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მათემატიკოსთა კავშირი
**Georgian
Mathematical Union**

აჭარის ავტონომიური რესპუბლიკის
განათლების, კულტურისა და
სპორტის სამინისტრო
**Ministry of Education,
Culture and Sport of Adjara
Autonomous Republic**

ბათუმის შოთა რუსთაველის
სახელმწიფო უნივერსიტეტი
**Batumi Shota Rustaveli
State University**

საქართველოს მათემატიკოსთა კავშირის XIV საერთაშორისო კონფერენცია

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BOOK OF ABSTRACTS

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რედაქტორები: გურამ გოგიშვილი, მაია ჯაფოშვილი

Editors: Guram Gogishvili, Maia Japoshvili

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Abstracts of Plenary Talks

The Idea of Complex Conjugacy (an Analogue of the Idea of Riemann Surfaces in Differential Geometry and Real Analysis)

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In short, the idea is to establish relationships between the surfaces M in R^3 and some complex functions $w(z)$ assoshiated with M; accordingly the idea is to apply complex functions $w(z)$ in studying surfaces M. In particular, real functions $u(x, y)$ also constitute a surface; thus the idea also applies to real functions.

In some ways, this idea is similar to the idea of Riemann surfaces.

Application of this idea leads to some results in differential geometry. In particular, analogues of the main theorems of classical Nevanlinna theory (in complex analysis) are obtained that are valid for generalized minimal surfaces (in geometry).

We also pose some problems that, in my opinion, can lead to a new crossroads between complex analysis, differential geometry and real analysis.

Quasi-Stationary Optical Gaussons with Maximum Intensity

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This presentation is on the dynamics of optical solitons with log-law nonlinearity, also known as optical *Gaussons*, in the presence of perturbation terms. The governing model is the nonlinear Schrodinger's equation that carries logarithmic form of nonlinear refractive index. A few perturbation terms, with maximum intensity, are included to get a better understanding of the Gausson transmission dynamics. These include multi-photon absorption, Raman scattering, self-steepening effects, self-frequency shift, nonlinear dissipation, saturable amplifiers, and others. The multiple-scale perturbation analysis, together with the Wentzel–Kramers–Brillouin–Jeffrey's (WKBJ) hypothesis are implemented to retrieve the quasi-stationary optical Gaussons solution. The WKBJ definition of the phase that is introduced reveals a couple of resonant conditions which cannot be otherwise recovered.

Mathematical Modeling of Genome Organization and Algorithms of Gene Prediction

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Over the past four decades, a new area of applied mathematics called bioinformatics has emerged alongside with the unfolding genomic revolution. Grounded in established chapters of probability, statistics, and optimization, this new field has developed efficient methods for analyzing vast new biological data, such as complete genomes. One of the main questions of bioinformatics is determining protein-coding regions within a continuous sequence of genomic DNA. The difficulty of the gene finding problem depends on complexity of genome organization as well as types and volumes of external information. Mathematical representation of genomes by Markov chains and hidden Markov models turned out to be useful for formulating and solving the gene finding problem as a pattern recognition (computing posterior probability of a function, protein-coding or non-coding, given DNA sequence) and an optimization problem. Finding the optimal parse of a genomic sequence into a sequence of fragments of different types could be done by application of a special type of the Viterbi algorithm for a hidden semi-Markov model parametrized by supervised or unsupervised training. Additional biological sequences (primary structures of RNA and proteins) have become abundant in recent years. They can effectively reduce the space of possible genome parses (annotations) without losing the biologically correct one. This talk provides an overview of the theoretical aspects of several frequently used methods of gene finding whose developments were led by researchers originating from the Republic of Georgia, particularly from Batumi (1-6; https://genemark.bme.gatech.edu/).

Acknowledgments

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On the Recent Results in the Theory of Operators in General Morrey-Type Spaces

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The theory of boundedness of classical operators of real analysis, such as maximal operator, fractional maximal operator, Riesz potential, singular integral operator etc, from one weighted Lebesgue space to another one is by now well studied. For the overwhelming majority of the values of the numerical parameters necessary and sufficient conditions on the weight functions ensuring boundedness have been found.

These results have good applications in the theory of partial differential equations. However, it should be noted that in the theory of partial differential equations, alongside with weighted Lebesgue spaces, general Morrey-type spaces also play an important role, but until recently there were no results, containing necessary and sufficient conditions on the weight functions ensuring boundedness of the aforementioned operators from one general Morrey-type space to another one (apart from the cases in which this follows directly from the appropriate results for weighted Lebesgue spaces). The case of power-type weights was well studied, but for general Morrey-type spaces only sufficient conditions were known.

In the last decade necessary and sufficient conditions for the case of general Morrey-type spaces have been found, but for a comparatively restricted range of the numerical parameters.

In this area there are many open questions which may be of particular interest to experts in studying such problems for weighted Lebesgue spaces.

In the talk a survey of results, containing necessary and sufficient conditions for boundedness of main operators of real analysis, will be given, and open problems will be discussed in detail.

The talk is based on the results obtained together with my numerous co-authors in this area: Algeria (M. A. Senouci), Antigua and Barbuda (D. J. Joseph), Azerbaijan (H. Guliyev, V. Guliyev, R. Mustafaev), Czech Republic (A. Gogatishvili), India (P. Jain), Indonesia (D. Hakim), Iran (A. Ghorbanalizadeh), Israel (E. Liflyand), Italy (P. D. Lamberti, M. Lanza de Cristoforis), Japan (E. Nakai, Y. Sawano, T. Sobukawa), Kazakhstan (N. Bokayev, D. Chigambaeva (D. Darbayeva), N. Kydyrmina, D. Matin, E. Nursultanov, R. Oinarov), Russia (M. L. Goldman, T. R. Khairullin, T. V. Tararykova), Turkey (A. Serbetci).

The Finite Hilbert Transform Taking Values in the Zygmund Space $L_{\exp}(-1,1)$

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We present a study of the action of the finite Hilbert transform defined on $L^\infty(-1,1)$ and taking its values in the Zygmund space $L_{\exp}(-1,1)$. This is a reciprocal situation to the recent investigation of the finite Hilbert transform defined on the Zygumd space $LlogL(-1,1)$ and taking its values in $L^1(-1,1)$. The fact that both $L^\infty(-1,1)$ and $L_{\exp}(-1,1)$ fail to be separable generates new features not present in previous studies.

Frontiers of Biomathematics: the Impact of Biology on Mathematics

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In my talk I will concentrate on success stories of what biology and medicine have done for mathematics, which will include concrete biomedical models and new successful mathematical theories developed for the study of these biomedical models. This will show that not only does mathematics have an impact on biology, but also that biology and medicine have an important role in the development of both pure and applied mathematics. In particular, during the talk, I will try to debunk the commonly believed fact that mathematical biology is very unsatisfying for pure mathematicians, who usually are interested in discovering fundamental and universal structural relationships.

It is worth to note that, fundamental questions arising from natural sciences, engineering and economics, and, especially during the last decade, also in biology and medicine, have always inspired mathematicians to search for new mathematical structures and methods. In my talk I will show how real life science problems, especially multiscale models arising in biology and medicine lead to a new class of partial differential equations (PDEs) or systems the analysis of which and the methods of their study, as well as their long-time spatio-temporal dynamics, is not well understood so far both quantitatively and qualitatively.

Indeed there have been very few or no systematic attempts to formulate the underlying dynamics of appropriate biomedical models in terms of degenerate parabolic/hyperbolic/elliptic PDEs comprising simultaneously several degeneracies that take into account essential features of complex biological/ medical issues. To this end we will develop new methods of infinite dimensional dynamical systems and will introduce new quantitative and qualitative characteristics responsible for their long-time dynamics.

Moreover, we will discuss how a deep knowledge of pure and applied mathematics will be crucial for success stories, when we deal with biomedical models in vitro, in vivo and in silico.

From Quarks and Gluons to Exotic Hadrons

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With recent experimental evidence for tetraquarks and pentaquarks, hadron spectroscopy is nowadays a cutting-edge area of research with the LHC among its central facilities. Hadrons are bound states of quarks and gluons in QCD, the theory of the strong interaction, but their spectrum and structure are much richer than the naïve quark model suggests and governed by nonperturbative phenomena such as confinement and dynamical mass generation. I will make a survey through some open questions in QCD, with an emphasis on the structure of exotic hadrons and multiquarks, and connect them with the key underlying phenomena of mass generation for quarks and gluons. I will highlight recent advances with functional methods, which allow one to compute hadron properties from first principles. Systematic improvements in this approach have made it possible to address a wide range of problems from multiquark spectroscopy to form factors, parton distributions, the anomalous magnetic moment of the muon, and the QCD phase diagram.

Logical Reasoning Capabilities in Maude 3.4

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Maude is a language and a system based on rewriting logic. It is a mathematical modeling language thanks to its logical basis and its initial model semantics. Maude can be used in three, mutually reinforcing ways: as a declarative programming language, as an executable formal specification language, and as a formal verification system. Logical reasoning capabilities have been recently added to Maude. This talk gives an overview of the different unification and narrowing techniques available in Maude 3.4, focusing on some of the programming, modeling, and verification aspects of Maude. Current applications of these logical capabilities in areas such as protocol verification and model checking are also presented.

Periodic Functions, Toroidal Groups, Generalized Jacobians and Fractional Ideals of Non-Totally Real Number Fields

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In 1889, a prize, sponsored by King Oscar II of Sweden, was offered to investigate two-variate complex functions with four real-independent periods. The prize was won by Poincaré, but Appell gave, as well, a solution which gained the second place, and was inspiring for further research: the solutions left the question of a two-variate complex function with three real-independent periods unexplored, and it was Cousin, a student of Poincaré, who proved the existence of such a function, and showed its main properties. Such a periodic function defines a toroidal group, given by the quotient of $\mathbb{C}^2$ by the lattice spanned by the three periods.

Quasi-Abelian varieties were introduced in 1947 by Severi exactly in the context of periodic functions, and were later studied by Rosenlicht as generalizations of the classic Jacobians of a curve. Rosenlicht was awarded the Frank Nelson Cole Prize in Algebra 1960 for this construction. In 1973 Andreotti and Gherardelli associated a toroidal group of dimension n and real rank $n+1$ to any non-totally real number field of degree $\delta = n+1$, and they showed, conversely, that tensoring with $\mathbb{Q}$ the endomorphism ring of a toroidal group of dimension n and real rank $n+1$, one obtains a non-totally real number field of degree $\delta \leq n+1$. More generally, Y. Abe showed that, given a non-totally real number field K of degree $\delta = r_1 + 2r_2$ and a Minkowski map $\mu : K \to \mathbb{C}^{r_1+r_2}$, the image $\mu(O_K)$ is a lattice Λ of complex rank $r_1 + r_2$ and real rank δ, such that the quotient group $\mathbb{C}^{r_1+r_2}/\Lambda$ is a toroidal group, and furthermore proved that for any toroidal group T which does not contains toroidal subgroups, the ring $\mathrm{End}_0(T)$ is a division algebra.

In this lecture we study this relationship between nontotally real number fields K and toroidal groups T, as well as meromorphic periodic functions, exploiting a representation of T as the generalized Jacobian $J_L(C)$ of a suitable elliptic curve C. We show that the toroidal Lie group $G = \mathbb{C}^2/\Lambda$, where Λ is the lattice generated by $(1,0)$, $(0,1)$ and (τ, ϑ), with τ not in $\mathbb{R}$, is isomorphic to the generalized Jacobian J_L of the complex elliptic curve C with modulus τ, defined by any divisor class $L = (M) + (N)$ of C fulfilling $M - N = \big(\wp(\vartheta) : \wp'(\vartheta)\big) \in C$. We consider in detail the cubic and quartic cases. In these cases, we write down the relations between the minimal polynomial of a suitable primitive element of K and the parameters defining the generalized Jacobian $J_L(C)$ corresponding to the toroidal group associated with the ring of integers. Furthermore, for the cubic case, we give an explicit description of the m-torsion of T in the geometric correspondence of T with $J_L(C)$, as image of a fractional ideal of K. The results are contained in the papers [1] and [2].

Acknowledgments

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Auto-Tuning High-Performance Programs with Model Checking in Promela

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We combine research approaches that traditionally have been disjoint:

(1) model checking as used in formal verification of programs,

(2) auto-tuning as often used in high-performance computing.

Auto-tuning [3] frameworks optimize parallel programs by finding the optimal values of the performance-critical parameters – so-called tuning parameters – for a particular high-performance architecture and input data size. As there are many parameters influencing program's performance, finding the optimal parameter configuration is a hardly manageable task even for experts. Auto-tuning automates this process, but it is often time- and resource-consuming.

We apply model checking [1] for accelerating auto-tuning by using a counterexample constructed during the verification of the optimality property of the program which says that the program cannot be finished in some predefined time. We describe in detail a proof-of-the-concept implementation of our approach for programs written in OpenCL [4] – the standard for programming modern high-performance architectures. We use the popular SPIN [5] verification tool as a model checker. We descrribe the execution of the abstract OpenCL program on the abstract OpenCL platform with the SPIN model representation language Promela. For scalability we also apply abstracting reduction and swarm verification [2]. We demonstrate how our approach to auto-tuning based on model checking works for a particular application use case – the problem of computing the minimal value in a very big integer array.

Acknowledgments

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An Introduction to Dimension Theory of Topological Spaces – Zero-Dimensional Extensions of Topologies

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Dimension Theory of topological spaces has a rich history in Topology and contains many important chapters, including the studies of different notions of dimensions such as the covering dimension, the small inductive dimension, the large inductive dimension and the dimension Dind of A. V. Arhangelskii. This talk is based on this significant fact and especially it is separated in the following two axes:

- We present a report on the first steps of birth and development of Dimension Theory in Topology.
- We present an extension of Topological Dimension Theory, providing the new notions of zero-dimensional extensions of topologies with respect to the small inductive dimension and the covering dimension. Thus, we succeed to study the topology of an arbitrary topological space as an intersection of zero-dimensional extensions of it.

Part of this talk has been presented in the research paper titled *"Zero-Dimensional Extensions of Topologies"* by D. Georgiou, Y. Hattori, A. Megaritis and F. Sereti, which has been accepted for publication to the journal *"Topology and its Applications"*.

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Applications of Linear and Multi-Linear Restriction to Uncertainty Principles and Exact Signal Recovery

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We are going to show that in the presence of non-trivial restriction estimates, the Fourier uncertainty principle can be improved, with applications to exact signal recovery. We are also going to show that multi-linear restriction estimates can be used to obtain increasingly better signal recovery conditions when a signal is sent multiple times. Connections with frame theory will be discussed as well. This is joint work with Azita Mayeli (CUNY).

About Banach–Mazur's Problem from the 1930s. Classical and New Results

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One of the famous unsolved problems in Functional Analysis asks (Banach–Mazur's problem (1932)) if every infinite-dimensional Banach space can be mapped by a continuous linear operator onto an infinite-dimensional separable Banach space. This problem is known under the name *Separable quotient problem.* For many concrete Banach spaces the answer is positive, for example, reflexive Banach spaces, or even weakly compactly generated Banach spaces. Argyros, Dodos and Kanellopoulos, proved that every dual Banach space has a separable quotient. On the other hand, Rosenthal showed that all Banach spaces $C(X)$ of continuous (real-valued) functions on X have a quotient isomorphic to c_0 or l_2. We provide several useful methods to examine which Banach spaces admit a separable quotient, and the same problem will be discussed for spaces $C_p(X)$ with the pointwise topology. The talk gathers also quite new results. A connection with Efimov compact spaces X will be also discussed.

The Wonderful Wizardry of Tomography

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Tomographic imaging is nowadays a major medical diagnostics technique. It is also widely used in industry for non-destructive testing, homeland security, geology, oil prospection, etc. For a mathematician, it is an inexhaustible treasure box of beautiful, hard and important problems, without solving which none of the above mentioned applications would be possible. Practically all areas of mathematics appear there: functional and harmonic analysis, geometry (differential, integral, algebraic), probability theory, and surely partial differential equations and numerical analysis, just to name a few.

The talk, directed to non-experts, will provide a brief survey of topics, challenges, and techniques of this area.

Recent Advances in Logic-Based Generalization

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Generalization of two or more objects (terms, trees, forests, graphs, formulas, proofs, programs, ...) is an object, which keeps common features of the given objects and shows how they differ. It is a fundamental concept for tasks where one is interested in extracting a common general pattern from the given concrete examples. Anti-unification is a logic-based method to compute generalizations. It was introduced in the 1970s, motivated by an application in inductive reasoning. In recent years, there has been growing interest in anti-unification due to novel applications in various subareas of artificial intelligence and computer science, where anti-unification-based techniques have been successfully applied in analogical reasoning, logic-based learning, term set compression, program analysis and synthesis, programming by examples, software code clone detection, automated program repair, chatbot development, etc.

In this talk, which is based on [1], we will give a survey on recently developed generalization algorithms in first-order and higher-order theories, consider their applications, and discuss some interesting research problems in this area.

Acknowledgments

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Angelic Spaces: Eberlein–Šmulyan Type Results for Abelian Topological Groups

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The flamboyant name of "angelic spaces" is atributed to a class of topological spaces in which different definitions of compactness are equivalent. The essential model of angelic spaces is given by metrizale topological spaces, since compactness, countable compactness and sequential compactness coincide for any subset of a metrizale topological space. The Eberlein–Šmulyan Theorem establishes that such an equivalence also holds for the class of Banach spaces endowed with their weak topology.

Leaning on the notion of locally quasi-convex Abelian topological group, introduced in [6] (and essentially used later in [1]), in joint works [2–5] we have developed counterparts of some well-known theorems of Functional Analysis for the wider context of Abelian topological groups.

In this lecture we present some results which generalize the Eberlein–Šmulyan Theorem, as well as the Grothendieck Theorem on completeness, for certain classes of locally quasi-convex Abelian topological groups.

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Projection Constants for the Spaces of Multivariate Polynomials

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We discuss results on a fundamental constant of Banach space theory – the projection constant. We develop new methods that shed light on projection constants in classes of multivariate polynomials. The relative projection constant $\boldsymbol{\lambda}(X, Y)$ of a subspace X of a Banach Y is the smallest norm among all possible projections on Y onto X, and the projection constant $\boldsymbol{\lambda}(X)$ is the supremum of all relative projection constants of X taken with respect to all possible super spaces Y. We will present results on the exact and asymptotically optimal behaviour of the projection constants which depend on both the dimension of the space and the degree of the polynomials. We focus on Banach spaces of polynomials, including polynomials on compact topological group, analytic polynomials on polydiscs, Dirichlet polynomials on the complex plane, polynomials on euclidean spheres, and polynomials on Boolean cubes. The talk is based on joint works with A. Defant, D. Galicer, M. Mansilla, and S. Muro.

Imprimitive Points on Elliptic Curves

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t concerns points on elliptic curves over a number field with the property that the reduction of primes is never a generator for the reduced curve. Earlier work considered the case when the index of the reduction was always divisible by a fixed prime. The current project deals with other cases. The approach considers the affine representation modulo composite n on the 3 dimensional Z/nZ-vector space generated by the n-th roots of the points.

Uncertainty Principles for Fourier Transform

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We discuss several Fourier uncertainty principles including the subcritical case of the classical Cowling–Price principle.

Widths of Convex Sets and the Power of Adaption and Randomization

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We present bounds between different widths of convex subsets of Banach spaces, including Gelfand, Kolmogorov and Bernstein widths. Using this, and some relations between widths and minimal errors, we obtain bounds on the maximal gain of adaptive and randomized algorithms over non-adaptive, deterministic ones for approximating linear operators on convex sets.

We conclude with an overview of the new state of the art and a list of open problems.

The Domination Problem in Graphs and Solution Methods

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Set covering and domination problems constitute an important class of graph optimization problems. Given a graph $G = (V, E)$, $S \subseteq V$ is a dominating set if every vertex in $V \setminus S$ is adjacent to a vertex in set S. The domination problem is among hardest computationally intractable graph optimization problems. In this talk we will overview the domination problem, its variations and applications and the best known exact and approximation solution methods for the problem.

A Class of Multilinear Bounded Oscillation Operators on Measure Spaces and Applications

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In recent years, with the extensive application of dyadic analysis and sparse domination, the theory of multilinear operators has developed rapidly, particularly in the areas of weighted theory, extrapolation theorems, and characterizations of compactness. In this talk, we will mainly introduce some of our recent work, involving a class of Banach-valued multilinear bounded oscillatory operators on general measure spaces. We developed a weighted theory that encompasses classical multilinear Calderón–Zygmund operators as well as some other operators. We established three typical estimates: local exponential decay estimates, mixed weak-type estimates, and sharp weighted norm inequalities. Based on the extrapolation properties of abstract multilinear compact operators, we obtained a weighted compactness extrapolation theorem for commutators of multilinear operators on homogeneous spaces.

Topological Data Analysis with an Example in Social Networks

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As artificial intelligence improves, the detection of non-human users in social networks becomes a more difficult task. While not all non-human social network users are harmful, many are programmed to spread disinformation or cause civil unrest. Thus, determining whether a social network user is human is an important task. In this talk, we provide an introduction to topological data analysis and provide preliminary results on how these methods could be used to detect non-human users in Twitter.

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Abstracts of Sectional Talks

Development of an Electronic Library for Mobile Applications using AI Functions

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The purpose of our research is to develop an e-library mobile application using AI functions. Traveling is a tiring process and it can last several hours. The goal of our research is to make it easier for people to find the book they want through a mobile application. In the database, it is possible to search for books by both Georgian authors and foreign authors, which is helped by some functions of Artificial Intelligence (AI) Databricks.

The Databricks Data Intelligence Platform enables access to data using AI and Machine Learning (ML). Based on the precise data, it is possible to build an electronic model of the library, in which people with the same interest will unite.

To process large texts, it is possible to use Google Cloud Natural Language AI, an ML language with algorithms and its extensive capabilities. The use of this service is characterized by efficiency and accuracy, as it is connected to cloud technology. Its disadvantages are: high price and less configurability/compatibility with Google Cloud.

The Graded Graph for Submodules

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In this paper, we study the graded annihilating graph for submodules, representing graded submodules as vertices connected by edges following a specific pattern. Our exploration leads us through the intricacies of this graph, uncovering insights into their connectivity, girth, bipartition, and completeness within graded modules, establishing connections between these graded annihilating graph submodules and their ungraded counterparts. We also extend prior work on finite girth conditions of annihilating graphs for modules. Our contributions include two comprehensive criteria for graded modules, which also apply to non-graded modules with trivial gradation.

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Q-Analysis of Connections of Hierarchical Systems using Fuzzy Sets

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The urban system is a collection of a finite number of objects: a collection of various groups of residents (P), a collection of buildings (B), and others. The structure of the urban system, on which various dynamic processes develop, is a set of mathematical relations $\Lambda = \{\lambda, \mu, \dots\}$ that exist between these sets $B, P, \dots$ Knowledge of the relations $\{\lambda, \mu, \dots\}$ allows us to detect the structural connection between them and follow the events developing in a long chain of connections between functional objects for various purposes in space-time. Typically, if there is a given connection between any elements of these sets, we use the incidence matrix corresponding to this connection.

In many cases, if the connection between some elements of two sets is insignificant (small) compared to other connections, then for the purpose of applying the Q-analysis method such a connection is considered zero. It is obvious that with further use of the model obtained under such an assumption, some information is lost, which can lead to incorrect results, possibly even to disaster.

The approach we present allows us to overcome this problem, in particular, to expand Q-analysis by introducing fuzzy sets.

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On the Topology of an Intersection of Quadric

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Let $\chi(n, m, d)$ denote the family of all possible values of the Euler characteristic of non-empty compact algebraic sets in R^n representable an intersection of m algebraic hypersurfaces of degree d. We present for the Euler characteristic a general estimate applicable to intersections of algebraic hypersurfaces of arbitrary degrees.

Theorem 1 *The doubled absolute value of the Euler characteristic of any compact intersection of arbitrary many real hypersurfaces of degree d in R^n does not exceed one plus the Petrovskii number $P(n+1; 2d+3)$.*

Theorem 2 *For any non-degenerate quadric map $Q : R^n \to R^m$ there are s real polynomials f_j in m variable, with s even, such that the spectrum $\chi(Q)$ coincides with the set of values of the sum of their sings over all points of R^n.*

The number of components and Euler characteristic for the fibers of a stable quadratic map in low dimension is also estimated.

Theorem 2 *For a stable quadratic map $Q : R^4 \to R^3$ the Euler characteristic of any fiber lies into $[-6; 3]$.*

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Numerical Simulation of Generated Heat in Metal Plate Placed Upon the Optical Waveguide using the 1D Simulation and Extending to 3D

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In optical integrated circuits, in some applications a metal layer is placed near an optical waveguide. When light propagates in a waveguide, a small portion of it interacts with the metal layer, warming it up. This phenomenon is a well-known light to heat conversion [1]. In this study, a 1D model was utilized to calculate the temperature increase in Titanium by solving the wave equation in the x-direction.

In the Transverse Mode (TM) of light, the principal field component is $H_y(x)$, with H_x and H_z both zero. The wave equation for TM mode is represented by the following equation

$$n(x)^2 \frac{\partial\left(\frac{1}{n^2}\frac{\partial H_y(x)}{\partial x}\right)}{\partial x} = \left(\beta^2 - n(x)^2 k_0^2\right) H_y(x). \tag{1}$$

For refractive index n of materials, apply the boundary condition $H_y = 0$ at both ends of the structure. The wave equation (1) has been numerically solved for various layer thicknesses in the SiO_2 substrate – Si waveguide system, including the SiO_2 layer, Titanium layer, and the final SiO_2 cover layer. Calculating the wave equation in the x-direction yields all components of the electric field $E_x(x)$, $E_y(x)$, $E_z(x)$ and the magnetic field $H_y(x)$ at every point along x. To determine absorbed light in the Titanium layer, all field components at each y and z point within the layer need to be computed. The absorption of light power is negligible compared to its propagation, so the z dependence is disregarded. By rotating the simulation axes around the waveguide center, the fields component at all y points in the Titanium layer has been located. In this regard, we perform various numerical simulations. Calculating the field components at the top and bottom of the Titanium layer allows for the calculation of absorbed light, which results in a temperature increase in the Titanium layer. The steady state temperatures of all points in the structure have been calculated by solving the heat transfer equation.

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Conditions for Automorphic Compositions on the Upper Half-Plane to be Universal

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A continuous linear operator $T : X \to X$ on a Fréchet space X is said to be *hypercyclic* if there is a vector $x \in X$ for which the orbit $\{x, Tx, T^2x, T^3x, \ldots\}$ is dense in X. More generally, a sequence of continuous linear operators $T_n : X \to X$ is said to be *universal* if there is a vector $x \in X$ for which the sequence $\{x, T_1x, T_2x, T_3x, \ldots\}$ is dense in X.

We focus on the case when $X = H(\mathbb{P}) =$ the Fréchet space of all analytic functions on the upper half-plane $\mathbb{P}$, and the operators are compositions $C_\sigma : H(\mathbb{P}) \to H(\mathbb{P})$ defined by $C_\sigma(f) = f \circ \sigma$, where each $\sigma : \mathbb{P} \to \mathbb{P}$ is an automorphism of the upper half-plane $\mathbb{P}$. Every automorphism σ of $\mathbb{P}$ is a linear fractional transformation

$$\sigma(z) = \frac{az + b}{cz + d},$$

where a, b, c, d are real values with $ad - bc = 1$. We are to give conditions for hypercyclicity and universality in terms of the coefficients a, b, c, d.

It turns out that $C_\sigma : H(\mathbb{P}) \to H(\mathbb{P})$ is hypercyclic if and only if $|a + d| \geq 2$, which surprisingly does not involve the two coefficients b and c. For a sequence of automorphisms

$$\sigma_n(z) = \frac{a_n z + b_n}{c_n z + d_n},$$

we show that $C_{\sigma_n} : H(\mathbb{P}) \to H(\mathbb{P})$ is universal if and only if

$$\limsup |a_n| + |b_n| + |c_n| + |d_n| = \infty.$$

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Almost Omega-Categoricity in Weakly o-Minimal Theories

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Let L be a countable first order language. Throughout this lecture we consider L-structures and suppose that L contains a binary relation symbol $<$ which is interpreted as a linear order in these structures. A subset A of a linearly ordered structure M is *convex* if for all $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$.

This lecture concerns the notion of *weak o-minimality* which was initially deeply studied by D. Macpherson, D. Marker and C. Steinhorn in [3]. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable (with parameters) subset of M is a union of finitely many convex sets in M. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures.

Definition ([1, 4]) Let T be a complete theory, and $p_1(x_1), \ldots, p_n(x_n) \in S_1(\varnothing)$. A type $q(x_1, \ldots, x_n) \in S_n(\varnothing)$ is said to be a $(p_1, \ldots, p_n)$-*type* if $q(x_1, \ldots, x_n) \supseteq \bigcup\limits_{i=1}^{n} p_i(x_i)$. The set of all $(p_1, \ldots, p_n)$-types of the theory T is denoted by $S_{p_1,\ldots,p_n}(T)$. A countable theory T is said to be *almost omega-categorical* if for any types $p_1(x_1), \ldots, p_n(x_n) \in S_1(\varnothing)$ there are only finitely many types $q(x_1, \ldots, x_n) \in S_{p_1,\ldots,p_n}(T)$.

Almost omega-categoricity is closely connected with the notion of Ehrenfeuchtness of a theory. So in [1] it was proved that if T is an almost omega-categorical theory with $I(T, \omega) = 3$ then a dense linear order is interpreted in T. In [2] it was established that Ehrenfeucht quite o-minimal theories are almost omega-categorical.

Theorem *Any weakly o-minimal theory of finite convexity rank having less than* 2^{ω} *countable models is almost omega-categorical.*

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On the Categories of Exponential R-Groups

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Let R be an arbitrary associative ring with unit. Using this ring we define a new category of R-groups in three different ways. To do this, we enrich the group language $L_{gr} = \{\cdot, ^{-1}, e\}$ as follows: $L_{gr}^R = L_{gr} \cup \{f_\alpha(g) \mid \alpha \in R\}$, where $f_\alpha(g)$ – unary operation, denoted by $f_\alpha(g) = g^\alpha$ for any $g \in G$. We will call a set G as R-group by Lyndon [3] if the operations $\cdot$, $^{-1}$, e, $f_\alpha(g)$ are defined on this set and the axioms are satisfied:

1. group axioms;
2. $g^1 = g$, $g^0 = e$, $g^{\alpha+\beta} = g^\alpha g^\beta$, $g^{\alpha\beta} = (g^\alpha)^\beta$, $(h^{-1}gh)^\alpha = h^{-1}g^\alpha h$.

In [4] A. G. Myasnikov and V. N. Remeslennikov introduced a new category M_R, with help of one additional axiom:

$$\forall\, g, h \in G \qquad [g, h] = e \;\longrightarrow\; (gh)^\alpha = g^\alpha h^\alpha,$$

where $[g, h] = g^{-1}h^{-1}gh$.

Clearly, all R-modules over the ring R satisfy the axiom (MR).

For nilpotent groups and binomial rings in [2] P. Hall introduced the category of R-groups, which is different from the category of M_R.

In the paper [1] the notion of a variety of R-groups with axiom (MR) and tensor completion in the variety were introduced. We consider the structure of free 2-step nilpotent R-groups for two special rings.

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Gender Challenges in Computer Science and STEM

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The gender imbalance in computer science and STEM (science, technology, engineering, and mathematics) fields indicates unequal representation and participation of different genders in these disciplines. This issue is highly relevant and affects education, employment, and various aspects of professional growth [1]. Addressing gender issues in computer science and STEM requires a collaborative community effort. The gender disparity in the tech sector is evident, with men dominating. Conducting research and analysis in this area, empowering women, and enhancing their knowledge level are essential steps to increase their motivation and promote career development. This, in turn, will create more opportunities in this sector, which is the fastest-growing in the world [2].

According to the conducted analysis, the number of female students in 2020-2024 is approximately 20% of the total number of enrolled students, although the trend is increasing. The decline in the trend after the pandemic is striking: before the pandemic, same figure was 22.1%, and after the pandemic, e.g. In 2020-2021, this number dropped from an average of 22.1% to 14.5%. However, the graduation success rate is much higher among female students – 70% versus 49%. The article provides detailed information on the analysis and results of the study based on numerical indicators. Today's gender-related challenges in technology focus on actively involving women in technology and science. Education remains one of the most critical social factors in achieving equality, including gender equality. The article provides an overview of factors contributing to gender imbalance in STEM and describes the problems and challenges associated with gender disparities in science and technology.

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The Structure of Upside-Down Linear Preservers of Majorization

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Let $\mathbb{R}_n$ be the set of all $1 \times n$ real vectors. For $x, y \in \mathbb{R}_n$, we say that x is majorized by y (written as $x \prec y$) if there exists a doubly stochastic matrix D such that $x = yD$. Also for $x, y \in \mathbb{R}_n$, we say that x is row-majorized by y (written as $x \prec_r y$) if there exists a row stochastic matrix R such that $x = yR$. In this paper we investigate the structure of upside-down linear preservers of majorization i.e. all linear operators $T : \mathbb{R}_n \to \mathbb{R}_n$ with the following properties:

- $x \prec y$ implies that $Ty \prec_r Tx$.
- If $x \prec_r y$ implies that $Ty \prec Tx$.
- $x \prec y$ implies that $Ty \prec Tx$.
- If $x \prec_r y$ implies that $Ty \prec_r Tx$.

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Identifying Key Relationships in a Data Set

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Living in the data age, it's only natural that we create and use more information than ever before. This data age has given birth to an entire industry called data science. The skills of integrated work with it have become one of the most valuable skills of the 21st century. Data science is fundamentally dependent on data, data sets, and structures. A dataset, in turn, consists of data associated with a set of objects, and each object is described by a set of attributes. Data structure is a fundamental component of any programming language that allows developers to organize and manipulate data efficiently. This article discusses how to implement data structures such as arrays, linked lists, queues, hashes, etc. The article is devoted to identifying key relationships, for which a data set is created that consists of a list of specific user groups, where each of them is assigned an identifier in the form of a numeric value and names using the corresponding operators. Code has been created that provides a list of user ID pairs, if such pairs exist. A network of identified connected pairs was formed, which was depicted schematically. If there are a large number of such pairs, a group of data called "data pairs" will be created in which the key will be the user ID and the value will be lists of IDs. In this case, it would be easier to find, for example, the average number of connections, the largest or smallest number of connections, etc. Such calculations provide a network metric of connections called degree centrality, which in many cases has the advantage of being easy to calculate.

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On the Existence of Bounded Solutions for Systems of Linear Impulsive Equations

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For the linear system of impulsive differential equations

$$\frac{dx}{dt} = P(t)x + q(t) \text{ for } t \in \mathbb{R} \setminus T, \quad x(\tau_l+) - x(\tau_l-) = G(\tau_l)x(\tau_l) + u(\tau_l) \ (l = 1, 2, \dots) \tag{1}$$

consider the problem of the bounded on $\mathbb{R}$ solution

$$\sup\{\|x(t)\| : t \in \mathbb{R}\} < +\infty, \tag{2}$$

where $P \in L_{loc}(\mathbb{R}; \mathbb{R}^{n\times n})$, $q \in L_{loc}(\mathbb{R}; \mathbb{R}^n$, $G \in B_{loc}(T; \mathbb{R}^{n\times n})$, $u \in B_{loc}(T; \mathbb{R}^n)$, $T = \{\tau_1, \tau_2, \dots\}$, $\tau_l \in \mathbb{R}$ $(l = 1, 2, \dots)$, $\tau_l \neq \tau_k$ if $l \neq k$ $(l, k = 1, 2,)$; $B_{loc}(T; \mathbb{R}^{n\times m})$ the set of all matrix-function $G : T \to \mathbb{R}^{n\times m}$ such that $\sum\limits_{\tau_l \in T \cap [a,b]} \|G(\tau_l)\| < +\infty$. The solutions are finding in the set of the vector-functions whose restrictions on the every closed interval from $\mathbb{R}$ are a continuous from the left absolutely continuous. Let $\det(I_n + G(\tau_l)) \neq 0$ $(l = 1, 2,)$, where I_n is the identity $n \times n$-matrix.

We give effective sufficient conditions for the existence of solution of the problem (1), (2).

For $t_i \in \mathbb{R} \cup \{-\infty; +\infty\}$ $(i = 1, \dots, n)$ we put $N_0(t_1, \dots, t_n) = \{i : t_i \in \mathbb{R}\}$. If $t_i = -\infty$ $(t_i = +\infty)$, then $\operatorname{sgn}(t - t_i) = 1$ for $t \in \mathbb{R}$ $(\operatorname{sgn}(t - t_i) = -1$ for $t \in \mathbb{R})$.

Theorem *Let* $1 + g_{ii}(\tau_l) \neq 0$ $(i = 1, \dots, n;\ l = 1, 2, \dots)$ *and let there exist* $t_i \in \mathbb{R} \cup \{-\infty; +\infty\}$ $(i = 1, \dots, n)$ *such that*

$$s_{ik} = \sup\left\{\left| \int_{t_i}^{t} |\gamma_i(t,\tau)|\,|p_{ik}(\tau)|\,d\tau + \sum_{t_i \leqslant \tau_l < t} |\gamma_i(t,\tau_l)|\,\left|(1 + g_{ii}(\tau_l))^{-1} g_{ik}(\tau_l)\right| \right| : t \in \mathbb{R}\right\} < +\infty,$$

$$\sup\left\{\left| \int_{t_i}^{t} |\gamma_i(t,\tau)|\,|q_{ik}(\tau)|\,d\tau + \sum_{t_i \leqslant \tau_l < t} |\gamma_i(t,\tau_l)|\,\left|(1 + g_{ii}(\tau_l))^{-1} u_i(\tau_l)\right| \right| : t \in \mathbb{R}\right\} < +\infty$$

for $(i \neq k;\ i, k = 1, \dots, n)$ *and* $\sup\{|\gamma_i(t, t_i)| : t \in \mathbb{R}\} < +\infty$ *for* $i \in N_0(t_1, \dots, t_n)$, *where* $\gamma_i(t, \tau)$ $(i = 1, \dots, n)$ *defined by the Cauchy problem. Let, moreover,* $r(S) < 1$, *where* $S = (s_{ik})_{i,k=1}^{n}$, $s_{ii} = 0$ $(i = 1, \dots, n)$. *Then for every* $c_i \in \mathbb{R}$ $(i \in N_0(t_1, \dots, t_n))$ *system (1) has at last one a bounded on* $\mathbb{R}$ *solution satisfying the condition* $x_i(t_i) = c_i$ *for* $i \in N_0(t_1, \dots, t_n)$.

The analogous questions are investigated in [1] for the generalized ordinary differential systems, and in [2] for ordinary differential systems.

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Boundary-Domain Integral Equations for Helmholtz BVPs with Non-Smooth Coefficients on Lipschitz Domain

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In this paper, boundary-domain integral equations (BDIEs) based on a parametrix and associated with the Dirichlet and Neumann boundary value problems (BVPs) for Helmholtz equation with a variable Hölder-continuous coefficients on Lipschitz domain are considered. The PDE right-hand side belongs to the Sobolev (Bessel potential) space $H^{s-2}(\Omega)$ or $\widetilde{H}^{s-2}(\Omega)$, $\frac{1}{2} < s < \frac{3}{2}$, when neither strong classical nor weak canonical co-normal derivatives are well defined. Using the approach developed in [2], we extend the results obtained in [1]. Equivalence of the BDIEs to the original BVP, BDIE solvability, solution uniqueness/non-uniqueness, and the Fredholm property and invertibility of the BDIE operators are investigated in appropriate Sobolev spaces.

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United Boundary-Domain Integro-Differential and Integral Equations to the Dirichlet Problem for a Compressible Stokes System with Variable Viscosity

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In this paper, the Dirichlet boundary value problem (BVP) for a compressible Stokes system of partial differential equations (PDEs) with variable viscosity is considered in a bounded three dimensional domain. Using an appropriate parametrix (Levi function), the problem is reduced to the united boundary-domain integro-differential equation (BDIDE) or to a domain integral equation supplemented by the original boundary condition, thus constituting a boundary-domain integro-differential problem (BDIDP). Solvability, solution uniqueness and equivalence of the BDIDE/BDIDP to the original BVP as well as invertibility of the associated operators are analysed in appropriate Sobolev (Bessel potential) spaces.

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On the Estimating Poisson Regression Function Using Bernstein Polynomials

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The estimate for the Poisson regression function is constructed using the Bernstein polynomial. The question of its consistency and asymptotic normality is studied. Testing hypothesis is constructed on the form of the Poisson regression function. The question of consistency of the constructed test is studied.

Application of Grid-Characteristic Method to Characteristic Problems for One Class of Quasi-Linear Equations

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For linear differential equations solving the Characteristic problem means to find a solution to the equation by given values on arcs of characteristics of different families. These arcs have one common point and their tangents are different at this point. As for nonlinear equations, families of characteristic curves depend on the sought solution and therefore are unknown in advance. For that reason, it is impossible to pose the characteristic problem (analogue of Goursat problem) for a nonlinear equation in the same way we do it for a linear one. In this work we present a numerical method to solve one class of quasi-linear equations [3]. For each equation, one of the families of characteristics is completely determined, while the other depends on the first derivatives of unknown solution and thus is not determined in advance. The type of equation is hyperbolic with possible parabolic degeneracy and this fact should be taken into account so that the problem is posed correctly (see for example [1]). For finding a numerical solution of the problem we use the Grid Characteristic Method, which provides a robust and accurate approach for solving hyperbolic PDEs (see for example [2]).

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V-Independent Sequences

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Let M be a saturated model of a complete theory T, A be a set such that M is $|A|^+$-saturated, and p, q are non-isolated one-types.

Definition 1 Let $p(\bar{(}x)$ be some (may be incomplete) n-type over a set $A \subseteq M$ in a model M of a theory T, and B be a set in the model M. A quasi-neighborhood of B in p is the set $QV_{p,M}(B)$ of all tuples $\overline{c} \in M$ such that there exist a tuple $\overline{b} \in B$ and a $(tp(\overline{b}/A), p)$-preserving formula $\phi(\overline{x}, \overline{y})$ with $M \models \phi(\overline{b}, \overline{c})$.

A quasi-neighborhood is called definable if there is a formula $\psi(\overline{x}, \overline{y})$ such that $QV_{p,\mathscr{M}}(\overline{a}) = \psi(\overline{a}, M)$.

Definition 2 Let T be a geometric theory, let $M \models T$ and let $\overline{a} = (a_0, a_1, \ldots, a_{n-1}) \in M^n$ and $B \subset M$ be such that $dim(\overline{a}/B) = n - 1$ but any subset of $a_0, \ldots, a_{n-1}$ is independent over B. We call such a tuple an algebraic n-gon.

Theorem *Let $\mathscr{M}$ be a saturated model of theory T, $A \subseteq M$, $p \in S(A)$, $\overline{a} \in p(M)$, and $QV_{p,\mathscr{M}}(\overline{a})$ is definable quasi-neighborhood such that $\overline{x} \in QV_{p,\mathscr{M}}(\overline{y})$ is not an equivalence relation. Then T has the strict order property.*

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Observations on the Number of Countable Models of Ordered Theories

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Let T be a countable complete theory in a countable signature Σ. The notation $I(T, \lambda)$ stands for the number of pairwise non-isomorphic models of the theory T of cardinality λ. One of the well-known problems in model theory, Vaught's conjecture, suggests that there is no theory T with $\aleph_0 < I(T, \aleph_0) < 2^{\aleph_0}$. Substantial results have been obtained in this direction, including confirmation of the conjecture for various classes of linearly ordered theories [1–5]. Nonetheless, Vaught's conjecture in general has yet to be confirmed or disproved.

We show that, when studying the aforementioned question, restricting to linearly ordered theories in general does not give any significant advantages. This is justified by the next theorem.

Theorem 1 ([6]) *Let $\mathfrak{M} = \langle M; \Sigma\rangle$ be a countable structure of a countable signature Σ. Then there exists a countable linearly ordered structure M^* such that*

$$I(Th(M), \aleph_0) = I(Th(M^*), \aleph_0).$$

Corollary ([6]) *If there exists a theory T of a countable signature such that $I(T, 2^{\aleph_0}) = \aleph_1$, then there exists a linearly ordered theory T^* of a countable signature such that $I(T^*, \aleph_0) = \aleph_1$.*

In a linearly ordered structure $\mathfrak{M}$ each 1-formula $\Theta(x)$ can be assigned an equivalence relation E_Θ, whose equivalence classes are maximal convex subsets of the set $\Theta(M)$. A relation of linear order naturally occurs on the set of these classes.

Theorem 2 *Let $\mathfrak{M}$ be a countably saturated model of a small linearly ordered theory T, $\overline{a} \in M$, $\Theta(x, \overline{a})$ be a 2-formula such that for each $n < \omega$ there exist $m_n \geq n$ and a discretely ordered chain of convex E_Θ-classes of length m_n. Then $I(T^*, \aleph_0) = 2^{\aleph_0}$.*

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Complex Cobordism Modulo Spherical Cobordism

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The talk is based on [1], where we proved that the ideal in complex cobordism ring MU^* generated by the polynomial generators $S = (x_1, x_k, k \geq 3)$ of c_1-spherical cobordism ring W^*, viewed as elements in MU^* by forgetful map is prime.

By using the Baas–Sullivan theory of cobordism with singularities and proving that all the Mironov obstructions to the commutativity vanish we define a commutative, complex oriented cohomology theory $MU^*_S(-)$, complex cobordism modulo c_1-spherical cobordism, with the coefficient ring identical to the quotient ring MU^*/S.

Then any subsequence $\Sigma \subseteq S$ is also regular in MU^* and therefore gives a multiplicative complex oriented cohomology theory $MU^*_\Sigma(-)$.

In particular we prove that the generators of $W^*[1/2]$ can be specified as follows.

(i) For $\Sigma = (x_k, k \geq 3)$ the corresponding cohomology is identical to the Abel cohomology, previously constructed by Busato (see [1]).

(ii) Another example [2] corresponding to $\Sigma = (x_k, k \geq 5)$ gives $MU^*[1/2]/\Sigma$, the coefficient ring of the universal Buchstaber formal group, i.e., rationally it is identical to the Krichever–Hoehn complex elliptic genus.

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On Khavin Subsets of Complex Plane

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Dedicated to Levan Zhizhiashvili (1934–2006)

Motivated from the work [1, p. 502] we call a proper non-empty subset S of the complex plane $\mathbb{C}$ *a Khavin subset* if for every $z \in \mathbb{C} \setminus S$ there exists a continuous function $\Gamma_z : S \to \mathbb{R}$ such that

$$s - z = |s - z| e^{i\Gamma_z(s)} \quad \forall\, s \in S.$$

We plan to discuss a proof of the following, rather unexpected, assertion: the unit circle

$$\mathbb{S} = \big\{ s \in \mathbb{C} : \ |s| = 1 \big\}$$

is not a Khavin subset of $\mathbb{C}$.

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An Inverse System Approach of Map and its Application in (Co)Homology Theory

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The main aim of this report is to investigate continuous maps from the standpoint of geometric topology and algebraic topology. The applications of obtained results include the new axiomatic approaches of Čech spectral (co)homology theories of compact Hausdorff spaces without using the relative (co)homology groups. The axiomatic approach of (co)homology theory goes back to S. Eilenberg and N. Steenrod [3]. In 1945 they gave the axiomatic characterizations of simplicial homology theory for finite poluhedral pairs and Čech spectral (co)homology theory for pairs of compact Hausdorff spaces [3]. In 1987 T. Watanabe used the achievements of shape theory to axiomatize the Čech spectral (co)homology theories based on all normal coverings of arbitrary topological spaces. Besides, in 1995 T. Miyata used the notions of uniform shape theory to axiomatize the Čech spectral (co)homology theory for pairs of uniform spaces.

The axiomatic characterizations considered by above mentioned outhors are formulated in theorems of the relative homology groups, the induced homomorphisms, and the boundary homomorphisms. There is a natural question. Can we characterize the Čech spectral (co)homology theory defined on the given category of spaces by axioms non using the relative (co)homology groups.

In the report [1, 2] we will give approximation theorems of maps.

The well-known Čech, Vietoris and Chogoshvili definitions of spectral Čech and spectral singular homology groups yield the functors $Mor_{\check{C}} : \boldsymbol{Mor}_{\boldsymbol{Comp}} \to \boldsymbol{pro-Mor}_{\boldsymbol{Pol_f}}$, $Mor_V : \boldsymbol{Mor}_{\boldsymbol{Comp}} \to \boldsymbol{pro-Mor}_{\boldsymbol{Pol_f}}$, $Mor_C : \boldsymbol{Mor}_{\boldsymbol{Top}} \to \boldsymbol{inj-Mor}_{\boldsymbol{CW_f}}$ [1].

The chogoshvili definition of projective homology groups also yields the functor $Mor_C : \boldsymbol{Mor}_{\boldsymbol{Comp}} \to \boldsymbol{pro-Mor}_{\boldsymbol{Pol_f}}$.

We prove the following theorems

Theorem 1 *Let $h_* = \{h_{*n}\}_{n\in Z}$ be a homological sequence in the sense of Hu [4] and $h_{*n} : \boldsymbol{Comp} \to \boldsymbol{Ab}$ be an extension of homology functors $H_{*n} : \boldsymbol{Pol_f} \to \boldsymbol{Ab}$, $n \in Z$, which are satisfy the axioms of Hu. If $h_* = \{h_{*n}\}_{n\in Z}$ family satisfies the continuity axiom, then it satisfies all axiom of Hu.*

Theorem 2 *There exists one and only one partially exact homology theory in the sense of Hu [4] on the category $\boldsymbol{Comp}$ of compact Hausdorff spaces with coefficients in abelian group G which satisfies the axiom of identity, composition, homotopy, suspension, dimension, and continuity.*

Theorem 3 *There exists one and only one exact cohomology theory in the sense of Hu on the category $\boldsymbol{Comp}$ of compact Hausdorff spaces with coefficients in abelian group G which satisfies the axioms of identity, composition, homotopy, suspension, dimension, and continuity.*

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On Mazurkiewicz Type Sets

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A set $B \subset R^2$ is called a Mazurkiewicz type set (or n-point set) with respect to the family of straight lines in R^2 if $\operatorname{card}(B \cap l) = n$ for any straight line l lying in R^2.

The famous theorem of Mazurkiewicz [4] initiated a number of research works related to the existence of certain versions of Mazurkiewicz type sets and the study of different aspects of such sets. The analysis of the mentioned type of sets was given from different points of view: geometrical, topological and measure-theoretical (see, e.g., [1–3]).

The talk discusses some aspects of a Mazurkiewicz type sets and presents the following statement.

Theorem 1 *Let r_1 and r_2 be any two distinct positive real numbers. Let F_1 be the family of all circumferences in the Euclidean plane with radius r_1, and let F_2 be the family of all circumferences in the Euclidean plane with radius r_2. Then there exists a subset S of the plane such that S is a Mazurkiewicz type set with respect to the family F_1 and is not a Mazurkiewicz type set with respect to the family F_2.*

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On Some Topological Properties of Luzin Set

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In the paper [2] Keneth Kunen introduced the Luzin spaces and their connections to several set-theoretical objects. We shall call a Luzin space (I) a T_2 space X such that:

(1) Every nowhere dense set in X is countable (i.e., X is a Luzin set in X)

(2) X has at most countably many isolated points.

(3) X is uncountable.

Let us remark, that:

(a) If X is a Luzin set in Y and Y has at most countably many isolated points, then X is a Luzin space;

(b) If X is a Luzin space so is every uncountable subspace;

(c) If X is a Luzin space, X is hereditarily Lindelof.

According to the other definition (see, [1]), a Luzin space (II) is a topological space X such that there exists no nonzero σ-finite Borel measures on X vanishing at all singletons in X.

In the presented talk we will discuss the point sets, which are Luzin spaces (I) and are not Luzin spaces (II), and conversely.

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On Estimation of Sums of Certain Finite Systems of Vectors with Applications

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According to one of our result, if we have m many $x_1, x_2, \ldots, x_m$ radius-vectors in R^n, then the following two statements are true:

1. If
$$\|x_1\| \leq 1, \|x_2\| \leq 1, \ldots, \|x_m\| \leq 1$$
and $0 \in conv\{x_1, \ldots, x_m\}$, then
$$\|x_1 + x_2 + \cdots + x_m\| \leq m - 1;$$

2. If
$$\|x_1\| = 1, \|x_2\| = 1 \ldots, \|x_m\| = 1$$
and $0 \in conv\{x_1, \ldots, x_m\}$, then
$$\|x_1 + x_2 + \cdots + x_m\| \leq m - 2;$$

In the talk some of applications of this result to one discrete optimization problem and one physical problem are presented.

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On the Homological Approach in Burau Representation Theory

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The reduced Burau representation is a natural action of the braid group B_n on the first homology group $H_1(\widetilde{D}_n;\mathbb{Z})$ of a suitable infinite cyclic covering space $\widetilde{D}_n$ of the n–punctured disc D_n. It is known that the Burau representation is faithful for $n \leq 3$ and that it is not faithful for $n \geq 5$. We use forks and noodles homological techniques and Bokut–Vesnin generators to analyze the problem for $n = 4$. We present a Conjecture implying faithfulness and a Lemma explaining the implication. We give some arguments suggesting why we expect the Conjecture to be true [3].

Moreover, using the technique developed by authors, it is reproved that there are braids α and β in the braid group B_4, such that the group $\langle \alpha, \beta \rangle$ is a fee subgroup [5], which contains the kernel K of the Burau map $\rho_4 : B_4 \to GL(3, \mathbb{Z}[t, t^{-1}])$ [1, 3, 4]. Therefore, as a corollary, obtained that the problem of faithfulness of the (reduced) Burau representation for $n = 4$ is equivalent to the problem of whether certain two matrices $A = \rho(\alpha)$ and $B = \rho(\beta)$ generate a free group of rank two. Moreover, we give a simple proof that A^3, B^3 is a free group of rank two [2].

In addition, we have shown that K is subgroup of $G = \langle \tau, \Delta \rangle$, where τ and Δ are fourth and square roots of the generator θ of the center Z of the group B_4. Consequently, we will write elements of K in terms of τ^i, $i = 1, 2, 3$ and Δ. Moreover, we will show that the quotient group G/Z is isomorphic to the free product $Z_4 * Z_2$.

The talk partially is based on joint works with co-author Pawel Traczyk.

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Using Problem Solving to Teach Mathematical Concepts

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The Future Scientists Center in Israel offers unique programs for talented and gifted youth, enabling them to integrate into the exiting world of research and academia, while still in high school.

In the talk I will describe a problem based course, that I prepared and taught in one of the programs, Odyssey, that introduced 9th – grade gifted students to university mathematics. The topics covered in the course included Number Theory, Group Theory, Set Theory, Graph Theory and Matrix Theory.

Problem Solving is an excellent way to teach mathematical concepts. This will be demonstrated by problems on one-to-one maps, permutations and determinants.

Positive Solutions to Semilinear Dirichlet Problems with General Boundary Data

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We give a probabilistic representation of the solution to a semilinear elliptic Dirichlet problem with general (discontinuous) boundary data. The boundary behaviour of the solution is in the sense of the controlled convergence initiated by A. Cornea. Uniqueness results for the solution are also provided.

The talk is based on a joint work with A. Teodor (Bucharest).

Probabilistic Inductive Logic Programming Techniques

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Inductive Logic Programming (ILP) attempts to provide a formal framework for inductively learning a hypothesis from the given positive and negative examples. The obtained hypothesis must *cover* all positive and no negative examples. There are different representation languages for the examples and number of definitions for the *cover* relation. This pair obtains the *setting* for ILP. The most famous settings are: *learning from entailment*, *learning from interpretations*, and *learning from proofs* [1, 2].

The *learning from entailment* is a process where the examples are definite (ground) clauses and a hypothesis to be obtained covers the examples w.r.t. some background theory. In the *learning from interpretations*, the examples are Herbrand interpretations, meaning that they carry out much more information. In practice, learning from interpretations is computationally easier process. Finally, when learning from proofs, the examples are ground proof-trees that might have various forms.

In most of real-world applications it is usually required to deal with uncertainty such as missing or noisy information, but ILP can not do it explicitly. Thus, its extension, Probabilistic ILP was proposed that aims at a formal framework for statistical relational learning. The probabilistic ILP combines together probabilistic reasoning, first-order logic and machine learning.

In this talk we describe how the above mentioned ILP settings can be extended to the probabilistic case. Usually, there are two approaches: either clauses are probabilistic logic formulae, or the cover relation is probabilistic. We elaborate on this matter, explain the basic notations, review some of the key concepts and show the usage by some examples.

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Some Properties of Unranked Probabilistic Logic

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The idea of combining both logical and probabilistic methods into a single framework influenced the development of several formalisms and programming tools. So far, the probabilistic logic formalisms that were studied are either propositional or permit only individual variables. Recently, probabilistic logic theories and systems that use sequence variables and flexible arity (unranked) function and predicate symbols have been emerging.

In this talk we discuss some properties of an unranked probabilistic first-order logic. The syntax of the logic permits unranked symbols and sequence variables, and its semantics is based on the well-known Kripke frames, where accessibility relations are replaced by probability measures.

The most important properties of logic are soundness and completeness. While soundness is "trivial" property, the completeness is a more complicated one. There are usually two forms of completeness: the weak completeness – a formula is consistent iff it is satisfiable; or the strong completeness – a set of formulas is consistent iff it is satisfiable. Clearly, the weak completeness follows from the strong one, but not vice versa. In classical logics these properties are equivalent due to the compactness theorem – a set of formulas is satisfiable iff every finite subset of it is satisfiable. But, according to [3], in probabilistic logics compactness usually fails and even more, the strong completeness is not available in some of them.

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On Optimal Embedding of the Space of Generalized Fractional-Maximal Functions in Rearrangement-Invariant Spaces

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Let $R \in (0;\infty]$. A function $\Phi : (0;R) \to R_+$ belongs to the class $B_n(R)$ if the following conditions hold: Φ decreases and is continuous on $(0;R)$. There exists a constant $C \in R_+$ such that

$$\int_0^r \Phi(\rho)\rho^{n-1}\,d\rho \le C\Phi(r)r^n, \quad r \in (0,R). \tag{1}$$

Let $\Phi : R_+ \to \mathbb{R}$. The generalized fractional-maximal function $M_\Phi f$ is defined for the function $f \in E(\mathbb{R}^n) \cap L_1^{loc}(\mathbb{R}^n)$ by the equality $(M_\Phi f)(x) = \sup\limits_{r>0} \Phi(r) \int\limits_{B(x,r)} |f(y)|\,dy$. Let E is rearrangement-invariant space (briefly: RIS [1]). The space of generalized fractional maximal functions $M_E^\Phi = M_E^\Phi(R^n)$ as the set of the functions u, for which there is a function $f \in E(R^n)$ such that $u(x) = (M_\Phi f)(x)$, $\|u\|_{M_E^\Phi} = \inf\{\|f\|_E :\ f \in E(R^n),\ M_\Phi f = u\}$. The cones: $K_1 := \{h \in L^+(\mathbb{R}_+) :\ h(t) = u^*(t),\ t \in \mathbb{R}_+,\ u \in M_E^\Phi\}$.

Theorem 1 *Let $\Phi \in B_n(\infty)$. The embedding $M_E^\Phi(\mathbb{R}^n) \hookrightarrow X(\mathbb{R}^n)$ is equivalence to the next embedding $K_1 M_E^\Phi(\mathbb{R}_+) \mapsto \widetilde{X}(\mathbb{R}_+))$.*

Theorem 2 *Let $\Phi \in B_n(\infty)$. The optimal RIS $X_0 = X_0(\mathbb{R}^n)$ for embedding (1) is defined by following norm:*

$$\|f\|_{\widetilde{X}_0(0,\infty)} = \sup_{g^*} \left\{ \int_0^\infty f^*(\tau)g^*(\tau)\,d\tau :\ g \in L_0(0,\infty), \sup_{\int_0^t h(s)\,ds \le \int_0^t g^*(s)\,ds} \left\| \int_t^\infty \Phi(s^{1/n})sh(s)\,ds \right\|_{E'} \le 1 \right\}.$$

Note that in the [2, 3]*, the generalized Riesz potential was considered.*

References

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Compactness of the Commutator of the Multilinear Calderon–Zygmund Singular Integral Operator in Global Morrey-Type Spaces

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In this paper we give sufficient conditions for compactness of Commutators for singular integrals $[b,T]$ from global Morrey-type space $GM^{w_1(\cdot)}_{p,\theta}$ to $GM^{w_2(\cdot)}_{p,\theta}$.

Let $1 \le p \le \infty$, w be a measurable non-negative function on $(0,\infty)$ not equivalent to zero. The global Morrey-type space $GM^{w(\cdot)}p,\theta \equiv GM^{w(\cdot)}_{p,\theta}(\mathbb{R}^n)$ is defined as the set of all functions $f \in L^{loc}_p(\mathbb{R}^n)$ with a finite norm: $\|f\|_{GM^{w(\cdot)}_{p\theta}} \equiv \sup_{x\in R^n}\big(\int\limits_0^\infty |w(r)\|f\|_{L_p(B(x,r))}|^\theta\, dr\big)^{\frac{1}{\theta}} < \infty$, where $B(x,r)$ is an open ball centred at the x origin with radius $r>0$.

Let $S^{n-1} = x \in \mathbb{R}^n :\ |x| = 1$ be the unit sphere in $\mathbb{R}^n$ with the area measure $d\sigma$. Suppose that Ω satisfies the following conditions:

(i) Ω is a homogeneous function of degree zero on $\mathbb{R}^n\{0\}$, i.e., $\Omega(\mu x) \equiv \Omega(x)$ for any $\mu > 0$ and $x \in \mathbb{R}^n\{0\}$.

(ii) Ω has mean zero on S^{n-1}, i.e., $\int\limits_{S^{n-1}} \Omega(x')\, d\sigma(x') = 0$

(iii) $\Omega \in Lip(S^{n-1})$, i.e., $|\Omega(x') - \Omega(y')| \le |(x') - (y')$ for any $(x'),(y') \in S^{n-1}$.

Moreover, here and in the sequel, we assume that $\Omega \neq 0$. The multilinear Calderon–Zygmund singular integral operator T is defined as follows:

$$T^{(m)}f(x) = \int_{\mathbb{R}^n} \frac{\Omega(x-y_1)\Omega(x-y_2)\cdots\Omega(x-y_m)}{|(x-y_1)+(x-y_2)+\cdots+(x-y_m)|^{mn-\alpha}}\, f_1(y_1)f_2(y_3)\cdots f_m(y_m)\, dy.$$

For a function $b \in L_{loc}(\mathbb{R}^n)$, let M_b denote the multiplication operator $M_bf = bf$, where f is a measurable function. Then, the commutator for multilinear Calderon–Zygmund singular integral operator T and the operator M_b is defined by:

$$[b,T^{(m)}](f)(x) = \int_{\mathbb{R}^n} \frac{[b(x)-b(y_1),\ldots,b(x)-b(y_m)]\Omega(x-y_1)\ldots\Omega(x-y_m)f_1(y_1)\ldots f_m(y_m)}{|(x-y_1)+(x-y_2)+\cdots+(x-y_m)|^{mn-\alpha}}\, dy.$$

By $VMO(\mathbb{R}^n)$ we denote the BMO-closure $C^\infty_0(\mathbb{R}^n)$, where $C^\infty_0(\mathbb{R}^n)$ the set of all functions from $C^\infty(\mathbb{R}^n)$ with compact support.

Theorem 1 *Let $1 < p < \infty$, $1 < \theta_1, \theta_2 < \infty$, $b \in VMO(\mathbb{R}^n)$, functions $w_1 \in \Omega_{p,\theta_1}$, $w_2 \in \Omega_{p,\theta_2}$ satisfy the condition*

$$\int_r^\infty \ln\Big(e + \frac{t}{r}\Big)\, \frac{ess\inf\limits_{t<s<\infty} w_1(s)}{t}\, dt \le Cw_2(r).$$

Let the operator $[b,T^{(m)}]$ is bounded on $L_p(\mathbb{R}^n)$. Then the commutator $[b,T^{(m)}]$ is a compact operator global Morrey-type space from $GM^{w_1(\cdot)}_{p,\theta}$ to $GM^{w_2(\cdot)}_{p,\theta}$.

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On Nilpotent Power MR-Groups

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The notion of a power MR-group, where R is an arbitrary associative ring with unity, was introduced by R. Lyndon, A. G. Myasnikov and V. N. Remeslennikov gave a more precise definition of an R-group by introducing an additional axiom. In particular, the new notion of a power MR-group is a direct generalization of the notion of an R-module to the case of noncommutative groups. In the present paper, central series and series of commutants in MR-groups are introduced. Three variants of the definition of nilpotent power MR-groups of step n are discussed. It is proved that, for $n = 1, 2$, all these definitions are equivalent. The question on the coincidence of these notions for $n > 2$ remains open. Moreover, it is proved that the tensor completion of a 2-step nilpotent MR-group is 2-step nilpotent.

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Numerical Solution and Applicability of β-Conformable Time Fractional Derivative Equations with Artificial Neural Networks

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The neural network method for solving differential equations has recently achieved significant advances in solving fractional differential equations. ANN is a mathematical model inspired by biological nervous systems. In this model, the loss function comprising initial-boundary conditions is produced using changeable parameters (weights and deviations). A space fractional differential equation is also expressed as an optimization problem in this article. To demonstrate the accuracy of the method utilized, numerical examples with proven analytical solutions are described, including numerical results, graphs, weights, and biases. The graphical and tabular results are also thoroughly examined. The average squared errors for various neuron, learning rate, and training steps values are shown nential function method.

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On the Issue of Teaching Irrational Equations

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An irrational equation is one equation that contains a variable (unknown) under the sign of a radical. An irrational equation may have a finite number of solutions, may have no solution at all, or may have an infinite number of solutions. All radicals (roots) in an irrational equation are treated as arithmetic, and all solutions represent numbers from the set of real numbers. When solving an irrational equation, it is necessary to ensure that the equivalence condition of the equations is not violated, an additional (extra) root does not appear, or the roots are not lost.

The main goal of teaching irrational equations is to familiarize students with the definition of an irrational equation, the domain of an equation, the concept of an extraneous root, and the methods of solving irrational equations: reducing an irrational equation to a rational one, sorting, introducing a new variable, and solving special cases. Care must be taken when making transformations; in order not to lose the roots, it is necessary to check the obtained roots. Roots should be checked in the given equation, not in any intermediate one.

At first, students should get used to solving the simplest irrational equations.

Equations of the type of $\sqrt{A(x)} = B(x)$ and $\sqrt{A(x)} = \sqrt{B(x)}$ belong to the simplest irrational equations.

On q-Sturm–Liouville Operator with Periodic Boundary Conditions

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In this study, we consider q-Sturm–Liouville operator

$$l(y) := \frac{-1}{\mathfrak{q}} D_{\mathfrak{q}^{-1}} D_{\mathfrak{q}} y(x) + r(x)y(x) = \lambda y(x), \quad x \in [0, b],$$

subject to the periodic boundary conditions

$$V_1(y) = y(b) - y(0) = 0,$$
$$V_2(y) = D_{\mathfrak{q}^{-1}} y(b) - D_{\mathfrak{q}^{-1}} y(0) = 0$$

with periodic boundary conditions. where $\lambda \in \mathbb{C}$ is called an eigenvalue, $r(x) \in C([0, b))$ is real-valued function. An asymptotic expression of the solution is obtained. With the help of this asymptotic representation, an asymptotic solution of the characteristic equation is presented. An application of the Rouche theorem, asymptotic expressions of eigenvalues are obtained.

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Rearrangement Universality of the Dirichlet Type Series

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In paper [1] we showed that for any complex number $s, 0 < \mathrm{Re}\,(s) \leq 1$, the Dirichlet series

$$\sum_n \frac{1}{n^s} \text{ and } \sum_n \frac{(-1)^{n-1}}{n^s}$$

are universal in $\mathbb{C}$, i.e. their sum ranges under the rearrangements coincide with the whole complex number field $\mathbb{C}$.

In this talk we plan to discuss the question of validity of a similar result for the Dirichlet type series

$$\sum_n \frac{1}{\lambda_n^s} \text{ and } \sum_n \frac{(-1)^{n-1}}{\lambda_n^s},$$

where $(\lambda_n)_{n\in\mathbb{N}}$ is an increasing sequences of positive real numbers with some additional growth property.

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Mathematical Models of the Propagation of World Religions

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Mathematical modeling of social and political processes, such as globalization, assimilation of peoples, bipolar or multipolar arrangement of the world, is of significant scientific and practical interest [1–4]. These processes are greatly influenced by the spread and interaction of the main world religions: Christianity (33,9 %), Islam (22,7 %), Hinduism (13,9 %), Budism and Chinese folk religions (13,2 %). At the same time, these religions and their carriers, in addition to mutual operations, are subjected to great open and secret pressure from atheists (11,5 %), who, as a rule, own the main financial and political resources. The remaining 4,8 % of the world's population are adherents of small religions that do not have a significant impact on world processes.

As the first mathematical model, a nonlinear three-dimensional dynamic system is considered, describing the interaction of the two main world religions of Christianity and Islam, the dynamics of the number of their carriers, as well as the impact on them of powerful groups consisting of atheists united in various communities, including transnational consortia affecting world processes.

In two special cases of constant coefficients of a nonlinear dynamic system, the first integrals were found and the three-dimensional dynamic system was reduced to a two-dimensional one, which in some cases was investigated.

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Mathematical Model of Detonation Wave Propagation in a Nonhomogeneous Gravitational Gas Sphere

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The mathematical modeling of astrophysics processes are one of the most actual problems of modern applied mathematics. To solve many problems of astrophysics, it is necessary to study the dynamics of gaseous bodies interacting with a gravitational field [1–3]. The work considers a nonavtomodel problem about the central explosion of nonhomogeneous gas body (linear function of star density drop from central core to the surface) bordering vacuum, which is in equilibrium in its own gravitational field. We have previously proposed an asymptotic thin impact layer method that is used to solve the problem [4]. The solution of the problem in the vicinity behind the detonation shock wave (the fracturing surface of the first kind) is sought in the form of a singular asymptotic decomposition by a small parameter. Analytically, the main (zero) approximation for the law of motion and the thermodynamic characteristics of the medium was accurately found. The Cauchy problem for zero approximation of the law of motion of the detonation shock wave is solved exactly, in the form of elliptic integrals of the first and second kind. The asymptotics of the zero approximation of the law of the detonation shock wave is found at the moment and during the time coming on the surface of the object. Also the time of coming on the surface is found.

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On One Nonlinear Fourth-Order Integro-Differential Parabolic Equation

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In [1] reduction of the well-known Maxwell system [5] of differential equations to the form of second order integro-differential equations was first performed. Some qualitative and structural properties of solutions of such type integro-differential models are established in many works. For more detail information see, for example, [2, 4] and references therein. The presented work discusses a natural mathematical generalization of the scalar analog of the above-mentioned integro-differential model. In particular, the following fourth-order integro-differential equation is investigated

$$\frac{\partial u}{\partial t}+\frac{\partial^2}{\partial x^2}\left\{\left[1+\left(\int\limits_0^t\left(\frac{\partial^2 u}{\partial x^2}\right)^2 d\tau\right)\frac{\partial^2 u}{\partial x^2}\right]\right\}=0.$$

The stability and uniqueness of the solution of the corresponding initial-boundary value problem is investigated. Finite-difference scheme is constructed and studied. Numerical solutions were approximated using machine learning algorithms [3, 6].

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A Universal Design of Teaching and Differentiated Approach

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The primary objective of the national curriculum is to steer the student-centered learning process, with the student being the focal point of teaching and learning. Learning should be an active process of knowledge construction. Teachers should base all their decisions on the needs of the students. As stated by NC, "Every student is unique with their individual physical and mental characteristics, abilities, emotions, interests, personal experiences, academic needs, and learning styles, which must be taken into account during teaching and learning. Learning should occur in a positive and orderly environment."

This paper underscores the importance of teachers mastering effective teaching-learning strategies, understanding the concepts of Universal Design for Learning (UDL) and a differentiated approach, and recognizing the similarities and differences between them. It is crucial that teachers do not conflate UDL with a differentiated approach, as emphasized in materials from the US-AID Education Program. Examples of UDL and differentiated approaches in the teaching-learning process are provided.

Top Quark Rare Decays in the BSM Scenarios

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Success of the Standard Model (SM) do not weaken theoretical arguments in favour of New Physics (NP), which is anticipated at the TeV scale. There are profound experimental and theoretical reasons to think that the SM is incomplete. One the experimental side, these are very small but non-vanishing neutrino masses, the presence of dark matter, dark energy and baryon asymmetry of the universe. Theoretical problems include the gauge hierarchy problem, replication of fermion families and hierarchical pattern of fermion mass matrix, CP-conservation in QCD. The task to find and identify NP seems to us to be as a most important challenge for High Energy Physics. Many people in scientific society await new discovery beyond the SM (BSM) at the scale around 1 TeV. Before showing in the direct production processes, BSM effects could manifest themselves in the rare decays induced by flavour changing neutral (FCNC) transitions. The importance of such a possibility is hard to be overestimated. To differ various NP scenarios, we are in need to investigate their influence on the aromatodynamics. FCNC processes are strongly suppressed in the SM. In the quark sector FCNC processes first arise on the one loop level and are suppressed by Glashow-Iliopoulos-Maiani (GIM) mechanism [2]. Some of such processes are experimentally observed and are in quite good agreement with the SM predictions. As for top quark sector, the FCNC processes are very small in the SM. So, even single experimental observation for any top rare processes could be breakthrough beyond the SM physics. In some extensions of the SM there could be several mechanisms for enhancements of FCNC processes, including top quark FCNC phenomena. One of such mechanism could be arisen in case whem GIM suppression changes its quadratic nature by linear one [3]. This happens when particles running in the loops have comparable masses. Such a situation is realized in the universal extra dimension (UED) models [1]. In the presented paper we have estimated additional to the standard model (SM) contributions to the top quark FCNC decays in the framework of the model with one extra dimension. The estimates of performed calculations show that it is possible to find extra dimension's trace in the top quark rare FCNC decays.

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Transmission Problems for a Second Order Differential Equation on a Hypersurface with Lipschitz Boundary in the Generic Bessel Potential Spaces

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We are given a hypersurface $\mathscr{C} \subset \mathbb{R}^3$ with the Lipschitz boundary $\Gamma := \partial\mathscr{C}$, containing angular points $c_1, \dots, c_n$. The surface is divided by a finite number of curves $\mathscr{T}_1, \dots, \mathscr{T}_m$ in non-intersecting domains $\mathscr{C}_1, \dots, \mathscr{C}_{m+1}$ and in each domain $\mathscr{C}_k$ is given Laplace–Beltrami equation with lower order perturbations $\Delta_{\mathscr{C}} u + \mathbf{P}_k(\mathscr{D})u = f_k$, $k = 1, \dots, m+1$. The Dirichlet, Neumann and mixed type BVPs are considered on the outer boundary Γ, while on curves $\mathscr{T}_1, \dots, \mathscr{T}_m$ are prescribed transmission conditions. The BVP is treated in a non-classical setting, when solutions are sought in the generic Bessel potential spaces with exponential weights $\mathbb{GH}^s_p(\mathscr{C}, \rho)$, $s > 1/p$, $1 < p < \infty$,

$$\rho(t) := \prod_{j=1}^{n} |t - c_j|^{\beta_j}.$$

First we get rid of transmission conditions and transmission curves and reduce problem to the Fredholm equivalent Boundary Integral Equation (BIE) on the boundary of the surface $\Gamma = \partial\mathscr{C}$. Second we apply the localization and reduce the obtained BIE to the investigation of the Model BIE corresponding to Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domains $\Omega_{\alpha_j} \subset \mathbb{R}^2$, $j = 1, 2, \dots, n$, associated to the angular points $c_1, c_2, \dots, c_n$. Third is investigated the model BIE in the generic Bessel potential spaces with weight $\mathbb{GH}^s_p(\Omega_{\alpha_j}, t^{\beta_j})$. For this we reduce further the BIE to a Fredholm equivalent Mellin convolution integral equations in the generic Bessel potential spaces on a semi-infinite axes with weight $\mathbb{GH}^{s-1/p}_p(\mathbb{R}^+, t^{\beta_j})$. Explicit criteria of Fredholm property of the initial BVPs are obtained. In contrast to the same BVPs in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathscr{C})$, the Fredholm property in the generic Bessel potential spaces $\mathbb{GH}^s_p(\mathscr{C}, \rho)$ with weight is independent of the smoothness parameter s. We also list explicit singularities of solutions to the mixed-transmission BVP in the neighbourhood of knots, where boundary has angular points or Dirichlet–Neumann boundary conditions collide.

The presentation is based on the joint work with Medea Tsaava.

Fuzzy Extension of Lambda Calculus

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In this talk we propose a fuzzy extension of the lambda calculus [1] to support approximate reasoning. In particular, we define fuzzy types for the terms, which means terms can have types with some degree $d \in [0, 1]$ and introduce typing rules. Such a type system allows term reduction with some degree $d \in (0, 1]$ and naturally generalizes pure lambda calculus where reduction is only allowed when the fuzziness degree between types is 1. Then we show that fuzzy types are preserved by reduction (subject reduction), We also prove, that the lambda calculus with fuzzy types enjoys confluence property,

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The Solar Wind – Earth Magnetic Field Interactions and its Effect on the Climate Change

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Earth's climate is determined by complex interactions among the Sun, oceans, atmosphere, cryosphere, land surface and biosphere. The Sun is the principal driving force for Earth's weather and climate. The influence of solar activity on the Earth's global surface is determined due to temperature variation, which in turn drives the instabilities and is expressed via turbulent effects. Standard approaches to identify such connections are often based on correlations between the appropriate time series. Here we present a novel method Granger causality, which can infer/detect relationships between any two fields. We compare Solar activity – climate connections via magnetic turbulence identified by correlation and Granger causality at different timescales.

Algorithm for Wiener–Hopf Factorization of Polynomial Matrices with Monomial Determinant

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For a polynomial $r \times r$ matrix function

$$S(t) = \sum_{k=0}^{N} C_k t^k, \tag{1}$$

$C_k \in \mathbb{C}^{r\times r}$, $k = 0, 1, \dots, N$, its (right) Wiener-Hopf factorization is called the representation (see, e.g., [2]):

$$S(t) = S_+(t) \begin{pmatrix} t^{\varkappa_1} & 0 & \cdots & 0 \\ 0 & t^{\varkappa_2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & t^{\varkappa_r} \end{pmatrix} S_-(t),$$

where

$$S_+(t) = \sum_{k=0}^{N} A_k t^k, \quad S_-(t) = \sum_{k=0}^{N} B_k t^{-k},$$

$A_k, B_k \in \mathbb{C}^{r\times r}$, $k = 0, 1, \dots, N$, $\det S_+(t) \neq 0$ for $t \in \mathbb{T}_+ = \{z \in \mathbb{C} : \ |z| < 1\}$, $\det S_-(t) \neq 0$ for $t \in \mathbb{T}_- = \{z \in \mathbb{C} : \ |z| > 1\} \cup \{\infty\}$, and $\varkappa_j \in \mathbb{Z}$, $j = 1, 2, \dots, r$. The latter integers are called (right) partial indices of S.

We propose a new algorithm for the Wiener–Hopf factorization of polynomial matrices (1) which have monomial determinants: $\det S(t) = t^n$, $n \in \mathbb{N}$. The algorithm for 2×2 matrices has been reported in [1].

Acknowledgments

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Boundary Contact Problems with Regard to Friction of Couple-Stress Viscoelasticity for Inhomogeneous Anisotropic Bodies (Non-Coercive, Quasi-Static Cases)

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Quasi-statical boundary contact problems of couple-stress visco-elasticity for inhomogeneous anisotropic bodies with regard to friction are investigated. The non-coercive case is considered when a friction force acts on the entire boundary of a viscoelastic body. The posed boundary-contact problem is equivalently reduced to a spatial variational inequality. After factorization of the main spaces with respect to the vector space of the function of generalized rigid displacements, the necessary condition for the existence of solutions is easily obtained. this condition, under some additional assumptions, becomes a sufficient condition for the existence of solutions. It is easy to prove that the form of energy, to which we add a term depending on the small parameter ε, after which it becomes strictly coercive. After obtaining some a priori estimates, we move to the limit, when ε approaches zero. The limit function satisfies the conditions of the boundary-contact problem. In the main spaces, the problem has an infinite number of solutions, each two of which differ in the vector of the generalized rigid displacement.

The Best Intelligent Systems for Idea Management and their Practical Integration

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In an era where innovation and knowledge are critical to competitive advantage, effective idea management systems (IMS) are essential for promoting creativity, capturing key insights, and easing strategic decision-making [2]. The paper examines the landscape of intelligent systems developed for concept management, with a focus on their architecture, major features, and the technologies that power them, such as artificial intelligence (AI), machine learning (ML), and natural language processing [3]. We present a complete evaluation of innovative IMS, assessing their benefits, limits, and practical obstacles connected with their implementation. We explain how organizations may smoothly integrate these solutions into their existing workflows using case studies and empirical data, thereby improving cooperation, innovation, and productivity. Our findings indicate that the successful integration of intelligent IMS is dependent not just on technological competence, but also on the system's alignment with corporate culture and practices [1]. The paper finishes with best practices and advice for organizations looking to implement and integrate advanced IMS, emphasizing how these systems may shift idea management from a reactive to a proactive and strategic function.

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On Approximate Solution of One Two-Dimensional System of Nonlinear Partial Differential Equations

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The system of nonlinear partial differential equations, that arises in the process of vein formation of young leaves [7] is considered. Investigation and numerical solution of such type systems are discussed in many papers (see, for example, [1–6] and references therein). In our note, two methods [2, 3] of numerical solution of corresponding initial-boundary value problems are used and comparison of numerical experiments of these methods is done [4].

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A Characterisation of Lie Algebras Using Ideals and Subalgebras

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Inspired by the categorical characterisation of Lie algebras amongst all varieties of non-associative algebras using *algebraic exponents* [2], the following result was proved in [1]:

Theorem 1 *Suppose that $\mathfrak{M}$ is a non-trivial variety of non-associative algebras over a field of zero characteristic satisfying the following two conditions:*

- *every subalgebra of every free algebra is free,*
- *for every ideal I in every algebra, I^2 is also an ideal.*

Then $\mathfrak{M}$ is the variety of Lie algebras.

A variety satisfying the first condition is called *Nielsen-Schreier*, whilst a variety satisfying the second condition is said to be a 2-*variety*.

In this talk we will first focus on giving a wide and intuitionistic idea about this result, together with its motivation and origins. Then, we will explain the methods used to prove it, which include homological and computational algebra, together with Gröbner bases for operads.

Joint work with Vladimir Dotsenko (Université de Strasbourg).

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Quantum Chromodynamics of Nucleons in the Representation of Complex Probabilistic Processes

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Shortly after the postulation of quarks by Zweig and Gell–Mann in 1964 and the experimental confirmation of these subnucleon formations [1,2] in 1969, Feynman, Ravndal and Kislinger (FRK) in 1971 proposed a relativistic three-quark model of nucleons [3] to study the internal structure and state of these dynamical systems and explain a number of thier important properties. The idea of this model is that all three quarks effectively interact with each other through the potential of a four-dimensional harmonic oscillator, which does not allow the system to decay into individual free quarks. Despite the obvious progress in describing the internal motion of nucleons within the framework of this model, we have to state that it is not realistic enough, since continuous processes of gluon exchange between quarks are ignored. To overcome this difficulty, we considered the problem of self-organization of a dynamical quark system immersed in a color gluon thermostat in the framework of the mathematical representation of a complex probabilistic process [4] that satisfies an equation of the Kline–Gordon–Langevin type. Obviously, this is a natural and essential generalization of the FRK nucleon model. In this work, using the hidden symmetry of the internal motion of a dynamical system, we have developed a mathematical approach that allows us to construct the wave function of the nucleon in the form of a two-fold integral representation, taking into account the relaxation of the quark system in a color gluon thermostat. It should be noted that the generalized model can be especially useful for studying the state of nucleons under critical conditions, which takes place, for example, in massive and dense stellar formations, such as neutron stars, etc.

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From the History of Trigonometric Functions

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The word "trigonometry" comes from the Greek words "triangle", and "measuring". The literal meaning is "the science of measuring triangles". The term appears for the first time in the title of the book of the German theologian and mathematician Bartholomeus Pitiscus, "Trigonometria sive de solution trangularum tractatus brevis et precpicuus" (1595). Science itself was formed in ancient times, together with astronomy.

The earliest trigonometric function was the chord (this was in the second century BC), which corresponded to an arc. It was for this function that the first trigonometric tables, which were needed in astronomy, were built.

In the V-XII centuries, Indian mathematicians and astronomers, for the first time in history, considered half of a chord instead of a full chord, which corresponds to the modern understanding of the concept of sine. They called the size of half of the chord "Arkajiva", which meant half of the bowstring; that is, the "Arkajiva" was $\sin x$. In addition to $\sin x$, the Indians considered the value $1 - \cos x$, which they called the "Komajiva", as well as the value $\cos x$, the "Kotijiva".

When the Arabs translated the books from Sanskrit, the "Arka" was removed from the "Arkajiva" and the "Jiva" remained, and the word "Jiva" was matched with "Jiba", but due to the peculiarities of the Arabic language, it was pronounced as "Jaib", and the "Jaib" literally meant "bosom". When translating from Arabic to Latin, Europeans matched the word "jaib" with the Latin word "sinus", which has the same meaning. As for trigonometric functions such as tangent, cotangent, secant, and cosecant, they were strictly defined by the Iranian mathematician Abul Wafa. He considered these functions on the trigonometric circle.

The modern names of these functions were introduced by European mathematicians in the XV-XVII centuries. In particular, the term "tangent" (in Latin, "tangent" as well) was introduced by Regiomontani (Johann M ller) in the 15th century. In the 16th century, Thomas Finke introduced the term "secant", and in the 17th century, Edmund Guenther introduced the words "cosine" and "cotangent" (the prefix "co" denotes complementum).

The modern notations $\sin x$ and $\cos x$ for sine and cosine were introduced in 1739 by Johann Bernoulli, and Leonard Euler finally established them in mathematics. The term "trigonometric functions" was introduced by Georg Simon Klugel in 1770.

In today's school, a big place is given to trigonometry in both the geometry and algebra courses. It is necessary that the students, instead of memorizing the trigonometric formulas, fully master their content and be certain about the essence of these formulas.

Some Functional Properties of Generalized Solutions of Two-Dimensional Elliptic System

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On the plane $\mathbb{C}$ of the complex variable z, the elliptic system of Carleman–Bers–Vekua equations is considered, the complex form of which is

$$\frac{\partial w}{\partial \overline{z}} + aw + b\overline{w} = 0, \quad \partial \overline{z} \equiv \frac{1}{2}\left(\frac{\partial}{\partial x} + i\,\frac{\partial}{\partial y}\right) \tag{1}$$

(see[1, 2]). Here a and b are fixed (given) coefficients. The solution will be understood in a generalized sense. The functional properties of the solutions of this equation are determined by its coefficients. We consider regular coefficients a and b, that is, it is implied that $a, b \in L_{p,2}(\mathbb{C})$, $p > 2$. It is known that under the mentioned conditions, the functional properties of the solutions of equation (1) are almost identical to the classical case, that is, many principles of classical complex analysis are preserved in one or another modified form for the solution of the equation (1) In addition, it should be emphasized that some principles of classical complex analysis are still not known for equation (1). In this paper, the main focus is on the following issue: for the solution of the equation (1), the so-called the concept of rating is introduced; rating is a real non-negative number or ∞, based on the rating the structure of the generalized analytic function is determined in the neighborhood of the point at infinity. Some characteristic properties of generalized analytic functions of finite rating are determined and that among the solutions of finite rating there are no solutions with in some sense exotic structure, which are among the solutions of infinite rating.

Acknowledgments

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Boundary Value Problem of Statics of the Second Gradient Theory of Thermoelasticity for a Ball

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The static case of the second gradient theory of thermoelasticity is considered. The representation formula of the general solution of a homogeneous system of differential equations of the second gradient theory of thermoelasticity is obtained using four harmonic and four metaharmonic functions. Using this formula, an explicit solution to the Dirichlet type boundary value problem of the second gradient theory of thermoelasticity for a ball is constructed in the form of an absolutely and uniformly convergent series.

On Sub-Gaussianity in Finite and Infinite-Dimensions

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We compare the existing concept of sub-Gaussian random elements and point out some, still un-answered, questions.

Our talk is based mainly on [1] and [2].

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On the Precision of Mathematical Concepts' Defining

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When teaching mathematics or working on mathematical texts, it is especially important to clearly formulate the content of concepts, to define the concept, so that the mathematical theory built on it does not face opposition during the further use of this concept.

For example, by the various definitions "polygon is a closed two-dimensional figure composed of strait line segments that meet at their endpoints". This definition does not excludes the case, when adjacent sides of the polygon lie on the same straight line. But this case must be excluded.

On the other hand, when determining polygonal chains (broken lines), the presence of adjacent sides of the chain lie on the same straight line is often excluded, while when determining the length of a certain curve, the case when a polygonal chain inscribed in a curve may have just such sides is not excluded.

We analyze definitions of some other mathematical concepts and the existence of corresponding possible difficulties.

Ptolemy's Theorem for Cyclic Quadrilaterals and its Consequences

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Ptolemy's Theorem/Inequality establishes a relationship between the product of the lengths of the diagonals of any convex quadrilateral and the sum of the products of the lengths of the opposite sides of the same quadrilateral. Although this theorem/inequality is not encountered by students in the standard school course, we think that if we introduce the aforementioned theorem and its inverse to students of VIII-IX-X classes of public or private schools and solve some relatively simple practical problems related to this topic, we will show them how conveniently it will be good to be able to solve Ptolemy's Theorem/Inequality quite difficult problems, because it is likely that this activity will cause positive emotions and additional interest in them.

Our article presents the essence of Ptolemy's Theorem/Inequality and provides several ways to prove it. Some interesting practical tasks are also discussed. For example, how we can demonstrate the truth of the Pythagorean theorem in right triangles in a different way, showing students that the distance from any point of the circumcircle of an equilateral triangle to any one vertex of the triangle is equal to the sum of the distances to the other two vertices, as well as how we can determine the diagonal and the ratio of side lengths, how to prove the truth of interesting trigonometric equations in a regular heptagon, etc.

We hope that our article will be interesting and useful for both mathematics teachers and students.

About Faber–Krahn Inequality for Divergence Form Elliptic Operators

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We obtained estimates for first eigenvalues of the Dirichlet boundary value problem for elliptic operators in divergence form (i.e. for the principal frequency of non-homogeneous membranes) in bounded domains satisfying quasihyperbolic boundary conditions. The suggested method is based on the quasiconformal composition operators on Sobolev spaces and their applications to constant estimates in the corresponding Sobolev–Poincaré inequalities. We also prove a variant of the Rayleigh–Faber–Khran inequality for a special case of these elliptic operators.

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Connection Formulas Between Mersenne and Fibonacci Numbers using Trudi's Formula

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In this note, we evaluate determinants of several families of Toeplitz–Hessenberg matrices having various translates of the Mersenne numbers as their nonzero entries. By Trudi's formula, these determinant identities we write equivalently as identities with multinomial coefficients (see [1–3] for more details).

Let $(F_n)_{n\geq 0}$ be the Fibonacci sequence satisfying the recurrence relation $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$, with initial conditions $F_0 = 0$, $F_1 = 1$. Let $(M_n)_{n\geq 0}$ be the sequence of Mersenne numbers defined by $M_n = 2^n - 1$ for $n \geq 0$.

In Theorem 1, we present formulas that describe the relationship between Mersenne and Fibonacci numbers.

Theorem 1 *If $n \geq 1$, then*

$$\sum_{\sigma_n=n} s_n(t) M_0^{t_1} M_1^{t_2} \cdots M_{n-1}^{t_n} = F_{2(n-1)},$$

$$\sum_{\sigma_n=n} s_n(t) M_2^{t_1} M_3^{t_2} \cdots M_{n+1}^{t_n} = 2^{n-1} F_{2(n+1)},$$

$$\sum_{\sigma_n=n} \frac{s_n(t)}{(-3)^{|t|}} M_0^{t_1} M_2^{t_2} \cdots M_{2(n-1)}^{t_n} = -\sum_{k=0}^{n-1} \binom{n-1}{k} F_{2k},$$

$$\sum_{\sigma_n=n} \frac{s_n(t)}{3^{|t|}} M_2^{t_1} M_4^{t_2} \cdots M_{2n}^{t_n} = 2^{n-1} F_{2n},$$

$$\sum_{\sigma_n=n} \frac{s_n(t)}{(-3)^{|t|}} M_4^{t_1} M_8^{t_2} \cdots M_{4n}^{t_n} = -5 \cdot 4^{n-1} F_{2n},$$

$$\sum_{\sigma_n=n} \frac{s_n(t)}{(-7)^{|t|}} M_0^{t_1} M_3^{t_2} \cdots M_{3(n-1)}^{t_n} = -\frac{1}{3} \sum_{k=0}^{n-1} \binom{n-1}{k} F_{4k},$$

where

$$\sigma_n = t_1 + 2t_2 + \cdots + nt_n, \;\; |t| = t_1 + \cdots + t_n, \;\; \text{and} \;\; s_n(t) = \frac{|t|!}{t_1! \cdots t_n!}$$

with $t_i \geq 0$.

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Decidability of Generalized 3-valued Post Algebras

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We introduce the new class $\mathbf{P}_{\mathbf{3}}^{\omega}$ of generalized 3-valued Post algebras that form a variety. An algebra $(A, \vee, \wedge, \oplus, \odot, \neg, 0, \frac{1}{2}, 1)$ is P_3^{ω}-algebra if $(A, \oplus, \odot, \neg, 0, 1)$ is an S_2^{ω}-algebra defined by Komori [1], i.e. MV-algebra satisfying the identity $(3(x^2))^2 = 2(x^3)$, $(A, \vee, \wedge, 0, 1)$ is a distributive bounded lattice, satisfying the following identities:

$$\frac{1}{2} \oplus \frac{1}{2} = 1, \quad \frac{1}{2} \odot \frac{1}{2} = 0, \quad \frac{1}{2} \odot (x \wedge \neg x) = 0,$$
$$\frac{1}{2} \oplus (x \vee \neg x) = 1, \quad \neg \frac{1}{2} = \frac{1}{2}.$$

Notice, that the $(0, \frac{1}{2}, 1, \vee, \wedge, \oplus, \odot, \neg, 0, \frac{1}{2}, 1))$ is an example of generalized 3-valued Post algebra with the following operations:

$$x \vee y = \max(x, y), \quad x \wedge y = \min(x, y),$$
$$x \oplus y = \min(1, x + y), \quad x \odot y = \max(0, x + y - 1), \quad \neg x = 1 - x,$$

which is functionally equivalent to the 3-element Post algebra P_3. Indeed, it is enough to express the cyclic negation

$$\sim x = \left(\frac{1}{2} \odot x\right) \vee (\neg x \odot \neg x).$$

We develop the theory of generalized S_2^{ω}-algebras, that are a subvariety of MV-algebras, with the additional constant $\frac{1}{2}$. The equational definition of generalized 3-valued Post algebras is given. A dual category of generalized 3-valued Post algebras is described. It is shown that the theory of generalized 3-valued Post algebras is decidable.

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Two-Scale Topology Optimization – Modeling, Analysis and Numerical Results

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Subject of my presentation is a novel approach for optimizing both the macroscopic shape and the porous mesoscopic structure of components. In the first part of my presentation I will introduce the concept of phase field based topology optimization. The second part of my presentation is devoted to two-scale topology optimization. The key feature here is the introduction of an additional local volume control (LVC), which allows to adjust the desired spatial scales. The main novelty is that the radius of the LVC may depend both on space and a local stress measure. This allows for creating optimal topologies with heterogeneous mesostructures enforcing any desired spatial grading and accommodating stress concentrations by stress dependent pore size. I will present some analytical results for the resulting optimal control problem and conclude with numerical results showing the versatility of our approach for creating optimal macroscopic designs with tailored mesostructures. Joint work with Moritz Ebeling-Rump and Robert Lasarzik, WIAS, see [1, 2].

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Dual Integral Transform in the Class of Laplace

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In this paper, a general dual integral transform (GDIT) in class of Laplace is presented which covers most or even all type of dual integral transforms in the family of Laplace transform. Many theorems related to this approach are proved. The obtained results show that proposed GDIT is more efficient and useful to handle such these kinds of equations. There are various types of integral transform method such as: single-, dual- and triple- Laplace transform method single- and dual- Sumudu transform method dual Sumudu–Laplace transform method single- and dual- Elzaki transform method single- and dual- Aboodh transform method single the general integral transform etc. for solving partial differential equations.

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Some Theorems and New Result Related to the Triple Laplace Transform

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In this paper, the new triple Laplace transform is defined. The main results and theorems on the new triple Laplace transform are investigated. The theory of fractional differential equations based on new triple Laplace transform is developed in this work. Fractional calculus has attracted many researchers in the last decades. The impact of this fractional calculus on both pure and applied branches of science and engineering have been increased. Many researchers started to approach with the discrete versions of this fractional calculus benefiting to get many aspects in the real life. the conformable calculus and their new properties have been analyzed for real valued multivariable functions. conformable gradient vectors are defined, and a conformable sense Clairaut's Theorem have been proven. the researchers have worked on the linear ordinary and partial differential equations based on the conformable derivatives. Namely, two new results on homogeneous functions involving their conformable partial derivatives are introduced, specifically, homogeneity of the conformable partial derivatives of a homogeneous function and the conformable version of Euler´s Theorem. The conformable Laplace transform was initiated in and studied and modified in. The conformable Laplace transform is not only useful to solve local conformable fractional dynamical systems but also it can be employed to solve systems within nonlocal conformable fractional derivatives. Finally, it is also a remarkable fact a large number of studies in the theory and application of fractional differential equations based on this new definition of derivative, which have been developed in a short time.

Acknowledgments

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Clark–Ocone Type Formulas for Some Past-Dependent Brownian Functionals

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We consider some path-dependent Brownian functionals and derive constructive formulas for the stochastic integral representation, first generalizing the Clark–Ocone type formulas obtained earlier by Jaoshvili–Purtukhia (2005).

Let a Brownian Motion $B = (B_t)$, $t \in [0,T]$, be given on a probability space $(\Omega, \Im, P)$, and let $\Im_t := \Im_t^B = \sigma\{B_u : \ 0 \le u \le t\}$. For any non-negative integer n, consider the following path-dependent Brownian functional: $F(n) := \big(\int\limits_0^T B_s\,ds\big)^{2n+1}$. Let us denote $F^-(n) := [F(n)]^-$.

Theorem 1 *Let f and its generalized derivative ∂f be a square-integrable functions with weight function $\exp\{-\frac{x^2}{2}\}$ and $\xi \in D_{1,2}$, then the following stochastic integral representation is valid:*

$$f(\xi) = Ef(\xi) + \int_0^T E[\partial f(\xi) D_t\xi | \Im_t]\,dB_t \quad (P-a.s.).$$

Theorem 2 *For any non-negative integer n the following stochastic integral representation is valid:*

$$F^-(n) = EF^-(n) - (2n+1)\sum_{k=0}^{2n}\int_0^T (T-t) C_{2n}^k \sigma^k \eta^{2n-k} I_k^-(\sigma,\eta)\Big|_{\eta=\int_0^t (T-s)\,dB_s} dB_t,$$

where

$$I_{2k-1}^-(\sigma,\eta) = -\varphi\Big(\frac{\eta}{\sigma}\Big)\sum_{i=0}^{k-1}\frac{(2k-2)!!}{(2i)!!}\Big(\frac{\eta}{\sigma}\Big)^{2i},$$

$$I_{2k}^-(\sigma,\eta) = (2k-1)!!\Big[1-\Phi\Big(\frac{\eta}{\sigma}\Big)\Big] - \varphi\Big(\frac{\eta}{\sigma}\Big)\sum_{i=1}^{k}\frac{(2k-1)!!}{(2i-1)!!}\Big(-\frac{\eta}{\sigma}\Big)^{2i-1},$$

$\sigma^2 = (T-t)^3/3$ and Φ (resp. φ) standard normal distribution function (resp. standard normal distribution density function).

Corollary *The following stochastic integral representation is true:*

$$\Big(\int_0^T B_s\,ds\Big)^- = \sqrt{\frac{T^3}{6\pi}} + \int_0^T (T-t)\Big[1-\phi\Big(\sigma^{-1}\int_0^t (T-s)\,dB_s\Big)\Big]\,dB_t.$$

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Performance Analysis for Homomorphic Encryption . Techniques Based on Paillier Encryption

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The increasing demand for secure data storage and processing in cloud environments has prompted the exploration of advanced encryption techniques. Homomorphic encryption, a new approach, enables specific computations on encrypted data without the need for decryption. This eliminates the exposure of sensitive information during processing, thereby enhancing security and privacy.

Conventional encryption methods, although effective, necessitate decryption for any data manipulation. This process can be burdensome and ineffective, particularly in cloud-based applications where data analysis is essential. Paillier encryption, a form of homomorphic encryption, presents a promising solution. It allows for addition and multiplication operations directly on encrypted data, facilitating computations within the cloud while upholding data confidentiality.

This study delves into the performance evaluation of homomorphic encryption techniques based on Paillier encryption. It compares the performance attributes of Paillier encryption with those of traditional encryption methods, emphasizing the benefits of homomorphic capabilities for cloud security and data processing. Through performance evaluation, the study examines factors such as encryption/decryption times, key size requirements, and applicability to various cloud storage and processing scenarios. The results contribute to the expanding body of research on homomorphic encryption and its significance in securing and enabling efficient data manipulation within the cloud.

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On the Some Boundary Value Problems for the Equation of String Oscillation

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In this paper, for the inhomogeneous equation of string oscillation, necessary and sufficient conditions for the solvability of Dirichlet and Neumann type problems with inhomogeneous boundary conditions are established. If these conditions are fulfilled, solutions to the corresponding problems are given in quadratures. In particular, it is shown that the corresponding homogeneous problems have an infinite number of linearly independent solutions that can be found explicitly. The correctness of one version of Zaremba problem is also proven and its solution in quadratures is given.

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Quasi-Optimal Rule of Testing Directional Hypotheses and its Application to Big Data

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The problem of testing directional hypotheses is examined using the consideration of the basic and alternative hypotheses in pairs, allowing implementing computation easily and faster with guaranteed reliability. The concept of mixed directional false discovery rate (*mdFDR*) is used for the decision rule optimality. The fact of guaranteeing the quality of a decision (in the developed approach) at the desired level is proved theoretically and is demonstrated practically by applied examples. The developed method is enhanced for testing multiple hypotheses that guarantees the restriction of the total *mdFDR* on the desired level. It is also shown that the proposed method can be used for solving the problems of testing intersection-union and union-intersection hypotheses also. The proposed method is adapted to testing large numbers of the subsets of individual hypotheses in testing multiple hypotheses that saves computational time and resources. Reliability and convenience of the developed method for big data are also demonstrated.

Emotion Recognition on the Basis . of Human Body Movements using Artificial Intelligence Methods

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Recent research has reported substantial progress in machine-learning based emotion recognition from outward behavior. These approaches, however, usually rely on facial expressions or speech and are thus not applicable to material with low resolution, without audio, or filmed from large distance or challenging angles (e.g., CCTV footage). Therefore, we aim to develop an approach to emotion recognition from full body movements. To this end, we use the MPIIEmo dataset which contains videos of dyadic interactions with emotional content. The dataset consists of 224 video sequences filmed from eight viewpoints each. For feature extraction, we used ByteTrack which successfully created bounding boxes and IDs for the depicted persons. Based on these bounding boxes, we were able to create crops for feature extraction. To increase the stability of the crops, we smoothed the coordinates using a moving-window median correction. Furthermore, we annotated the videos to have a ground truth for supervised learning algorithms. Based on this preliminary work, we plan to extract features using DinoV2 and use those features within several machine-learning approaches such as contrastive learning.

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Queue Length's Analysis for an M/G/1 System in Random Environment

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Analytical challenges in many cases for modern queuing theory are effectively addressed within the framework of queuing systems operating in a random environment [1, 3].

We treat here such kind of a queuing system.

In M/G/1 system the server is influenced by a random environment (a random process) $\mu(t)$ with a state set $I = \{0, 1, 2\}$, and changes its state based on the state of the environment. In other words, if $\mu(t) = i$, then the server is in state i. State 0 is special, indicating that the server cannot begin service even if there are customers in the queue due to various reasons.

When the system is busy, it operates in individual service cycles or sequences of cycles, with probabilistic characteristics independent of cycle labels.

When the server is idle, the random environment μ(t) is modeled as a birth-death process with state set $I = \{0, 1, 2\}$ and transition intensities α_i from $i \to i+1$ $(i = 0, 1, 2)$ and β_i from $i \to i-1$ $(i = 1, 2)$. When the system is empty, the birth-death process acts as the random environment, while when the system is busy, the random environment only affects the initial service probabilistic characteristics-functions – $H_{ij}(x)$.

By definition, $H_{ij}(x)$ is the conditional probability that the service time of customer is less than x and the state of the server or the process $\mu(t)$ at the end of service time is j, provided that the server was in the state I at the beginning of the service [2, 3].

In present paper an analytical semi-Markov model of above system is constructed and studied in terms of operational calculus. Using a novel probabilistic method as an alternative approach [3–5].

Laplace transform of generating function for queue length is derived.

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Evaluation in EEG Emotion Recognition: State-of-the-Art Review and Unified Framework

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Emotion recognition from EEG signals has become a rapidly growing area of research in recent years. To fairly compare proposed approaches and to track the field's progress, commonly unified evaluation protocols are essential. In a comprehensive literature review of EEG-based emotion recognition articles published between 2018 and 2023 we uncover that the field lacks such protocols, making it impossible to reliably determine the state-of-the-art approach. Analyzing 231 papers, we show that inconsistencies between evaluation protocols are due to ground truth definition, evaluation metrics selection, data splitting types (e.g., subject-dependent or subject-independent) and different datasets. To overcome these challenges and foster the field's progress, we give recommendations for a unified evaluation protocol and present EEGAIN, a novel open source software framework for researchers. This innovative tool enables efficient evaluation of new methods and datasets, offering researchers the capability to detect state-of-the-art results and compare their work in this field. EEGAIN includes standardized methods for data pre-processing, data splitting, evaluation metrics, and the ability to load the 6 most relevant datasets in EEG emotion recognition with only a single line of code. In addition, we have assessed and validated EEGAIN using 6 most popular datasets (Mahnob-HCI, Deap, Dreamer, Amigos, SEED, SEED IV) in this field on 4 most common publicly available methods (TSception, EEGNet, Deepconvnet, Shallowconvnet). As such, our work is a significant step to make research on EEG emotion recognition more reproducible and comparable, thereby accelerating the overall progress of the field.

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Nonlinear Mathematical Model of Immunopathogenesis of Rheumatoid Arthritis Considering the Role of Mediator of Inflammation

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Based on the equations described in [1, 2], we created a new mathematical model of the immunopathogenesis of rheumatoid arthritis, which takes into account the role of the important protein IL6. This model is the main prerequisite for developing a treatment model for rheumatoid arthritis.

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Dynamic Study of Overhead Crane

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Nowadays, when throughout the world construction volume reached a huge scale, the industry is developing at an increasing pace, transportation links between the different countries are increasing, trade is developing, etc., efficiency increase is of major importance in the fulfillment of cargo handling, loading-unloading and warehousing activities, as this works significantly facilitate the development of country's economics. All of the aforesaid stipulates increasing attention and requirements for the reliability of cargo handling vehicles and the durability of their constituent elements. Main characteristics – reliability and durability of overhead crane are considerably stipulated by respective indicators of their running gear. Namely, rigidity of operational-constructive indicators of bearing capacity of rapidly wearing components and wheels and metal construction of the driving unit. It is to be noted that the severe natural phenomena should be taken into account. In particular, due to a snowstorm in 2010 and in September 2021, gantry cranes were damaged at the container terminal belonging to "Georgian Trans Expedition – Poti" LLC in Poti. Provided about the elimination of the mentioned risks. Thus, increasing the durability of crane running wheels (unified elements for different types of cranes) is an important and urgent task for the development of country's industry.

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A Litterature Survey on the Effect of Scanning Strategy on Temperature Gradients in SLM

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My aim in this conference is to give a literature survey on the effect of scanning strategy (laser trajectory) on temperature gradients in selective laser melting process where I will mention some works in which the authors have tested the effect of laser scanning strategy in SLM with specific and random paths. Then, I will discuss a laser path optimization model given by Alam–Nicaise–Paquet, giving some ideas on the mathematical study of this model using optimal control theory (see [1–3]).

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Endomorphisms of a Separable Primary Group

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By using the finite topology of the endomorphism ring, the form of action of the endomorphisms of the abelian separable primary group on the elements of a socle of the group is shown. Applications of the obtained results are also discussed.

Information Security Model in the Case of Two Sources of Misinformation

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The paper discusses the issue of information security modeling in society under the conditions of sources of misinformation. Defined levels of information security in society. A dynamic system is chosen to describe the spread of misinformation in society and the fight against it. In contrast to works [1, 2], this article discusses not one, but two sources of disinformation, one of which functions directly within society, and the other affects society members from outside society. Through the analysis of the built dynamic system, the degree of public vulnerability is studied, resulting from different levels of misinformation and the intensity of fighting against them.

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On Almost Invariant Bernstein Type Sets

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In the presented talk we introduce the notion of the almost invariant sets and existence a partition of the space $\mathbf{R^N}$ into almost invariant Bernstein type sets, where $\mathbf{N}$ denotes the set of all natural numbers.

Let E be the ground set which is equipped with a transformation group G and $Z \subset E$. We shall say that Z is almost G-invariant (in the set-theoretical sense) if, for each transformation $g \in G$, we have

$$card\,(g(Z) \triangle Z) < card\,(E)$$

About of above-mentioned definition see for example, [1, 3, 4].

It is known that there exists a partition $\{X_i : i \in I\}$ of the $\mathbf{n}$-dimensional Euclidean space $\mathbf{R^N}$ into almost invariant Bernstein type sets, whose corresponding cardinal number satisfies following relation $2 \leq card\,(I) \leq \mathbf{c}$, where $\mathbf{c}$ denotes the cardinality of the continuum (see [2, 5]).

Theorem *There exists a partition of the space $\boldsymbol{R^N}$ into almost invariant Bernstein type sets.*

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A Note on Scalable K-g-Frames

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In this talk, we study the construction of Parseval K-g-frames from a given K-g-frame by scaling the elements of the K-g-frame with the help of diagonal operators and such frames are termed scalable K-g-frames. We prove some properties of scalable K-g-frames and construct new scalable K-g-frames from a given K-g-frame. Further, we obtain equivalent conditions for the scalability of K-g-frames and the K-frames induced by K-g-frames. Finally, it is shown that the direct sum of two scalable K-g-frames is again a scalable K-g-frame for some suitable bounded linear operator K.

Joint work with Varinder Kumar and Sapna Malhotra.

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On the Novel Solutions of 2D Navier–Stokes Equation

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We consider incompressible steady fluid flow in 2D infinite areas. The velocity components of the flow satisfy the nonlinear Navier–Stokes equations (NSE) with the suitable boundary conditions [1–12]. The novel exact solutions of NSE are obtained in some specific cases.

Besides, the solutions obtained by means of the conformal mapping methods are discussed [3, 4].

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Crossed Modules, Cohomology and Wells Exact Sequence for Leibniz n-Algebras

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In this talk, we present the notion of crossed modules for Leibniz n-algebras and show their equivalence with internal categories in the category of Leibniz n-algebras. Then we interpret the set of equivalence classes of crossed extensions as the second cohomology of Leibniz n-algebras developed in [2]. Furthermore, we present the investigation of the extensibility problem of a pair of derivations associated with an abelian extension of Leibniz n-algebras using the cohomology of Leibniz n-algebras, and derive an exact sequence of the Wells type connecting various vector spaces of derivations.

The results of this research are presented in the papers [1] and [3].

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Dual Symmetries of Dense Three and Two-Color QCD and Some QCD-Like NJL Models

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The massless NJL model extended by the diquark interaction channel is considered in the work. We study its phase structure at zero temperature and in the presence of baryon μ_B, isospin μ_I, chiral μ_5 and chiral isospin μ_{I5} chemical potentials in the mean-field approximation.

In the mean-field approximation, the $(\mu_B, \mu_I, \mu_5, \mu_{I5})$-phase portrait of this *massless* NJL model has been investigated in the papers [1, 2], where it was shown that only two non-trivial phases are allowed for dense quark matter: (i) the chiral symmetry breaking (CSB) phase and (ii) the charged pion condensation (PC) one. Moreover, it was established that CSB and charged PC phases are dually conjugated to each other. It means that at fixed μ_B and μ_5 these phases are arranged mirror-symmetrically with respect to the line $\mu_I = \mu_{I5}$ on the mean-field (μ_I, μ_{I5})-phase portrait of the model, and so on. A more detailed influence of this dual symmetry on the phase structure of quark matter considered in the framework of the simplest massless NJL model has been investigated in Refs. [2, 3].

Moreover, it follows from our analysis that chiral μ_5 chemical potential promotes the formation of CSC phase in dense quark matter. And together with μ_{I5} it can generate the charged PC phase even at $\mu_I = 0$.

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On Some Properties of Small Sets

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Let $(G, \cdot)$ be an arbitrary uncountable group and let Y be a subset of G.

We shall say that $Y \subset G$ is G-absolutely negligible set in G if for each countable family $\{g_i : i \in I\}$ of elements from G, there exists a countable family $\{h_j : j \in J\}$ of elements from G, such that

$$\bigcap_{j\in J}\Big(h_j \cdot \Big(\bigcup_{i\in I}(g_i \cdot Y)\Big)\Big) = \varnothing.$$

About this definition see [1, 2].

Theorem *Let $(G, \cdot)$ be an uncountable group and let $card(G)$ be a regular cardinal. Then, for each $X \subset G$ with $card(X) = card(G)$, there exists a G-absolutely negligible set $Y \subset G$ such that $X \cdot Y = G$.*

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Using Reverse Monte Carlo to Populate Data Lakes and Marts

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In today's reality, data is an important asset for any type of business as all businesses willingly or unwillingly participate in the modern digital age [3]. Data collection and subsequent analysis support strategic decision-making and improved operational efficiency. The paper introduces and discusses the various data sources used in data warehouses and describes the evolution from data lakes to data marts. The use of big data in business analytics and the crucial role of lakes and data warehouses in this process are considered [1]. Describes various data sources for data warehouses and a new application of the reverse Monte Carlo method for populating data sources [2]. A possible procedure for simplifying the data entry process when populating data sources is described. This method aims to improve the efficiency and accuracy of data sets by using stochastic modeling to process large volumes of data with minimal manual intervention. By automating repetitive tasks and using advanced data validation and transformation algorithms, this approach reduces the time and resources required for data preparation.

The new interpretation of the method aims to overcome the problems associated with traditional data collection processes, which are often time-consuming and error-prone. The method automates the data entry process by using stochastic modeling to validate and transform data. This reduces manual intervention, improves data accuracy and improves overall efficiency.

The findings are based on real-world diagrams and case studies in various industries, which will help us in future research and better analyze and solve problems in this field.

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Well-Posedness of the Governing Equations for Nonlinear Elastic Model in which Both Stress and Strain Appear Linearly

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The response of a body described by a nonlinear elastic constitutive relation, in which both stress and strain appear linearly, is studied. The constitutive relation stems from a class of implicit relations between the histories of the stress and the relative deformation gradient. A-priori thresholding is enforced through the material moduli that ensures that the displacement gradient remains small. Moreover, the thresholding procedure ensures that the solution does not blow-up in finite time. The resulting mixed variational problem consists of a quasi-static equilibrium equation and the nonlinear material response supported by mixed Dirichlet–Neumann boundary conditions. The problem is studied for well-posedness within the theory of coercive and maximal monotone graphs. The theoretical results are supported by computer simulation of representative examples solved numerically with respect to monotone loading both with and without thresholding.

Acknowledgments

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Approximate Calculations of Singular Integrals with Piecewise Differentiating Density

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We consider the singular integral

$$S(\varphi; t_0) \equiv \frac{1}{\pi i} \int\limits_L \frac{\varphi(t)}{t - t_0}\, dt, \quad t_0 \in L,$$

that is widely used in engineering practice. Namely, the actual problems of the applied type of mechanics (aerodynamics, crack theory, electrodynamics, elasticity theory) will be reduced to the equations containing such integrals.

In $S(\varphi; t_0)$: L is piecewise closed contour; $\varphi(t)$ is a continuous function on the arcs $C_q C_{q+1} \subset L$ (implied here the smallest arc with ends $C_q C_{q+1}$, where the transition from C_q to C_{q+1} corresponds to going around the circle L in the positive direction) with its derivatives of order r, moreover, the derivative of $\varphi^{(r)}(t)$ satisfies Helder's condition; C_j $(j = 1, 2, \ldots, p)$ are the corner points of circle L and, in general, $\varphi'(C_j - 0) \neq \varphi'(C_j + 0)$. We denote the functions of such a class by $H_\alpha^{(r)}(L; C_1, C_2, \ldots, C_p)$.

For the singular integral $S(\varphi; t_0)$, the approximate quadrature formula $S_{n;C_1,C_2,\ldots,C_p}(\varphi; t_0)$ is constructed and it is proved that when $\varphi \in H_\alpha^{(r)}(L; C_1, C_2, \ldots, C_p)$ fair is the estimate

$$\left|S(\varphi; t_0) - S_{n;C_1,C_2,\ldots,C_p}(\varphi; t_0)\right| \leq \frac{C \ln n}{n^{r+\alpha}} \quad (n > 1),$$

where C is a constant that does not depend on n and t_0.

Acknowledgments

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A Rigorous Mathematical Analysis to Explore the Density of Tumour Cells and Immune Response (TNF)

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A model for cancerous growths has been developed in this work, especially on solid tumours, in which growth primarily comes from cellular proliferation. Consider a procedure for cancer therapy which consists of an interaction between immune response (immune cells) and tumour cells without any specific drug. The purpose of this research work is to establish a rigorous mathematical analysis of the model and to explore the density/concentration of tumour cells and immune response (TNF). The result suggests that although TCD is capable of the growth of tumors the immune response is blocked to direct tumor growth. Two factors need to be considered for such predictions: net growth rate and infiltrative ability in this model. The result suggests that although TCD is responsible for the growth of tumours, the immune response is stopped to direct tumour growth.

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Exploring Some Applications of Probability Logics

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In this paper, we review some applications of probabilistic logic. We start exploring their role in Artificial Intelligence, where probability logics enhance decision-making processes under uncertainty and improve the robustness of machine learning models. The use of probability logics in information theory is also examined, highlighting their utility in optimizing data transmission and error correction. Through these diverse applications, we underscore the essential role of probability logics in advancing both theoretical research and practical implementations across multiple fields. Finally, we consider some limitations and open challenges in applying probability logics to complex real-world.

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School and University – One Unit, One Goal

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In recent years, there has been a decline in interest in fundamental sciences. For young people, the pursuit of science is becoming less attractive. The interest of talented young people in scientific research is very necessary for the country. In addition, we consider it very important to introduce school curriculum innovations that will contribute to increasing the effectiveness of general educational institutions, to the student's understanding of the importance of this or that subject and interest. The inclusion in the state scientific grant competition "Science starts at school – research with the participation of students" announced by the National Science Foundation of the University of Georgia served this purpose. The purpose of this competition was to raise the quality of education of students of general educational institutions, to bring education and research closer together, as well as to involve universities in the school process in order to understand the general problems of teaching, to integrate research into the learning process, to promote the development of research skills among students, and to increase the motivation of teachers. Within the framework of the competition, in 2018, 2019 and 2023, the University of Georgia, together with 42 public schools of physics and mathematics named after I. Vekua, which has more than half a century of experience in working with talented students, conducted successful work. In the project, in which more than 50 students of IX-XII grades of this school participated, the issues were worked out: "Diophantine equations" (2018), "Functional methods in set theory" (2019), "Using linear methods of higher algebra in economic problems" (2023). The University of Georgia is one of the private higher education institutions in the Georgian academic space, where scientific research in mathematics and physics is successfully carried out. The university strives to attract as many young people as possible to science and technology, those who show an interest in mathematical and technical disciplines. For this purpose, the university often conducts lectures for students in mathematics and physics, in order to popularize science, organizes field events in schools of Tbilisi and various districts of the Republic, holds conferences with the participation of students, organizes exhibitions and other things, which were also carried out within the framework of the above-mentioned project.

On Boolean Topos Constructions by Freyd and Pataraia and their Generalizations

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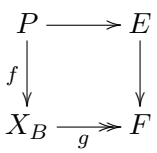
In [3], A. Pitts proved a somewhat surprising result, showing that for each variable p and formula ϕ in IPC there exists a formula $A_p\phi$ (effectively computable from ϕ), containing only variables distinct from p which occur in ϕ, and such that for all formulas ψ not involving p, $\vdash \psi \Rightarrow A_p\phi$ if and only if $\vdash \psi \Rightarrow \phi$.

He showed that a model of IPC^2 can be constructed with an algebra of truth values isomorphic to any given Heyting algebra. In [3] he also asked whether his result can be generalized further to higher order calculi. Speaking in category theoretic terms, this asks whether every Heyting algebra can be realized as the algebra of all subobjects of the terminal object in a topos.

For the case when the Heyting algebra in question is in fact Boolean, the affirmative answer is the contents of Exercise 9.11 in [1] and P. Johnstone attributes it to P. Freyd. D. Pataraia came up with an alternative construction of what turns out to be an equivalent topos $\mathscr{L}_B$. Namely, using the Stone duality for Boolean algebras, he considered certain explicitly described subcategory of local homeomorphisms over the Stone space $X = X_B$ dual to the Boolean algebra B.

To demonstrate that for a given Boolean algebra B the topos $\mathscr{F}_B$ by Freyd and the topos $\mathscr{L}_B$ by Pataraia are isomorphic, Jibladze considered a third, intermediate category $\mathscr{M}_B$. The objects of this category are pullbacks of the form shown on the right, where $E \to F$ is any map between finite discrete topological spaces, and g is any surjective continuous map from the Stone space X_B of B to F.



In the present work, we generalize the constructions by Freyd and Pataraia and M. Jibladze. We apply the resulting generalization to some classes of Heyting algebras beyond the classes of Boolean algebras and complete Heyting algebras.

The main theorem in this development says the following:

Theorem *For a given Boolean algebra B, the toposes $\mathscr{F}_B$, $\mathscr{L}_B$ and $\mathscr{M}_B$ described above are equivalent to the subcategory $\mathscr{C}$ of coherent objects in the category of sheaves* $\mathbf{Sh}(X_B)$ *over the Stone space X_B associated with the Boolean algebra B.*

Later in the talk, we will discuss some generalizations of this result to other classes of Heyting algebras.

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Chebyshev's Theorem for Sequences with Chain Dependence

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On the probability space (Ω, F, P), a two-component stationary (in the narrow sense) sequence $\{\xi_i, Y_i\}_{i\geq 1}$ is considered. The control sequence $\{\xi_i\}_{i\geq 1}(\xi_i : \Omega \to \{s_1, s_2, \ldots, s_r\})$ is a finite, homogeneous regular Markov chain. $\{Y_i\}_{i\geq 1}$ is sequence with a chain dependence. It is shown that when the conditional variances $\sigma^2(\alpha) = D\{Y_1|\xi_1 = s_\alpha\}$, $\alpha = 1, 2, \ldots, r$ are finite, then for the sequence $\{Y_i\}_{i\geq 1}$ when $n \to \infty$ the following convergence

$$\frac{1}{n}\sum_{i=1}^{n} Y_i - \frac{1}{n}\sum_{i=1}^{n} EY_i \xrightarrow{p} 0$$

occurs.

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Boosting Bound States in the Chiral Effective Field Theory

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In the Weinberg's theory of "Nuclear forces from chiral Lagrangians" the perturbation theory (PT) is developed for the nucleon potentials. Although these potentials are being calculated nowadays in a rigorous systematic way to the including 5-th order of the ChPT, still there is a part of the nuclear studies which needs the same rigorous treatment. It concerns accounting for the motion of the bound states involved in scattering processes. A PT is derived for the ab initio high accuracy nuclear studies which enables one a systematic account for this bound states' motion.

A Characterization of Dual Catenaries Problem in Dual Spaces

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In this paper, we investigate non-degenerate dual surfaces in 3-dimensional de Sitter dual space S_1^3, specifically focusing on dual minimal surfaces with zero constant mean curvature. We examine the dual de Sitter space and explore the property that the generating dual curve of the dual catenoid is variationally characterized as the shape of a dual hanging chain. Additionally, we address the question: Can the dual generating curves of rotational dual minimal surfaces of dual S_1^3 be characterized from a variational viewpoint? We study the dual catenary problem, characterizing dual catenaries and exploring solutions to the hanging dual chain problem. Our research includes investigating spacelike or timelike dual curves in dual S_1^2 and dual surfaces in dual space S_1^3. We also examine how to define the center of mass of a dual curve in dual S_1^2. Finally, we characterize the dual catenary problem in the dual spherical, dual hyperbolic, and dual parabolic cases.

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Assessing Metro Station Efficiency using Queueing Theory

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The operation of the metro is essential. Above all, passenger transportation safety is a top priority, and ensuring a comfortable and secure experience for passengers at the station is equally important, which depends on the waiting time for the train. The main goal of the work is to study passenger flow at a metro station in Kyiv and to find the optimal interval between trains by using the tools of queueing theory. This problem is applicable to practical, real-world situations and significantly impacts efficiency of passenger movement. Our approach is based on a model of tandem queues for the metro station and a model for estimating trains loading. One of the queueing systems in our tandem model is the checkpoints in the station, and another is the station itself. Providing such a model unable us to solve a lot of problems. This gave us the opportunity to point out the condition of the overloaded regime of system nodes, number of service devices, calculate the number of passengers who can be at the station at the same time (maximum and safe-comfortable) and the optimal (safe and/or comfortable) interval between trains. The model's flexibility allows for recalculating performance measures with changing passenger flow rates and system parameters. Using the R programming language, we simulate performance under varying passenger flow intensities, improving station capacity and reducing waiting times. So, first, given estimated passengers flow into the station, we calculate the optimal number of devices at the subway checkpoints, so that with this minimum number, the queue of passengers would not accumulate. That is, to make passenger service comfortable. Since the metro station under consideration already has a set number of validators, we can compare how the overall experience of staying at the station would change by simulating the flow of passengers with different numbers of devices. Next important performance measure, that can be calculated, is the interval between trains. It means an interval such that the number of passengers, who will arrive at the platform within it, does not exceed the average number of people that can comfortably fit on the train. To model train loading, we introduce two additional functions. The first is a load factor of the train. This coefficient depends on the time of day and on the intensity of passenger traffic corresponding to this time of day, as well as on the train waiting time τ. The second factor is the percentage of passengers who get off the carriages at the station. This value depends on the time of day and on a specific station, and in our case, it is a constant value. Under given values of the parameters of the system the optimal interval between trains is for $\lambda = 36$ passengers/min: $\tau \leq 2.51294$.

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A Note on Generalized Functions

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The purpose of this talk is to show that finite order Schwartz distributions can be very easily constructed from continuous functions by means of two simple commutative algebra techniques, namely, localization and factorization.

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Data Analysis

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Data analysis is a way which connects experimentally obtained data to the scientific theory. Data analysis does not provide a definite (unambiguous) solution of a particular scientific problem but gives it with some degree of confidence. The book discusses the main directions of the data analysis, the basics of the probability theory, statistical analysis methods, uncertainty analysis, time series analysis, multivariate analysis and Monte Carlo method.

Chapter 1 is devoted to the basics of the probability theory, random variable and the distribution function concepts, their properties and transformation. In the statistics section (Chapter 2) except of the hypothesis testing and the parameter estimation methods, some special methods are given (histogramming, bootstrap, method of moments etc.). The uncertainty analysis section (Chapter 3) involves a conventional linear model for error propagation. Also some less common methods are considered. Particular attention is paid to methods of estimating the systematic uncertainties. An overview of specific physical experiments, experimental setup properties, statistical and systematical uncertainties sources is given. Chapter 4 discusses the time series analysis and forecasting methods, the models of stationary and non-stationary time series, their spectral, orthogonal and wavelet transformations. In Chapter 5, all major sophisticated methods of multivariate analysis are discussed. Such as the principal component analysis, factor analysis, artificial neural networks, decision trees, kernel method, genetic algorithm, Kalman filter, multivariate time series analysis, and the blind analysis. In Chapter 6 Monte Carlo method is presented. A special attention is given to the random variable generators, multidimensional numerical integration methods and the analysis methods using Markov chains. In the chapter the concept of the statistical model and the simulation issues according to the model are discussed. The synchrotron beam dynamics simulation is given as an example of the deterministic model. The simulation of physical processes of elementary particles interaction with a medium are given using the positron emission tomography and the hadron therapy models.

The book provides the glossary of conventional terms of analysis with translations, in some cases with explanation. The book contains references to most advanced textbooks, as well as an incomplete list of internet resources from which modern data analysis software and relevant documentation can be obtained. Most of the methods and discussed topics are provided with examples, including more than 250 figures and over 2000 formulae.

The book is addressed mainly to students and researchers interested in state-of-the-art methods of experimental and financial data analysis and forecasting.

The book was published in 2022 by Free and Agricultural Universities Press. The publication was partially supported by National Scientific Foundation.

Fuzzy Probability – One of the Ways to Remove Uncertainty

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In science, it took a lot of effort to deviate from the classical understanding of probability. Following the transition from classical to axiological (subjective) probability, the role of experts has increased, the influence of subjective preferences of experts on assessment has increased.

In general, quantification should be consistent with natural language terms. If such an assessment is made as a consensus based on the opinions and choices of a number of experts, then it becomes important for modeling an economic object, together with the data on the given object. When modeling under uncertainty, it is appropriate and useful to use the formalisms of fuzzy set theory. Even the latest kind of probability fails categorically in the main thing: to describe the essence of the subjective action of the person who perceives the world and makes decisions.

Fuzzy numbers (variety of fuzzy sets) are ideal for planning time factors when their future estimation is complicated (ambiguous, there is no sufficient probabilistic basis). Thus, all the scenarios about this or that individual factor can be reduced to one set of scenarios in the form of triangular numbers, where three points are distinguished: the minimum possible, the most expected and the maximum possible value of the factor. At the same time, the weight of individual scenarios in the set of scenarios is formalized as a "close to the average" triangular membership function of assigning level of the factor to the fuzzy set.

We can return probabilistic descriptions to our scientific everyday life as probability distributions with phase parameters. The impreciseness of the distribution of parameters is caused by the fact that there is no statistical selection of observations in the classical sense, and we use the scientific category of quasi-statics for analysis. In such an approach, the triangular parameters of the distribution are determined based on the procedure for determining the degree of reliability. In this way, the way for the synthesis of probabilistic and phase-set description was revealed.

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Pines and Future

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I am privileged to teach high-achieving students in ATID Education Network. I also teach in Oranim Academic College of Education. ATID means in Hebrew FUTURE and ORANIM means PINES. Hence, the name of the paper. In the talk, I will share my teaching experience with the audience. I will give examples of problems from the course "Problem Solving, Problem Posing and Problem Choosing", that was given in Oranim in a graduate program for primary school math teachers. One of the problems on a cards game that introduces the binary system and the pigeonhole principle was also given to my high school students. I will also describe a task for my high school students in which they collaborated in small groups in solving problems, posing new problems, and explaining the solutions to the other groups.

Commutators of Sublinear Operators in Grand Variable Exponent Morrey Spaces and Applications to PDEs

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The goal of our talk is to present the boundedness of commutators of sublinear operators in grand variable exponent Morrey spaces introduced in [5] in the constant exponent case, and in [4] in variable exponent setting. The operators and spaces are defined on quasi-metric measure spaces with doubling measure. The obtained results are applied to study regularity properties of solutions of second order partial differential equations with discontinuous coefficients in the frame of grand variable exponent Morrey spaces. We refer to [1–3] for previous investigations in this direction.

This talk is based on the investigation jointly with M. A. Ragusa.

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Some Kinematic Approaches to Solving Geometric Problems

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In modern integrative learning approaches, leveraging knowledge and skills from various disciplines to solve problems within a specific field is highly valued. Since ancient times, researchers have paid attention to the possibility of using non-mathematical means in teaching mathematics. First of all, we are talking about the application of physical ideas. In the physics and mathematics educational literature, techniques have been developed with the help of which certain statements and formulas of a purely mathematical nature are obtained as an additional "by-product" of the discussion of a physical problem. Moreover, in recently published educational literature and scientific and methodological articles one can find many approaches, when ideas taken from the "arsenal" of physics and especially mechanics are effectively used as a means of solving mathematical problems.

Kinematics, describing in a mathematical way the various motions of bodies, by its nature is one of the most mathematized sections of mechanics and cannot have "feedback" with mathematics. Given this possibility, the purpose of this work is to present possible applications of certain kinematic approaches to solving geometry problems.

First, based on the method of discussing the motion of points of a geometric figure, an approach to solving some geometry proof problems is discussed. Using the specified kinematic method, the solution of a number of generalized versions of well-known geometric problems, including the Napoleon problem, is considered. The preliminary knowledge that the learner needs to have for learning this kinematic method is also presented.

In the second part of the report, we discussed the issue of determining the radii of curvature of various curves (parabola, ellipse, and cycloid) using the kinematics of moving points. To determine the radius of curvature, elementary kinematic formulas and the method of changing the reference frame were used.

Based on teaching experience, some arguments are also given to substantiate the importance of the presented kinematic approaches in teaching geometry.

Acknowledgments

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Solving Fractional Models Using Artificial Neural Networks

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Due to the non-local behavior of fractional derivative it is more suitable to model real-world problems in an elegant way. But, this property makes the existence and obtaining of the solution of fractional differential and integral equations an exciting challange. On the other hand artificial neural network (ANN) is one of the popular areas of artificial intelligence (AI) research and also an abstract computational model based on the organizational structure of the human brain. In this study we consider the use of ANN in fractional models. In solving fractional differential equations by ANN, no desired values are known and the output of the model can be generated by training only. As per the existing training algorithm, the architecture of a neural model is problem dependent and the number of nodes, etc., is taken by trial-and-error method where the training depends upon the weights of the connecting nodes.

Histogram Clustering and Edge Detection in Grayscale Digital Images

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Edge detection is a major task in image processing and analysis. The gray level distribution (the histogram) of a particular object or region in an image is usually near constant and different from that of other neighboring objects. Suppose a sliding window runs through the entire image, and consider the local histogram of such a sliding window. This histogram changes its composition as it passes from one object to the other. More precisely, the histogram shows higher complexity (higher variety of gray levels) just when the sliding window is straddling the two objects.

The authors have exploited this idea to detect edges[1], using Shannon entropy H as a measure of the complexity or variety of the histogram. However, H tends to reach high values (saturate) when dealing with a large scale of gray levels in the presence of textures or degradations such as noise or blurring. So, a corrected information measure is used, the clustered entropy CH, based on clustering local histograms. CH has a zero value for quasi-homogeneous regions and reaches high values for regions containing edges. Some mathematical properties of CH are studied, a comparison between CH and H is done, and some comparative experiments of edge detection are shown.

Acknowledgments

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The Third Cohomology Group of a Monoid and Admissible Abstract Kernels

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We define the product of admissible abstract kernels of the form $\Phi\colon M \to \frac{End(G)}{Inn(G)}$, where M is a monoid, G is a group, and Φ is a monoid homomorphism. Identifying C-equivalent abstract kernels, where C is the center of G, we obtain that the set $\mathscr{M}(M,C)$ of C-equivalence classes of admissible abstract kernels inducing the same action of M on C is a commutative monoid. Considering the submonoid $\mathscr{L}(M,C)$ of abstract kernels that are induced by special Schreier extensions, we prove that the factor monoid $\mathscr{A}(M,C) = \frac{\mathscr{M}(M,C)}{\mathscr{L}(M,C)}$ is an abelian group. Moreover, we show that this abelian group is isomorphic to the third cohomology group $H^3(M,C)$.

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Radius Properties of Harmonic Mappings Fixing with the Analytic Part as Convex

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In this article, we study harmonic mappings of form $f = h + \overline{g}$ defined in the unit disk, where g and h satisfy certain prescribed conditions and the analytic part is fixed as Ma and Minda class of convex functions. Certain sharp radius results for univalency, close-to-convex, fully starlikeness, fully convexity are established. We also calculate the radius of uniformly starlikeness and convexity for these functions. Several results enhance the well-known radius result.

About the Specifics of using the Open Book Method at School

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One of the important parts of organizing the educational process is the control system. Much has been said and written about methods for testing the quality of students' knowledge, but research in this direction is always relevant. There is no clear opinion regarding the frequency and form of diagnosing the quality of students' knowledge – it depends on the subject, schedule, age of students and many other subjective or objective circumstances.

When testing students' knowledge using the "open book" method, the use of textbooks and notebooks is allowed. In itself, this approach is not a didactic novelty. However, with proper preparation of the task, this form of monitoring is not only a means of testing academic skills and knowledge level, but also an effective tool for motivating students.

The author of the report shares with listeners her own experience of using the "open book" method – discusses in detail the frequency of such activities and the features of writing assignments at all three levels of school: primary (age of students 11-12 years), basic (13-15 years) and secondary (16-18 years). The observed results of using the described method are presented.

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About One Method of Solving the Schrödinger Equation and its Application in Modeling the Electronic Properties of Quantum Nanostructures

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In this talk, we explore numerical techniques for modeling and determining the electronic spectrum and structure of quantum nanostructures. We focus on solving the stationary Schrödinger equation using a finite difference scheme. To determine the eigenvalues and eigenfunctions, we develop Danilevsky's method tailored for systems with three-diagonal matrices. Our approach is validated through a series of numerical experiments conducted with various input parameter values, demonstrating the robustness and efficiency of the proposed methods. Results can be used for investigation and optimization of the electronic properties of quantum heterostructures.

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On Simultaneously Reflective and Coreflective Full Subcategories of an (Enriched) Functor Category

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We give a short proof of a result asserting that a simultaneously reflective and coreflective full subcategory of an (enriched) functor category is again a functor category.

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On Monoid Factorization

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Factorizations of monoids are studied. Two necessary and sufficient conditions in terms of so-called descent 1-cocycles for a monoid to be factorized through two submonoids are found. A full classification of those factorizations of a monoid whose one factor is a subgroup of the monoid is obtained. The relationship between monoid factorizations and non-abelian cohomology of monoids is analyzed. Some applications of semi-direct product of monoids are given

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Rubio de Francía's Extrapolation in Grand Banach Function Spaces

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In the papers [1] and [3] the authors introduced grand Banach function space (grand BFS briefly) $E^{p),\theta}$. In this space the norm is defined as follows:

$$\|f\|_{E^{p),\theta}} := \sup_{0<\varepsilon<p-1} \varepsilon^{\frac{\theta}{p-\varepsilon}} \|f\|_{E^{p-\varepsilon}},$$

where E is a BFS, $1 < p < \infty$, $\theta > 0$. The space $E^{p),\theta}$ is again a BFS.

Our aim is to deliver recent results regarding Rubio de Francía's extrapolation theorem in the space $E^{p),\theta}$. Two-weight extrapolation will also be discussed in the case when E is a rearrangement invariant BFS.

Some of these results are were announced in [2].

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Boundedness Criteria for the Multilinear Riemann–Liouville Operators

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Let us denote by R_α multilinear Riemann–Liuoville integral operator given by the formula:

$$R_\alpha^{(m)}(f_1,\dots,f_m)(x)=\int\limits_{(0,x)^m}\frac{f_1(x-t_1)\cdots f_m(x-t_m)}{(t_1+\cdots+t_m)^{m-\alpha}}\,dt_1\cdots dt_m,\ \ x>0,$$

where $\alpha>0$. Taking $\alpha=m$ we have the multilinear Hardy operator $H^{(m)}:=R_m^{(m)}$. Two-weight criteria for $H^{(m)}$ in the bilinear case (i.e. for $m=2$) were derived in [1].

The operator $R_\alpha^{(\theta)}$ is the one-sided variant of the multilinear fractional integral operator:

$$\mathscr{I}_\gamma^{(m)}(f_1,\dots,f_m)(x)=\int\limits_{(\mathbb{R}^n)^m}\frac{f_1(y_1)\cdots f_m(y_m)}{(|x-y_1|+\cdots+|x-y_m|)^{mn-\gamma}}\,dy_1\cdots dy_m,\ \ x\in\mathbb{R}^n,\ \ 0<\gamma<nm.$$

We refer also to [2] for R_α.

We present necessary and sufficient conditions on a weight function v defined on $\mathbb{R}_+$ governing the inequality

$$\|R_\alpha^{(m)}(f_1,\dots,f_m)\|_{L_v^q(\mathbb{R}_+)}\le C\prod_{k=1}^m\|f_k\|_{L^{p_k}(\mathbb{R}_+)}.$$

In the linear case ($m=1$) this characterization was obtained in [3], and independently in [4]. The same problem in Lorentz spaces is studied as well.

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Nonparametric Estimation in Random Coefficient Regression Model

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In this talk the problem of estimating the joint probability density function of random coefficients of a linear regression model is considered. It is known that in such models using the estimation of conditional characteristic function combined with the regularized Fourier inversion formula provides the estimator of bivariate density function of random coefficients. Under assumptions that coefficients of a regression function are non-negative random variables, the moment-based procedure for estimating unknown density function of coefficients is proposed.

Proper Linear Commutant Preserving Maps

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Let G be a 2-torsion free unital generalized matrix algebra, and ϕ be a linear commutant preserving mapping on G. We give sufficient conditions under which ϕ is proper and this result is applied for triangular algebras.

Definition Let us begin with the definition of generalized matrix algebras given by a Morita context. Let $\mathscr{R}$ be a commutative ring with identity. A *Morita context* consists of two unital $\mathscr{R}$-algebras $\mathscr{A}$ and $\mathscr{B}$, two bimodules ${}_{\mathscr{A}}\mathscr{M}_{\mathscr{B}}$ and ${}_{\mathscr{B}}\mathscr{N}_{\mathscr{A}}$, and two bimodule homomorphisms called the pairings $\Phi_{\mathscr{M}\mathscr{N}} : \mathscr{M} \underset{\mathscr{B}}{\otimes} \mathscr{N} \to \mathscr{A}$ and $\Psi_{\mathscr{N}\mathscr{M}} : \mathscr{N} \underset{\mathscr{A}}{\otimes} \mathscr{M} \to \mathscr{B}$ satisfying the following commutative diagrams:

$$\begin{array}{ccc} \mathscr{M} \underset{\mathscr{B}}{\otimes} \mathscr{N} \underset{\mathscr{A}}{\otimes} \mathscr{M} & \xrightarrow{\Phi_{\mathscr{M}\mathscr{N}}\otimes I_{\mathscr{M}}} & \mathscr{A} \underset{\mathscr{A}}{\otimes} \mathscr{M} \\ \downarrow{\scriptstyle I_{\mathscr{M}}\otimes\Psi_{\mathscr{N}\mathscr{M}}} & & \downarrow{\scriptstyle \cong} \\ \mathscr{M} \underset{\mathscr{B}}{\otimes} \mathscr{B} & \xrightarrow{\cong} & \mathscr{M} \end{array} \quad \text{and} \quad \begin{array}{ccc} \mathscr{N} \underset{\mathscr{A}}{\otimes} \mathscr{M} \underset{\mathscr{B}}{\otimes} \mathscr{N} & \xrightarrow{\Psi_{\mathscr{N}\mathscr{M}}\otimes I_{\mathscr{N}}} & \mathscr{B} \underset{\mathscr{B}}{\otimes} \mathscr{N} \\ \downarrow{\scriptstyle I_{\mathscr{N}}\otimes\Phi_{\mathscr{M}\mathscr{N}}} & & \downarrow{\scriptstyle \cong} \\ \mathscr{N} \underset{\mathscr{A}}{\otimes} \mathscr{A} & \xrightarrow{\cong} & \mathscr{N} \end{array} .$$

Let us write this Morita context as $(\mathscr{A}, \mathscr{B}, \mathscr{M}, \mathscr{N}, \Phi_{\mathscr{M}\mathscr{N}}, \Psi_{\mathscr{N}\mathscr{M}})$. If $(\mathscr{A}, \mathscr{B}, \mathscr{M}, \mathscr{N}, \Phi_{\mathscr{M}\mathscr{N}}, \Psi_{\mathscr{N}\mathscr{M}})$ is a Morita context, then the set

$$\begin{bmatrix} \mathscr{A} & \mathscr{M} \\ \mathscr{N} & \mathscr{B} \end{bmatrix} = \left\{ \begin{bmatrix} a & m \\ n & b \end{bmatrix} \;\middle|\; a \in \mathscr{A},\ m \in \mathscr{M},\ n \in \mathscr{N},\ b \in \mathscr{B} \right\}$$

is an $\mathscr{R}$-algebra under matrix-like addition and matrix-like multiplication, where at least one of M and N is non-zero. Such an $\mathscr{R}$-algebra is usually called a *generalized matrix algebra* of order 2 and is denoted by

$$\mathscr{G} = \mathscr{G}(\mathscr{A}, \mathscr{M}, \mathscr{N}, \mathscr{B}) = \begin{bmatrix} \mathscr{A} & \mathscr{M} \\ \mathscr{N} & \mathscr{B} \end{bmatrix}.$$

Theorem 1 *Let $\mathscr{M}$ or $\mathscr{N}$ are faithful bimodules in $\mathscr{G}$, and $\pi_{\mathscr{A}}(Z(\mathscr{G})) = Z(\mathscr{G})$ and $\pi_{\mathscr{B}}(Z(\mathscr{G})) = Z(\mathscr{B})$. Then the linear mapping $\Phi : \mathscr{G} \to \mathscr{G}$ is commutant preserving mapping if and only if $\Phi(X) = \lambda X + \mu(X)$ for all $X \in \mathscr{G}$, where $\lambda \in Z(\mathscr{G})$ and $\mu : \mathscr{G} \to Z(\mathscr{G})$ is a linear mapping.*

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On Projective Class Group of Crossed Products

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Let R be a Dedekind ring, K its quotient field and $R[\pi,\rho]$, $K[\pi,\rho]$ crossed products [1]. We will consider those finitely generated projective $R[\pi,\rho]$-modules such that $K\otimes P$ is $K[\pi,\rho]$-free. Let $\mathbf{P}_0$ be the category of such projectives and let $P_0(R[\pi,\rho])$ be the Grothendieck group of $\mathbf{P}_0$. Let $P_0^{'}(R[\pi,\rho])$ be the subgroup of $P_0(R[\pi,\rho])$ generated by all $[F]$ with F free over $R[\pi,\rho]$.

Definition The special projective class group [3] is defined to be

$$C_0(R[\pi,\rho]) = P_0(R[\pi,\rho])/P_0^{'}(R[\pi,\rho]).$$

The special projective class group coincides with the usual projective class group $C(R[\pi,\rho])$ as defined in [2].

Theorem *Let R be a ring of algebraic integers. Then $C(R[\pi,\rho])$ is finite.*

Let $G(R[\pi])$ the Grothendieck group of finitely generated $R[\pi]$-modules. In fact $G(R[\pi])$ is a ring.

Proposition *$P_0(R[\pi,\rho])$ is a module over $G(R[\pi])$.*

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Cross-Disciplinary Insights in AI and Neuroscience

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Artificial Intelligence (AI) and Neuroscience are fields with a shared interest in replicating and understanding cognitive functions. Parallels are often drawn between the processes in the human brain and AI models, such as learning, attention, vision, and memory. This presentation starts with a review of the history of designing artificial systems and which areas of neuroscience they have drawn inspiration from, while also pointing out the differences between the similar concepts in these two fields. Examples are given as to how attention, saliency, and other biologically inspired ideas are modeled in computer vision. A distinction is made between how humans learn and memorize information and how deep learning and reinforcement learning models do it. Despite the challenges of differing goals and methods across disciplines, some modern neuroscience findings applicable to future AI research are highlighted. Conversely, the idea that deep learning can offer theories to model and understand how the biological brain works is discussed.

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On Cofinitely $\oplus - g$-Supplemented Modules

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In this work, every ring has a unity and every module is a unitary left module. Let M be an R-module. If every cofinite submodule of M has a g-supplement that is a direct summand in M, then M is called a cofinitely $\oplus - g$-supplemented module. It is clear that every $\oplus - g$-supplemented module is cofinitely $\oplus - g$-supplemented. It is also clear that every $\oplus$-supplemented module is cofinitely $\oplus - g$-supplemented.

Proposition 1 *Every hollow module is cofinitely $\oplus - g$-supplemented.*

Proposition 2 *Let M be an R-module and $RadM = M$. Then M is cofinitely $\oplus - g$-supplemented.*

Proposition 3 *Let M be a cofinitely $\oplus - g$-supplemented module. If every nonzero submodule of M is essential in M, then M is cofinitely supplemented.*

Proposition 4 *Let M be a finitely generated R-module. Then M is cofinitely $\oplus - g$-supplemented if and only if M is $\oplus - g$-supplemented.*

Proposition 5 *Let M be a finitely generated cofinitely $\oplus - g$-supplemented module. Then M is g-supplemented.*

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Log-Concavity of Infinite Product Generating Functions

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The talk presents some results motivated by Lehmer's conjecture [5] that Ramanujan's tau function constituted by the Fourier coefficients of the 24th power of Dedekind's eta function never vanishes. Generally, when Dedekind's eta function is raised to a power with exponent x it turns out that the Fourier coefficients are polynomials in x. They satisfy a recurrence relation. Even in a more general form we can provide a bound on x outside of which these never vanish [1, 3]. In some cases, including Dedekind's eta function, this seems to be the best possible. In its general form it provides relations for example to orthogonal polynomials and Eisenstein series [2].

In the 1970s Nicolas [4] proved that the coefficients $p_d(n)$ defined by the generating function

$$\sum_{n=0}^{\infty} p_d(n)\, q^n = \prod_{n=1}^{\infty} (1-q^n)^{-n^{d-1}}$$

are log-concave for $d = 1$. Recently, Ono, Pujahari, and Rolen [5] have extended the result to $d = 2$. Note that $p_1(n) = p(n)$ is the partition function and $p_2(n) = \mathrm{pp}(n)$ is the number of plane partitions. We explore properties for $p_d(n)$ for general d. Let $n \geq 6$. Then $p_d(n)$ is almost log-concave for n divisible by 3 and almost strictly log-convex otherwise.

The talk includes joint work with A. Abdesselam (University of Virginia) [1] and B. Heim (University of Cologne/RWTH Aachen University) [1–3].

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On a Sufficient Condition of Solvability of a Diophantine Equation

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For natural numbers $n > 1$, $p_i > 0$, $i = 1, \dots, n$ and $L \geq \sum_{k=1}^{n} p_k$ we shall discuss a sufficient condition for solvability in natural numbers of the Diophantine equation

$$\sum_{k=1}^{n} p_k x_k = L.$$

The talk is based on [1] and [2].

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Mathematical Model of Combined Treatment of Rheumatoid Arthritis Considering IL6

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Using nonlinear differential equations, based on the models proposed in [1, 2], we developed a new mathematical model that describes the treatment of rheumatoid arthritis. The model explores the functional dynamics of cartilage destruction during disease progression. The Cauchy problem is posed and we obtain an exact solution.

Acknowledgements

The project is financed by the Shota Rustaveli National Science Foundation of Georgia. Grant # STEM 22-360.

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Fracture Criteria for Matrix and Fibers in Unidirectional Polymeric Composites

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Matrix fracture criterion

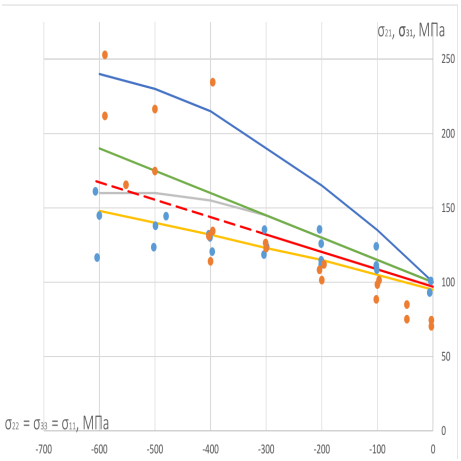
$$F_*^{\pm} = \sup_{\alpha^{\pm}} F^{\pm}, \tag{1}$$

$$F^{\pm} \equiv \sigma_{nn}(\alpha^{\pm}) + \sqrt{\left[m_{nt}^{\pm}\sigma_{nt}(\alpha^{\pm})l\right]^2 + \left[m_{nl}^{\pm}\sigma_{nl}(\alpha^{\pm})l\right]^2} = Y^{\pm}.$$

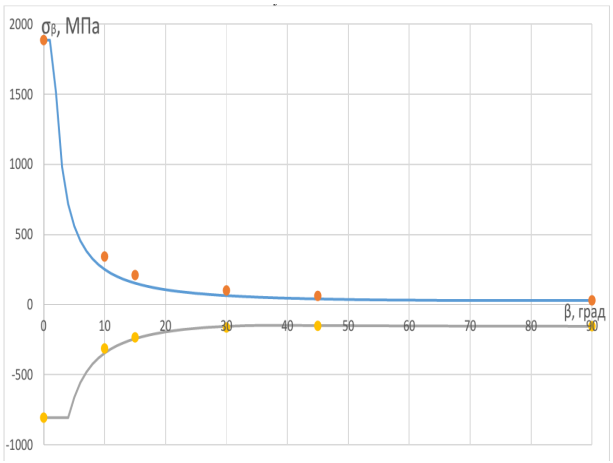
Fibers fracture criterion

$$\sigma_{mm}^{\pm} = S_{11}^{\pm} \ \text{ for } \ |\beta^{\pm}| \leqslant |\beta_*^{\pm}| < 5_{exp}^{o}. \tag{2}$$

Failure of the T300/PR319 matrix according to the criteria (1) and Puck (2013), Pinho (2013), Carrere (2012), Cuntze (2012)



(Experiment • – Shin, Pae, 1992)



(Experiment • – Kawai, 2014)

Fiber failure according to the criterion (2) and matrix fracture according to the criterion (1) of carbon fiber T700S/2592.

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Approximation and Transfer of Properties Between Translation Invariant Convex Differentiation Bases

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We prove a transfer of differential properties within the framework of non-negative functions from a translation invariant convex density basis $\mathbf{B}$ to a translation invariant convex basis $\mathbf{H}$ provided each set H forming $\mathbf{H}$ can be approximated by some set B forming $\mathbf{B}$ in the sense that the estimate $|H \triangle B| \leq c|H \cap B|$ is valid, where c is a positive constant not depending on H. An application of this transference result shows that the study of differential properties in the class of non-negative functions for translation invariant convex density bases can be reduced to that of for translation invariant Busemann–Feller density bases formed of multi-dimensional rectangles.

The talk is based on the paper [1] joint with Irakli Japaridze.

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On the Solvability of a System of First-Order Differential Equations in a Non-Cylindrical Domain

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Various problems for systems of first order differential equations in bounded and unbounded domains have been considered by many authors. In this paper, systems of first order differential equations are considered in a non-cylindrical domain. The problem is investigated using D. S. Dzhumabaev's parameterization method [1].

Suppose $\varphi(x)$ is continuous in R. In domain $\Omega = [0, \varphi(x)] \times [0, \omega]$ consider next problem

$$\frac{\partial V}{\partial t} = A(t,x)V + \Phi(t,x), \quad t \in [0, \varphi(x)], \quad x \in [0, \omega] \tag{1}$$

$$V(0,x) = V(\varphi(x), x) \tag{2}$$

where the matrix $A(t,x) = [a_{ij}(t,x)]_{i,j=1}^{n}$ and n-vector function $\Phi(t,x)$ are continuous in $\Omega = [0, \varphi(x)] \times [0, \omega]$ and satisfies condition

$$|a_{ii}(t,x)| \geq \sum_{i \neq j}^{n} |a_{ij}(t,x)| + \theta(t,x), \quad i = \overline{1,n} \tag{3}$$

where $\theta(t,x) \geq \theta_0 > 0$ are continuous function in Ω and θ_0-constant.

Theorem *Let the matrix $A(t,x)$ is satisfied condition (3) and the function $\Phi(t,x)$ is continuous in Ω. Then the problem (1), (2) has a unique solution and is implemented in the next evaluation*

$$\|V(t,x)\| \leq \left\| \frac{F(t,x)}{\varphi(x)\theta(t,x)} \right\|, \quad x \in [0,\omega], \quad t \in [0, \varphi(x)].$$

where $\|V(t,x)\| = \max\limits_{(t,x)\in\Omega} |V(t,x)|$.

Acknowledgments

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Conditions for the Regularity of a Solution of One Fourth-Order Differential Equation with Unbounded Coefficients

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Fourth order differential equations are used in various applied problems. The solvability of boundary value problems for such equations in linear and nonlinear cases has been well studied (see, for example, [1–3]). Much fewer works are devoted to the case of an infinite domain; we note the article [4], where the Cauchy problem for a fourth-order linear equation of bending waves is considered. However, in the above works, coefficients of the equation are assumed to be either constant or bounded functions. We consider the following equation

$$L_0 y = y^{(4)} + p(x)y^{(2)} + q(x)y = F(x), \tag{1}$$

where $x \in \mathbb{R} = (-\infty, \infty)$, and $F \in L_2(\mathbb{R})$. We assume that the coefficients p and q are smooth functions, but they may not be bounded.

For continuous functions $\rho(s)$ and $v(s) \neq 0$ we denote

$$\gamma_{\rho,v,2} = \max\Big\{ \sup_{x>0} \|\rho\|_{L_2(0,x)} \|sv^{-1}(s)\|_{L_2(x,\infty)}, \sup_{t<0} \|\rho\|_{L_2(t,0)} \|sv^{-1}(s)\|_{L_2(-\infty,t)} \Big\}.$$

We will discuss the following statement.

Theorem *Suppose that p is twice continuously differentiable, and q is a continuous function and the following relations $p \geq 1$, $\gamma_{1,\sqrt{p},2} < \infty$ and $\gamma_{q,p,2} < \infty$ are fulfilled. Then for each $F \in L_2(\mathbb{R})$ there exists a solution to equation (1) and it is unique. Furthermore, if $\sup\limits_{x,\eta\in\mathbb{R}:\ |x-\eta|\leq 1} \frac{p(x)}{p(\eta)} < \infty$, then for the solution y the following inequality holds*

$$\|y^{(4)}\|_2 + \|py^{(2)}\|_2 + \|qy\|_2 \leq C\|F\|_2,$$

where $\|\cdot\|_2$ is the norm in $L_2(\mathbb{R})$.

Acknowledgments

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On Compositions of Some Singular Distributions

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In this paper we concern the compositions of some singular distributions by using the method mainly related to the finite part of divergent integral, in fact we consider the composition of the Dirac delta function and an infinitely differentiable function having any number of distinct multiple roots.

Definition Let F be distributions in $\mathscr{D}'$ and let f be infinitely differentiable function. We say that the distribution $F(f(x))$, the neutrix composition of F and f, exists and is equal to $h(x)$ on the interval (a, b) if the neutrix limit

$$\mathrm{N}\!-\!\lim_{n\to\infty}\left[\int_{-\infty}^{\infty} F_n(f(x))\phi(x)\,dx\right] = \langle h(x), \phi(x)\rangle$$

for all ϕ in $\mathscr{D}$ with support contained in the interval (a, b), where $F_n(x) = (F * \delta_n)(x)$, and N is he neutrix having domain $N' = \{1, 2, \dots, n, \dots\}$, range the real numbers with negligible functions which are finite linear sums of the functions $n^{\lambda}\ln^{r-1} n, \ln^r n$ $(\lambda > 0,\ r = 1, 2, \dots)$ and all functions which converge to zero in the usual sense as n tends to infinity.

Theorem *Let $f(x)$ be an infinitely differentiable function having multiple root at x_0 with multiplicity s on the open interval (a, b). Then the k-th power of the composition $\delta(f(x))$ of Dirac delta function and f exists in the sense of Fisher's definition.*

Acknowledgments

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Kneser Problem for Two-Dimensional Nonlinear Singular Differential Systems

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In the interval $]0, +\infty[$, we consider the initial value problem

$$u_k' = f_k(t, u_1, u_2) \ \ (k = 1, 2), \tag{1}$$

$$\lim_{t \to 0} u_1(t) = c, \tag{2}$$

where $f_k :]0, +\infty[\times \mathbb{R}^2 \to \mathbb{R}$ $(k = 1, 2)$ are continuous functions, and c is a real number other than zero.

A solution $(u_1, u_2) :]0, +\infty[\to \mathbb{R}^2$ to system (1) is said to be **Kneser solution** if

$$u_1(t) \geq 0, \ \ u_2(t) \leq 0 \ \text{ for } \ t > 0,$$

or

$$u_1(t) \leq 0, \ \ u_2(t) \geq 0 \ \text{ for } \ t > 0,$$

and it is said to be **vanishing at infinity** if

$$\lim_{t \to +\infty} u_k(t) = 0 \ \ (k = 1, 2).$$

We have established optimal in a certain sense conditions guaranteeing, respectively, the existence of at least one (unique) Kneser solution to problem (1), (2), and the fact that this solution is vanishing at infinity. They cover the cases where the function f_2 in the time variable has non-integrable singularity at the initial point of the interval under consideration, i.e. the case, where

$$\int_0^1 |f_2(t, x_1, x_2)| \, dt = +\infty \ \text{ for } \ x_1 \neq 0, \ \ x_2 \neq 0.$$

For the Emden–Fowler differential system

$$u_k' = p_k(t) |u_{3-k}|^{\lambda_k} \operatorname{sgn}(u_{3-k}) \ \ (k = 1, 2) \tag{1'}$$

with continuous coefficients $p_k :]0, +\infty[\to]0, +\infty[$ and positive exponents λ_k $(k = 1, 2)$, the theorems proven by us yield the following

Proposition *For the existence of a unique Kneser solution to problem* (1′), (2)*, it is necessary and sufficient that the condition*

$$\int_0^1 p_1(t) \left(\int_t^1 p_2(s) \, ds \right)^{\lambda_1} dt < +\infty$$

be satisfied. For this solution to be vanishing at infinity, it is sufficient, and in the case of $\lambda_1 \lambda_2 \geq 1$ *it is also necessary, that one of the following three conditions be satisfied:*

$$\int_1^{+\infty} p_k(t) \, dt = +\infty \ \ (k = 1, 2);$$

$$\int_1^{+\infty} p_1(t) \, dt < +\infty, \quad \int_1^{+\infty} p_2(t) \left(\int_t^{+\infty} p_1(s) \, ds \right)^{\lambda_2} dt = +\infty;$$

$$\int_1^{+\infty} p_2(t) \, dt < +\infty, \quad \int_1^{+\infty} p_1(t) \left(\int_t^{+\infty} p_2(s) \, ds \right)^{\lambda_1} dt = +\infty.$$

Chromatic Congruences and Bernoulli Numbers

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For every natural number n and a fixed prime p, we prove a new congruence for the orbifold Euler characteristic of a group. The p-adic limit of these congruences as n tends to infinity recovers the Brown–Quillen congruence. We apply these results to mapping class groups and using the Harer–Zagier formula we obtain a family of congruences for Bernoulli numbers. We show that these congruences in particular recover classical congruences for Bernoulli numbers due to Kummer, Voronoi, Carlitz, and Cohen.

Acknowledgments

The work is supported by the EPSRC grant EP/X038424/1 – *"Classifying spaces, proper actions and stable homotopy theory"*.

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A Numerical Algorithm of Solving a Nonlinear Integro-Differential String Equation and its Error

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An initial boundary value problem for the Kirchhoff equation

$$w_{tt} = \varphi\Big(\int_0^\pi w_x^2\,dx\Big)w_{xx}$$

describing the oscillation of a string is considered. It is assumed that $\varphi(z) \in C^p[0,\infty)$, $\varphi(z) \geq \alpha > 0$, where p is equal to 1 or 2, while the coefficients $a_i^{(l)}$ of expansions into a Fourier sine-series, $i = 1, 2, \ldots$, of the initial functions $w^l(x)$, $l = 0, 1$, satisfy the inequality $|a_i^{(l)}| \leq \omega i^{l-(p+s+2.5)}$, where ω and s are positive numbers. To find an approximate solution a numerical algorithm is used, which consists of three parts – Galerkin's method, the Crank–Nicolson difference scheme and a Picard type iteration process. The algorithm error is estimated.

The Drinfeld Centre and Gottlieb Group of a Crossed Module

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Crossed modules are algebraic models of homotopy 2-types. By definition, a crossed module G_* is a group homomorphism $\partial\colon G_1 \to G_0$ together with an action of G_0 on G_1 satisfying some properties. The most important invariants of the crossed module G_* are the group $\pi_0(G_*) = Coker(\partial)$ and the $\pi_0(G_*)$-module $\pi_1(G_*) = \ker(\partial)$.

Given a crossed module $\partial : G_1 \to G_0$, we construct a braided crossed module, which we call the centre of the crossed module G_* and denote by $Z_*(G_*)$.

We show that the braided monoidal category corresponding to the braided crossed module $Z_*(G_*)$ is isomorphic to the Drinfeld centre of the monoidal category corresponding to G_*.

The essential invariants of $Z_0(G_*)$ are closely related to low dimensional group cohomology. In fact, we describe the invariants $\pi_i(Z_*(G_*))$, $i = 0, 1$, in terms of group cohomology involving the groups $\pi_0(G_*)$ and $\pi_1(G_*)$.

It should be pointed out that in the 80's Norrie also introduced the notion of a centre of a crossed module, but our notion differs from hers. Our centre can be shown to be homotopy invariant, unlike hers.

The main result is to establish a connection between this definition and the Gottlieb group of the classifying space of the crossed module.

Modern Confirmation Theory

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In mathematics, a proof is deductive reasoning used to justify a statement. In such reasoning, one can use existing, previously substantiated provisions, and this reasoning strictly obeys the laws of formal logic. In mathematics, the following methods of proof are discussed: direct proof, mathematical induction, opposition, and the technique of supposing the opposite, experimenting. Assertion theory is used to test the correctness of a statement or to find the validity of a statement. It is usually easier to check the validity of an existing permit than to seek a new one. In addition to formal logic, it often requires intuition and creativity. Therefore, we can say that a mathematical proof combines both analysis and synthesis. Proof theory is often called a mathematical experiment. Most often, a mathematical experiment is used through calculations on a computer. For example, you can cite a calculation using Fermat's theorem that occurred before a certain date. In addition to classical methods in proof theory, it is necessary to find other types of methods, theorems, lemmas, etc. to confirm. One such issue for discussion may be the use of association during approval. Association is a legally formalized connection between events, facts and objects, which is reflected in the human mind and remembered in memory. Assoc. Other theorems during the existence of the connection can be understood associatively and subsequently proven either in the form of a generalization or in the form of a simplification.

Association concepts are introduced in various fields:

- Association (logic) – replacement of complex forms of structure, functioning and interaction with simple, generally understandable forms without distorting the meaning;
- Association (object-oriented programming) – a relationship between classes of objects that allows one instance of an object to call another;
- Association (statistics) – a connection between two measurable quantities, a synonym for correlation;
- File association is a feature of modern operating systems;
- Associative memory is a special type of computer memory for quick access to content and more.

In our case, we are talking about the use of associations in proving theorems; if we have existing transformations from one formula to another formula and the sequence of their transformations, then the remaining transformations should be constructed similarly to one transformation. One author of these theses, K. Gogichaishvili used the association during the proof of Fermat's little theorem. If you consider other mathematical problems similarly, then you can find problems that can be solved similarly to the proof of other problems. For example, the Algorithm of the construction of the Tower of Hanoi is necessary to connect with the algorithm of the task of assembling the Rubik's cube. Sometimes mathematicians of classical thinking oppose the use of other non-traditional methods along with existing ones.

This is an attempt to open a discussion about whether it is possible to use associations in mathematics.

The Prediction Model of Parallel Data

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The most popular existing forecasting model is Bayesian, mainly used to prove asynchronous hypotheses. We presented a new model (building a forecasting model with parallel data) that further improves forecasting accuracy. To increase the accuracy of the forecast, the algorithm uses "parallel data" – "parallel probability", which allows you to choose two or more combinations of forecast models, whose "compound" probability is much better than each model [2].

We explained what "necessary" and "sufficient" models are [1]. The "necessary" predicted models are models whose forecasts always include real events, and "sufficient" predicted models – these models of which are always produced. An algorithm was given for the creation of a hybrid model, from a combination of two or more models to get a higher probability forecast. We also discussed "enough" models and algorithms on how to make such "sufficient" models that combine all elegant events, that is, such "sufficient" models "necessary".

Thus, you can get a "sufficient" or almost close forecast model with the necessary "necessary" models, and, by combining "sufficient" models, we get the "necessary" or close model.

If there are models that do not guess this event (but not "enough"), then such models are removed from our database. In the same way, when discussing "sufficient" models, if you have a forecast of excessive events, such a model can be excluded from the abundance of "sufficient" models.

We get the necessary model from "sufficient" models that will be "sufficient" and "necessary" at the same time. In addition, we combine enough models to get the "necessary" model.

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Short-Time Gabor Transform in Graph Signal Processing

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In recent years, graph signal processing (GSP) has become an explosive research area, resulting in the generalization of key signal processing ideas for signals that are defined over irregular domains described by graphs. In this talk, we study the construction of the Graph Gabor Transform (GGT) and the Short Time Graph Gabor Transform (STGGT) by using oversampling schemes. In fact, we generalize the classical windowed Fourier analysis in signal processing to the graph settings. Further, we obtain the necessary and sufficient conditions for the existence of dual Gabor frames in graphs, and that ensures the reconstruction property of GGT. Also, we study the perfect reconstruction property of STGGT through the construction of frames for signals in graphs. STGGT enables sparse representation on graphs by tight Gabor frames, and that ultimately plays an important tool in compressed sensing using majorization-minimization.

Joint work with Shankar.

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Limiting Weak Type Behaviors for Rough Operators, and Characterizations

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To find the optimal (smallest) positive constant for the weak type norm inequality attracts a lot of attention and it usually called the best constants problem. Great achievements have been made in lower dimensions, but for higher dimension, it becomes more difficult. Efforts have also been made to study the limiting weak type behaviors for the left side of the inequality whenever λ tends to zero, which yields a lower bound for the optimal constant as a by product. We show that the limiting weak type behaviors hold for Calderón–Zygmund singular integral operators, Hardy–Littlewood maximal operators, Calderón commutators, and Marcinkiewicz integrals with rough $L\log L(\mathbb{S}^{n-1})$ kernels, which essentially improved results of Janakiraman, Ding, Lai, Wu et al. with Dini-type kernels. Two characterizations of the limiting weak type behaviors are also given through Daubechies wavelets.

Theorem 1 *Let $\Omega \in L\log L(\mathbb{S}^{n-1})$. Then for all $0 \le f \in L^1(\mathbb{R}^n)$, it holds that*

$$\lim_{\lambda\to 0^+} \lambda\Big|\Big\{x\in\mathbb{R}^n:\ |\mathscr{T}_\Omega(f)(x)|>\lambda\Big\}\Big| = \frac{\|\Omega\|_{L^1(\mathbb{S}^{n-1})}}{n}\,\|f\|_{L^1(\mathbb{R}^n)},$$

$$\lim_{\lambda\to 0^+} \lambda\Big|\Big\{x\in\mathbb{R}^n:\ \Big||\mathscr{T}_\Omega(f)(x)|-\frac{|\Omega(x)|}{|x|^n}\,\|f\|_{L^1(\mathbb{R}^n)}\Big|>\lambda\Big\}\Big| = 0,$$

where $\mathscr{T}_\Omega$ represents the Calderón–Zygmund singular integral operator T_Ω (with vanishing kernels), Hardy–Littlewood maximal operator M_Ω, Marcinkiewicz integrals μ_Ω multiply $\sqrt{2}$ (with vanishing kernels).

Theorem 2 *Let $\Omega \in L\log L(\mathbb{S}^{n-1})$ and satisfy the cancellation condition of order 1, $A \in \mathrm{Lip}(\mathbb{R}^n), \nabla A \in L^\infty(\mathbb{R}^n)$ and $\lim\limits_{|x|\to\infty} A(x)/|x|$ exists for a.e. $x' = x/|x| \in \mathbb{S}^{n-1}$. Then for all $0 \le f \in L^1(\mathbb{R}^n)$, it holds for the Calderón commutators $T_{\Omega,A}$ that $\lim\limits_{\lambda\to 0^+} \lambda\big|\big\{x\in\mathbb{R}^n:\ \big||T_{\Omega,A}(f)(x)|-\frac{|\Omega(x)|}{|x|^n}\frac{|A(x)|}{|x|}\,\|f\|_{L^1(\mathbb{R}^n)}\big|>\lambda\big\}\big| = 0$. Furthermore, if $\lim\limits_{|x|\to\infty} A(x)/|x| = \phi(x') \in L^\infty(\mathbb{S}^{n-1})$ for a.e. $x'\in\mathbb{S}^{n-1}$, then $\lim\limits_{\lambda\to 0^+} \lambda\big|\big\{x\in\mathbb{R}^n:\ |T_{\Omega,A}(f)(x)|>\lambda\big\}\big| = \frac{\|\Omega\phi\|_{L^1(\mathbb{S}^{n-1})}}{n}\,\|f\|_{L^1(\mathbb{R}^n)}$.*

Acknowledgments

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Rewriting Logic with Quantitative Relations

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Symbolic computation is one of the cornerstones of mathematics and computer science. It can describe mathematical reasoning, computational processes and the like just by manipulating symbols, independently of their meaning. Rewriting theory is a subfield of symbolic computation that is used to study equational reasoning, computation models, operational semantics of programming languages, etc.

Rewriting logic is a formalism that is using rewrite rules to do the equational reasoning. Recent development of Artificial Intelligence and probabilistic reasoning methods makes it necessary to replace exact equality with quantitative or approximate counterparts [2].

In this talk we discuss extension of a rewriting logic with quantitative relations. In such logic, term equalities are replaced by quantitative equations. The later is an expression of the form $s \overset{\varepsilon}{=} t$, where ε is a distance between the terms s and t. In other words, s and t are equal with error ε. We describe basic properties of the extended rewriting logic, as it is important to preserve the confluence and avoid the distance trivialization phenomenon where distance collapses to equivalence [1].

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Modeling & Simulating the Stochastic Dynamics of Games with Three Pure Strategies

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Typically, randomness in evolutionary games is included by equipping the discrete deterministic replicator dynamics with Moran noise processes. In this talk, we extend this approach to obtain, from first principles, continuous evolution equations subject to white noise excitation. The modeling of these stochastic replicator dynamics will follow the approach applied in biological systems subject to system-internal random fluctuations. Our focus of study are evolutionary games with three pure strategies as they can be found by playing rock-scissor-paper or, in nature, by the territorial dynamics of the side-blotched lizards Uta stansburiana. Simulations of the stochastic game dynamics in its phase space together with discussions of the evolutionary stability of the equilibria will round-up the presentation.

LoRA-Like Parameter-Efficient Transfer Learning for CNNs

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Parameter-efficient fine-tuning methods, including adapters, have gained significant attention in the field of transformer models. However, their origins trace back to Convolutional Neural Networks (CNNs). Since Rebuffi et al. [2] introduced this approach, numerous variations have emerged for specialized downstream tasks. Despite this proliferation, a systematic comparison of these variations in a more general setting has been lacking.

Concurrently, various tensor decomposition techniques have surfaced as parameter reduction strategies for CNN architectures [1]. These methods share fundamental concepts with low-rank weight approximations, an approach that has also gained traction in transformer models. This commonality naturally leads to the generalization of LoRA-like adaptation schemas. However, the diversity of decomposition methods necessitates individual study and performance evaluation across a wide range of downstream tasks.

In this presentation, we offer a systematic exploration and side-by-side comparison of parameter-efficient fine-tuning methods for CNNs. Our comprehensive analysis encompasses various techniques, their theoretical underpinnings, and their practical performance across diverse applications.

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Measurement of the Excess Inflation in the Balassa Samuelson Model in the Countries of South Caucasus

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Catching-up economies are experiencing an excess inflation, as revealed by the Balassa–Samuelson hypothesis. In this study we give an estimate for the BS effect, by applying the model to Armenia, Azerbaijan, and Georgia throughout a three-decade period starting in 1993 and ending in 2022. Our results confirm the persistence of the BS effect, in consistence with the assumptions of the inflation targeting framework of these countries' monetary authorities. The theory of the Balassa–Samuelson effect also implies that within their floating exchange rate regimes, these countries are experiencing price and wage adjustments, as described in the analysis of their monetary policy.

Algorithm for Semigroup Bases and its Implication on the Bases of Classes of Semigroup

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With comparison of a characterisation about vector spaces, theorems on independent generating sets of semigroup that vary from those about independent-generating sets in vector spaces, are formulated. There may exist maximal independent set of elements in a semigroup which is not generating set. It reveals that there may exists independent set in a semigroup which cannot be expanded to a generating set, and that adding an element independent from an independent set may make the independent set dependent in a semigroup. These results become tool for use in the bases-selecting algorithms. Interesting graphical illustrations are given to explain the basis selecting algorithm.

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About the ABET-Program in Linear Algebra

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ABET (The Accreditation Board for Engineering and Technology) is an American non-governmental accrediting organization whose goal is to support educational programs related to modern needs in the field of engineering and technology in higher education institutions.

In Georgia, programs of the above-mentioned kind were created in collaboration with San Diego State University in the USA. Several educational programs have received ABET accreditation in Georgia, including the undergraduate program "Biomedical Engineering" for English-speaking students of the Georgian Technical University. In the present work, we review the course "Linear Algebra" included in this program.

Thematically, this linear algebra course is not fundamentally different from the traditional linear algebra courses, but the approaches are different here – the course is less formalized, the axiomatic part is reduced to a minimum. Abstract concepts are introduced step by step after considering relevant examples.

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The Secondary Cohomology Operations and Loop Space Cohomology

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Motivated by the loop space cohomology we construct the secondary operations on the cohomology $H^*(X;\mathbb{Z}_p)$ to be a Hopf algebra for a simply connected space X. The loop space cohomology ring $H^*(\Omega X;\mathbb{Z}_p)$ is calculated in terms of generators and relations. This answers to A. Borel's decomposition of a Hopf algebra into a tensor product of the monogenic ones in which the heights of generators are determined by means of the action of the primary and secondary cohomology operations on $H^*(X;\mathbb{Z}_p)$. An application for calculating of the loop space cohomology of the exceptional Lie group F_4 is given.

Analytical Integration in Galerkin Boundary Element Methods

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In this talk, we introduce explicit formulas for the calculation of matrix entries within the context of Galerkin boundary element methods for the numerical solution of boundary value problems in 3D. The computation involves integrating singular kernel functions over pairs of surface panels, which becomes challenging when these panels intersect. While coordinate transformations can eliminate singularities, the use of numerical integration remains expensive since the integrals are still four-dimensional. Our alternative approach makes use of analytical integration for the standard Galerkin discretization of the Laplace equation, focusing on piece-wise constant and linear boundary elements on flat triangles. We demonstrate that employing the Duffy transformation yields regularized integrals, which admit closed and exact formulas. This method enables the accurate computation of matrix entries while significantly reducing computational costs compared to full numerical integration. We validate the accuracy of the new formulas and showcase their efficiency in numerical experiments.

Basics of Designing Individual Orthopedic Boot-Tree by for Children with Cerebral Palsy

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It should be noted that in the process of treating cerebral palsy, orthopedic means have a high degree of recommendation. A significant novelty of the research consists in obtaining the approximate shapes of transverse-vertical sections of the orthopedic boot-tree by means of the lines of the solution to singular Dirichlet boundary value problem. We consider singular Dirichlet boundary value problem:

$$u''(t) + \frac{a}{t}\,u'(t) - \frac{a}{t^2}\,u(t) = f(t, u(t), u'(t)), \tag{1}$$

$$u(t) = 0, \quad u'(t) = 0 \tag{2}$$

where $a \in (-\infty; 1)$, f satisfies the Carathéodory's local condition potency, $[0, t] \times D$ – potency, $D = (0; +\infty) \times R$.

The solution to the problem (1), (2) is presented in the form of equations

$$u(t) = \frac{t^3}{2} - \frac{1}{3}\,ct^{-2} - \Big(1 - \frac{1}{3}\,c\Big)t + \frac{1}{2}\,, \tag{3}$$

$$u(t) = \Big(-\frac{1}{3} - \frac{1}{3}\,c\Big)t + \frac{1}{3}\,c \cdot \frac{1}{t^2} + \frac{2}{3}\,t - \frac{t^2}{2} + \frac{t^4}{6}\,. \tag{4}$$

By means of the integral curves of the solutions of equations (3) and (4), the construction of the geometric forms of the transverse vertical sections of the orthopedic boot-tree.

Thus, by means of the integral curves of the solution to singular Dirichlet boundary value problem, it is possible to describe the transverse-vertical shapes of the orthopedic boot-tree in high degree of accuracy. Also, it allows changing the transverse-vertical shapes of boot-tree in unlimited numbers, during the transfer from one size to another size. The latter is particularly relevant in the production of orthopedic footwear, when we are dealing with patients having a deformed and pathological foot.

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On Hilbert–Schmidt Frames in Quaternionic Hilbert Spaces

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In this talk, we have introduced and studied the Hilbert Schmidt frames (HS-frames) in quaternionic Hilbert spaces. Various characterization and representation of Hilbert Schmidt frames in quaternionic Hilbert spaces have been studied. Further, we have examined the existence of synthesis, analysis and frames operators and investigated their properties for Hilbert Schmidt frames in quaternionic Hilbert spaces. Finally, we have introduced and studied the dual of Hilbert Schmidt frames in quaternionic Hilbert spaces.

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The Contact Problem for Piecewise-Homogeneous Viscoelastic Plate with Elastic Patch

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The contact problem for a piecewise-homogeneous viscoelastic plate reinforced with a semi-infinite elastic inclusion (patch) is considered. The patch meets the interface of two materials at a right angle and is loaded with normal forces. The problem is reduced to a two-dimensional singular integro-differential equation with a fixed singularity. Using the methods of the theory of analytic function, a Carleman type problem for a strip is investigated and the Volterra's integral equation is solved in explicitly. The normal contact stresses along the contact line are determined and the behavior of the contact stresses in the neighborhood of singular points is established.

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Least Squares Time-Varying Parameter Estimation with Least Deviations from Attraction Points and its Recurrent Representation for Some Classes of Systems Under Non-Classical Assumptions

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The estimation problem of non-stationary parameters that can slowly change over time is considered for some classes of systems in a situation where the classical assumptions that guarantee the uniqueness of parameter estimate may be violated. In addition, it is supposed that for a vector of unknown parameters, its attraction point is known at each moment of time.

To solve the problem of unknown parameter estimation, it is proposed to use the method of least squares with a variable forgetting factor, modifying it for this formulation of the task. It turned out that in order to obtain an explicit form of representation of such an estimate, it is necessary to use the Moore–Penrose pseudo-inversion operator of matrices.

The goal is to obtain explicit and recurrent forms of representation of the desired estimate for some classes of systems. And in order to control the quality of the constructed mathematical model, propose a recurrent algorithm for recalculating the corresponding weighted sum of squared residuals.

The results are obtained for the regression model, linear and bilinear discrete dynamic systems, as well as for a discrete nonlinear dynamic system, but linear with respect to unknown parameters.

Let us present the problem statement for the last class of systems. Consider the estimation problem of slowly time-varying parameter vector α with its known attraction point $\alpha_*(k)$ at any moment k for a discrete dynamic system with such a state equation

$$x(k+1) = F(x(k), u(k), k)\alpha + g(x(k), u(k), k) + \xi(k), \quad k \in \mathbb{N},$$

where $x(k)$, $u(k)$ – observable state and control vectors, $F(x(k), u(k), k)$, $g(x(k), u(k), k)$ – given matrix and vector functions, $\xi(k)$ – disturbance vector.

The set of all least squares estimates with variable forgetting factor $\lambda(k)$ ($\lambda(k) \in (0, 1]$, $k \in \mathbb{N}$) for this system under non-classical assumptions when this estimate may be not unique is defined as $\operatorname{Arg\,min}_\alpha Q(\alpha, N)$, where

$$Q(\alpha, N) = \sum_{k=1}^{N} w(k, N)\|\xi(k)\|^2, \quad w(k, N) = \begin{cases} \prod\limits_{i=k}^{N-1} \lambda(i), & \text{if } k = 1, \ldots, N-1, \\ 1, & \text{if } k = N. \end{cases}$$

As the desired unique estimate on the above-mentioned set of estimates, it is proposed to take an estimate $\widehat{\alpha}(N)$ by the method of least squares with a variable forgetting factor $\{\lambda(k)\}_{k=1}^{\infty}$ and the least deviation from the given attraction point $\alpha_*(N)$ at any moment N.

An explicit form of representation for $\widehat{\alpha}(N)$ is obtained by using the Moore-Penrose matrix pseudo-inversion operator. Recurrent forms of representation for recalculating $\widehat{\alpha}(N)$ and the corresponding weighted sum of squared residuals $q(N) = Q(\widehat{\alpha}(N), N)$ are also derived.

The advantage of the proposed recurrent estimation algorithms for all the above-mentioned classes of systems is the absence of the need to use either the Moore–Penrose matrix pseudo-inversion operation or even the usual matrix inversion operation.

Decay Estimates for the Degenerate Differential Equations

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The main goal is to study the following degenerate differential equation

$$\partial^{\alpha}_{0+,t}u(t,x) - a(t)\mathscr{A}(u(t,x)) = 0, \quad (t,x) \in \mathbb{R}_+ \times \Omega := \Omega_+,$$

with initial-boundary conditions. Here $0 < \alpha \leq 1$, $a(t) \in L^1_{loc}(\mathbb{R}_+)$, $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$, and $\partial^{\alpha}_{0+,t}$ is the Caputo fractional derivative [1, p. 97]. $\mathscr{A}(u)$ is linear or nonlinear operator.

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Coulomb Interaction in Semiconductor Quantum Wells

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In this paper we discuss the possibility of Coulomb interaction engineering in Semiconductor quantum wells, which represent quasi-$2D$ structures with one dimension $\sim nm$. In such structures the motion of particles are confined in one dimension, which significantly affects the Coulomb interaction and influences exciton (electron-hole pair) stability and spectrum, as well as electron/hole binding energy in donor/acceptor impurities, and consequently, optical spectra and electrical conductivity. We solved eigenvalue and eigen vector problem for Schrodinger equations for two functional form of potential energy: $1/r$ and $\ln(r/r_0)$ (r_0 is a scaling constant). The perturbation theory and finite difference method are used. The calculations are carried out for different well thickness and composition.

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On the Uniqueness in the Inverse Problems of Potential Theory

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Let $K(x,y)$ be fundamental solution to the Laplace operator in R^n $(n=2,3)$ and

$$V_{\partial\Omega}(\mu)=\int\limits_{\partial\Omega}K(x,y)\mu(y)\,dS_y$$

be single-layer potential with a density μ.

Uniqueness problem. Let Ω_1 and Ω_2 be two bounded domains of R^n and μ be some density function defined on $\partial\Omega_1\cup\partial\Omega_2$ Moreover, let $V_{\partial\Omega_1}(\mu)=V_{\partial\Omega_2}(\mu)$ for all $x\in R^n\setminus\overline{(\Omega_1\cup\Omega_2)}$. By this condition we have to define the location of the domains Ω_1 and Ω_2 with respect to each other (do these domains coincide or not).

This problem, in general case, is not uniquely solvable. Under some additional restrictions we prove the following uniqueness theorems.

Theorem 1 *Let Ω_1 and Ω_2 be two bounded piecewise smooth domains in R^2 with $\Omega_\infty^{(1,2)}-$ connected unbounded component of $R^n\setminus\overline{(\Omega_1\cup\Omega_2)}$. Let there exist a point $x_0\in\partial\Omega_\infty^{(1,2)}$ and a number $r>0$ such that $\sigma_0^r\cap\overline{\Omega_1}=\varnothing$, where $\sigma_0^r=\{x:\ |x-x_0|<r\}\cap\partial\Omega_\infty^{(1,2)}$. Assume that for all $r'\le r$ the arc $\sigma_0^{r'}$ contains a segment of some line L and $\sigma_0^{r'}$ does not belong to L. If $\mu\in C^1\overline{(\Omega_1\cup\Omega_2)}$, $\frac{\partial\mu}{\partial L}=0$ and $\mu(x_0)\neq 0$, then $V_{\partial\Omega_1}(\mu)\not\equiv V_{\partial\Omega_2}(\mu)$ on $\Omega_\infty^{(1,2)}$.*

Theorem 2 *Let Ω_1 and Ω_2 be two bounded piecewise smooth domains in R^3 with $\Omega_\infty^{(1,2)}$-connected unbounded component of $R^n\setminus\overline{(\Omega_1\cup\Omega_2)}$. Let there exist a point $x_0\in\partial\Omega_\infty^{(1,2)}$ and a number $r>0$ such that $\sigma_0^r\cap\overline{\Omega_1}=\varnothing$, where $\sigma_0^r=\{x:\ |x-x_0|<r\}\cap\partial\Omega_\infty^{(1,2)}$. Assume that for all $r'\le r$ the subsurface $\sigma_0^{r'}$ contains a part of some plane P and $\sigma_0^{r'}$ does not belong to P. If $\mu\in C^1\overline{(\Omega_1\cup\Omega_2)}$, $\frac{\partial\mu}{\partial L}=0$ for a line L parallel to P and $\mu(x_0)\neq 0$, then $V_{\partial\Omega_1}(\mu)\not\equiv V_{\partial\Omega_2}(\mu)$ on $\Omega_\infty^{(1,2)}$.*

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Almost Everywhere Convergence of Subsequences of Partial Sums of Fourier Series with Respect to Vilenkin Systems

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The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves (for details see the book [1]). There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, combined with martingale theory.

In [2], using a novel technique in martingale theory, a new proof was proposed for an analogy of the famous Carleson-Hunt theorem for Fourier series with respect to the Vilenkin system for any $f \in L_p$, for any $p > 1$. Moreover, it was also proved the analogy of Kolmogorov theorem and was found a new construction of an integrable function $f \in L_1$ such that partial sums of Vilenkin-Fourier series diverges everywhere.

This talk is devoted to characterize some subsequences of natural numbers such that partial sums of Vilenkin-Fourier series with such indices of any integrable function $f \in L_1$ converges almost everywhere to this function (for details see [3]).

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On the Boundary Behaviour of Blaschke–Djerbashyan Canonical Product in the Unit Disk

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Let p be a natural number and (a_n) be a sequence of complex numbers such that $0 < |a_n| \leq |a_{n+1}| < 1$ for every $n \in \mathbb{N}$,

$$\lim_{n\to\infty} |a_n| = 1 \ \text{ and } \ \sum_{n=1}^{\infty}(1-|a_n|)^{p+1} < +\infty.$$

Denote by

$$\mathscr{B}_{p+1}(z,(a_n)) = \prod_{n=1}^{\infty}\Big(1-\frac{1-|a_n|^2}{1-\overline{a}_n z}\Big)\exp\Big(\sum_{k=1}^{p}\frac{1}{k}\Big(\frac{1-|a_n|^2}{1-\overline{a}_n z}\Big)^k\Big)$$

the Blaschke–Djerbashyan canonical product corresponding to p and (a_n).

For a sequence of complex numbers (a_n) denote by $E_{(a_n)}$ the set of all limit points of (a_n).

The following theorem is true.

Theorem *Let $p \in \mathbb{N}$. The following two conditions are equivalent:*

(1) *A subset E of the unit circle is closed and nowhere dense on the unit circle;*

(2) *There exists a sequence of complex numbers (a_n) with $0 < |a_n| \leq |a_{n+1}| < 1$ for every $n \in \mathbb{N}$,*

$$\lim_{n\to\infty} |a_n| = 1 \ \textit{ and } \ \sum_{n=1}^{\infty}(1-|a_n|)^{p+1} < +\infty$$

such that $E_{(a_n)} = E$ and for every point $e^{i\theta}$ on the unit circle there exists the radial limit $\lim\limits_{r\to 1}\mathscr{B}_{p+1}(re^{i\theta},(a_n))$, the product $\mathscr{B}_{p+1}(e^{i\theta},(a_n))$ is convergent, and the equality

$$\lim_{r\to 1}\mathscr{B}_{p+1}(re^{i\theta},(a_n)) = \mathscr{B}_{p+1}(e^{i\theta},(a_n))$$

is true.

The above theorem generalizes the result of Colwell [1] related with the case of Blaschke products.

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Extension of τ Logic with Probabilistic Operators

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We extend Bourbaki τ logic [1] with probabilistic operators to define notions of approximate proof, approximate definition, approximate contracting proof, and the like. Namely, the language of the logic τ is extended by adding some metasymbols, auxiliary symbols and probabilistic operators. The main purpose of the modified theory is a computer realization of the mathematical research.

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The Solvability and Regularity Results of Dynamical Mixed Type Boundary-Transmission Problems with Interior Cracks of the Thermo-Electro-Elasticity Theory without Energy Dissipation

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In the paper are studied dynamical mixed type interaction problems with interior cracks between the thermo-elastic and thermo-electro-elastic homogeneous isotropic bodies. The model under consideration is based on the Green-Naghdi theory of thermo-electro-elasticity without energy dissipation. This theory permits propagation of thermal waves at finite speed. Using the Laplace transform, potential theory and the theory of pseudodifferential equations, the existence and uniqueness theorems are proved, and obtained optimal regularity of solutions.

Unconditional Convergence of General Fourier Series

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S. Banach, in particular, proved that for any function, even $f(x) = 1$, where $x \in [0,1]$, the convergence of its Fourier series with respect to the general orthonormal systems (ONS) is not guaranteed. In this talk, we find conditions for the functions φ_n of an ONS (φ_n), under which the Fourier series of functions $f \in Lip_1$ are unconditionally convergent almost everywhere. The obtained results are the best possible. We also prove that any ONS contains a subsystem such that the Fourier series of any function $f \in Lip_1$ are unconditionally convergent a.e. on $[0,1]$. Additionally, we have shown that the solutions for these types of problems for the general ONS are trivial for classical ONS trigonometric, Haar and Walsh systems.

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One Approach to Solve Probability Problems

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On the one hand probability problems are interesting for the students in middle and high schools. On the other hand, these problems are difficult to solve despite the fact that the underlying principles are based on set theory. According to empirical experience this challenge can be overcome by teaching the students to rephrase the given of the problem. Let us discuss an example. There are 4 red and 7 black marbles in the jar. What is the probability of randomly drawing two black marbles simultaneously?

Students often get confused with such a given. However, if we rephrase the problem in the following way: we draw one marble and then the other (without replacement), then students can guess that the probability of getting the first black marble is 7/11 and the probability of getting the second black marble is 6/10. Moreover, they have already encountered the 'AND' operation multiple times in set theory exercises or while solving counting problems and know that mathematically 'AND' translates into multiplication. Thus, they can figure that the probability is $\frac{7}{11} \cdot \frac{6}{10} = \frac{21}{55}$. Problems related to dice can also be phrased in the similar manner. Students can connect that rolling a pair of dice or rolling one die twice are similar events and produce the same result mathematically.

Let us discuss another example. There are 4 red and 7 black marbles. Two marbles are drawn simultaneously. Find the probability of drawing two marbles of different colors.

It is best to teach the students to rephrase the problem in the following manner: first we draw one marble and then the other, Thus, the student now understands that there are two favorable outcomes: I is black and II is red OR I is red and II is black. The probability equals

$$\frac{7}{11} \cdot \frac{4}{10} + \frac{4}{11} \cdot \frac{7}{10} = \frac{56}{110} = \frac{28}{55}.$$

Having had the experience with set theory and counting problems, students are already familiar with the OR operation. They know or will learn, that mathematically it translates into addition.

In conclusion, several interesting probability problems will be presented at the conference to demonstrate the effectiveness of the above approach.

The Role of Patterns in Improving Performance in the .NET Environment

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Application performance is a critical component of modern software development, directly affecting customer satisfaction. Software designed with business needs in mind must have efficient response times to ensure smooth operations. These operations often involve interacting with databases and performing various manipulations, making optimal database interaction a crucial factor in software effectiveness. One approach to simplifying database interactions is using Object-Relational Mapping (ORM) [2].

The aim of this paper is to find the best method for improving software performance on the .NET platform, particularly focusing on which ORM developers should use to implement CRUD operations (Create, Read, Update, Delete) for efficient software development. The study focuses on the .NET MinimalAPI service.

Three ORMs are discussed: Entity Framework, Dapper, and ADO.NET. For each ORM, 9 services were developed, and specific manipulations were carried out to assess performance. The experiments highlight the advantages of both separate and combined implementations of the ORMs. The results clearly emphasize the benefits of using the Command Query Responsibility Segregation (CQRS) architectural pattern, which separates command and query responsibilities. Quantitative analysis [1] of the research results showed that services implementing CQRS have 1.6 times higher throughput, 10.72 % fewer errors, and 2.2 times faster response times compared to services without this architectural pattern.

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Logical-Probabilistic Modeling of Risks and Quantitative Assessment of Safety

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In designing and operating systems for various purposes, significant emphasis is placed on risk management, which relies on quantitative assessment data of risk factors. One common method of risk assessment is the scoring method, where risks are "assigned" points and then prioritized. This approach uses a matrix to calculate the points, determined by the product of risk impact and the probabilities of risk occurrence. According to the authors, this method raises many questions, one of the most important being how to determine the probability of risk occurrence. This paper proposes an alternative approach to risk assessment based on logical-probabilistic methods. To evaluate the system's risks and safety indicators, a method is used that describes the scenario of the system's transition into a hazard state with a logical function. The logical elements of this function are risk-initiating events and initiating conditions. The most crucial aspect is accurately defining risk-initiating events and conditions, including identifying logical connections and constructing a logical function for the scenario of the system's transition to a hazard state. By applying logical-probabilistic methods, the logical function of the scenario of the system's hazard state is converted into a probabilistic model. This allows for probabilistic assessments of expected risks and safety. This method helps identify the system's weak points, quantitatively assess system security, optimize risk management, etc.

The Flow Between two Co-Axial Tube

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Laminar entry flow (flow in the inlet length) of a viscous incompressible fluid between two co-axial cylinders is studied on the basis of linearized Navier-Stokes equations. It is assumed that the velocity of the fluid is uniform at the entrance cross section. Formulas for the longitudinal components of the velocity and pressure gradient are obtained and inlet length is calculated.

On the Weighted Integrability of the Series with Monotonic Coefficients with Respect to the Multiplicative Systems

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In this work we consider the weighed integrability of the sum of the series with respect to the multiplicative systems with monotonic coefficients. Conditions are obtained for weight functions that ensure that the sum of such series belongs to the weighted Lebesgue space. The main theorems are proved without the condition that the generator sequence is bounded; in particular, it can be unbounded. In the case of boundedness of the generating sequence, the proved theorems imply an analogue of the well-known Hardy–Littlewood theorem on trigonometric series with monotone coefficients. Some of the results obtained are presented in [2]. Weighted integrability for the sum of series with respect to multiplicative systems was considered in [3]. The weighted integrability of the sum of trigonometric series with generalized monotone coefficients was studied in [1].

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Econometric Forecasting During Crisis Periods Using Unrestricted Mixed-Data Sampling Regression

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The use of econometric models for forecasting is becoming more difficult and less reliable in crisis times (and also during steep recoveries). It is difficult to specify ex-ante models with the inclusion of many lags of the explanatory variables, in order to generate economic forecasts for the policymakers. In this paper, we investigate the mixed-data sampling (MIDAS) regression models, more specifically, its unrestricted versions – UMIDAS-AR and UMIDAS-ARMA. The special feature of MIDAS models lies in the fact that they can handle data of different frequencies at the same time, so, for example, monthly and quarterly data are integrated into the forecasts at the same time. The UMIDAS-AR and UMIDAS-ARMA models provide additional flexibility by allowing the inclusion of autoregressive and moving average components, which improves forecast accuracy and robustness, especially in volatile economic environments.

By analysing the components of the different models from the literature, we propose recommendations for their use in nowcasting and forecasting.

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On a Central Algorithm for Calculation of the Inverse of the Multidimensional Harmonic Oscillator in the Spaces of Orbits

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We continue using our results about orbitization of quantum mechanics for multidimensional Hamiltonian $\mathscr{H}_p$ on the Euclidean space $\mathbb{R}_p$, which corresponds to the observable "energy". The notions of finite orbit of quantum harmonic oscillator $\mathscr{H}_p$ at the states ψ of quantum mechanical systems $\operatorname{orb}_n(\mathscr{H}_p,\psi) = (\psi,\mathscr{H}_p\psi,\dots,\mathscr{H}_p^n\psi)$, orbit $\operatorname{orb}(\mathscr{H}_p,\psi) = (\psi,\mathscr{H}_p\psi,\dots,\mathscr{H}_p^n\psi,\dots)$, Hilbert spaces of finite orbits $D(\mathscr{H}_p^n)$, $n\in\mathbb{N}_0$, Frechet–Hilbert spaces of all orbits $D(\mathscr{H}_p^\infty)$, corresponding to $\mathscr{H}_p$ self-adjoint finite orbital operators $\mathscr{H}_{pn} : D(\mathscr{H}_p^n)\to D(\mathscr{H}_p^n)$, corresponding to $\mathscr{H}_p$ self-adjoint orbital operator $\mathscr{H}_p^\infty : D(\mathscr{H}_p^\infty)\to D(\mathscr{H}_p^\infty)$ in Frechet–Hilbert space are studied. As well as for approximate solution of the equation $\mathscr{H}_{pn}(\operatorname{orb}_n(\mathscr{H}_p,u)) = \operatorname{orb}_n(\mathscr{H}_p,f)$, where $\mathscr{H}_{pn}(\operatorname{orb}_n(\mathscr{H}_p,u)) = \operatorname{orb}_n(\mathscr{H}_p,\mathscr{H}_pu)$, in the Hilbert space $D(\mathscr{H}_p^n)$ (resp. for the equation $\mathscr{H}_p^\infty(\operatorname{orb}(\mathscr{H}_p,u) = \operatorname{orb}(\mathscr{H}_p,f))$ in the Frechet–Hilbert space $D(\mathscr{H}_p^\infty)$ central spline algorithm is constructed. When $n=0$, a classical case is obtained. The case $p=1$ is considered in [1].

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To the Reduction, Spread and Perspective of Nonlinear Complex Analysis for Some Problems of Theoretical Physics

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We consider some theoretical physics problems (for example, according to Ph. Morse, H. Feshbach [1]) in the unified placement [2]. Emphasis of [1] is dictated by the fact that in this work, estimation, analysis or evaluation of a "number" , an event or a fact descriptive phenomena is presented by essential Laplacian and Poisson operators. For three-dimensional Problems (with respect to the independent variable) of continuum mechanics (including J. C. Maxwell system), by means of a unified representation, we consider following:

A. Methods of reduction (strictly theoretical along with empirical methods). Adopted models clearly capture the inadequacies of classical mathematical physics with respect to differential equation system type, a new physical fact detection, or in the direction of discovering some other different property;

B. The extension of research and algorithms by extending the complex analysis to essentially nonlinear integro-differential systems;

C. Construction of relatively simple numerical algorithms, convergent schemes, stable processes, using reduction to Laplace and Poisson operators.

As an illustration, the corresponding numerical realization of some nontrivial tasks, together with G. Buzhghulashili, will be presented. For example, we consider following BVP in 2D case:

$$\Delta u - u \operatorname{div} u = f, \quad x \in \Omega, \quad u\big|_{\partial\Omega} = g,$$

where Ω is square domain.

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The Combined Effect of Elastic Deformation and Viscosity Variation on a Ferrofluid Based Rough Short Bearing: A Comparison of Porous Structures

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The study investigates the effect of elastic deformation in a porous, rough, short bearing lubricated with a ferrofluid, considering the variation in viscosity. Tipei's pressure-temperature expression is used to assess the impact of viscosity changes, while the N-R model is applied to describe the magnetic fluid flow.

A modified Reynolds-type equation for pressure distribution is derived, incorporating elastic deformation, viscosity variation, porosity, and aspect ratio parameters. Subsequently, the load calculation is performed.

Tabular comparison of Load Carrying Capacity (LCC) with respect to various bearing parameters indicate that increasing magnetization, viscosity variation, and aspect ratio parameters can enhance LCC, whereas an increase in elastic deformation and porosity parameters tends to decrease LCC.

On the Importance of Teaching the Elements of Number Theory in Middle and High School

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The pedagogical as well as psychological benefits of an increase in number theory study for students of middle and high schools are investigated.

On Pure Linear Orderings of Morley o-Rank 1

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Recall that when the number of types is less than $2^{|A|}$, for each set A of cardinality λ, we have *stability* in λ. Combining stability and o-minimality we obtain o-stability. In the same way we can define Morley o-rank and o-degree of a formula. We aim to give a complete description of the pure ordered structures of o-rank Morley 1 and o-degree Morley n for each $n > 1$.

Definition 1 (B. Baizhanov, V. Verbovskiy [1]) Linear ordered structure $\mathfrak{M}$ is called *o-stable in* λ, if for any any subset $A \subseteq \mathfrak{M}$ such that $|A| \leq \lambda$ and for any arbitrary cut s in $\mathfrak{M}$ there exist the biggest λ complete types over A which are consistent with cut s. Theory T is called *o-stable in* λ if every model of T is o-stable. Theory T is called *o-stable*, if there is λ, such that T is λ stable.

(1) We say that the Morley o-rank of a formula $\phi(x)$ inside a cut $\langle C, D\rangle$ in $\mathscr{M}$ is equal to or greater than 1 and write $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) \geq 1$ if $\{\phi(x)\} \cup \langle C, D\rangle$ is consistent.

(2) $RM_{\langle C,D\rangle,\mathscr{M}}(\phi)(x) \geq \alpha + 1$ if there are infinitely many pairwise inconsistent formulas $\psi_i(x)$ such that $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x) \wedge \psi_i(x)) \geq \alpha$.

(3) If α is a limit ordinal, then $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) \geq \alpha$ if $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) \geq \beta$ for all $\beta < \alpha$.

(4) $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) = \alpha$ if $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) \geq \alpha$ and $RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) \ngeq \alpha + 1$.

Similarly, we can determine the Morley o-degree of a formula inside a cut.

Definition 2 (V. Verbovskiy, [2]) Let $\mathscr{M}$ be a totally ordered structure. We say that the Morley o-rank of a formula $\phi(x)$ inside $\mathscr{M}$ equals $o\text{-}RM_{\mathscr{M}}(\phi(x)) = \sup\{RM_{\langle C,D\rangle,\mathscr{M}}(\phi(x)) : \langle C, D\rangle$ is a cut in $\mathscr{M}\}$. Let T be a complete theory, expanding the theory of linearly ordered set. We say that the Morley o-rank of a formula $\phi(x)$ in T equals $o\text{-}RM(\phi(x)) = \sup\{RM_{\mathscr{M}}(\phi(x)) : \mathscr{M} \models T\}$.

Lemma 1 *A structure of the form $\mathbb{Q} + \mathbb{Q} \times F_2$ has Morley o-degree 3, as well as $\mathbb{Q} \times F_2 + \mathbb{Q}$, $\omega + \mathbb{Q} \times F_2$, and $\mathbb{Q} \times F_2 + \omega^*$.*

Let us define $\mathscr{O}_{1,2}$ as the set of all finite ordered sums of the form $C_1 + \cdots + C_m$, where each C_i is elementary equivalent to some element of G_2, where there are no two consecutive elements of the form $\mathbb{Q}$ and $\mathbb{Q} \times F_2$ as well as the following sums: $\omega + \mathbb{Q} \times F_2$, $\omega^* + \omega + \mathbb{Q} \times F_2$, and $\mathbb{Q} \times F_2 + \omega^*$, $\mathbb{Q} \times F_2 + \omega^* + \omega$.

Lemma 2 *$o\text{-}RM(\mathscr{M}) = 1$ and $o\text{-}DM(\mathscr{M}) \leq 2$ if and only if $\mathscr{M} \equiv \mathscr{N}$ for some $\mathscr{N} \in \mathscr{O}_{1,2}$.*

Similarly, we give a description of pure orderings up to elementary equivalence for each finite o-degree, provided that Morley o-rank of the ordering is 1.

Acknowledgments

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A Combination of Ant Colony Optimization and Simulated Annealing for Solving the Multiple Traveling Salesman Problem

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The multiple traveling salesman problem (MTSP) is one of the most important combinational optimization problems that has nowadays received much attention because of its real application in industrial and service problems [2]. The aim of this paper is to introduce a hybrid two-phase algorithm called elite MEAS for solving the MTSP which can be explained as the problem of designing collection of tours from one depot to a number of customers [1]. At the first stage, the MTSP is solved by the elite ant system (EAS), and at the second stage, the simulated Annealing (SA) is used for improving solutions. This process avoids the premature convergence and makes better solutions. Extensive computational tests on standard instances from the literature confirm the effectiveness of the presented approach.

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The Numerical Solution of the Dirichlet Generalized and Classical Harmonic Problems for Irregular n-Sided Pyramidal Domains by the MPS

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In this paper is described how the method of probabilistic solutions (MPS) can be applied to numerical solving the Dirichlet generalized and classical harmonic problems for irregular n-sided pyramidal domains, where the term "generalized" indicates that a boundary function has a finite number of first kind discontinuity curves. In the considered case: edges of the pyramid are in a role of the mentioned curves; its base is a convex polygon and a project of the vertex of the pyramid lies in the base. The suggested algorithm for the numerical solution of boundary problems consists of the following main steps:

(a) application of the MPS, which in its turn is based on a computer modeling of the Wiener process;

(b) finding the intersection point of the path of the simulated Wiener process and the pyramid surface;

(c) development of a code for the numerical realization and checking the accuracy of calculated results;

(d) calculating the meaning of a sought function at any chosen point.

For illustration two examples are considered. The results of numerical experiments are presented and discussed.

On the Representation of the "either ..., or ..." Conjunction in School Mathematics Textbooks and National Examination Tests

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It is known that the disjunctive conjunction "either ..., or ..." in propositional logic represents an exclusive disjunction. However, in the grammar book [1] it is written: "either ..., or ..." is used to indicate that one of the named identical members will be selected (to the exclusion of others)". Unfortunately, this is not reflected in school mathematics textbooks, despite the fact that the conjunction "either ..., or ..." is defined in the preparatory books for entrants and recognized by specialists [1, 2].

It will be shown the negative results that the neglect of this issue causes in the tests of the national exams.

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Boolean Algebras with Exclusive Disjunction and Symmetric Difference

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The report discusses the Boolean algebra defined by the operations of exclusive disjunction and conjunction in propositional logic. Also Boolean algebra defined by symmetric difference and intersection operations in set theory is considered. The connections between them are discussed. Paradoxical reasoning regarding De Morgan's duality laws is given.

The "Z-Criteria" of Hypotheses Testing for Statistical Structure in Banach Space of Measures

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Definition 1 An object $\{E, S, \mu_h, h \in H\}$, where $\{\mu_h, h \in H\}$ is a family of probability measures on (E, S), is called a statistical structure.

Definition 2 We will say that the Hausdorff topological space E is a Radon space if every Borel probability measure on E is a Radon measure.

Let H be the set hypotheses and $B(H)$ be σ-algebra of subsets of H which contains all finite subsets of H.

Definition 3 We will say that the orthogonal statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ admits "Z-Criteria" criteria for hypothesis testing if there exists at least one measurable mapping $\delta : (E, S_1) \to (H, B(H))$, such that

$$\overline{\mu}_h(\{x : \ \delta(x) = h\}) = 1, \ \ \forall\, h \in H.$$

Theorem *Let $M_B = \oplus_{h\in H} M_B(\overline{\mu}_h)$, $card\, H \leq c$ be the Banach space of measures, E be a Radon complete separable metric space,*

$$S_1 = \bigcap_{h\in H} dom\,(\overline{\mu}_h)$$

is a Borel σ-algebra on E. Then, in order for Borel orthogonal statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ to admit "Z-Criteria" for hypothesis testing it is necessary and sufficient that the correspondence $f \longleftrightarrow l_f$ defined by equality

$$\int_E f(x)\overline{\mu}_h\,(dx) = l_f(\overline{\mu}_h), \ \ \overline{\mu}_h \in M_B, \ \ l_f \in M_B^*$$

was one-to-one.

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Riesz Transforms Associated with the Neumann Laplacian

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In this study, we establish the sharp weighted estimates, local decay estimates and weighted endpoint estimates for Riesz transforms associated with the Neumann Laplacian R_{Δ_N} and its commutator $C_b(R_{\Delta_N})$. It is worth highlighting that the weighted class corresponding to the operator Δ_N is significantly larger than the classical Muckenhoupt weighted class and includes non-doubling weights. Furthermore, as part of our investigation, we construct the Rubio de Francia extrapolation theory for commutators and compact operators. By employing the A_{p,Δ_N} extrapolation technique, we prove the weighted compactness of the commutator $C_b(R_{\Delta_N})$.

Theorem 1 *Let $1 < p < \infty$ and $w \in A_{p,\Delta_N}$. Then $C_b(R_{\Delta_N})$ is compact on $L^p(w)$ if and only if $b \in \mathrm{CMO}_{\Delta_N}(\mathbb{R}^n)$.*

We would like to establish a compact extrapolation below to show Theorem 1.

Theorem 2 *Assume that T, $\widetilde{T}$ are linear operators satisfying*

$$Tf = \widetilde{T}(f_{+,e})\mathbf{1}_{\mathbb{R}^n_+} + \widetilde{T}(f_{-,e})\mathbf{1}_{\mathbb{R}^n_-},$$

with the following properties:

(i) *there exists $p_0 \in (1,\infty)$ such that $\widetilde{T}$ is bounded on $L^{p_0}(u)$ for all $u \in A_{p_0}$.*

(ii) *there exists $p_1 \in (1,\infty)$ such that $C_{\widetilde{b}}(\widetilde{T})$ is compact on $L^{p_1}(v)$ for some $v \in A_{p_1}$, whenever $\widetilde{b} \in \mathrm{c}(\mathbb{R}^n)$.*

Then, $C_b(T)$ is compact on $L^p(w)$ for all $1 < p < \infty$, $w \in A_{p,\Delta_N}$, and $b \in \mathrm{CMO}_{\Delta_N}(\mathbb{R}^n)$.

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On the Existence of the Solution to the Goursat Problem for Loaded System Hyperbolic Equations

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In the region $\Omega = [0,T] \times [0,\omega]$, we consider the Goursat problem for a system of loaded differential equations of hyperbolic type second order

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t) + C_0(x,t)u(x_0,t) + f(x,t),$$
$$u(0,t) = \psi(t), \quad t \in [0,T],$$
$$u(x,0) = \varphi(x), \quad x \in [0,\omega].$$

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