail: lenka.mihokovic@fer.hr

Consider bivariate mean M, i.e. a function $M\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ which assumes values between minimum and maximum of its variables, which is additionally symmetric and homogeneous of degree one.

Let M, N, K, be three homogeneous symmetric bivariate means and let

$$\mathcal{R}(K,N,M)(s,t) = K\big(N(s,M(s,t)),N(M(s,t),t)\big).$$

$\mathcal{R}$ is also called the resultant mean-map of K, M and N. Observing various functional equations involving the resultant mean-map yields the following notions. A symmetric mean M is said to be:

- *stable* (*balanced*), if $\mathcal{R}(M,M,M) = M$;
- (K,N)-*stabilizable*, if $M(s,t) = \mathcal{R}(K,M,N)(s,t)$ for two nontrivial stable means K and N;
- (K,N)-*stabilized*, if $M(s,t) = \mathcal{R}(K,N,M)(s,t)$ for two nontrivial stable means K and N.

For two nontrivial stable comparable means the (strict) sub-stabilizability and super-stabilizability concept can be introduced with the appropriate inequality sign in stabilizability relation. For many classical means, relations of this kind are known but there are still some open problems as can be seen in [3] and [4].

The main goal ([1]) is to derive the complete asymptotic expansion

$$R(x-t,x+t) = \mathcal{R}(K,N,M)(x-t,x+t) \sim \sum_{m=0}^{\infty} a_m^R t^{2m} x^{-2m+1}, \ \ x\to\infty,$$

and also the asymptotic expansions of stable, stabilizable and stabilized mean. Based on these results, we show how to obtain the necessary conditions for mean N to be simultaneously (K,M) and (M,K)-stabilizable, for mean M to be simultaneously (K,N) and (N,K)-stabilized and for mean M to be simultaneously (K,N)-stabilizable and (K,N)-stabilized.

With respect to known asymptotic expansions of parametric means from [2] and [3] it will be shown how the obtained coefficients are used to solve the problem of identifying stable means within classes of parametric means under consideration, how to disprove some mean is stabilizable or stabilized and how to obtain best possible parameters such that given mean is stabilizable with some pair of parametric means.

References

[1] L. Mihoković, Asymptotic expansions of stable, stabilizable and stabilized means with applications. Preprint arXiv:2303.05217; `https://arxiv.org/abs/2303.05217`.

[2] N. Elezović and L. Vukšić, Asymptotic expansions and comparison of bivariate parameter means. *Math. Inequal. Appl.* **17** (2014), no. 4, 1225–1244.

[3] M. Raïssouli, Stability and stabilizability for means. *Appl. Math. E-Notes* **11** (2011), 159–174.

[4] M. Raïssouli and J. Sándor, Sub-stabilizability and super-stabilizability for bivariate means. *J. Inequal. Appl.* **2014**, 2014:28, 13 pp.

[5] L. Vukšić, Seiffert means, asymptotic expansions and inequalities. *Rad Hrvat. Akad. Znan. Umjet. Mat. Znan.* **19(523)** (2015), 129–142.

About Maximum Principles for Weak Solutions of Some Parabolic Systems

Sergey E. Mikhailov

Department of Mathematics, Brunel University London, Uxbridge, UK
E-mail: sergey.mikhailov@brunel.ac.uk

Maximum principles for solutions parabolic equations constitute a traditional part of PDE analysis. It is well developed for classical solutions of scalar parabolic equations with constant coefficients and these results also generalised to weak solutions of scalar elliptic and parabolic equations with variable coefficients, see, e.g., [1, Chapter III, Theorem 7.2]. The estimates of the essential maximum of weak solutions of parabolic systems are also available although with a constant depending on the system coefficients, cf., e.g., [1, Chapter VII, Theorem 2.1].

In this contribution, by employing special test functions, some sharper versions of the maximum principle for weak solutions of several linear parabolic variable-coefficient systems have been proved. The considered systems include non-stationary convection-reaction-diffusion systems as well as the Stokes, Oseen and Brinkman systems. The obtained maximum principles for weak solutions can be employed to prove global existence of solutions of some nonlinear parabolic systems, cf. [3], where a maximum principle for strong solutions of the Burgers system has been used for this.

The talk is based on paper [2].

References

[1] O. A. Ladyzhenskaya, V. A. Solonnikov and N. N. Ural'tseva, *Linear and Quasilinear Equations of Parabolic Type.* (Russian) Translated from the Russian by S. Smith, Translations of Mathematical Monographs, Vol. 23 American Mathematical Society, Providence, R.I., 1968.

[2] S. E. Mikhailov, On Maximum Principles for Weak Solutions of Some Parabolic Systems. In: *Integral Methods in Science and Engineering*, C. Constanda et al. (Eds.), Springer, 2023, Chapter 18 (*to appear*); `https://doi.org/10.1007/978-3-031-34099-4_18`.

[3] B. C. Pooley and J. C. Robinson, Well-posedness for the diffusive 3D Burgers equations with initial data in $H^{1/2}$. *Recent progress in the theory of the Euler and Navier-Stokes equations*, 137–153, London Math. Soc. Lecture Note Ser., 430, *Cambridge Univ. Press, Cambridge*, 2016.

Optimal Control in the Models Similar to Neural Networks

Miranda Mnatsakaniani

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: Miranda.mnatsakaniani@atsu.edu.ge

In the paper the results of setting and resolving a problem of optimal control for models of Makarenko is considered. Mathematical formalization of such a problem is reduced to investigation of the problems of optimal control for discrete dynamical system similar to neural networks. The theorem about existence of optimal control with application of the Krotov's method on the base of a sufficient condition of optimality is proved.

References

[1] A. Makarenko, *Sustainable Development and Principles of Social Systems Modeling.* Generis Publisher, Kyshyniv, 2020.

Lyapunov Dimension Formula of n-Generalized Henon Map

Vitali Muladze, Giorgi Rakviashvili

School of Business, Technology and Education, Ilia State University, Tbilisi, Georgia
E-mails: vitali.muladze.1@iliauni.edu.ge; giorgi.rakviashvili@iliauni.edu.ge

Definition We consider the n-generalized Henon map F

$$\begin{cases} x' = 1 - 2a_1x^2 - 2a_2x^4 - \cdots - 2a_nx^{2n} + by, \\ y' = bx. \end{cases}$$

Lemma *Let*

$$f(x) = 1 - a_1x^2 - a_2x^4 - \cdots - a_nx^{2n} - bx, \tag{1}$$

where $a_1, a_2, \dots, a_n, b \geq 0$, and $a_n \neq 0$. Then $f(x)$ has one negative (let it be x_-) and one positive real roots, the other roots are conjugate complex numbers and

$$f'(x_-) = -2a_1x_- - 4a_2x_-^3 - \cdots - 2na_nx_-^{2n-1} - b > 0.$$

Theorem *Assume $a_1, a_2, \dots, a_n \geq 0$, $0 < b < 1$, $a_n \neq 0$, the bounded set K contains all stationary points of F and $F(K) = K$. If some additional assumptions are satisfied, then*

$$\dim_L K = 1 + \frac{1}{1 - \ln b^2 / \ln \alpha_1(x_-)}\,,$$

where $\dim_L K$ is the Lyapunov dimension of K and

$$\alpha_1(x) = \sqrt{(a_1x + 2a_2x^3 + \cdots + 2na_nx^{2n-1})^2 + b^2} + \big|a_1x + 2a_2x^3 + \cdots + 2na_nx^{2n-1}\big|.$$

References

[1] G. A. Leonov, Strange attractors and classical stability theory. *Nonlinear Dyn. Syst. Theory* **8** (2008), no. 1, 49–96.

Mathematical Experiments in the Process of Studying Probability and Statistics

Roin Nakaidze

Maksvalta Public School of Shuakhevi Municipality, Adjara, Georgia
E-mail: roininakaidze@gmail.com

The ability of a teacher to impart knowledge of information technology is evolving on a daily basis. As a result, it is crucial to understand and incorporate computer programs into the learning process. Teachers can take advantage of the chances provided by this program to improve the learning process with their students.

My innovative findings can be utilized to solve multiple issues in the mathematics class and, additionally, to conduct mathematical experiments, which boosts student motivation and contributes to knowledge generalization.

The team of GeoGebra creates such a perfect, interactive and educational environment. It is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one engine.

Furthermore, the GeoGebra program itself, which is an innovative approach in the modern educational system, can be used for the process of studying probability and statistics. The teacher can conduct computer-aided experiments to demonstrate the importance of using the program, which will increase their enthusiasm.

In this presentation, I will go over some issues that clearly demonstrate the relationship between the relative frequency and theoretical probability. GeoGebra resources will be presented, the majority of which were recently uploaded to the GeoGebra official page and can be used by any teacher. The process of creating some of the materials has been recorded and made available on YouTube.

An Alternative Potential Method for Mixed Steady State Elastic Oscillation Problems

David Natroshvili

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: natrosh@hotmail.com

We consider an alternative approach to investigate three-dimensional exterior mixed boundary value problems (BVP) for the steady state oscillation equations of the elasticity theory for isotropic bodies. The unbounded domain occupied by an elastic body, $\Omega^- \subset \mathbb{R}^3$, has a compact boundary surface $S = \partial\Omega^-$, which is divided into two disjoint parts, the Dirichlet part S_D and the Neumann part S_N, where the displacement vector (the Dirichlet type condition) and the stress vector (the Neumann type condition) are prescribed respectively. Our new approach is based on the classical potential method and has several essential advantages compared with the existing approaches. We look for a solution to the mixed boundary value problem in the form of a linear combination of the single layer and double layer potentials with densities supported on the Dirichlet and Neumann parts of the boundary respectively. This approach reduces the mixed BVP under consideration to a system of boundary integral equations, which contain neither extensions of the Dirichlet or Neumann data nor the Steklov–Poincaré type operator involving the inverse of a special boundary integral operator, which is not available explicitly for arbitrary boundary surface. Moreover, the right-hand sides of the resulting boundary integral equations system are vector-functions coinciding with the given Dirichlet and Neumann data of the problem in question. We show that the corresponding matrix integral operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies unconditional unique solvability of the mixed BVP in the class of vector-functions belonging to the Sobolev space $[W^1_{2,loc}(\Omega^-)]^3$ and satisfying the Sommerfeld–Kupradze radiation conditions at infinity.We also show that the obtained matrix boundary integral operator is invertible in the L_p-based Besov spaces and prove that under appropriate boundary data a solution to the mixed BVP possesses C^α-Hölder continuity property in the closed domain $\overline{\Omega}^-$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.

The talk is based on collaboration with Maia Mrevlishvili and Zurab Tediashvili.

A Generalization of $\oplus$-Cofinitely Supplemented Modules

Celil Nebiyev[1], Hasan Hüseyin Ökten[2]

[1] *Department of Mathematics, Ondokuz Mayıs University, Samsun, Turkey*
E-mail: cnebiyev@omu.edu.tr

[2] *Technical Sciences Vocational School, Amasya University, Amasya, Turkey*
E-mail: hokten@gmail.com

In this work, cofinitely $\oplus - g$-supplemented modules are defined and some properties of these modules are investigated. Every module is a unitary left R-module over the ring R with unity, in this work. It is clear that every $\oplus$-cofinitely supplemented module is cofinitely $\oplus - g$-supplemented. Because of this, cofinitely $\oplus - g$-supplemented modules are more general than $\oplus$-cofinitely supplemented modules.

Definition 1 *Let M be an R-module. If every cofinite submodule of M has a g-supplement that is a direct summand in M, then M is called a cofinitely $\oplus - g$-supplemented module.*

Proposition 2 *Every $\oplus - g$-supplemented module is cofinitely $\oplus - g$-supplemented.*

Proposition 3 *Let M be a finitely generated R-module. If M is cofinitely $\oplus - g$-supplemented, then M is $\oplus - g$-supplemented.*

Proposition 4 *Every cofinitely $\oplus - g$-supplemented module is cofinitely g-supplemented.*

Acknowledgments

This research was in part supported by grants from Ondokuz Mayıs University (Project # PYO.EGF.1901.22.002).

References

[1] H. Çalışıcı and A. Pancar, $\oplus$-confinitely supplemented modules. *Czechoslovak Math. J.* **54(129)** (2004), no. 4, 1083–1088.

[2] A. Harmancı, D. Keskin and P. F. Smith, On $\oplus$-supplemented modules. *Acta Math. Hungar.* **83** (1999), no. 1-2, 161–169.

[3] B. Koşar, Cofinitely g-supplemented modules. *British J. Math. Computer Sci.*, **14** (2016), no. 4, 1–6.

[4] B. Koşar, C. Nebiyev and N. Sökmez, g-supplemented modules. Translation of *Ukraïn. Mat. Zh.* **67** (2015), no. 6, 861–864; *Ukrainian Math. J.* **67** (2015), no. 6, 975–980.

[5] C. Nebiyev and H. H. Ökten, Some properties of $\oplus$-g-supplemented modules. *Miskolc Math. Notes* **24** (2023), no. 1, 335–341.

[6] C. Nebiyev and H. H. Ökten, Cofinitely $\oplus - G$-supplemented modules. *IX International Conference of the Georgian Mathematical Union (Batumi–Tbilisi, Georgia, September 3-8, 2018)*, Book of Abstracts, pp. 178–179, 2018.

[7] R. Wisbauer, *Foundations of Module and Ring Theory. A Handbook for Study and Research.* Revised and translated from the 1988 German edition. Algebra, Logic and Applications, 3. Gordon and Breach Science Publishers, Philadelphia, PA, 1991.

Bounds on the Zeros of Recursively Defined Polynomials

Markus Neuhauser[1,2]

[1]*Kutaisi International University, Kutaisi University Campus, Kutaisi, Georgia*
[2]*RWTH Aachen University, Aachen, Germany*
E-mail: markus.neuhauser@kiu.edu.ge

The talk presents some results motivated by Lehmer's conjecture [5] that Ramanujan's tau function constituted by the Fourier coefficients of the 24th power of Dedekind's eta function never vanishes. Generally, when Dedekind's eta function is raised to a power with exponent x it turns out that the Fourier coefficients are polynomials in x. They satisfy a recurrence relation. Even in a more general form we can provide a bound on x outside of which these never vanish [1,3]. In some cases, including Dedekind's eta function, this seems to be the best possible. In its general form it provides relations for example to orthogonal polynomials and Eisenstein series [2].

The talk includes joint work with B. Heim (University of Cologne/RWTH Aachen University), R. Tröger (GUtech), and A. Weisse (MPIM Bonn) [4].

References

[1] B. Heim and M. Neuhauser, On the growth and zeros of polynomials attached to arithmetic functions. *Abh. Math. Semin. Univ. Hambg.* **91** (2021), no. 2, 305–323.

[2] B. Heim and M. Neuhauser, Polynomials and reciprocals of Eisenstein series. *Int. J. Number Theory* **17** (2021), no. 2, 487–497.

[3] B. Heim, M. Neuhauser and R. Tröger, Zeros of recursively defined polynomials. *J. Difference Equ. Appl.* **26** (2020), no. 4, 510–531.

[4] B. Heim, M. Neuhauser and A. Weisse, Records on the vanishing of Fourier coefficients of powers of the Dedekind eta function. *Res. Number Theory* **4** (2018), no. 3, Paper no. 32, 12 pp.

[5] D. H. Lehmer, The vanishing of Ramanujan's function $\tau(n)$. *Duke Math. J.* **14** (1947), 429–433.

Construction of Equistable Holes in the Case of an Axisymmetric Problem

Nana Odishelidze

Department of Computer Science, Department of Theoretical Informatics, Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia

E-mail: nana.odishelidze@tsu.ge

The construction of equistable hole structures in the case of an axisymmetric problem is considered. The optimal stress distribution is achieved by choosing the appropriate boundary. For each external loading on the plate's outer boundary, appropriate construction for its equistable holes are built and tangential normal stresses are determined on it. The unknown equistable part of the boundary and a stressed state of the body are defined. Computer mathematical systems [1] MATLAB and [2] Mathcad are used for calculations and constructions.

References

[1] S. Attaway, *Matlab: A Practical Introduction to Programming and Problem Solving.* Fourth Edition. Elsevier, Amsterdam–Oxford-New-York, 2017.

[2] 2. R. Larsen, *Introduction to Mathcad 15.* 3rd Edition, Pearson, 2010.

Almost Everywhere Convergence of Nets of Operators and Weak Type Maximal Inequalities

Giorgi Oniani

School of Mathematics and Computer Science, Kutaisi International University
Kutaisi, Georgia
E-mail: giorgi.oniani@kiu.edu.ge

We will present extensions of the weak type maximal principles of Stein and Sawyer to nets of operators defined on classes of functions on general measure spaces (possibly of infinite measure), including the case of locally compact groups. An applications to differentiation of integrals, multiple Fourier series and multi-parameter ergodic averages will be given. The talk is based on the article [1].

References

[1] G. Oniani, Almost everywhere convergence of nets of operators and weak type maximal inequalities. *Fund. Math.*, 2023; DOI: 10.4064/fm272-2-2023.

On the Mellin–Gauss–Weierstrass Operators in the Weighted Lebesgue Spaces

Fırat Özsaraç

Department of Mathematics, Kırıkkale University, Kırıkkale, Turkey
E-mail: firat_ozsarac@hotmail.com

In this presentation, we introduce the modulus of smoothness of function f in the weighted Lebesgue space, and then we give some properties of it. By means of this, the rate of convergence is obtained. Also, we express some pointwise convergence results for a family of the linear Mellin type operators. In particular, we gain the pointwise convergence at any Lebesgue point of function f. Articles in the literature on the subject can be seen in [1–11].

References

[1] A. Aral, T. Acar and S. Kursun, Generalized Kantorovich forms of exponential sampling series. *Anal. Math. Phys.* **12** (2022), no. 2, Paper no. 50, 19 pp.

[2] C. Bardaro and I. Mantellini, A note on the Voronovskaja theorem for Mellin-Fejer convolution operators. *Appl. Math. Lett.* **24** (2011), no. 12, 2064–2067.

[3] C. Bardaro and I. Mantellini, Pointwise convergence theorems for nonlinear Mellin convolution operators. *Int. J. Pure Appl. Math.* **27** (2006), no. 4, 431–447.

[4] L. Angeloni and G. Vinti, Convergence and rate of approximation in $BV^{\varphi}(\mathbb{R}^N_+)$ for a class of Mellin integral operators. *Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl.* **25** (2014), no. 3, 217–232.

[5] L. Angeloni and G. Vinti, Variation and approximation in multidimensional setting for Mellin integral operators. *New perspectives on approximation and sampling theory*, 299–317, Appl. Numer. Harmon. Anal., *Birkhäuser/Springer, Cham*, 2014.

[6] C. Bardaro and I. Mantellini, Voronovskaya-type estimates for Mellin convolution operators. *Results Math.* **50** (2007), no. 1-2, 1–16.

[7] C. Bardaro and I. Mantellini, Approximation properties for linear combinations of moment type operators. *Comput. Math. Appl.* **62** (2011), no. 5, 2304–2313.

[8] C. Bardaro and I. Mantellini, Asymptotic behaviour of Mellin–Fejer convolution operators. *East J. Approx.* **17** (2011), no. 2, 181–201.

[9] C. Bardaro and I. Mantellini, On the iterates of Mellin-Fejer convolution operators. *Acta Appl. Math.* **121** (2012), 213–229.

[10] C. Bardaro and I. Mantellini, On Mellin convolution operators: a direct approach to the asymptotic formulae. *Integral Transforms Spec. Funct.* **25** (2014), no. 3, 182–195.

[11] C. Bardaro, I. Mantellini and G. Schmeisser, Exponential sampling series: convergence in Mellin–Lebesgue spaces. *Results Math.* **74** (2019), no. 3, Paper no. 119, 20 pp.

On the Approximation of the Solution for a Kirchhoff's-type Equation Describing the Dynamic Behavior of a String

Archil Papukashvili[1,2], **Giorgi Papukashvili**[3], **Jemal Peradze**[4], **Meri Sharikadze**[1]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *School of Science and Technology, The University of Georgia, Tbilisi, Georgia*

[3] *Vladimir Komarov Tbilisi School of Physics and Mathematics # 199, Tbilisi, Georgia*

[4] *Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: archil.papukashvili@tsu.ge, gagapapukashvili@gmail.com, j_peradze@yahoo.com, meri.sharikadze@tsu.ge

The presented discourse serves as a direct extension of the research papers [1, 2], which delve into the investigation of an initial-boundary value problem associated with Kirchhoff's integro-differential equation. This mathematical model effectively characterizes the dynamic behaviour of a string. To seek an approximate solution for this problem, a combined approach involving a Galerkin method, a stable symmetric finite difference scheme, and a Picard-type iterative method is employed. In article [1], the algorithm is tested using a simple test example, providing the error solely for the difference method. However, this work considers a more complex test example that allows us to assess the errors for both the difference method and the Galerkin method. Numerical computations are performed to validate the proposed approach, and the resulting findings are presented in both tabular and graphical formats.

References

[1] G. Papukashvili and J. Peradze, A numerical solution of string oscillation equation. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **23** (2009), 80–83.

[2] J. Peradze, A numerical algorithm for the nonlinear Kirchhoff string equation. *Numer. Math.* **102** (2005), no. 2, 311–342.

On the Kernel of the Gysin Homomorphism on Chow Groups

Rina Paucar[1], **Claudia Schoemann**[2]

[1] *Instituto de Matemáticas y Ciencias Afines, Universidad Nacional de Ingeneria Lima, Perú*

E-mail: rina.paucar@imca.edu.pe

[2] *Laboratoire Gaati, Université de La Polynésie Française, BP 6570-98702 FAAA Polynésie Française*

E-mail: claudia.schoemann@upf.pf

Let S be a connected smooth projective surface over $\mathbb{C}$. Let Σ be the complete linear system of a very ample divisor D on S and let $d = \dim(\Sigma)$. For any closed point $t \in \Sigma \cong \mathbb{P}^{d*}$, let H_t be the hyperplane in $\mathbb{P}^d$ corresponding to t, $C_t = H_t \cap S$ the corresponding hyperplane section of S, and r_t the closed embedding of C_t into S. Let Δ_S be the discriminant locus of Σ parametrizing singular hyperplane sections of S and $U = \Sigma \setminus \Delta_S$ its complement of smooth hyperplane sections of S. Let $\mathsf{CH}_0(S)_{\deg=0}$ and $\mathsf{CH}_0(C_t)_{\deg=0}$ be the Chow groups of 0-cycles of degree 0 on S and C_t respectively. In this paper we prove that for C_t a smooth hyperplane section of S the kernel G_t of the Gysin homomorphism r_{t*} from $\mathsf{CH}_0(C_t)_{\deg=0}$ to $\mathsf{CH}_0(S)_{\deg=0}$ induced by r_t is a countable union of translates of an abelian subvariety A_t inside the Jacobian J_t of the curve C_t. We also prove that there is a c-open subset U_0 in U such that $A_t = 0$ for all $t \in U_0$ or $A_t = B_t$ for all $t \in U_0$, where B_t is an abelian subvariety of J_t.

Acknowledgments

The work was supported by the *"Programa de Doctorado en Universidades Peruanas"* CG-176-2015-FONDECYT.

References

[1] K. Banerjee and V. Guletskii, Etale monodromy and rational equivalence for 1-cycles on cubic hypersurfaces in $\mathbb{P}^5$. (Russian) *Mat. Sb.* **211** (2020), no. 2, 3–45; translation in *Sb. Math.* **211** (2020), no. 2, 161–200.

[2] C. Voisin, *Hodge Theory and Complex Algebraic Geometry.* II. Translated from the French by Leila Schneps. Cambridge Studies in Advanced Mathematics, 77. Cambridge University Press, Cambridge, 2003.

The Total Error of a Numerical Algorithm for a Timoshenko Plate System

Jemal Peradze

Department of Mathematics, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: jemal.peradze@tsu.ge

A boundary value problem for a nonlinear system of ordinary differential equations

$$\begin{gathered}
u'' + \frac{1}{2}\,(w'^2)' = 0,\\
k_0^2\,\frac{Eh}{2(1+\nu)}\,(w'' + \psi') + \frac{Eh}{1-\nu^2}\,\Big[\Big(u' + \frac{1}{2}\,w'^2\Big)w'\Big]' + f(x) = 0,\\
\frac{h^2}{6(1-\nu)}\,\psi'' - k_0^2(w' + \psi) = 0
\end{gathered}$$

modeling the symmetric static displacement of the Timoshenko plate [1] is considered. To approximate the solution the Green functions, the Galerkin method and the Jacobi–Cardano iteration process [2] are used. The total error of the algorithm is estimated.

References

[1] J. Lagnese and J.-L. Lions, *Modelling Analysis and Control of Thin Plates*. Recherches en Mathématiques Appliquées [Research in Applied Mathematics], 6. Masson, Paris, 1988.

[2] J. Peradze, On an iteration method of finding a solution of a nonlinear equilibrium problem for the Timoshenko plate. *ZAMM Z. Angew. Math. Mech.* **91** (2011), no. 12, 993–1001.

Understanding Relationships for Multivariate Data Using Copulas and Stochastic Differential Equations

Edmundas Petrauskas[1], **Petras Rupšys**[2]

[1]*Department of Forest Sciences, Vytautas Magnus University, Kaunas, Lithuania*
E-mail: edmundas.petrauskas@vdu.lt

[2]*Department of Mathematics and Statistics, Vytautas Magnus University Kaunas, Lithuania*
E-mail: petras.rupsys@vdu.lt

A copula is mathematical function that combines the marginal distributions into a joint multivariate cumulative probability distribution. Copula models have many advantages for applications in almost every discipline. First, many studies in applied areas collect longitudinal, multivariate and discrete data, for which the amount of measurements of individual variables does not match. Second, they allow fitting any univariate marginal distributions that need not be coming from the same family of distributions. Third, they reduce complexity compared to existing multivariate probabilistic models as the number of dimensions increases. Fourth, they provide a framework for generating many different relationships between variables. In this study for estimating five-dimensional dependencies we used a Gaussian copula approach, when the dynamics of individual variables are described by a stochastic differential equation with mixed effect parameters. For parameter estimation was used a semiparametric maximum pseudo-likelihood estimator procedure, which was characterized by a two-step technique, namely, separately estimating the parameters of the marginal distributions and the parameters of the copula. This study introduced a normalized multivariate interaction information measure based on differential entropy to compare newly derived relationships between state variables. Theoretical findings are illustrated using real dataset.

Acknowledgments

The authors would like to express their appreciation for the support from the Lithuanian Association of Impartial Timber Scalers.

Cohomology with Coefficients in Stacks

Teimuraz Pirashvili

Institute of Mathematics of the University of Georgia, Tbilisi, Georgia
E-mail: prtmrz@gmail.com

This talk will be based on my joint work with Mamuka Jiblazde. We will explain 2-dimensional analogue of homological algebra and its applications. In particular we will discuss on cohomology with coefficients in stacks. Our stacks takes values in the 2-category of symmetric monoidal (but not strictly monoidal) groupoids.

The Aims of Constructing Technological Alphabets of Georgian and Abkhazian Languages and the Action Plan to Establish Studying Program "Digital Humanities and Computational Linguistics" at the Georgian Technical University

Konstantine Pkhakadze

Center for Cultural Protection and Technological Development of Georgian State Languages at Georgian Technical University, Tbilisi, Georgia
E-mail: gllc.ge@gmail.com

At the presentation we will shortly overview the aims of construction Georgian and Abkhazian technological Alphabets and, after, we will clearly prove the direct connection of these aims with the aims of establishing studying program "Digital Humanities and Computational Linguistics" at the Georgian Technical University. Also, at the presentation we will prove that the establishment of this studying program at the Georgian Technical University are directly connected:

1. With the aims of the protection the Georgian and Abkhazian Languages and Identities;
2. With the aims to enter United Georgia in the European Union with the Georgian and Abkhazian languages and traditions;
3. With the aims of the United Program (Strategy) of the Georgian State Languages, which was finally proved in 27 December, 2021 by Prime Minister of Georgia Irakli Garibashvili.

References

[1] K. Pkhakadze, The technological alphabet of the Georgian language – one of the main Georgian challenges of the XXI century. *Proceedings of the Parliamentary Conference "Georgian Language – Challenges of the 21st Century"*, 2013, 98–105.

[2] K. Pkhakadze, G. Chichua, M. Chikvinidze, D. Kurtskhalia and Sh. Malidze, Open Letter to Georgian Parliament, Government, National Academy of Sciences and Georgian and Abkhazian Society i.e. Key Principles of the Unified program of Complete technological supporting of the official language of Georgia i.e. In the Future Cultural World with the Completely Supported Georgian and Abkhazian Languages. *Journal "Georgian Language and Logic"*, 2017-2018, no. 11, 121–164.

[3] K. Pkhakadze, M. Chikvinidze, G. Chichua, Sh. Malidze, D. Kurtskhalia, C. Demurchev and N. Okroshiashvili, In the European Union with Georgian and Abkhazian Languages – Aims and Problems of Complete Technology Support of Georgian and Abkhazian Languages. *Bull. Georgian Natl. Acad. Sci.* **14** (2020), no. 3, 36–42.

[4] K. Pkhakadze, M. Chikvinidze, G. Chichua, Sh. Malidze, D. Kurtskhalia, C. Demurchev, N. Okroshiashvili and B. Mikaberidze, In the European Union with The Georgian and Abkhazian Languages – Aims, Problems, Results, and Recommendations of the Complete Technological Support of the Georgian and Abkhazian Languages. *AMIM* **25** (2020), no. 2, 147–173.

Convergence and Numerical Experiments of a Three-layer Semi-Discretization Approach for the Nonlinear Kirchhoff-Type Dynamic String Equation with Time-Varying Coefficients

Jemal Rogava[1,2], **Zurab Vashakidze**[3,4]

[1]*Department of Partial Differential Equations, Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

[2]*Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: jemal.rogava@tsu.ge

[3]*Institute of Mathematics, The University of Georgia (UG), Tbilisi, Georgia*

[4]*Department of Numerical Mathematics and Modelling, Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

E-mails: z.vashakidze@ug.edu.ge, zurab.vashakidze@gmail.com

In this talk, we shall delve into an initial-boundary value problem associated with the Kirchhoff-type nonlinear dynamic string equation. This equation features coefficients that change over time and has been discussed in detail in the paper [1]. Our main goal is to develop a method for discretizing time that can effectively estimate the solution to the initial-boundary value problem. To achieve this objective, we apply a symmetrical three-layer semi-discrete approach that focuses on the temporal variable. Within this method, the nonlinear term is assessed at the midpoint node. By using this technique, we can calculate numerical solutions at each step of time by inverting linear operators. As a result, we end up with a set of second-order linear ordinary differential equations. We have proved that this proposed approach locally converges and demonstrates a quadratic convergence pattern in relation to the time step size through the local time interval. Lastly, we performed several numerical experiments using the proposed algorithm to tackle various test issues. The numerical outcomes we obtained align well with the theoretical conclusions.

Acknowledgments

The second author of this work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant # FR-21-301, project title: *"Metamaterials with Cracks and Wave Diffraction Problems"*].

References

[1] J. Rogava and Z. Vashakidze. On Convergence of a Three-layer Semi-discrete Scheme for the Nonlinear Dynamic String Equation of Kirchhoff-type with Time-dependent Coefficients. *preprint* arXiv:2303.10350, 2023. https://doi.org/10.48550/arXiv.2303.10350.

Taking Over Maritime Ecosystems: Modelling Fish-Jellyfish Deterministic & Randomized Dynamics

Florian Rupp

Kutaisi International University, Kutaisi, Georgia
E-mail: florian.rupp@kiu.edu.ge

We present the results of a complete phase space analysis for a deterministic two-dimensional predator-prey model representing the key dynamics of fish-jellyfish interactions. Progressing through a series of bifurcations the biological phenomenon of jellyfish blooming is illustrated in this model and thus the taking over of the maritime ecosystem by jellyfish. We then turn to the question of how stochasticity in parameters and equations effects the emerge of blooming. Therefore, we first randomize the essential bifurcation parameter to model/ simulate and discuss stochastic environmental impact and, second, use first principle Markovian birth/ death processes to set-up a system of stochastic differential equations to model/ simulate and discuss the effects of intrinsic noise. In both cases, knowledge about the dynamics of the underlying deterministic system is indispensable and, in particular, in the intrinsic noise case a sooner occurrence of blooming can be observed.

References

[1] F. Rupp, Taking over maritime ecosystems: modelling fish-jellyfish deterministic and randomized dynamics. *Adv. Math. Sci. Appl.* **30** (2021), no. 2, 281–304.

[2] F. Rupp and J. Scheurle, The dynamics of the jellyfish joyride: mathematical discussion of the causes leading to blooming. *Math. Methods Appl. Sci.* **38** (2015), no. 16, 3408–3420.

[3] F. Rupp and J. Scheurle, Analysis of a mathematical model for jellyfish blooms and the Cambric fish invasion. *Discrete Contin. Dyn. Syst.* **2013**, Dynamical systems, differential equations and applications. 9th AIMS Conference. Suppl., 663–672.

On the Theory of Binary Lie Algebras

Liudmila Sabinina

Research Center of Science, Autonomous University of the State of Morelos
Cuernavaca, Mexico
E-mail: liudmila@uaem.mx

In 1955 A. I. Malcev found the defining identities of binary Lie algebras as a the tangent algebras of local analytic diassociative loops, i.e. loops such that every two elements generate subgroups. We will discuss the properties of binary Lie algebras and will present some recent results on BL algebras with identities. Our talk is based mostly on joint work with M. Rasskazova and A. Grishkov.

A Delayed Model for Lysogenic and Lytic Cycle of Bacteria-Bacteriophage Interaction

Saroj Kumar Sahani

Department of Mathematics, South Asian University, New Delhi, India
E-mail: sarojkumar@sau.ac.in

In this article, we introduce a delayed model of Bacteria-Bacteriophage interaction incorporating the two life cycles followed during the lysis of bacteria. A Bacteriophage is a virus that infects bacteria and often follows two life cycles. The lytic path cycle is very common, and the lysogenic cycle is the other less common. This infection process always has a time lag. The latency period is the most widely used in the mathematical model of the bacteria-bacteriophage model. Another time lag is involved in the lysogenic phase before bacteria goes into the lytic phase and then to lysis. The underlying mathematical model is analysed by local stability analysis to study the local behaviour of the solution. A Hopf bifurcation is studied for the existence of possible periodic oscillation. The numerical analysis has been performed to check the validity of conditions and predict the model equation's long-term dynamics.

Acknowledgments

The work is supported by South Asian University.

References

[1] E. Beretta and Y. Kuang, Modeling and analysis of a marine bacteriophage infection. *Math. Biosciences* **149** (1998), no. 1, 57–76.

[2] S. Gakkhar and S. K. Sahani, A time delay model for bacteria bacteriophage interaction. *J. Biol. Systems* **16** (2008), no. 03, 445–461.

[3] B. R. Levin and J. J. Bull, Population and evolutionary dynamics of phage therapy. *Nature Reviews Microbiology* **2** (2004), 166–173.

[4] B. R. Levin, F. M. Stewart and L. Chao, Resource-limited growth, competition, and predation: a model and experimental studies with bacteria and bacteriophage. *The American Naturalist* **111** (1977), no. 977, 3–24.

[5] A. Rabinovitch, I. Aviram and A. Zaritsky, Bacterial debris-an ecological mechanism for coexistence of bacteria and their viruses. *J. Theor. Biol.* **224** (2003), no. 3, 377–383.

[6] S. K. Sahani and S. Gakkhar, A mathematical model for phage therapy with impulsive phage dose: model for phage therapy. *Differ. Equ. Dyn. Syst.* **28** (2020), no. 1, 75–86.

The Sufficient Conditions for Insolvability of Some Diophantine Equations of Higher Degrees

Eteri Samsonadze

Department of General Mathematics, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: eteri.samsonadze@outlook.com

This work generalizes our results presented in the previous, XII International Conference of the Georgian Mathematical Union, and deals with the Diophantine equations

$$\sum_{i=1}^{m} x_i^n = b \ \text{ and } \ \sum_{i=1}^{m} x_i^n = bc^n$$

with $n \geq 2$, nonnegative integers $x_1, x_2, \dots, x_m, b$, and natural c, and also with the equation

$$\sum_{i=1}^{m} x_i^n = z^n$$

with $n, m \geq 2$ and $x_1, x_2, \dots, x_m, z \in \mathbb{N}$. In particular, the sufficient conditions for insolvability of the equations

$$\sum_{i=1}^{m} x_i^n = b \ \text{ and } \ \sum_{i=1}^{m} x_i^n = b(p_1^{s_1} p_2^{s_2} \cdots p_l^{s_l})^n,$$

are given provided that p_i are prime numbers, $s_i \in \mathbb{N}$, and there are numbers k_i's with $\varphi(p_i^{k_i}) \mid n$ and $p_i^{k_i} \geq 3$ $(i = 1, 2, \dots, l)$; here φ is the Euler's totient function. Moreover, it is proved that the latter equation has no solutions with natural x_i's if $0 \leq b < m \leq p_i^{k_i} - 1$ $(i = 1, 2, \dots, l)$. Besides, it is proved that the equations

$$x_1^n + x_2^n = (p^s)^n \ \text{ and } \ x_1^n + x_2^n = (p_1^{s_1} p_2^{s_2} \cdots p_l^{s_l} p^s)^n$$

have no solutions with natural x_1 and x_2 if $n \geq 3$, p is any prime, $s_i, s \in \mathbb{N}$, and there are natural numbers k_i's with $\varphi(p_i^{k_i}) \mid n$ and $p_i^{k_i} \geq 3$ $(i = 1, 2, \dots, l)$.

It is also proved that if the equality

$$x_1^n + x_2^n + \cdots + x_m^n = z^n$$

holds for some $x_1, x_2, \dots, x_m, z \in \mathbb{N}$, where m is an even natural number and there are natural numbers p and k such that p is odd and prime, $\frac{\varphi(p^k)}{2} \mid n$ and $m \leq p^k - 3$, then at least one of the numbers $x_1, x_2, \dots, x_m, z$ is divisible by p. But if $m \in \mathbb{N}$, $x_1, x_2, \dots, x_m$, z are coprime numbers, and there is a natural k with $\varphi(p^k) \mid n$ and $2 \leq m \leq p^k - 1$, then $p \nmid z$ and precisely one of the numbers $x_1, x_2, \dots, x_m$ is not divisible by p. Applying this fact, the sufficient conditions for some x_i's in a solution (in the case of its existence) of the following equations to be divisible by resp. 2, 3, 5 are obtained:

$$\sum_{i=1}^{m} x_i^{2^k n} = z^{2^k n}, \quad \sum_{i=1}^{m} x_i^{3^k n} = z^{3^k n}, \quad \sum_{i=1}^{m} x_i^{5^k n} = z^{5^k n}.$$

These sufficient conditions, in particular, imply that if $x_1^{2n} + x_2^{2n} = z^{2n}$ $(n \in \mathbb{N})$, where x_1, x_2, z are coprime numbers, then $z \equiv \pm 1 \mod 6$ and $30 \mid x_1 x_2 z$.

All the results of this work are obtained employing only the elementary methods.

On the Joint Course of Linear Algebra and Analytic Geometry

Guram Samsonadze

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: g.samsonadze@gtu.ge

The course of linear algebra and analytic geometry is being taught in many universities. The course is often structured in a manner such that students perceive it as the mechanical union of two different subjects since the interrelation between these subjects is not clearly exposed, and the geometric aspects of linear algebra are less considered. In our opinion, the syllabus of the course has to contain topics emphasizing the unity of all the course material.

As is well-known, the axioms of a linear space reflect the properties of geometric vectors, while the axioms of an affine space reflect the properties of the interrelation between points and vectors in elementary geometry. This interrelation is based on the fact that one can introduce the operation of the addition of a geometric point and a vector: the sum of a point A and a vector $\vec{u}$ is defined as the terminal point B of the vector $\vec{AB} = \vec{u}$.

A non-empty set S is called an affine space associated with a vector space E if one has an addition operation $S \times E \to S$ that satisfies the following conditions:

(i) for any $a \in S$ and any $x, y \in E$, one has $a + (x + y) = (a + x) + y$;

(ii) for any $a \in S$, one has $a + 0 = a$;

(iii) for any $a, b \in S$, there exists a unique $x \in E$ such that $a + x = b$.

The elements of the set S are called points; the vector x from the condition (iii) is denoted by the symbol $\vec{ab}$. Note that the condition (i) implies that $\vec{ab} + \vec{bc} = \vec{ac}$ for any $a, b, c \in S$.

Any vector space E can be viewed as an affine space associated with itself: the points in it are precisely the vectors of E, while the sum of a point and a vector is their sum in E. Then the vector $\vec{ab}$ is equal to the difference of the vectors b and a.

If one fixes a point o in an affine space S, then any of its points can be identified with the radius vector $\vec{oa}$ of this point. In that case, the operation of the addition of a point and a vector is the usual addition operation of vectors in the vector space. Such an identification of points with vectors is called the vectorization of an affine space.

The notion of an affine space makes it possible to generalize many concepts of analytic geometry. Incorporating such topics in the joint course of linear algebra and analytic geometry, we will help students to perceive this subject as a single whole and will make it more interesting to them.

Isochrones Method and Feynman's Lifeguard Problem

Natia Sazandrishvili

Ilia State University, Tbilisi, Georgia
E-mail: natia.sazandrishvili.1@iliauni.edu.ge

We give an analytic solution to a generalization of Feynman's lifeguard problem formulated as a cooperative game in which the first object A (lifeguard) is able to move in two adjacent media G, S with maximal speeds $w > 0$ and $v < w$, respectively, and the second object B (swimmer) is able to move in S in a fixed direction with constant speed $u < v$.

Our solution is based on the concept of isochrone $I_A(T)$ defined as the set of points which can be reached by object A at the moment $T > 0$ but cannot be reached at any earlier moment of time. More precisely, we determine the isochrone's exact shape of lifeguard A in the case where the first medium G (ground) is the closed lower half-plane and the second S (sea) is the open upper half-plane. Namely, we give explicit parametric equations of lifeguard's isochrone $I_A(T)$ using its representation as the envelope of a system of circles provided by Huygens principle, and prove that, for any $T > 0$, it is a convex piecewise differentiable curve. This enables us to obtain an analytic formula for the minimal rescue time in the case where B moves in a given direction with the constant speed u, and describe the exact shape of the optimal trajectory of lifeguard A. Furthermore, minimizing the minimal rescue time over the unit circle of all possible directions of B we find the optimal collective strategy of both actors A and B, which yields an analytic solution of the problem considered.

We will also formulate several further generalizations of Feynman's lifeguard problem which admit explicit analytic solution using the representation of isochrones as envelopes.

Acknowledgement

This research was supported by the grant PHDF-22-3325 of Shota Rustaveli National Science Foundation of Georgia (SRNSFG).

On Mathematical Aspects of the Theory of Topological Insulators

Armen Sergeev

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia
E-mail: sergeev@mi-ras.ru

The role of topology in the theory of condensed matter first became clear in the study of the quantum Hall effect starting from the papers by Loughlin and Thouless et al. From the physical point of view the topological invariance is equivalent to the adiabatic stability.

A key role in the classification of topological objects in the theory of solid states is played by the study of their symmetry groups. The description of possible symmetry types goes back to Kitaev who proposed a classification of topological insulators based on the investigation of their symmetry groups and their representations.

In our talk we pay main attention to the topological insulators invariant under time reversion. Such systems are characterized by having the wide energy gap stable under small deformations. An example of these systems is provided by the quantum spin Hall insulator which has a non-trivial topological $\mathbb{Z}_2$-invariant introduced by Kane and Mele.

Designing of Orthopedic Insoles for Children with Cerebral Palsy

Merab Shalamberidze[1], **Zaza Sokhadze**[2]

[1] *Department of Design and Technology, Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: merab.sh@hotmail.com

[2] *Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: z.soxadze@gmail.com

Cerebral palsy develops as a result of irreversible damage to the brain and can happen before birth, during birth or within the first years of child's life. It should be noted that in the process of treating cerebral palsy, orthopedic means have a high degree of recommendation.

An important novelty of the research consists in the fact that the description of the geometric shapes of the locally over-pressured areas of patients' pedograms was described by means of integral curves of the solutions to Dirichlet singular boundary differential equations. Application of the mentioned method makes it possible to describe the geometric shapes of the curves of the locally over-pressured areas of individual orthopedic insoles with great accuracy.

Below is the Dirichlet singular boundary differential equation:

$$u''(t)+\frac{a}{t}\,u'(t)-\frac{a}{t^2}\,u(t)=f(t,u(t),u'(t)), \tag{1}$$

$$u(t)=0, \quad u'(t)=0, \tag{2}$$

where $a\in(-\infty;1)$, f satisfies the local Carathéodor condition for a set, $[0,t]\times D$, $D=(0;+\infty)\times R$.

The solution of the problem $(1),(2)$ is presented in the form of equations (3) and (4):

$$u(t)=\frac{t^3}{2}-\frac{1}{3}\,ct^{-2}-\Big(1-\frac{1}{3}\,c\Big)t+\frac{1}{2}\,, \tag{3}$$

$$u(t)=\Big(-\frac{1}{3}-\frac{1}{3}\,c\Big)t+\frac{1}{3}\,c\,\frac{1}{t^2}+\frac{2}{3}\,t-\frac{t^2}{2}+\frac{t^4}{6}\,. \tag{4}$$

By means of the integral curves of the solutions of equations (3) and (4), a description of the geometric shapes of the curves of the locally over-pressured areas on the pedograms was made. Based on the mathematical algorithm, the software package was developed to describe the above curves. Individual orthopedic insoles were manufactured on a CNC-controlled milling machine taking into account the locally over-pressured areas. It provides improved quality of patient care and prevention of foot injuries.

Research in this direction is of topical importance, especially when it concerns children with cerebral palsy.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) (Grant # FR-22-1515).

Hadron Center of the Kutaisi International University

Revaz Shanidze[1,2]

[1]*Kutaisi International University, Kutaisi, Georgia*
[2]*Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: revaz.shanidze@kiu.edu.ge

Hadron Center of the Kutaisi International University in Georgia is currently under construction. The center will be equipped with 2 superconducting synchrocyclotrons IBA S2C2, providing proton beams, with maximum kinetic energy of 230 MeV. One of these accelerators is a part of IBA single gantry Proteus©ONE system for proton therapy, while the other one is the main device for the new research infrastructure at Kutaisi International University. The Hadron Center is funded by the International Charity Foundation Cartu, the largest charity foundation in Georgia. The opening of the center is planned for the end of 2025. Research with the proton beams in the 70–230 MeV energy range is foreseen in multiple disciplines, including basic and applied nuclear physics, radiation biology, medical physics, and material science. As the only cyclotron-based research center and cancer treatment facility in Georgia and South Caucasus, the development of the Hadron Center into an international hub for research and cancer treatment with the protons beams is foreseen.

The Addition Theorem for Two-Step Nilpotent Torsion Groups

Menachem Shlossberg

School of Computer Science, Reichman University, Herzliya, Israel
E-mail: menachem.shlossberg@post.runi.ac.il

The Addition Theorem for the algebraic entropy of group endomorphisms of torsion abelian groups was proved in [2]. Later, this result was extended to all abelian groups [1] and, recently, to all torsion finitely quasihamiltonian groups [3]. In contrast, when it comes to metabelian groups, the additivity of the algebraic entropy fails [4]. Continuing the research within the class of locally finite groups, we prove that the Addition Theorem holds for two-step nilpotent torsion groups.

References

[1] D. Dikranjan and A. Giordano Bruno, Entropy on abelian groups. *Adv. Math.* **298** (2016), 612–653.

[2] D. Dikranjan, B. Goldsmith, L. Salce and P. Zanardo, Algebraic entropy for abelian groups. *Trans. Amer. Math. Soc.* **361** (2009), no. 7, 3401–3434.

[3] A. Giordano Bruno and F. Salizzoni, Additivity of the algebraic entropy for locally finite groups with permutable finite subgroups. *J. Group Theory* **23** (2020), no. 5, 831–846.

[4] A. Giordano Bruno and P. Spiga, Some properties of the growth and of the algebraic entropy of group endomorphisms. *J. Group Theory* **20** (2017), no. 4, 763–774.

On the Teaching of Taylor's Formula in Higher Education

Levan Sulakvelidze

School of Science and Technology, University of Georgia, Tbilisi, Georgia
E-mail: Levan.sulakvelidze@gmail.com

The talk will present some ideas about teaching Taylor's formula in calculus. The interest in this issue is due to the fact that it allows to understand some issues of both elementary and higher mathematics with a unified approach.

References

[1] W. Briggs, L. Cochran and B. Gillett, *Calculus for Scientists and Engineers: Early Transcendentals*. Pearson Education, Inc. Boston, 2013.

On Shangua's SLLN

Vaja Tarieladze

Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University
Tbilisi, Georgia
E-mail: v.tarieladze@gtu.ge

Dedicated to Alexandre Shangua (1953-2014)

We will discuss A. Shangua's results related to Kolmogorov's and Prokhorov's strong laws of large numbers. His contribution to [1] will be covered as well.

References

[1] Z. Ergemlidze, A. Shangua and V. Tarieladze, Sample behavior and laws of large numbers for Gaussian random elements. *Georgian Math. J.* **10** (2003), no. 4, 637–676.

Representation of the Sides of a Right Triangle in Terms of the Radius of the Circle Inscribed in the Triangle

Lasha Tavartkiladze

Batumi Shota Rustaveli State University, Batumi, Georgia
E-mail: tavartkiladzelasha35@gmail.com

In the report we will describe our formulas for representing the hypotenuse c and legs a and b of the right triangle according to the radius r of the circle inscribed in the triangle:

$$c=\frac{2r^2}{K}+2r+K, \quad b=\frac{2r^2}{K}+2r, \quad a=2r+K,$$

where K is any positive number. The issue of obtaining Pythagorean triples from these formulas will be discussed too.

On Segmental Variation of Blaschke–Djerbashyan Canonical Product

Giorgi Tetvadze, Lili Tetvadze, Lamara Tsibadze, Iuri Tvalodze

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: giorgi.tetvadze@atsu.edu.ge

Let f be a complex-valued function on the unit disk $D = \{z \in \mathbb{C} : \ |z| < 1\}$ and $\theta \in [0, 2\pi)$. The function f is said to have *finite segmental variation at a point* $e^{i\theta}$, if for every point a from D, the line segment joining a and $e^{i\theta}$ is mapped by f into a rectifiable curve.

Segmental variations for Blaschke products was studied by Cargo [1]. We study the similar topic for more general Blaschke–Djerbashyan type products. Namely, we prove the following

Theorem *Let $\theta \in [0, 2\pi)$, $p \in \mathbb{N}$ and let (a_n) be a sequence from the unit disk D such that $0 < |a_n| \le |a_{n+1}| < 1$ $(n \in \mathbb{N})$,*

$$\lim_{n\to\infty} |a_n| = 1, \quad \sum_{n=1}^{\infty} (1 - |a_n|)^p = \infty, \quad \sum_{n=1}^{\infty} (1 - |a_n|)^{p+1} < \infty$$

and

$$\sum_{n=1}^{\infty} \Big(\frac{1 - |a_n|}{|e^{i\theta} - a_n|} \Big)^{p+1} < \infty.$$

Then Blaschke–Djerbashyan canonical product

$$B_{p+1}(z, (a_n)) = \prod_{n=1}^{\infty} \Big(1 - \frac{1 - |a_n|^2}{1 - \overline{a_n} z} \Big) \exp\Big(\sum_{k=1}^{p} \frac{1}{k} \Big(\frac{1 - |a_n|^2}{1 - \overline{a_n} z} \Big)^k \Big)$$

has finite segmental variation at the point $e^{i\theta}$.

References

[1] G. T. Cargo, The segmental variation of Blaschke products. *Duke Math. J.* **30** (1963), 143–149.

Sequent Calculus for Unranked Probabilistic Logic

Lali Tibua[1,2]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *Georgian National University (SEU), Tbilisi, Georgia*

E-mail: ltibua@gmail.com

Since the early days of Artificial Intelligence logical and probabilistic methods have been independently used in order to solve tasks that require some sorts of intelligence. Probability theory deals with the challenges posed by uncertainty, while logic is more often used for reasoning with perfect knowledge. Considerable efforts have been devoted to combining logical and probabilistic methods in a single framework, which influenced the development of several formalisms and programming tools. All probabilistic logic formalisms studied so far permit only individual variables, that can be instantiated by a single term. On the other hand, theories and systems that use also sequence variables (these variables can be replaced by arbitrary finite, possibly empty, 12 sequences of terms) and unranked symbols (function and/or predicate symbols without fixed arity) have emerged. The unranked term is a first-order term, where the same function symbol can occur in different places with different number of arguments. Unranked function symbols and sequence variables bring a great deal of expressiveness in language. Therefore, it is actual to study extension of probabilistic logic with sequence variables and flexible-arity function and predicate symbols.

In this talk we discuss sequent calculus for unranked probabilistic logic. We show that the calculus is sound and complete.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia under the project # FR-22-4254.

Some Properties of the Sequence of Linear Functionals on the Space V

Vakhtang Tsagareishvili[1], **Giorgi Tutberidze**[2]

[1]*Department of Mathematics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: cagare@ymail.com

[2]*Department of Mathematics, The University of Georgia, Tbilisi, Georgia*
E-mail: g.tutberidze@ug.edu.ge

The talk is devoted to investigating the sequence of linear functions in the space of finite variation functions. We prove that under certain conditions, this sequence is bounded. We also show that these results are sharp. In particular, the obtained results can be used to study the issues of convergence in the general Fourier series. Moreover, the obtained conditions are effective for bounded orthonormal systems.

References

[1] S. Banach, *Théeorie des Opérations Linéaires*. Matematyczne, Warsaw, 1932.

[2] L. Gogoladze and G. Tsagareishvili, Optimal convergence factors for general Fourier coefficients. *Georgian Math. J.* **30** (2023), no. 4, 515–521.

[3] V. Tsagareishvili, Some particular properties of general orthonormal systems. *Period. Math. Hungar.* **81** (2020), no. 1, 149–157.

[4] V. Tsagaraeishvili and G. Tutberidze, Some problems of convergence of general fourier series. *J. Contemp. Math. Anal., Armen. Acad. Sci.* **57** (2022), 369–379.

About Solution of a Nonlinear sIntegro-Differential Timoshenko Dynamic Beam Equation

Zviad Tsiklauri

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: zviad_tsiklauri@yahoo.com

The equation which describes the oscillation of a beam by the Timoshenko theory, is considered in [1–3]. In the present paper is introduced an approximate algorithm for the problem, and study the accuracy of its iterative part.

References

[1] E. Henriques de Brito, A nonlinear hyperbolic equation. *Internat. J. Math. Sci.* **3** (1980), no. 3, 505–520.

[2] G. P. Menzala and E. Zuazua, Timoshenko's beam equation as a limit of a nonlinear one-dimensional von Kármán system. *Proc. Roy. Soc. Edinburgh Sect. A.* **130** (2000), no. 4, 855–875.

[3] M. Tucsnak, Semi-internal stabilization for a non-linear Bernoulli–Euler equation. *Math. Methods Appl. Sci.* **19** (1996), no. 11, 897–907.

A non-abelian group of congruent numbers

Ruslan Tsinaridze

Department of Mathematics, Batumi Shota Rustaveli State University, Batumi, Georgia
E-mail: r.tsinaridze@bsu.edu.ge

One of the oldest problems is the Congruent Numbers Problem. An n-natural number is called a congruent number if there is a right triangle with rational sides whose area is equal to n. This problem is related to elliptical curve. The following theorem is true: an n-natural number is congruent if and only if the elliptic curve $y^2 = x^3 - n^2x$ has a non-trivial rational solutions. The following results are obtained: an algebraic operation is defined on sets of all congruent numbers; It is shown that this operation define a non-abelian group; The connection between this group and the 3-dimensional Special Linear group $Sl(3; Z)$ is established; In particular, it is shown that the non-abelian group of all congruent numbers is a subgroup of the 3-dimensional Special linear group.

References

[1] J. Coates and A. Wiles, On the conjecture of Birch and Swinnerton–Dyer. *Invent. Math.* **39** (1977), no. 3, 223–251.

[4] K. Conrad, The congruent number problem. *The Harvard College, Mathematics Review* **2** (2008), no. 2, 58–74;
https://legacy-www.math.harvard.edu/ knill/various/eulercuboid/HCMR_Fall2008.pdf#page=60.

[2] J. B. Tunnell, A classical Diophantine problem and modular forms of weight 3/2. *Invent. Math.* **72** (1983), no. 2, 323–334.

Developed MHD Flows in Channels at Existence of Pointed Geometry External Magnetic Field

Mariam Tsutskiridze[1,2], **Varden Tsutskiridze**[3]

[1]*MS Student, Caucasus University, Tbilisi, Georgia*
[2]*MS Student of Grenoble University, Grenoble, France*
E-mail: Tsutskiridzegrenobleem@gmail.com

[3]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mail: btsutskirid@yahoo.com

Are considered the fully developed flows of a viscous incompressible isotropic-conducting fluid in a channel of rectangular cross section in the presence of a transverse magnetic field

$$\vec{B} = \frac{B_0}{a}\,(-\vec{e}_y y + \vec{e}_z z).$$

Is shown that, at high Hartmann numbers, a zone of increased velocities can form in the vicinity of the channel axis. The flow in a plane of slot has a paradoxical property in this connection: the flow rate increases with an increase in the Hartmann number. The reason for this lies in the fact that the limiting transition "takes" to infinity a region with an infinitely large EMF, and the region where the flow occurs in pump mode is considered. In conclusion, some other flows in inhomogeneous fields of pointed geometry are considered.

Small Models of a Jonsson Spectrum

Indira Tungushbayeva, Aibat Yeshkeyev

The Faculty of Mathematics and Information Technologies, Karaganda Buketov University Qaragandy, Kazakhstan

E-mails: intng@mail.ru; aibat.kz@gmail.com

Let L be a first-order language, T be a Jonsson theory (see [1, 2]) in L, $K \subseteq E_T$. Let us consider a Jonsson spectrum $JSp(K)$ [3] that is defined as follows: $JSp(K) = \{T \mid \; T$ is a Jonsson theory and for all $A \in K \;\; A \models T\}$.

We introduce a cosemanticness ($\bowtie$) relation [2] on $JSp(K)$. Cosemanticness is an equivalence relation, so we obtain the factor set $JSp(K)_{/\bowtie}$. $[T']$ is a cosemanticness class of a theory T', when $T' \in JSp(K)$.

Definition 1 A is called (Γ_1, Γ_2)-atomic model of a theory T, if A is a model of T and, for any n, each n-tuple from A satisfies some formula from Γ_1 that is complete for Γ_2-formulas.

Definition 2

1. A is called Σ-nice-algebraically prime model of a theory T, if A is a countable model of T and, for any model B of T, for any $n \in \omega$, and for any $a_0, \dots, a_{n-1} \in A$, $b_0, \dots, b_{n-1} \in B$, if $(A, a_0, \dots, a_{n-1}) \Longrightarrow_\exists (B, b_0, \dots, b_{n-1})$, then, for any $a_n \in A$, there exists some $b_n \in B$ such that $(A, a_0, \dots, a_n) \Longrightarrow_\exists (B, b_0, \cdots_n)$.
2. A is called Σ^*-nice-algebraically prime model of a theory T, if A is a countable model of T and, for any model B of T, for any $n \in \omega$, and for all $a_0, \dots, a_{n-1} \in A$, $b_0, \dots, b_{n-1} \in B$, if $(A, a_0, \dots, a_{n-1}) \equiv_\exists (B, b_0, \dots, b_{n-1})$, then, for any $a_n \in A$, there exists some $b_n \in B$ such that $(A, a_0, \dots, a_n) \equiv_\exists (B, b_0, \cdots_n)$.

Definition 3 A cosemanticness class $[\Delta] \in JSp(K)_{/\bowtie}$ is called κ-categorical, if, for any $\Delta \in [\Delta]$, Δ is κ-categorical.

Theorem 1 *Let $[\Delta] \in JSp(K)_{/\bowtie}$, $[\Delta]$ be a class complete for $\exists$-sentences, A be a countable model from K. Then (1) $\Longrightarrow$ (2) and (2) $\Longleftrightarrow$ (3), where*

1. *A is (Σ, Σ)-atomic;*
2. *A is Σ^*-nice-algebraically prime;*
3. *A is existentially closed and Σ-nice-algebraically prime.*

Theorem 2 *Let $[\Delta] \in JSp(K)_{/\bowtie}$, $[\Delta]$ be a class complete for $\exists$-sentences. Then the following conditions are equivalent:*

1. *$[\Delta]^*$ is ω-categorical, where $[\Delta]^*$ is the center (see [2]) of the class $[\Delta]$;*
2. *$[\Delta]$ is ω-categorical.*

Acknowledgments

This work is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant # AP09260237).

References

[1] K. J. Barwise, *Handbook of Mathematical Logic.* (Russian) Nauka, Moscow, 1982.

[2] A. R. Yeshkeyev and M. T. Kassymetova, *Jonsson Theories and Their Classes of Models.* (Russian) KarGU, Qaragandy, 2016.

Recent Results on Approximation by Fractional Integral type Sampling Series

Metin Turgay

Department of Mathematics, Selçuk University, Konya, Türkiye
E-mail: metinturgay@yahoo.com

In this work, we present sampling type series based on fractional integral and it is dedicated to examining the approximation properties of such a series. The second part of this work explores the weighted approximation of the family of operators. Finally, we provide concrete examples of kernels that satisfy the obtained results, accompanied by numerical tables and graphical representations.

References

[1] T. Acar, O. Alagöz, A. Aral, D. Costarelli, M. Turgay and G. Vinti, Convergence of generalized sampling series in weighted spaces. *Demonstr. Math.* **55** (2022), no. 1, 153–162.

[2] T. Acar, O. Alagöz, A. Aral, D. Costarelli, M. Turgay and G. Vinti, Approximation by sampling Kantorovich series in weighted spaces of functions. *Turkish J. Math.* **46** (2022), no. 7, 2663–2676.

[3] C. Bardaro, G. Bevignani, I. Mantellini and M. Seracini, Bivariate generalized exponential sampling series and applications to seismic waves. *Constr. Math. Anal.* **2** (2019), no. 4, 153–167.

[4] P. L. Butzer and R. J. Nessel, *Fourier Analysis and Approximation.* Volume 1: *One-Dimensional Theory.* Pure and Applied Mathematics, Vol. 40. Academic Press, New York–London, 1971.

[5] A. D. Gadziev, The convergence problem for a sequence of positive linear operators on unbounded sets, and Theorems analogous to that of P. P. Korovkin. (Russian) *Dokl. Akad. Nauk SSSR* **218** (1974), no. 5, 1001–1004; translation in *Sov. Math., Dokl.* **15** (1974), 1433–1436.

[6] N. İspir, On modified Baskakov operators on weighted spaces. *Turkish J. Math.* **25** (2001), no. 3, 355–365.

[7] S. Kursun, A. Aral and T. Acar, Riemann–Liouville fractional integral type exponential sampling Kantorovich series. 2023 (*Submitted*)

[8] N. Mahmudov and M. Kara, Approximation properties of the Riemann–Liouville fractional integral type Szász–Mirakyan–Kantorovich operators, *J. Math. Inequalities* **16** (2022), no. 4, 1285–1308.

[9] M. Turgay, A. Aral and T. Acar, Approximation by Riemann–Liouville Fractional Integral Type Generalized Sampling Kantorovich Series. 2023 (*Submitted*)

Orbitization of Quantum Mechanics and Interpretation of its Notions

Duglas Ugulava, David Zarnadze

Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia

E-mail: duglasugu@yahoo.com; d.zarnadze@gtu.ge

In this report we review results about *orbitization* of quantum mechanics [2] and obtained result we call *orbital quantum mechanics*. For Hamiltonian $\mathcal{H}$ of the quantum harmonic oscillator, which corresponds to the observable *"energy"*, the notions of finite orbit $\operatorname{orb}_n(\mathcal{H},\psi)=(\psi,\mathcal{H}\psi,\dots,\mathcal{H}^n\psi)$, orbit $\operatorname{orb}_n(\mathcal{H},\psi)=(\psi,\mathcal{H}\psi,\mathcal{H}^2\psi,\dots,\mathcal{H}^n\psi,\dots)$, Hilbert spaces of finite orbits $D(\mathcal{H}^n)$, $n\in\mathbb{N}_0$, Frechet–Hilbert spaces of all orbits $D(\mathcal{H}^\infty)$, corresponding to $\mathcal{H}$ self-adjoint finite orbital operator $\mathcal{H}_n$, defined as $\mathcal{H}_n\operatorname{orb}_n(\mathcal{H},\psi)=\operatorname{orb}_n(\mathcal{H},\mathcal{H}\psi)$, $n\in\mathbb{N}_0$, self-adjoint orbital operator $\mathcal{H}^\infty:D(\mathcal{H}^\infty)\to D(\mathcal{H}^\infty)$ are studied (when $n=0$ a classical case is obtained [2]). As well as for approximate solution of this operator equation $\mathcal{H}_n\operatorname{orb}_n(\mathcal{H},u)=\operatorname{orb}_n(\mathcal{H},f)$ (resp. for the equation $\mathcal{H}^\infty\operatorname{orb}_n(\mathcal{H},u)=\operatorname{orb}_n(\mathcal{H},f)$ central spline algorithm in Hilbert space $D(\mathcal{H}^n)$ (resp. in the Frechet–Hilbert space $D(\mathcal{H}^\infty)$ is constructed [1].

The quantum mechanical interpretations of these notions of orbital quantum mechanics are discussed: The n-th orbit $\operatorname{orb}_n(\mathcal{H},\psi)=(\psi,\mathcal{H}\psi,\cdots,\mathcal{H}^n\psi)$ of $\mathcal{H}$ at ψ is $n+1$ dimensional vector, whose k-th coordinate reflects the movement of the particle in the potential field by k times ($k=0,1,\dots,n$) action of Hamiltonian $\mathcal{H}$ on state ψ. The n-orbital operator $\mathcal{H}_n$ acts on all coordinates of the orbit. The solution of operator equation $\mathcal{H}_n(\operatorname{orb}_n(\mathcal{H},u))=\operatorname{orb}_n(\mathcal{H},f)$ gives us possibility to find orbit corresponding to $\operatorname{orb}_n(\mathcal{H},f)$. The same is valid also for the operator $\mathcal{H}^\infty$.

References

[1] D. Ugulava and D. Zarnadze, On a central algorithm for calculation of the inverse of the harmonic oscillator in the spaces of orbits. *J. Complexity* **68** (2022), Paper no. 101599, 12 pp.

[2] D. Zarnadze, Orbitization of Quantum Mechanics. *GESJ – Computer Sciences and Telecommunications*, no. 1(61), 2022, 59–64.

Strongly Best Approximation and Moore–Penrose Generalized Solution

Duglas Ugulava, David Zarnadze

Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia

E-mails: duglasugu@yahoo.com; d.zarnadze@gtu.ge

In this article the continuous injective linear operator $A : E \to F$ of a Frechet–Hilbert space E into Frechet–Hilbert space F with closed range $A(E) = G$ is considered. Let the topology of the space E (respectively, F) is generated with the non-decreasing sequence of Hilbertian norms $\{|\cdot|_n\}$ (respectively, $\{\|\cdot\|_n\}$). The notion of strongly best approximation element [2] is used for definition of the the strongly best approximation solution of the equation $Au = f$ in Frechet space F with respect to the metric d [1]. Since the operator A has closed range, then it is monomorphism and by equality $A_n x = Ax$ we can defined injective operator $A_n : (E, |\cdot|_n) \to (F, ||\cdot||_n)$ from pre-Hilbert space $(E, |\cdot|_n)$ into pre-Hilbert space $(F, \|\cdot\|_n)$ for any $n \in N$. It is proved the following

Theorem *Let $G = A(E)$ is closed in (F, d), $f \in F \setminus G$ and*

$$d(f, G) = \inf \big\{ d(f, A(u)),\ u \in E \big\} = r \in I_n = [2^{-n+1}, 2^{-n+2}[,\ \ n \geq 2$$

(for $n = 1$ the theorem is proved similarly). In order for f to have a strongly best approximation element g_0 in G, it is necessary and sufficient that $f \in \operatorname{Im} A + (\operatorname{Im} A)_n^{\perp}$. In this case there exist the generalized solution $A_n^+ f \in E$ of the equation $A_n u = f$ in the sense of Moore–Penrose for this $n \in N$ and $g_0 = A(A_n^+ f)$.

References

[1] D. N. Zarnadze, On a generalization of the least squares method for operator equations in some Fréchet spaces. (Russian) *Izv. Ross. Akad. Nauk Ser. Mat.* **59** (1995), no. 5, 59–72; translation in *Izv. Math.* **59** (1995), no. 5, 935–948.

[2] D. N. Zarnadze and D. K. Ugulava, Best approximations of functions on open intervals. (Russian) *Trudy Inst. Vychisl. Mat. Akad. Nauk Gruzin. SSR* **27** (1987), no. 1, 59–71.

The Development of Complex Analysis Method for Essentially Non-Linear Systems of DE and About its Numerical Analogies

Tamaz Vashakmadze

Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: tamazvashakmadze@gmail.com

In this report, it is extended the classical theory (for the linear case developed by E. Goursat, H. Weyl, J. L. Walsh, S. Bergman, G. V. Koloson, N. Muskhelishvili, L. Bers, I. Vekua and so on) of finding general solutions and some boundary value problems of partial differential equations by applying complex analysis. We developed [1] the method of solving system of essentially non-linear DEs when with Laplace and biharmonic operators, DEs containing composition for example of Laplace and Monge–Ampére operator,

$$\Delta[u,v]=\Delta[\partial_{11}u,\partial_{22}v]-2\partial_{12}u\partial_{12}v+\partial_{11}v\partial_{22}u$$

or with more complete members. The method gives possibility to solve some boundary value problems as well. Then this method will be applied to the solution of boundary value problems corresponding to refined theories in enlarged sense for elastic plates and shells. In this direction, in the terminology of real analysis are constructed full numerical analogies to refined theories are proved the convergence of corresponding iteration processes.

References

[1] T. Vashakmadze, On the theory and practice of thin-walled structures. *Georgian Math. J.* **28** (2021), no. 3, 483–498.

Logic in School Mathematical Education

Teimuraz Vepkhvadze

Department of Mathematics, Ivane Javakhishvili Tbilis State University
Tbilisi, Georgia
E-mail: t-vepkhvadze@hotmail.com

Logic is one of the oldest intellectual disciplines on human history. It dates back to Aristotle. Its significance is a generally accepted fact. The paper discussed the importance of incorporating elements of logic into basic and secondary school curricula.

The Composition of Rough Singular Integral Operators on Function Spaces

Qingying Xue

School of Mathematical Sciences, Beijing Normal University, Beijing, P.R. China
E-mail: qyxue@bnu.edu.cn

The composition of singular integral operators arises typically in the algebra of singular integral (Calderón and Zygmund, 1956) and the non-coercive boundary-value problems for elliptic equations. Considerable attention has been paid to the composition of singular integral operators. In this talk, we will focus on the behavior of the bounds of the composition for rough singular integral operators on the weighted space,on rearrangement invariant Banach function spaces and quasi-Banach spaces.

This is a joint work with (Guoen Hu, Xudong Lai and Jiawei Tan).

The Method of Probabilistic Solution for the Dirichlet Generalized Harmonic Problem in Irregular Pyramidal Domains

Mamuli Zakradze[1], **Zaza Tabagari**[1], **Manana Mirianashvili**[1], **Nana Koblishvili**[1], **Tinatin Davitashvili**[2]

[1] *Department of Computational Methods, Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*

E-mails: m.zakradze@gtu.ge; z.tabagari@gtu.ge; m.mirianashvili@gtu.ge; n.koblishvili@gtu.ge

[2] *The Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: tinatin.davitashvili@tsu.ge

The Dirichlet generalized harmonic problem for irregular n-sided pyramidal domains is considered with a boundary function that has a finite number of first kind discontinuity curves. In the considered report the edges of the pyramid are in a role of the mentioned curves. The algorithm for numerical solution of the boundary problem is constructed and consists of the following main steps:

(a) Application of the Method of Probabilistic Solution (MPS), which in its turn is based on a computer modeling of the Wiener process;

(b) Finding the intersection point of the path Wiener process simulation and the pyramid surface;

(c) Development of a code for the numerical realization and checking the accuracy of calculated results;

(d) Calculating the meaning of searching function at any chosen point.

For the illustration two examples are considered. Numerical results are presented and discussed.

The Teaching-Learning Process Based on the Principles of Constructivism in Mathematics Class

Manana Zivzivadze-Nikoleishvili

Department of Teaching Methods, Akaki Tsereteli State University
Kutaisi, Georgia
E-mail: manana.zivzivadze@atsu.edu.ge

The main goal of the third-generation national curriculum is to conduct a student-centered learning process that is based on a constructivist educational concept focused on personality development and defines five basic educational principles:

(1) activation of students' inner strengths;

(2) gradually building knowledge based on previous knowledge;

(3) Interconnection and organization of knowledge;

(4) Learning to learn;

(5) All three categories of knowledge: declarative, procedural, and conditional.

In the paper, a student-oriented teaching-learning process management activity sample is proposed: topic-geometric figures, question-Euler's formula, where the teaching-learning process is built with a constructivist approach. The stages of conducting the activity are described. It is clearly visible the activation of the student's internal forces, what prior knowledge is the basis of knowledge construction, how knowledge is organized and interconnected, learning to learn, and systems of exercises in a context familiar to the student, so that all three categories of knowledge are visible: declarative, procedural, and conditional.

Index