საქართველოს მათემატიკოსთა კავშირი
Georgian Mathematical Union

აჭარის ავტონომიური რესპუბლიკის განათლების, კულტურისა და სპორტის სამინისტრო
Ministry of Education, Culture and Sport of Adjara Autonomous Republic

ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტი
Batumi Shota Rustaveli State University

საქართველოს მათემატიკოსთა კავშირის XV საერთაშორისო კონფერენცია

ეძღვნება ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტის დაარსების 90 წლისთავს

თეზისების კრებული

XV International Conference of the Georgian Mathematical Union

Dedicated to the 90-th Anniversary of the Batumi Shota Rustaveli State University

BOOK OF ABSTRACTS

ბათუმი, 1 – 6 სექტემბერი
Batumi, September 1 – 6
2025

საორგანიზაციო კომიტეტი:

ალექსანდრე მესხი (თავმჯდომარე), ვლადიმერ ბალაძე, ანზორ ბერიძე, თენგიზ ბოკელავაძე, მიხეილ გაგოშიძე, გურამ გოგიშვილი, თინათინ დავითაშვილი (კონფერენციის მდივანი), ზურაბ ვაშაკიძე, ნუგზარ კერესელიძე, ლელა თურმანიძე (თავმჯდომარის მოადგილე), დალი მახარაძე, ლაზარე ნათელაშვილი, მიხეილ რუხაია, თემურ ჯანგველაძე (თავმჯდომარის მოადგილე).

Organizing Committee:

Alexander Meskhi (Chair), Vladimer Baladze, Anzor Beridze, Tengiz Bokelavadze, Tinatin Davitashvili (Conference Secretary), Mikheil Gagoshidze, Lazare Natelashvili, Guram Gogishvili, Temur Jangveladze (Deputy Chair), Nugzar Kereselidze, Dali Makharadze, Mikheil Rukhaia, Lela Turmanidze (Deputy Chair), Zurab Vashakidze.

ლოკალური საორგანიზაციო კომიტეტი:

ლელა თურმანიძე (თავმჯდომარე), ედელია დემურია (თავმჯდომარის მოადგილე), ფრიდონ დუმბაძე, სალომე კახიძე, ლიზა მალაკმაძე, მარიამ ფუტკარაძე.

Local Organizing Committee

Lela Turmanidze (Chair), Edelia Demuria (Deputy Chair), Pridon Dumbadze, Salome Kakhidze, Liza Malakmadze, Mariam Phutkaradze.

სამეცნიერო კომიტეტი:

მიხეილ ამაღლობელი, მალხაზ ბაკურაძე, ვლადიმერ ბალაძე, ანზორ ბერიძე (თანათავმჯდომარე), როსტომ გეწაძე, გურამ გოგიშვილი, ლუკას გრაფაკოსი, თეიმურაზ დავითაშვილი, მასსიმო ლანცა დე კრისტოფერის, როლანდ დუდუჩავა, ბესიკ დუნდუა, ლაშა ეფრემიძე, თეიმურაზ ვეფხვაძე, დავით კაპანაძე, ვახტანგ კვარაცხელია, ალექსანდრე კვინიხიძე, ალექს კირთაძე, ტარიელ კიღურაძე, ჰამლეტ მელაძე, სერგეი მიხაილოვი, მარიუს მიტრეა, ელიზბარ ნადარაია, დავით ნატროშვილი (თანათავმჯდომარე), როლანდ ომანაძე, გიორგი ონიანი, მიხეილ რუხაია, ილია სპიტკოვსკი, ვაჟა ტარიელაძე, გიორგი ტეფნაძე, შაქრო ტეტუნაშვილი, თეიმურაზ ფირაშვილი, კონსტანტინე ფხაკაძე, ომარ ფურთუხია, თორნიკე ქადეიშვილი, ევგენი შარგოროდსკი, ნუგზარ შავლაყაძე, თემურ ჩილაჩავა, ოთარ ჭკადუა, ალექსანდრე ხარაზიშვილი, გიორგი ხიმშიაშვილი, ემზარ ხმალაძე, გიორგი ჯაიანი, თემურ ჯანგველაძე.

Scientific Committee:

Mikheil Amaglobeli, Malkhaz Bakuradze, Vladimer Baladze, Anzor Beridze (co-Chair), Temur Chilachava, Otar Chkadua, Teimuraz Davitashvili, Roland Duduchava, Besik Dundua, Lasha Ephremidze, Rostom Getsadze, Guram Gogishvili, Loukas Grafakos, George Jaiani, Temur Jangveladze, Tornike Kadeishvili, David Kapanadze, Alexander Kharazishvili, George Khimshiashvili, Emzar Khmaladze, Tariel Kiguradze, Aleks Kirtadze, Vakhtang Kvaratskhelia, Aleksander Kvinikhidze, Massimo Lanza de Cristoforis, Hamlet Meladze, Sergey Mikhailov, Marius Mitrea, Elizbar Nadaraya, David Natroshvili (co-Chair), Roland Omanadze, George Oniani, Teimuraz Pirashvili, Konstantine Pkhakadze, Omar Purtukhia, Mikheil Rukhaia, Eugene Shargorodsky, Nugzar Shavlakadze, Ilia Spitkovsky, Vaja Tarieladze, George Tephnadze, Shakro Tetunashvili, Teimuraz Vepkhvadze.

რედაქტორები: გურამ გოგიშვილი, მაია ჯაფოშვილი

Editors: Guram Gogishvili, Maia Japoshvili

Contents

Abstracts of Sectional Talks 39

Abstracts of Plenary Talks

Simultaneous Variables Selection and Prediction: Wisdom or Folly

Syed Ejaz Ahmed

Brock University, Canada
E-mail: sahmed5@brocku.ca

In high-dimensional statistics settings where the number of features is greater than observations, many penalized regularization strategies were studied for simultaneous variables selection and prediction. Penalty estimation strategy yields good results when the model at hand is assumed to be sparse. However, in many real scenarios models may include strong, weak, and sparse signals. In this setting penalized methods may not distinguish predictors with weak signals and sparse signals and may treat weak signals as sparse signals. Thus, the prediction based on such a selected submodel may not be preferable due to selection and estimation bias. To overcome this difficulty, we suggest a high-dimensional post-shrinkage estimation strategy to improve the prediction performance of selected submodel. Such a high-dimensional post-shrinkage estimator (HDPSE) is constructed by shrinking a model with strong and weak signals in the direction of a candidate submodel with strong signals only. We demonstrate that the proposed HDPSE performs relatively better than its competitors. Interestingly, it improves the prediction performance of the selected submodel estimation be penalized methods. The relative performance of the proposed HDPSE strategy is appraised by both simulation studies and the real data analysis.

Nonstandard Weights and Uniform Weighted Norm Inequalities

Alexandre Almeida

Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal
E-mail: jaralmeida@ua.pt

We consider power weights with nonstandard growth and discuss their connection within generalized Muckenhoupt classes. Moreover, we introduce related variable exponent function spaces and discuss the role of uniform weighted norm inequalities in the study of mapping properties of some classical operators, including maximal operators and fractional integral operators.

This is based on joint work with H. Rafeiro (Al Ain, UAE).

Ackowledgements

This work was partially supported by the Center for Research and Development in Mathematics and Applications (CIDMA), under the Portuguese Foundation for Science and Technology (FCT, https://ror.org/00snfqn58) Multi-Annual Financing Program for R & D Units.

Solutions to Degenerate Elliptic Equations: Existence, Boundedness, Regularity

David Cruz-Uribe, OFS

Department of Mathematics, The University of Alabama, Tuscaloosa, USA
E-mail: dcruzuribe@ua.edu

It is a classical result, due to Trudinger, Nash, Moser, de Georgi, and others, that if Q is a uniformly elliptic matrix, and $f \in L^2(\Omega)$, then there exists a weak solution u of the Dirichlet problem,

$$\begin{cases} -\mathrm{Div}\,(Q\nabla u) = f & \text{for } \ x \in \Omega, \\ u = 0 & \text{for } \ x \in \partial\Omega. \end{cases}$$

If we further assume that $f \in L^q(\Omega)$, $q > \frac{n}{2}$, then solutions are bounded functions and satisfy

$$\|u\|_{L^\infty(\Omega)} \le C\|f\|_{L^q(\Omega)}.$$

This result is sharp in the sense that if $q = \frac{n}{2}$, then there exists $f \in L^{\frac{n}{2}}(\Omega)$ such that this inequality fails even for the Laplacian ($Q = I$). Finally, the solutions are locally Hölder continuous: given a ball B such that $2B \subset \Omega$, there exists $0 < \alpha < 1$ such that $u \in C^\alpha(B)$. Corresponding results hold if we consider the differential equation with lower order terms.

In this talk we will discuss results from a large project to systematically extend this theory to the degenerate elliptic equation

$$\begin{cases} -v^{-1}\mathrm{Div}\,(Q\nabla u) = f & \text{for } \ x \in \Omega, \\ u = 0 & \text{for } x \in \partial\Omega, \end{cases}$$

where Q is no longer uniformly elliptic but satisfies the degenerate ellipticity condition

$$w(x)|\xi|^2 \le \langle Q\xi, \xi\rangle \le v(x)\xi|^2, \ \ \xi \in \mathbb{R}^n.$$

We will discuss existence, uniqueness, and boundedness of solutions to this equation and the corresponding equation with lower order terms. A great deal of work has gone into determining the minimal hypotheses required to establish these results. Central has been the existence of a global degenerate Sobolev inequality

$$\bigg(\int\limits_\Omega |\varphi|^{\sigma p} v\,dx\bigg)^{\frac{1}{\sigma p}} \le \bigg(\int\limits_\Omega \big|\sqrt{Q}\,\nabla\varphi\big|^p\,dx\bigg)^{\frac{1}{p}},$$

where $\sigma \ge 1$ and φ is a smooth function of compact support. We will discuss very recent work using a generalization of Rubio de Francia extrapolation to prove such inequalities with minimal assumptions on Q and v.

Acknowledgments

This research is in collaboration with Scott Rodney, Cape Breton University, Sydney Canada, and Yusuf Zeren and his students Şeyma Çetin and Feyza Elif Dal at Yıldız Technical University, Istanbul Türkiye. It is supported by a Simons Foundation Travel Support for Mathematicians Grant; by NSF Grant # DMS-2349550; by TUBITAK 2211-E Domestic Direct Doctorate Scholarship Program; an NSERC development grant; and by TUBITAK 2501 Joint Research Program grant # 223N112.

Spectral Properties of Convolution Equations on the Submonoid $\mathrm{M} = [0, 1)$

Roland Duduchava[1,2]

[1] *Viktor Kupradze Institute of Mathematics, University of Georgia, Tbilisi, Georgia*

[2] *Andrea Razmadze Institute of Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: roldud@gmail.com

The interval $\mathbf{G} = (-1, 1)$ turns into a Lie group under the group operation $x \circ y := (x + y)(1 + xy)^{-1}$, $x, y \in \mathbf{G}$, the Fourier transformation $\mathscr{F}_{\mathbf{G}}$ and the invariant Haar measure $d\mu_{\mathbf{G}} := \frac{dt}{1-t^2}$. Then $\mathbf{M} = [0, 1)$ is a submonoid of $\mathbf{G}$ (has the same binary operation $x \circ y$) and we can induce the invariant Haar measure $d\mu_{\mathbf{M}}$ and the Fourier transformation $\mathscr{F}_{\mathbf{M}}$ from $\mathbf{G}$ to $\mathbf{M}$. The main object of the investigation is the Fourier convolution operator $W_{\mathbf{M},a} := r_+ W^0_{\mathbf{G},a} \ell_+$, which represents the restriction of the convolution $W^0_{\mathbf{G},a} := \mathscr{F}^{-1}_{\mathbf{G}} a \mathscr{F}_{\mathbf{G}}$ from $\mathbf{G}$.

An example of such convolution integro-differential equation on the submonoid $\mathbf{M} = [0, 1)$ is

$$\sum_{j=0}^{n} \left[a_j \mathfrak{D}^{m_j} u(t) - b_j \mathfrak{D}^{n_j} \int_0^1 k_j\Big(\frac{t-\tau}{1-t\tau}\Big) \frac{\mathfrak{D}^{\ell_j} u(\tau)\, d\tau}{1-\tau^2} \right] = f(t), \quad t \in \mathbf{M},$$

where $a_j, b_j \in \mathbb{C}$, $m_j, n_j, \ell_j \in \mathbb{N}$, $j = 1, \dots, n$, $k_j \in \mathbb{L}_1(\mathbf{G}, d\mu_{\mathbf{G}})$, $\mathfrak{D}u(x) := -(1 - x^2)\, \frac{d}{dx}\, u(x)$ is the generic differential operator, which is a convolution operator and $(\mathscr{F}_{\mathbf{G}} \mathfrak{D}) = -i\xi$, $\xi \in \mathbb{R}$.

The theory of convolution operators $W_{\mathbf{M},a}$ on the submonoid $\mathbf{M}$ is much more complicated, but more rich and important in applications (example of Wiener–Hopf equations on submonoid $\mathbf{M} = [0, \infty)$ of the Lie group $\mathbf{G} = (-\infty, \infty)$ is a good example). Convolution equation $W_{\mathbf{M},a}\varphi = f$ in the Generic Bessel potential space setting $f \in \mathbb{GH}^{s-r}_p(\mathbf{M}, d\mu_{\mathbf{M}})$, $\varphi \in \mathbb{GH}^s_p(\mathbf{M}, d\mu_{\mathbf{M}})$, $1 < p < \infty$, $s, r \in \mathbb{R}$, has non-trivial Fredholm index. The Fredholmity and solvability conditions for discontinuous symbols $a(\xi)$ depend on the parameters of the spaces.

We expose a full theory of such convolution integro-differential equations: Fredholm property and solvability criteria, index formula. Formula for solutions are available through the factorization of the symbol.

It is worth to mention that the celebrated equations of Prandtl, Tricomi and Lavrentjev–Bitsadze belong to the class of convolution equations on the Lie group $\mathbf{G} = (-1, 1)$.

The report is based on the works [1], [2] and some other recent publications.

Acknowledgments

The work was supported by the Rustaveli National Science Foundation of Georgia, Grant # FR-24-69.

References

[1] D. Cardona, R. Duduchava, A. Hendrickx and M. Ruzhansky, Generic Bessel potential spaces on Lie groups. *Tbilisi analysis and PDE seminar*, 43–54, Trends Math., Res. Perspect. Ghent Anal. PDE Cent., 7, *Birkhäuser/Springer, Cham*, 2024.

[2] R. Duduchava, Convolution equations on the submonoid $\mathbf{M} = [0, 1)$, *Boletín de la Sociedad Matemática Mexicana (BSMM)* (submitted).

Functions, Universal with Respect to the Classical Systems

Martin Grigoryan
Yerevan State University, Yrevan, Armenia, E-mail: gmarting@ysu.am

The present talk is devoted to studying the problem of the existence of functions (universal functions), whose Fourier series with respect to a given classical system are universal in a certain sense for different function classes. We will also discuss the problem of the existence of universal pairs and universal triads. Existence of functions and series which are universal in some or other sense has been extensively studied in the theory of functions of real or complex variable. First examples of universal functions were constructed by Birkhof in complex analysis setting (any entire function was shown as being representable in any disc by uniformly convergent translations of a universal function) and by Marcinkiewicz in real analysis setting (any measurable function was shown as being representable as the limit almost everywhere of some sequence of difference relations of a universal function. The concept of a universal series dates back to Men'shov and Talalyan. Recently in works ([1]–[10]) we defined different types of universal functions (almost universal, quasi-universal, universal in the sense of signs, universal in the sense of permutations, conditionally universal). and obtained some results on existence and description of the structure of functions (universal functions), whose Fourier series are universal (in some or other sense, for various function classes) for a given classical system. In particular holds

Theorem *There exists an integrable function U that is almost universal for the class of all measurable functions with respect to the trigonometric system.*

Acknowledgments

This work was supported by the RA MSECS Com. of Science, in the frames of the research project # 21AG-1A066.

References

[1] M. G. Grigoryan, On the universal and strong (L^1, L^∞)-property related to Fourier–Walsh series. *Banach J. Math. Anal.* **11** (2017), no. 3, 698–712.

[2] M. G. Grigoryan, Functions, universal with respect to the classical systems. *Adv. Oper. Theory* **5** (2020), no. 4, 1414–1433.

[3] M. G. Grigoryan, On universal Fourier series in the Walsh system. (Russian) *Sibirsk. Mat. Zh.* **63** (2022), no. 5, 1035–1051; translation in *Sib. Math. J.* **63** (2022), no. 5, 868–882.

[4] M. G. Grigoryan, On almost universal double Fourier series. (Russian) *Tr. Inst. Mat. Mekh.* **28** (2022), no. 4, 91–102; translation in *Proc. Steklov Inst. Math.* **319** (2022), suppl. 1, S129–S139.

[5] M. G. Grigoryan, On universal (in the sense of signs) Fourier series with respect to the Walsh system. (Russian) *Mat. Sb.* **215** (2024), no. 6, 3–28; translation in *Sb. Math.* **215** (2024), no. 6, 717–742.

[6] M. G. Grigoryan and L. N. Galoyan, On the universal functions. *J. Approx. Theory* **225** (2018), 191–208.

[7] M. Grigoryan and L. Galoyan, On Fourier series that are universal modulo signs. *Studia Math.* **249** (2019), no. 2, 215–231.

[8] M. G. Grigoryan and S. V. Konyagin, On Fourier series in the multiple trigonometric system. (Russian) *Uspekhi Mat. Nauk* **78** (2023), no. 4(472), 201–202; translation in *Russian Math. Surveys* **78** (2023), no. 4, 782–784.

[9] M. G. Grigoryan and A. A. Sargsyan, On the universal function for the class $L^p[0,1]$, $p \in (0,1)$. *J. Funct. Anal.* **270** (2016), no. 8, 3111–3133.

[10] A. Sargsyan and M. Grigoryan, Universal function for a weighted space $L^1_\mu[0,1]$. *Positivity* **21** (2017), no. 4, 1457–1482.

Lyapunov Functions and Contraction Metrics: Theory and Numerical Methods

Sigurður Freyr Hafstein

Faculty of Physical Sciences, University of Iceland, Reykjavík, Iceland
E-mail: shafstein@hi.is

Lyapunov functions and contraction metrics, which are a kind of Lyapunov functions on the tangent space, are an important tool in the qualitative analysis of dynamical systems. A Lyapunov function is the mathematical generalisation of the concept of dissipative- or free energy in physics. It is a real-valued function from the state-space of a dynamical system that is decreasing along all system trajectories. Hence, the system ends up at a local minimum of the Lyapunov function. We discuss the theory of conventional- and complete Lyapunov functions. Further, we discuss its extension to the tangent space resulting in a contraction metric. Finally, we discuss numerical methods to compute both Lyapunov functions and contraction metrics.

What is Model-Theoretic Algebra?

Deirdre Haskell[1,2]

[1]*Department of Mathematics and Statistics, McMaster University, Hamilton, ON, Canada*
E-mail: haskell@mcmaster.ca

[2]*Fields Institute for Research in Mathematical Sciences, Toronto, ON, Canada*
E-mail: dhaskell@fieldsinstitute.ca

Model-theoretic algebra is a subdiscipline of mathematical logic, in which the tools of model theory are used to study particular algebraic structures. My goal in this talk is to outline the basic methods and foundational model-theoretic results and show how they can be used in the context of theories of fields with both an ordering and a valuation. I will put this in the context of a project I am working on to study valued fields with restricted analytic functions.

Acknowledgments

The work was supported by the Natural Sciences and Education Research Council of Canada.

References

[6] R. Cluckers, An introduction to b-minimality. *Logic Colloquium 2006*, 91–102, Lect. Notes Log., 32, *Assoc. Symbol. Logic, Chicago, IL*, 2009.

[1] P. Cubides Kovacsics and D. Haskell, Real closed valued fields with analytic structure. *Proc. Edinb. Math. Soc. (2)* **63** (2020), no. 1, 249–261.

[2] J. Denef and L. van den Dries, p-adic and real subanalytic sets. *Ann. of Math. (2)* **128** (1988), no. 1, 79–138.

[5] L. Lipshitz and Z. Robinson, Overconvergent real closed quantifier elimination. *Bull. London Math. Soc.* **38** (2006), no. 6, 897–906.

[3] Ya. Peterzil and S. Starchenko, Expansions of algebraically closed fields in o-minimal structures. *Selecta Math. (N.S.)* **7** (2001), no. 3, 409–445.

[4] L. van den Dries and A. H. Lewenberg, T-convexity and tame extensions. *J. Symbolic Logic* **60** (1995), no. 1, 74–102.

Geometric Properties of Rough Curves Via Dynamical Systems: SBR Measure, Local Time, Rademacher Chaos and Number Theory

Peter Imkeller

Humboldt Universität zu Berlin, Berlin, Germany
E-mail: imkeller@mathematik.hu-berlin.de

We investigate geometric properties of graphs of Takagi and Weierstrass type functions, represented by series based on smooth functions. They are Hölder continuous, and can be embedded into smooth dynamical systems, where their graphs emerge as pullback attractors. It turns out that occupation measures and Sinai–Bowen–Ruelle (SBR) measures on their stable manifolds are dual by "time" reversal.

A suitable version of approximate self-similarity for deterministic functions allows to "telescope" small scale properties from macroscopic ones, and leads to representations of relevant functionals along dyadic expansions. As one consequence, absolute continuity of the SBR measure is seen to be dual to the existence of local time.

The investigation of questions of smoothness both for SBR and for occupation measures surprisingly leads us to the Rademacher version of Malliavin's calculus, Bernoulli convolutions, and into probabilistic number theory. The link between the rough curves considered and smooth dynamical systems can be generalized in various ways. For instance, Gaussian randomizations of Takagi curves just reproduce the trajectories of Brownian motion. Applications to regularization of singular ODEs by rough signals are envisaged.

This is joint work with O. Pamen (University of Liverpool and AIMS Ghana) and F. Proske (University of 10 Oslo).

Hausdorff and Dunkl–Hausdorff Operators for Monotone Functions and Monotone Weights

Pankaj Jain

Department of Mathematics, South Asian University, New Delhi, India
E-mail: pankaj.jain@sau.ac.in

In this talk, we shall discuss the $L^p_v(\mathbb{R}^+)$-$L^q_u(\mathbb{R}^+)$ boundedness of the Hausdorff operator

$$(H_\phi f)(x) := \int\limits_0^\infty \frac{\phi(y)}{y}\, f\Big(\frac{x}{y}\Big)\, dy$$

on the cone of non-increasing functions for $1 < p \le q < \infty$ as well as $1 < q < p < \infty$. We shall also consider the more general Dunkl–Hausdorff operator

$$(H_{\alpha,\phi} f)(x) := \int\limits_0^\infty \frac{\phi(y)}{y^{2\alpha+2}}\, f\Big(\frac{x}{y}\Big)\, dy.$$

and characterize its weighted $L^p(\mathbb{R}^+)$ boundedness for monotone weights.

Some Non-Correct Problems for a Partial Differential Equations

Otar Jokhadze

The Faculty of Exact and Natural Sciences,
Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: otar.jokhadze@tsu.ge

For the inhomogeneous equation of string oscillation, necessary and sufficient conditions for the solvability of Dirichlet and Neumann type problems with inhomogeneous boundary conditions are established. If these conditions are satisfied, the solutions to the corresponding problems are given in quadratures. In particular, it is shown that the corresponding homogeneous problems have an infinite number of linearly independent solutions that can be found explicitly. The correctness of one version of the Zaremba problem is also proven, and its solution in quadratures is given. The mixed problem for partial differential equations of the second order with involution is considered, the main part of which without involution is represented by a wave operator. Depending on the value parameter, standing in front of the member with involution, the question of the correctness of the problem for this equation is investigated. In an infinite strip for second-order hyperbolic equations with constant coefficients, a periodic problem is investigated. Two-parameter weighted spaces are introduced, depending on the behavior of the solution at infinity. Depending on the parameters of the weight space, different cases are considered: the problem has a unique solution, a solution exists, but the corresponding homogeneous problem has an infinite number of linearly independent solutions, the problem cannot have more than one solution and for its solvability it is necessary and sufficient that a certain orthogonality condition are satisfied, the problem is not solvable even according to Hausdorff. The influence of lower terms on the correctness of the statement of the characteristic Dirichlet–Goursat problems for a third-order linear hyperbolic equation is considered. It is shown that if the equation does not contain lower terms, then the problem is ill-posed, in particular, a homogeneous problem has an infinite number of linearly independent solutions, and if the equation contains at least one of the coefficients with lower term, then the problem is correctly posed in the corresponding characteristic domain. Using invariants of Riemann and general solutions of the Euler–Poisson–Darboux–Riemann equations, a new class of exact solutions of the Von Karman's equation in the nonlinear theory of gas dynamics is given.

A mixed problem with a nonlinear boundary condition for a semilinear wave equation is investigated. The uniqueness and the local and global solvability of the problem are studied depending on the character of the nonlinearities occurring both in the equation and in the boundary condition.

On Asplund Spaces $C(X)$ of Continuous Functions with the Compact-Open Topology

Jerzy Kąkol

Adam Mickiewicz University, Poznań, Poland
E-mail: kakol@amu.edu.pl

A famous theorem of Namioka and Phelps says that for a compact space X, the Banach space $C(X)$ of continuous real-valued functions on a compact space X is Asplund if and only if X is scattered. We extend this result to the space $C_k(X)$ of continuous real-valued functions endowed with the compact-open topology for several natural classes of non-compact Tychonoff spaces X. Illustrating examples will be provided. The weak Asplund property will be also discussed. The concept of Δ_1-spaces, recently introduced by Kakol, Kurka and Leiderman, will be used for this research.

Sensitivity Analysis for Maxwell's Equations in Electromagnetic Cavities

Pier Domenico Lamberti

Dipartimento di Tecnica e Gestione dei Sistemi Industriali,
University of Padova, Vicenza, Italy
E-mail: pierdomenico.lamberti@unipd.it

We consider the spectral problem for the stationary Maxwell's equations under homogeneous boundary conditions in a cavity and investigate how the eigenvalues depend on perturbations of the cavity's shape. The problem is reduced to the study of the curl-curl operator with homogeneous electric-type boundary conditions. We examine families of diffeomorphic domains and discuss the analytic dependence of the eigenvalues on the domain shape. In particular, we establish a Hadamard–Hirakawa-type formula for the shape derivatives and derive a Rellich–Pohozaev-type identity. Additionally, we address a related shape optimization problem and demonstrate that the Faber–Krahn inequality does not hold in this vectorial setting. Time permitting, we will also discuss similar problems concerning the dependence of the eigenvalues on other parameters, such as electric permittivity and magnetic permeability.

Acknowledgments

We acknowledge support from the project *"Perturbation problems and asymptotics for elliptic differential equations: variational and potential theoretic methods"* funded by the European Union – Next Generation EU and by MUR Progetti di Ricerca di Rilevante Interesse Nazionale (PRIN) Bando 2022 grant # 2022SENJZ3.

References

[1] D. Krejčiřík, P. D. Lamberti and M. Zaccaron, A note on the failure of the Faber–Krahn inequality for the vector Laplacian. *ESAIM Control Optim. Calc. Var.* **31** (2025), Paper no. 21, 10 pp.

[2] P. D. Lamberti, P. Luzzini and M. Zaccaron, On the optimization of Maxwell's eigenvalues as functions of the electric permittivity in a cavity. *Math. Methods Appl. Sci.* **47** (2024), no. 16, 12507–12525.

[3] P. D. Lamberti and M. Zaccaron, Shape sensitivity analysis for electromagnetic cavities. *Math. Methods Appl. Sci.* **44** (2021), no. 13, 10477–10500.

[4] P. D. Lamberti and M. Zaccaron, Spectral stability of the curlcurl operator via uniform Gaffney inequalities on perturbed electromagnetic cavities. *Math. Eng.* **5** (2023), no. 1, Paper no. 018, 31 pp.

Hyperbolic Lebesgue Constants

Elijah Liflyand

Department of Mathematics, Bar-Ilan University, Ramat-Gan, Israel
E-mail: liflyand@gmail.com

Let

$$\Big\{k \in \mathbb{Z}^n: \ \ 1 \le \prod_{j=1}^{n} |k_j|^{\gamma_j} \le R^{\gamma_1+\cdots+\gamma_n}\Big\},$$

where $\gamma_1, \dots, \gamma_n > 0$, be a "hyperbolic crosses" dilated homothetically as $R \to +\infty$. In our study, their Lebesgue constants are, as expected, always of power growth R^p, $p > 0$, maybe up to a logarithmic factor. What turned out to be surprising is that contrary to the expected $p = \frac{n-1}{2}$ in any case, p may become, for an appropriate choice of $\gamma_1, \dots, \gamma_n$, an arbitrary number larger than that fraction. In many cases, the estimates of the Lebesgue constants are sharp in the sense that those from above and from below differ from one another only by coefficients.

Derived Functors, Satellites, and Stabilizations

Alexander Martsinkovsky

Northeastern University, Boston, United States
E-mail: a.martsinkovsky@northeastern.edu

It is widely known that the study of rings benefits from studying modules over rings. Moreover, studying modules benefits from studying (additive) functors on modules with values in abelian groups. Basic examples of such functors are known to everybody: the tensor product functors and the Hom functors. There are many other functors, and, as it turns out, module categories can be recovered as quotients of functor categories. In particular, there is no loss of information, at least from a theoretical perspective, when a module category is replaced by the category of functors on that category. This critical discovery goes back to the works of Maurice Auslander in the 1960s.

It is common knowledge that modules are often studied by tools from homological algebra, particularly derived functors like Ext and Tor. It is less known that derived functors have close "relatives" called satellites. The general mathematical community is virtually unaware of yet another group of closely related concepts called stabilizations.

The aim of this talk is to tie all these constructs together into a new homological formalism called fundamental sequence. We shall see that each additive functor (covariant or contravariant) gives rise to a left and a right fundamental sequence. Using this tool, we will go back to module categories and establish results that hold for arbitrary modules over arbitrary rings. In particular, several formulas of Auslander that only hold for finitely presented modules will be extended to arbitrary modules. Similarly, universal coefficient theorems, which come with heavy assumptions (e.g., projective complexes with projective boundaries or flat complexes with flat boundaries), will be extended to arbitrary complexes without any restrictions.

Key Subgroups in Topological Groups

Michael Megrelishvili

Department of Mathematics, Bar-Ilan University, Ramat Gan, Israel
E-mail: megereli@math.biu.ac.il

Extending results of a joint work with E. Glasner (2021), we continue to study the Polish group $G := \mathrm{Aut}\,(\mathbb{Q}_0)$ of all circular order preserving permutations of $\mathbb{Q}_0$ with the pointwise topology, where $\mathbb{Q}_0 = \mathbb{Q}/\mathbb{Z}$ is the rational discrete circle. We show that certain subgroups H of $G := \mathrm{Aut}\,(\mathbb{Q}_0)$ are *inj-key* (i.e., H distinguishes weaker Hausdorff group topologies on G) but not co-minimal in G. This counterexample answers a question from a recent joint work with M. Shlossberg (2025) and is inspired by a question proposed by V. Pestov about Polish groups G with metrizable universal minimal G-flow $M(G)$. It is an open problem to study Pestov's question in its full generality. We are also going to discuss some additional open questions. The following concepts are important in this project:

1. Key and injectively-key subgroups (new minimality conditions in topological groups).
2. Circularly ordered dynamical G-systems.
3. Universal minimal dynamical G-systems $M(G)$.
4. Maximal G-compactifications of G-spaces.

References

[1] E. Glasner and M. Megrelishvili, Circular orders, ultra-homogeneous order structures, and their automorphism groups. *Topology, geometry, and dynamics – V. A. Rokhlin-Memorial*, 133–154, Contemp. Math., 772, *Amer. Math. Soc., [Providence], RI*, 2021.

[2] M. Megrelishvili, Key subgroups in the Polish group of all automorphisms of the rational circle. *Preprint* ArXiv:2410.17905, 2024; https://arxiv.org/abs/2410.17905.

[3] M. Megrelishvili and M. Shlossberg, Key subgroups in topological groups. *Forum Math.*, 2025; https://www.degruyterbrill.com/document/doi/10.1515/forum-2023-0418/html.

Isomorphism and Stable Isomorphism in "Real", "Quaternionic" and Equivariant K-Theory

Ralf Meyer

Mathematisches Institut, Universität Göttingen, Göttingen, Germany
E-mail: rmeyer2@uni-goettingen.de

This talk is about my recent joint article with Malkhaz Bakuradze.

I will present lower bounds on the rank of a "real" vector bundle over an involutive space, such that "real" vector bundles of higher rank have a trivial summand and such that a stable isomorphism for such bundles implies ordinary isomorphism. We prove similar lower bounds also for "quaternionic" bundles. These estimates have consequences for the classification of topological insulators with time-reversal symmetry. In the end, I explain why these results fail for equivariant K-theory and what kind of result still holds in that generality.

Acknowledgements

The authors were supported by the Shota Rustaveli NSF grant # FR-23-779.

Exponentiation in Groups: General Theory and Applications

Alexei Miasnikov[1], **Mikheil Amaglobeli**[2]

[1]*Stevens Institute of Technology, New-Jersey, USA*
E-mail: amiasnikov@gmail.com

[2]*Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: mikheil.amaglobeli@tsu.ge

Exponentiation in groups is an old and well-researched subject. The main theme here is to understand what a "non-commutative module" is in various classes of groups. Following Lyndon in 1994 V. Remeslennikov and Myasnikov introduced the notion of a group admitting exponentiation in an associative unitary ring R (now called R-groups). This is the most "freest and universal" exponentiation that works in all groups and it applies nicely to free and hyperbolic groups, free products with amalgamation and HNN extensions, etc. Jointly with M. Amaglobeli we started research on R-exponentiation in varieties of groups, in particular, nilpotent and solvable ones. However, if a group satisfies an identity the notion of exponentiation can be further adjusted to reflect more closely the nature of the group. Thus, in the class of nilpotent groups there is a famous P. Hall and A. Mal'cev's exponentiation that gives a perfect notion of a "nilpotent non-commutative module". In the class of "free-like" groups, in particular, torsion-free hyperbolic ones, there is a notion of a Magnus exponentiation, there are other types of exponentiations in groups. In this talk I will discuss relations between exponentiations, how they relate to each other, and to the most universal ones. I will also describe some new applications of the theory of exponentiation in groups to other areas of mathematics.

On Higher Hochschild Homology

Temur Pirashvili

University of Georgia, Tbilisi, Georgia
E-mail: t.pirashvili@ug.edu.ge

In this talk we give an overview and some new results, on Higher order Hochschild homology and its relation on homology of mapping spaces.

Convolution-Type Operators in Grand Lorentz Spaces

Humberto Rafeiro

Department of Mathematical Sciences, United Arab Emirates University, Al Ain, UAE
E-mail: rafeiro@uaeu.ac.ae

We introduce and study a novel grand Lorentz space – that we believe is appropriate for critical cases – that lies "between" the Lorentz–Karamata space and the recently defined grand Lorentz space from [1]. We prove both Young's and O'Neil's inequalities in the newly introduced grand Lorentz spaces, which allows us to derive a Hardy–Littlewood–Sobolev-type inequality. We also discuss Köthe duality for grand Lorentz spaces, from which we obtain a new Köthe dual space theorem in grand Lebesgue spaces.

The talk is based on joint work with E. D. Nursultanov and D. Suragan.

References

[1] I. Ahmed, A. Fiorenza and A. Hafeez, Some interpolation formulae for grand and small Lorentz spaces. *Mediterr. J. Math.* **17** (2020), no. 2, Paper no. 57, 21 pp.

Hybrid Finite Element Methods for Partial Differential Equations in Networks and Hypergraphs

Andreas Rupp

Department of Mathematics, Saarland University, Saarbrücken, Germany
E-mail: andreas.rupp@uni-saarland.de

We introduce a general, analytical framework to express and to approximate partial differential equations (PDEs) numerically on graphs and networks of surfaces – generalized by the term hypergraphs. To this end, we consider PDEs on hypergraphs as singular limits of PDEs in networks of thin domains (such as fault planes, pipes, etc.), and we observe that (mixed) hybrid formulations offer useful tools to formulate such PDEs. Thus, our numerical framework is based on hybrid finite element methods (mainly the class of hybrid discontinuous Galerkin (HDG) methods).

In particular, we notice the beneficial properties of HDG in graphs and consider, as an example, the numerical solution of Timoshenko beam network models, comprised of Timoshenko beam equations on each edge of the network, which are coupled at the nodes of the network using rigid joint conditions. Our discretization of the beam network model achieves arbitrary orders of convergence under mesh refinement without increasing the size of the global system matrix. As a preconditioner for the typically very poorly conditioned global system matrix, we employ a two-level overlapping additive Schwarz method (if the graph is dense).

Acknowledgments

The work was supported by the A. Rupp has been supported by the Research Council of Finland's grants # 350101, 354489, 359633, 358944, and the Business Finland's project number # 539/31/2023.

References

[1] M. Hauck, A. Målqvist and A. Rupp, Arbitrary order approximations at constant cost for Timoshenko beam network models. *Preprint* arXiv:2407.14388, 2024; `https://arxiv.org/abs/2407.14388`.

[2] A. Rupp, M. Gahn and G. Kanschat, Partial differential equations on hypergraphs and networks of surfaces: derivation and hybrid discretizations. *ESAIM Math. Model. Numer. Anal.* **56** (2022), no. 2, 505–528.

Geometric and Summing Properties of Operators on Banach Function Spaces: Optimal Extensions

Enrique A. Sánchez Pérez

Instituto Universitario de Matemática Pura y Aplicada,
Universitat Politècnica de València, València, Spain
E-mail: easancpe@mat.upv.es

Vector measure representations offer a flexible framework for extending operators on Banach function spaces. The talk begins with an overview of the general context in which these ideas are developed, together with the main theoretical results. As a concrete application, we examine the largest subspaces on which such extensions continue to satisfy a prescribed lower p-estimate. For a given vector measure m we define the space $V_p(m) \subset L^1(m)$ consisting of functions whose induced scalar measure has finite p-variation. The starting result shows that $V_p(m)$ is maximal for preserving lower p-estimates. As applications, we show that the integration operator associated with m is $(p,1)$-summing exactly when m itself has finite p-variation, and then $V_p(m) = L^1(m)$. We also show that positive operators on $(p,1)$-concave Banach lattices, operators acting on spaces of cotype p, and operators on 2-concave Banach lattices all have optimal domains that automatically enjoy a lower p- (or 2-) estimate. These results show that the family $V_p(m)$ provides the natural environment for restricting operators while simultaneously controlling both summability and geometric structure through finite p-variation.

Joint work with M. Mastyło from Poznań.

Acknowledgments

The author was supported by the R&D&I project/grant PID2022-138342NB-I00 funded by MCIN/AEI/10.13039/501100011033/ (Spain).

References

[1] M. Mastyło and E. A. S. Pérez, Lipschitz (q,p)-summing maps from $C(K)$-spaces to metric spaces. *J. Geom. Anal.* **33** (2023), no. 4, Paper no. 113, 24 pp.

[2] S. Okada, W. J. Ricker and E. A. Sánchez Pérez, *Optimal Domain and Integral Extension of Operators.* Acting in function spaces. Operator Theory: Advances and Applications, 180. Birkhäuser Verlag, Basel, 2008.

The Rise of AI in Statistical Machine Learning: Bliss or Blight?

Ashis SenGupta

Augusta University, USA and METU, Turkey
E-mail: amsseng@gmail.com

The reach and speed of Artificial Intelligence (AI) are fascinating and can be unequivocally a highly desirable tool for modern, time-critical research in modern Statistical Data Science. However, indiscriminate use of AI for emerging statistical research can be disastrous. Even with the help from prompt engineering and proper content feeding, AI may produce erroneous and misleading results, and sometimes may even fail miserably in deriving new methodologies. In this talk, a new area of Statistical Machine Learning (SML) involving Unsupervised Learning for Manifold Data, with emerging modern applications, will be presented. This will be used to illustrate some of the pitfalls of AI and indicate methods to detect those. It is argued that in this new era of Data Science, statisticians capable of developing new and rigorous methodologies have and will become increasingly indispensable as AI is at most in its infancy to create analytical methods not yet conceived. Thus, it is essential that AI and Statistical Data Science do complement each other for the advancement of Science towards the benefit of mankind.

Hardy–Leray Type Inequalities in Variable Lebesgue and Morrey Spaces

Durvudkhan Suragan

Department of Mathematics, Nazarbayev University, Astana, Kazakhstan
E-mail: durvudkhan.suragan@nu.edu.kz

In this talk, we discuss the problem of extending the classical Hardy–Leray inequality and related inequalities to the scale of variable Lebesgue and Morrey spaces.

This is joint work with David Cruz-Uribe (University of Alabama) and Arash Ghorbanalizadeh (IASBS).

On the Boundedness of Calderón Commutators

Qingying Xue

School of Mathematical Sciences, Beijing Normal University
Beijing, Republic of China
E-mail: qyxue@bnu.edu.cn

It is well known that the Calderón's commutator plays an important role in Harmonic analysis. In this talk, I will briefly review its developmental history and present some of our recent results, particularly the endpoint weak-type estimates for its maximal operator. This is a joint work with Guoen Hu, Xiangxing Tao and Xudong Lai.

Abstracts of Sectional Talks

Bessenrodt–Ono Inequalities for ℓ-Tuples of Pairwise Commuting Permutations

Abdelmalek Abdesselam[1], **Bernhard Heim**[2,4], **Markus Neuhauser**[3,4]

[1] *University of Virginia, Charlottesville, VA 22904-4137, USA*
E-mail: malek@virginia.edu

[2] *University of Cologne, Weyertal 86–90, 50931 Cologne, Germany*
E-mail: bheim@uni-koeln.de

[3] *Kutaisi International University, Kutaisi University Campus, Kutaisi, Georgia*
E-mail: markus.neuhauser@kiu.edu.ge

[4] *RWTH Aachen University, Aachen, Germany*

Let S_n denote the symmetric group. We consider $N_\ell(n) := \frac{|\hom(\mathbb{Z}^\ell, S_n)|}{n!}$ which also counts the number of ℓ-tuples $\pi = (\pi_1, \dots, \pi_\ell) \in S_n^\ell$ with $\pi_i \pi_j = \pi_j \pi_i$ for $1 \leq i, j \leq \ell$ scaled by $n!$. A recursion formula, generating function, and Euler product have been discovered by Dey [5], Wohlfahrt [9], Bryan and Fulman [4], and White [8]. Let $a, b, \ell \geq 2$. It is known by Bringmann, Franke, and Heim [3], that the Bessenrodt–Ono inequality $\Delta_{a,b}^\ell := N_\ell(a) N_\ell(b) - N_\ell(a+b) > 0$ is valid for $a, b \gg 1$ and by Bessenrodt and Ono [2] that it is valid for $\ell = 2$ and $a + b > 9$. In this talk we show that for each pair (a, b) the sign of $\{\Delta_{a,b}^\ell\}_\ell$ is getting stable [1]. In each case we provide an explicit bound.

The numbers $N_\ell(n)$ had been identified by Bryan and Fulman [4] as the nth orbifold characteristics, generalizing work by Macdonald [7] and Hirzebruch–Höfer [6] concerning the ordinary and string-theoretic Euler characteristics of symmetric products, where $N_2(n) = p(n)$ represents the partition function.

References

[1] A. Abdesselam, B. Heim, and M. Neuhauser, Bessenrodt–Ono inequalities for ℓ-tuples of pairwise commuting permutations. *Ann. Comb.* (to appear).

[2] C. Bessenrodt and K. Ono, Maximal multiplicative properties of partitions. *Ann. Comb.* **20** (2016), no. 1, 59–64.

[3] K. Bringmann, J. Franke and B. Heim, Asymptotics of commuting ℓ-tuples in symmetric groups and log-concavity. *Res. Number Theory* **10** (2024), no. 4, Paper no. 83, 19 pp.

[4] J. Bryan and J. Fulman, Orbifold Euler characteristics and the number of commuting m-tuples in the symmetric groups. *Ann. Comb.* **2** (1998), no. 1, 1–6.

[5] I. M. S. Dey, Schreier systems in free products. *Proc. Glasgow Math. Assoc.* **7** (1965), 61–79 (1965).

[6] F. Hirzebruch and T. Höfer, On the Euler number of an orbifold. *Math. Ann.* **286** (1990), no. 1-3, 255–260.

[7] I. G. Macdonald, The Poincaré polynomial of a symmetric product. *Proc. Cambridge Philos. Soc.* **58** (1962), 563–568.

[8] T. White, Counting free Abelian actions. Preprint arXiv:1304.2830, 2013; `https://arxiv.org/abs/1304.2830`.

[9] K. Wohlfahrt, Über einen Satz von Dey und die Modulgruppe. (German) *Arch. Math. (Basel)* **29** (1977), no. 5, 455–457.

New Challenge in the Educational Institutions Post-Pandemic Period

Inga Abuladze[1,2]

[1]*Faculty of Informatics and Control Systems, Georgian Technical University, Tbilisi, Georgia*
E-mail: i_abuladze@gtu.ge

[2]*Faculty of Business and Technologies, Business and Technology University, Tbilisi, Georgia*
E-mail: inga.abuladze@btu.edu.ge

Since the 21st century is era of globalization, online, distance and hybrid methods are considered to be a more acceptable form of teaching in the future. The use of artificial intelligence (AI) tools in the education system is a new challenges for both professors/teachers and students.

In this study author developed model how to make the educational process more flexible in the post-pandemic period using modern teaching methods include Artificial Intelligence (AI) technologies. Machine learning algorithms are used to solve this task, which will make the learning process more efficient and objective. Along with this, it will be adjusted to the ability of each student and will make the work of the teacher easier.

In the post-pandemic period, modern teaching methods are: face-to-face, online and hybrid teaching. Each method has its own advantages and disadvantages. In this study the Author reviews all teaching methods. After the COVID-19 era, hybrid and online teaching methods are a new challenges in educational institutions, representing a more acceptable form of teaching worldwide. The situation now is that many institutions continue online classes and, at the same time, blend them with face-to-face classes.

Along with online learning, hybrid teaching methods have become established at leading universities around the world. Hybrid teaching is still a necessary trend in the future teaching reform base on its multiple advantages. However, in order to improve the teaching outcomes, students participation and learning initiatives, it is imperative to undertake additional reforms in the future teaching process. Finally, hybrid education is one of the most interesting concepts of the 21st century education system.

Self-Explanation as a Methodology of Teaching Mathematics in Secondary and High Schools

Nikoloz Abzianidze[1], **Giorgi Baghaturia**[2], **Giorgi Ghlonti**[1]

[1] *International Black Sea University, Tbilisi,Georgia*
E-mails: nabzianidze@ibsu.edu.ge; gghlonti@ibsu.edu.ge

[2] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia*
E-mail: gi.baghaturia@gtu.ge

According to ideas of modern pedagogy, knowledge within any subject area can be divided into at least three categories:

- Conceptual knowledge;
- Procedural knowledge;
- Problem-solving skills.

Conceptual knowledge means understanding the concepts accumulated in the subject area and the relations between them. Procedural knowledge refers to the knowledge of procedures and algorithms for certain actions. Under problem-solving skills - ability of identification and formulation of problems, possibility of application of different models and strategies, and the ability to analyzing solutions are assumed.

Four theoretical schemas are known for that describer the interrelationship between conceptual and procedural knowledge:

1. First of all, concepts;
2. First of all, procedures;
3. Inactivation concept;
4. Iterative concept.

Currently the most common is an iterative concept, which implies the gradual deepening of each of the above-mentioned knowledge over time. It has become clear that one type of knowledge cannot be formed, fully, without the formation of the other, and that the assimilation of new knowledge requires from the student to be actively involved in the process of constructing his own knowledge.

One of the mechanisms of active learning is the so-called self-explanation. This is a constructive activity that allows the student to be deeply involved in the learning process and to monitor it effectively.

Design of a EWMA Control Chart by Adaptation of Smoothing Constant Based on a Function of Estimated Shift

Abdullah Ali H. Ahmadini

Department of Mathematics, College of Science Jazan University
Jazan, Saudi Arabia
E-mail: aahmadini@jazanu.edu.sa

This study introduces a novel Adaptive EWMA (AEWMA) control chart designed to monitor the mean of a normally distributed process with enhanced responsiveness. The proposed methodology dynamically adjusts the smoothing constant based on a proposed continuous function of the estimated mean shift derived from the EWMA statistic. The Monte Carlo simulations are conducted to assess the performance of the AEWMA chart across various magnitudes of process mean shifts, using run-length profiles as the primary evaluation metric. The results indicate that the AEWMA chart outperforms traditional methods in terms of detection efficiency. To demonstrate its practical applicability, the AEWMAchart is applied to a real-world manufacturing dataset specifically analyzing the flow width resistance of substrates. The findings highlight the efficiency of the proposed chart, making it a valuable tool for improving process monitoring and quality control in industrial environments.

Algebras of Binary Formulas and the Number of Countable Models

Gulnaz Akimbekova[1], **Beibut Kulpeshov**[2,3]

[1]*Kazakh-British Technical University, Almaty, Kazakhstan*
E-mail: akguka@gmail.com

[2]*Kazakh-British Technical University, Almaty, Kazakhstan*
[3]*Institute of Mathematics and Mathematical Modeling, Ministry of Education and Science Almaty, Kazakhstan*
E-mail: b.kulpeshov@kbtu.kz

The present lecture concerns the notion of *weak o-minimality* originally studied by H. D. Macpherson, D. Marker and C. Steinhorn in [4]. A subset A of a linearly ordered structure M is *convex* if for all $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, <, \dots \rangle$ such that any parametrically definable subset of M is a finite union of convex sets in M. Real closed fields with a proper convex valuation ring provide an important example of weakly *o*-minimal structures.

The convexity rank of a set in a weakly *o*-minimal theory was introduced in [3]. In particular, a theory has convexity rank 1 if there are no definable (with parameters) equivalence relations with infinitely many infinite convex classes.

Algebras of binary formulas are a tool for describing relationships between elements of the sets of realizations of an one-type at the binary level with respect to the superposition of binary definable sets. A *binary isolating formula* is a formula of the form $\varphi(x, y)$ such that for some parameter a the formula $\varphi(a, y)$ isolates a complete type in $S_1(\{a\})$. In recent years, algebras of binary formulas for weakly *o*-minimal theories have been studied in [1], [2].

The following theorem is a criterion for having at most countably many countable pairwise non-isomorphic models by a small binary weakly *o*-minimal theory of finite convexity rank.

Theorem *Let T be a small binary weakly o-minimal theory of convexity rank n for some $1 \leq n < \omega$, Γ_1 and Γ_2 be maximal pairwise weakly orthogonal families of irrational and quasirational 1-types over $\varnothing$ respectively. Then T has at most ω countable models iff both Γ_1 and Γ_2 are finite, and the algebra $\mathfrak{P}_p$ of binary isolated formulas is finite for any non-algebraic $p \in S_1(\varnothing)$.*

Acknowledgments

This research has been funded by Science Committee of Ministry of Science and Higher Education of the Republic of Kazakhstan (AP19674850, BR20281002).

References

[1] A. B. Altayeva, B. Sh. Kulpeshov and S. V. Sudoplatov, Algebras of distributions of binary isolating formulas for almost ω-categorical weakly *o*-minimal theories. (Russian) *Algebra Logika* **60** (2021), no. 4,

[2] D. Yu. Emel'yanov, B. Sh. Kulpeshov and S. V. Sudoplatov, Algebras of distributions of binary formulas in countably categorical weakly *o*-minimal structures. (Russian) *Algebra Logika* **56** (2017), no. 1, 20–54; translation in *Algebra Logic* **56** (2017), no. 1, 13–36.

[3] B. Sh. Kulpeshov, Weakly o-minimal structures and some of their properties. *J. Symbolic Logic* **63** (1998), no. 4, 1511–1528.

[4] D. Macpherson, D. Marker and Ch. Steinhorn, Weakly o-minimal structures and real closed fields. *Trans. Amer. Math. Soc.* **352** (2000), no. 12, 5435–5483.

Minimization of Investment Risks by the Randomization Method

Merab Akhobadze, Elguja Kurtskhalia

Department of Interdisciplinary Informatics,
Georgian Technical University, Tbilisi, Georgia
E-mails: m.akhobadze@gtu.ge; e.kurtskhalia@gtu.ge

Uncertainty, in general, denotes a certain qualitative form of the object under consideration. Its essence lies in the lack of information about the subject of observation, the process. It can be said that when studying economic processes, we deal with a special uncertainty, since not only observable "players" participate in the economic process, but its characteristics are also determined by many invisible players and subjective-objective factors.

The study of uncertain economic processes is carried out mainly on the basis of probability theory, mathematical statistics, and the theory of fuzzy sets. One of the best methods to reduce the impact of uncertainty on decision making is the randomization method [1], as the use of randomization often allows us to detect those properties of the process under study that cannot be detected by methods of fuzzy set theory and mathematical statistics.

It is obvious that the risks and opportunities associated with economic processes also determine the investment policy of investors. In the presented work, using fuzzy number theory and randomization methods, a methodology for the optimal implementation of investment policy under uncertainty has been developed.

References

[1] B. T. Pole and P. S. Shcherbakov, *Robust Stability and Control.* (Russian) Nauka, Moscow, 2002.

Fractional Bessel Differential Equation via Fractional Laplace Transform

Yazan Alhrout, Ma'mon Abu Hammad, Amjed Zraiqat

Al-Zaytoonah University of Jordan, Airport Rd., Amman, Jordan
E-mail: amjad@zuj.edu.jo

This paper explores the application of the conformable fractional Laplace transform to solve fractional Bessel differential equations. We establish a rigorous mathematical framework for the fractional Bessel differential equation and develop a systematic methodology for its solution using the conformable fractional Laplace transform. The analytical solutions are derived in terms of fractional Bessel functions, whose properties are investigated and compared with their classical counterparts. Numerical examples demonstrate the effect of the fractional order parameter on the behavior of solutions.

Random Polynomials from a Topological Viewpoint

Teimuraz Aliashvili[1], **Beka Tsertsvadze**[2]

[1] *Institute of Fundamental and Interdisciplinary Mathematics Study, Ilia State University, Tbilisi, Georgia*
E-mail: teimuraz.aliashvili@iliauni.edu.ge

[2] *Ilia State University, Tbilisi, Georgia*
E-mail: beka.tsertsvadze.3@iliauni.edu.ge

Random polynomials with independent identically distributed Gaussian coefficients are considered. In the case of random gradient endomorphism $F = (f, g) : R^2 \to R^2$ the mean topological degree is computed, and the expected number of complex points is estimated. In particular, the asymptotes of these invariants are determined as the algebraic degree of F tends to infinity. We also give the asymptotic of the mean writhing number of a standard equilateral random polygon with a big number of sides.

Theorem 1 *Let P be a Gaussian random polynomial in two variables of algebraic degree $m \geqslant 1$ with independent standard normal coefficients. Then the expectation $E(|\operatorname{Deg} P'|)$ of the absolute topological degree of its gradient P' is asymptotically equivalent to $\frac{2}{\pi}\log m$ as m tends to infinity.*

Theorem 1 *As $n \to \infty$ the function $E|W(n)|$ is asymptotically equivalent to $\sqrt{\frac{3}{16n\ln n}}$.*

References

[1] T. Aliashvili, On topological invariants of polygonal random knots. *Bull. Georgian Acad. Sci.* **172** (2005), no. 1, 13–16.

[2] M. Kac, On the average number of real roots of a random algebraic equation. *Bull. Amer. Math. Soc.* **49** (1943), 314–320.

[3] G. Khimshiashvili and A. Ushveridze, On the average topological degree of random polynomials. *Bull. Georgian Acad. Sci.* **159** (1999), no. 3, 385–388.

[4] R. Shahbazi, R. Raizada and Sh. Edelman, Similarity, kernels, and the fundamental constraints on cognition. *J. Math. Psych.* **70** (2016), 21–34.

[5] M. Shub and S. Smale, Complexity of Bezout's theorem. II. Volumes and probabilities. *Computational algebraic geometry (Nice, 1992)*, 267–285, Progr. Math., 109, *Birkhäuser Boston, Boston, MA*, 1993.

Extremal Values of the Ratio of Differences of Power Mean, Arithmetic Mean, and Harmonic Mean

Yagub Aliyev[1], **Narmin Aliyeva**[2]

[1]*School of IT and Engineering, ADA University, Baku, Azerbaijan*
E-mail: yaliyev@ada.edu.az

[2]*Faculty of Mechanics and Mathematics, Baku State University, Baku, Azerbaijan*
E-mail: nermineliyeva934@gmail.com

Let P_α be the power mean of nonnegative real numbers $x_1, \dots, x_n$:

$$P_\alpha(x_1, \dots, x_n) = \left(\frac{\sum\limits_{i=1}^{n} x_i^\alpha}{n} \right)^{\frac{1}{\alpha}},$$

except when (i) $\alpha = 0$ or (ii) $\alpha < 0$ and one or more of x_i are zero. If $\alpha = 0$, then we assume that $P_\alpha = G_n$, where $G_n = \sqrt[n]{\prod\limits_{i=1}^{n} x_i}$ is the geometric mean of numbers $x_1, \dots, x_n$. If $\alpha < 0$ and there is an i such that $x_i = 0$, then we assume that $P_\alpha = 0$. In particular, if $\alpha = -1, 1, 2$, then we obtain familiar harmonic, arithmetic, and quadratic means:

$$H_n = \frac{n}{\sum\limits_{i=1}^{n} \frac{1}{x_i}}, \quad A_n = \frac{\sum\limits_{i=1}^{n} x_i}{n}, \quad Q_n = \sqrt{\frac{\sum\limits_{i=1}^{n} x_i^2}{n}}.$$

In [1] and [2] the best constants for the inequalities $C_1 \le \frac{A_n - H_n}{Q_n - H_n} \le C_2$ are studied for the cases $n = 3, 4, 5$. In [1] and [2] it was shown that if $n = 3$, then $C_1 \le \frac{\sqrt{3}}{3}$ and $C_2 \ge \frac{\sqrt{6}}{3}$. Similarly, if $n = 4$, then $C_1 \le \frac{1}{2}$ and $C_2 \ge \frac{\sqrt{3}}{2}$. Finally, if $n = 5$, then $C_1 \le \frac{\sqrt{5}}{5}$ and $C_2 \ge \frac{2\sqrt{5}}{5}$. In the current paper, we generalize these results by considering all the cases $n \ge 3$ in a unified way. We also studied the more general question about the best constants for the inequalities $C_1 \le \frac{A_n - H_n}{P_\alpha - H_n} \le C_2$ for all possible values of α. In particular, the following result is obtained.

Theorem *If $n \ge 3$, then $C_1 \le \frac{1}{\sqrt{n}}$ and $C_2 \ge \sqrt{\frac{n-1}{n}}$.*

Acknowledgments

The work was supported by the ADA University Faculty Research and Development Fund and Baku State University.

References

[1] T. P. Mitev, Some new inequalities among the arithmetic, geometric, harmonic and quadratic means. (Bulgarian) *Mathematics and Informatics, Bulgarian Journal of Educational Research and Practice* **59** (2016), no. 6, 626–656.

[2] T. P. Mitev, Some Holder approximations among the arithmetic, harmonic and quadratic means. (Bulgarian) *Proceedings of University of Ruse* **58** (2019), Book 6.1, 22–28.

Sturm–Liouville Problems with an Eigenparameter in the Boundary Condition

Yagub Aliyev[1], **Narmin Aliyeva**[2]

[1]*School of IT and Engineering, ADA University, Baku, Azerbaijan*
E-mail: yaliyev@ada.edu.az

[2]*Faculty of Mechanics and Mathematics, Baku State University, Baku, Azerbaijan*
E-mail: nermineliyeva934@gmail.com

We consider the Sturm–Liouville problem:

$$-y'' + q(x)y = \lambda y, \ \ 0 < x < 1,, \tag{1}$$

$$y(0)\cos\beta = y(0)\sin\beta, \ \ 0 \le \beta < \pi, \tag{2}$$

$$(a\lambda + b)y(1) = (c\lambda + d)y'(1), \tag{3}$$

where λ is a spectral parameter, $q(x)$ is a real-valued and continuous function over the interval $[0,1]$ and a, b, c, d are real constants $a \neq 0$, $b \neq 0$.

For a non-zero solution $y(x,\lambda)$ of (1), (2), we define the characteristic function as

$$\omega(\lambda) = (a\lambda + b)y(1,\lambda) - (c\lambda + d)y'(1,\lambda) \ \ (\lambda \in \mathbb{C}).$$

If λ_k is a double eigenvalue $\lambda_k = \lambda_{k+1}$ or a triple eigenvalue $\lambda_k = \lambda_{k+1} = \lambda_{k+2}$ then the associated functions y_{k+1} and y_{k+2} corresponding to the eigenfunction y_k are defined as usual. Using these definitions we find the norm $\|y_k\|_2$, the inner products (y_{k+1}, y_k), (y_{k+2}, y_k), and other useful relations for the basis properties of the system of eigen and associated functions.

Acknowledgments

The work was supported by the ADA University Faculty Research and Development Fund and Baku State University.

References

[1] Ya. Aliyev, Necessary and sufficient conditions for basis properties of the system of root functions of Sturm–Liouville boundary value problems with eigenparameter dependent boundary conditions. *Analysis and Applied Mathematics*, 21–32, Trends Math., Res. Perspect. Ghent Anal. PDE Cent., 6, *Birkhäuser/Springer, Cham*, 2024.

[2] N. Kerimov and Ya. Aliyev, Minimality conditions for Sturm–Liouville problems with a boundary condition depending affinely or quadratically on an eigenparameter. *Advances in functional analysis and operator theory*, 1–12, Contemp. Math., 798, *Amer. Math. Soc., [Providence], RI*, 2024.

Fractional Analysis of Nonlinear Dynamics in Korteweg-de Vries-Burgers' and Modified Korteweg-de Vries Equations

Musawa Yahya Almusawa, **Hassan Almusawa**

Department of Mathematics, Jazan University, Jazan, Saudi Arabia
E-mails: malmusawi@jazanu.edu.sa; haalmuswa@jazanu.edu.sa

This paper investigates the fractional-order KdV-Burgers' and modified KdV (mKdV) equations using two advanced mathematical techniques: the iterative transform method (ITM) and the residual power series transform method (RPSTM). These methods are employed to solve nonlinear fractional-order differential equations, with the Caputo operator providing the framework for fractional differentiation. The study highlights the effectiveness of ITM and RPSTM in obtaining approximate analytical solutions for these complex equations, which play a significant role in modeling wave propagation, fluid dynamics, and nonlinear systems. The proposed methods demonstrate robust convergence, accuracy, and computational efficiency in addressing the complexities of fractional-order systems. Through numerical examples and graphical representations, the paper provides insight into the behavior of solutions for different fractional orders, offering valuable contributions to the fields of mathematical physics and applied mathematics.

References

[1] K. Diethelm and N. J. Ford, Analysis of fractional differential equations. *J. Math. Anal. Appl.* 265 (2002), no. 2, 229–248.

[2] K. S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations.* A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1993.

[3] K. B. Oldham and J. Spanier, *The Fractional Calculus. Theory and Applications of Differentiation and Integration to Arbitrary Order.* With an annotated chronological bibliography by Bertram Ross. Mathematics in Science and Engineering, Vol. 111. Academic Press [Harcourt Brace Jovanovich, Publishers], New York–London, 1974.

[4] I. Podlubny, *Fractional Differential Equations. An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of their Solution and Some of their Applications.* Mathematics in Science and Engineering, 198. Academic Press, Inc., San Diego, CA, 1999.

Rough Intuitionistic Fuzzy Filters in BE-Algebras

Kholood Mohammad Alsager

Department of Mathematics, College of Science, Qassim University
Buraydah, Saudi Arabia
E-mail: ksakr@qu.edu.sa

In this paper, we introduce the notion of rough intuitionistic fuzzy filters within the context of BE-algebras and explore their fundamental properties. To further advance this study, we define a set-valued homomorphism over BE-algebras, leading to the introduction of the concept of T-rough intuitionistic fuzzy filters. Additionally, we utilize the (α, β)-cut of an intuitionistic fuzzy set in BE-algebras as a tool to characterize and deepen our understanding of these new concepts. Through these characterizations, we aim to provide a comprehensive framework for the application of rough intuitionistic fuzzy filters in algebraic structures.

Keywords and phrases: Fuzzy set, Rough set, T-rough set, Intuitionistic fuzzy set, Lower approximation, Upper approximation, Fuzzy ideal, Rough intuitionistic Fuzzy ideal.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **20** (1986), no. 1, 87–96.

[2] T. Bej and M. Pal, Doubt intuitionistic fuzzy ideals in BCK/BCI-algebras. *International Journal of Fuzzy Logic Systems (IJFLS)* **5** (2015), no. 1, 1–13.

[3] S. B. Hosseini, N. Jafarzadeh and A. Gholami, T-rough ideal and T-rough fuzzy ideal in a semigroup. *Advanced Materials Research* **433-440** (2012), 4915–4919.

[4] Y. B. Jun and K. H. Kim, Intuitionistic fuzzy ideals of BCK-algebras. *Int. J. Math. Math. Sci.* **24** (2000), no. 12, 839–849.

[5] Z. Pawlak, Rough sets. *Internat. J. Comput. Inform. Sci.* **11** (1982), no. 5, 341–356.

Convergence Rate of ℓ^p-Energy Minimization on Graphs

Gideon Amir[1], **Fedor Nazarov**[2], **Yuval Peres**[3]

[1]*Department of Mathematics, Bar-Ilan University, Ramat Gan, Israel*
E-mail: Gideon.amir@biu.ac.il

[2]*Department of Mathematics, Kent State University, Kent, OH, USA*
E-mail: nazarov@math.kent.edu

[3]*Beijing Institute of Mathematics Sciences and Applications (BIMSA), Huairou district Beijing, China*
E-mail: yperes@gmail.com

We consider the following dynamics on a connected graph (V, E) with n vertices. Given $p > 1$ and an initial opinion profile $f_0 : V \to [0, 1]$, at each integer step $t \geq 1$ a uniformly random vertex $v = v_t$ is selected, and the opinion there is updated to the value $f_t(v)$ that minimizes the sum $\sum\limits_{w\sim v} |f_t(v) - f_{t-1}(w)|^p$ over neighbours w of v. The case $p = 2$ yields linear averaging dynamics, but for all $p \neq 2$ the dynamics are nonlinear. In the limiting case $p = \infty$ (known as *Lipschitz learning*), $f_t(v)$ is the average of the largest and smallest values of $f_{t-1}(w)$ among the neighbours w of v. We show that the number of steps needed to reduce the oscillation of f_t below ϵ is at most n^{β_p} (up to logarithmic factors in n and ϵ), where $\beta_p = \max(\frac{2p}{p-1}, 3)$; we prove that the exponent β_p is optimal. The phase transition at $p = 3$ is a new phenomenon. We also derive matching upper and lower bounds for convergence time as a function of n and the average degree; these are the most challenging to prove.

Derivative-Free Optimization of a ODE System

Karen Arzumanyan[1], **Emil Gazazyan**[1,2], **Hrachya Astsatryan**[1]

[1]*Institute for Informatics and Automation Problems of the National Academy of Sciences of the Republic of Armenia, Yerevan, Armenia*
E-mail: hrach@sci.am

[2]*Institute for Physical Research of the National Academy of Sciences of the Republic of Armenia, Ashtarak, Armenia*
E-mail: emil@quopt.net

We consider a dynamical system described by nine coupled first-order ordinary differential equations for the elements of the density matrix $\rho = (\rho_1, \rho_2, \dots, \rho_9)$:

$$\dot{\rho}(t) = F(\rho(t), \theta, t),$$

where $\theta = (\Omega_{01}, \Omega_{02}, \Delta_1, \Delta_2, \sigma_1, \sigma_2, t_{01}, t_{02})$ is the vector of control parameters, including the pulse amplitudes Ω_{01}, Ω_{02}, their temporal profiles $(\sigma_1, \sigma_2, t_{01}, t_{02})$, and the frequency detunings Δ_1, Δ_2, where

$$\Omega_j(t) = \Omega_{0j} \exp\Big(- \frac{(t - t_{0j})^2}{\sigma_j^2} \Big), \quad j = 1, 2.$$

The optimization task is to determine the parameter set θ such that one of the equations (e.g., $\dot{\rho}_k$) remains identically zero for all $t \in [t_0, t_f]$. This condition is interpreted as a structural constraint that eliminates unwanted dynamical channels and ensures the system remains in a desired state.

The loss function is defined as the integral quadratic norm: $L(\theta) = \int\limits_{t_0}^{t_f} |\dot{\rho}_k(t, \theta)|^2 \, dt$, and the problem reduces to finding $L(\theta^*) = \min\limits_{\theta \in \Theta} L(\theta)$, where $\Theta \subset \mathbb{R}^8$ is the admissible parameter domain bounded by lower and upper constraints.

The system is integrated numerically using high-order Runge–Kutta methods [1], [3] (such as the Dormand–Prince scheme), which provide high accuracy and stability. The minimization of the loss functional is performed with the derivative-free BOBYQA (Bound Optimization by Quadratic Approximation) optimization method [2], chosen due to the difficulty of analytically or numerically computing gradients arising from the implicit dependence of the solution on the parameters. To reduce the risk of convergence to local minima, a multistart strategy is employed, launching the optimization from multiple initial guesses uniformly distributed within the domain Θ.

The proposed approach demonstrates high numerical stability and accuracy in enforcing the condition $\dot{\rho}_k(t, \theta^*) \equiv 0$ over the entire time interval. It should be emphasized that this method is of particular importance for quantum physics, as it enables precise control of the evolution of quantum systems.

References

[1] W. Kutta, Beitrag zur näherungsweisen Integration totaler Differentialgleichungen. (German) *Zeitschr. f. Math.* **46** (1901), 435–452.

[2] M. JD. Powell, The BOBYQA algorithm for bound constrained optimization without derivatives. *Cambridge NA Report NA2009/06, University of Cambridge, Cambridge 26*, 2009, 26–46.

[3] C. Runge, Ueber die numerische Auflösung von Differentialgleichungen. (German) *Math. Ann.* **46** (1895), no. 2, 167–178.

Complexity of the Resolving Set of KK-MBF Functions

Levon Aslanyan, Hasmik Sahakyan

Institute for Informatics and Automation Problems, Yerevan, Armenia
E-mails: lasl@sci.am; hsahakyan@sci.am

The goal of reconstructing monotone Boolean functions (MBFs) is to reconstruct a hidden function based on limited information obtained from observations. The difficulty of this problem can vary depending on the possible constraints and the type of observations. This involves algorithmic challenges, such as determining the minimum set of observations required for an exact reconstruction of the function. The problem for general MBFs is complex, which motivates the study of subclasses with acceptable reconstruction complexity. Recently, a special subclass of MBFs arising from the theory of shadow minimization of finite set systems, known as the Kruskal–Katona class was introduced [1]. In this paper, we investigate deadlockresolving sets for this class and provide cardinality estimates. Due to the resolving property, the estimate will show the number of tests required for query-based recognition of KK-MBFs.

Definition 1 Boolean function $f : B^n \to \{0, 1\}$ is monotone if for every $\alpha, \beta \in B^n$, the relation implies $f(\alpha) \le f(\beta)$.

Vertices of B^n, where f equals 1 are called *units* of the function, while those where f equals 0 are called *zeros.* α^1 is a *lower unit* if $f(\alpha^1) = 1$, and $f(\alpha) = 0$ for all $\alpha \prec \alpha^1$. α^0 is an *upper zero* if $f(\alpha^0) = 0$ and $f(\alpha) = 1$ for all $\alpha^0 \prec \alpha$. $\min T(f)$ and $\max F(f)$ denote the sets of lower units and upper zeros, respectively. Let M_n denote the set of all monotone Boolean functions on B^n.

Definition 2 A subset $G(f, S)$ of B^n is called a *resolving set* for the pair (f, S), if for every $g \in f$ the condition $g(\alpha) = f(\alpha)$ for all $\alpha \in G(f, S)$, implies that $g = f$.

To reconstruct a function it is sufficient to find its values on some of its resolving sets. A resolving set is called a *deadlock resolving set* for (f, S), if no proper subset of it is resolving for (f, S). Every $f \in M_n$ has a unique deadlock resolving set, which is included in every its resolving sets. This set is: $G(f, M_n) = \min T(f) \cup \max F(f)$ [2]. We should note that this is not the case for other classes. We consider a special class of MBFs arising from the theory of shadow minimization of finite set systems and known as the Kruskal–Katona-MBF.

Definition 3 $f \in M_n$ is called a KK-MBF type function if zeros of f on the layers of B^n compose initial segments of the reverse-lexicographic order.

We obtain cardinality estimates for a deadlock resolving set of the KK-MBF class:

Theorem $2(n-3) \le |G(f, \text{KK-MBF})| \le 2(n-2)$.

Acknowledgments

The work was supported by the grant # 25SRNSF-1B022.

References

[1] L. Aslanyan, G. Katona and H. Sahakyan, Shadow minimization Boolean function reconstruction. *Informatica (Vilnius)* **35** (2024), no. 1, 1–20.

[2] V. K. Korobkov, Monotone functions of the algebra of logic. (Russian) *Problemy Kibernet.* **13** (1965), 5–28.

Hardy-Type Inequality Associated with a Class of Generalized Hypergroup in n-Dimensional Spaces $n \geq 1$

Miloud Assal

Departement of Mathematics, University of Carthage, Carthage, Tunisia
E-mail: miloud.assal@fst.rnu.tn

This paper aims to establish a Hardy-type inequality associated with a class of generalized Hypergroup in n-dimensional spaces $n \geq 1$ for critical exponent $\sigma_0 = (\sigma_{0,1}, \sigma_{0,2}, \dots, \sigma_{0,n})$ by using the atomic decomposition. Moreover, we extend this inequality to exponents $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$ belonging to a hypercube of $\mathbb{R}^n$ containing σ_0.

References

[1] L. De Carli, On the L^p-L^q norm of the Hankel transform and related operators. *J. Math. Anal. Appl.* **348** (2008), no. 1, 366–382.

[2] C. Fefferman and E. M. Stein, H^p spaces of several variables. *Acta Math.* **129** (1972), no. 3-4, 137–193.

[3] H.-C. Li, G.-T. Deng and T. Qian, Hardy space decomposition of L^p on the unit circle. *Complex Var. Elliptic Equ.* **61** (2016), no. 4, 510–523.

[4] R. Radha, Hardy-type inequalities. *Taiwanese J. Math.* **4** (2000), no. 3, 447–456.

Differential-Boundary Value Problems for Second-Order System of Hyperbolic Equations

Anar T. Assanova

Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
E-mail: anartasan@gmail.com

On the domain $\Omega = [0,T] \times [0,\omega]$ we consider the differential-boundary value problem for system of hyperbolic equations in the following form

$$\begin{aligned} \frac{\partial^2 w}{\partial t \partial x} = A(t,x)\,\frac{\partial w(t,x)}{\partial x} + B(t,x)\,\frac{\partial w(t,x)}{\partial t} + C(t,x)w(t,x) + f(t,x) \\ + A_0(t,x)\,\frac{\partial w(\gamma(t),x)}{\partial x} + B_0(t,x)\,\frac{\partial w(\gamma(t),x)}{\partial t} + C_0(t,x)w(\gamma(t),x), \qquad (1) \end{aligned}$$

$$\begin{aligned} \mathscr{P}_2(x)\,\frac{\partial w(0,x)}{\partial x} + \mathscr{P}_1(x)\,\frac{\partial w(t,x)}{\partial t}\Big|_{t=0} + \mathscr{P}_0(x)w(0,x) + \mathscr{S}_2(x)\,\frac{\partial w(T,x)}{\partial x} \\ + \mathscr{S}_1(x)\,\frac{\partial w(t,x)}{\partial t}\Big|_{t=T} + \mathscr{S}_0(x)w(T,x) = \varphi(x), \ \ x \in [0,\omega], \qquad (2) \end{aligned}$$

$$w(t,0) = \psi(t), \ \ t \in [0,T]. \qquad (3)$$

where $u(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x))'$ is unknown vector function, the $n \times n$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $A_0(t,x)$, $B_0(t,x)$, $C_0(t,x)$ and n vector function $f(t,x)$ are continuous on Ω; $\gamma(t) = \zeta_j$ if $t \in [\theta_j, \theta_{j+1})$, $j = \overline{0, N-1}$; $\theta_j \le \zeta_j \le \theta_{j+1}$ for all $j = 0, 1, \dots, N-1$; $0 = \theta_0 < \theta_1 < \dots < \theta_{N-1} < \theta_N = T$; the $(n \times n)$ matrices $P_i(x)$, $S_i(x)$, $i = 0,1,2$, and n vector function $\varphi(x)$ are continuously differentiable on $[0,\omega]$, the n vector function $\psi(t)$ is continuously differentiable on $[0,T]$.

We propose a new approach for solving problem (1)–(3) based on introducing functional parameters [1], which represent the values of the desired solution over the time-partitioned domain. By incorporating these parameters and new unknown functions [2], the original problem is reduced to an equivalent problem for system of hyperbolic equations. Conditions for an existence and uniqueness solution to problem (1)–(3) are established in the terms of solvability to the equivalent problem.

Acknowledgments

The work was supported by the Science Committee of the Ministry of Science and Higher Education the Republic of Kazakhstan (Grants # AP23485509 and # AP19675193).

References

[1] A. T. Assanova, Initial boundary value problem for partial differential-algebraic equations with parameter. *Math. Methods Appl. Sci.* **48** (2025), no. 5, 6180–6190.

[2] A. T. Assanova and D. S. Dzhumabaev, Well-posedness of nonlocal boundary value problems with integral condition for the system of hyperbolic equations. *J. Math. Anal. Appl.* **402** (2013), no. 1, 167–178.

Bernstein Polynomial-Based Estimation and Hypothesis Testing for Bernoulli Regression Models

Petre Babilua[1], **Elizbar Nadaraya**[1,2]

[1]*Department of Mathematics, Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

[2]*Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: petre.babilua@tsu.ge; elizbar.nadaraya@tsu.ge

We propose a Bernstein polynomial-based estimator for the Bernoulli regression function and investigate its theoretical properties, including uniform consistency and asymptotic normality. Unlike kernel estimators, the Bernstein approach eliminates boundary bias while preserving smoothness and interpretability. Leveraging this estimator, we construct hypothesis tests for verifying the shape of a Bernoulli regression function and for comparing two such functions from independent samples. The tests demonstrate strong asymptotic performance.

An Algorithm for Constructing an Approximate Solution to One Non-Linear Characteristic Problem

Giorgi Baghaturia[1], **Marine Menteshashvili**[1,2]

[1] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*

[2] *Sokhumi State University, Tbilisi, Georgia*

E-mails: gi.baghaturia@gtu.ge; m.menteshashvili@gtu.ge

In the paper [1], General integrals were constructed for some equations, which belong to one class of Hyperbolic-Parabolic mixed type second order PDEs –

$$(u_y^2 - u_y)u_{xx} - (2u_xu_y + u_y - u_x - 1)u_{xy} + (u_x^2 + u_x)u_{yy} = \Phi(x, y, u, u_x, u_y),$$

Where Φ is a given function of five variables, defined and continuous on the space of independent variables (x, y) and for all finite values of arguments u, u_x and u_y.

Both families of characteristics and the set of parabolic degeneracy depend on the unknown solution. For these equations we have studied the non-linear variant of characteristic Goursat problem and applying the general integral we proved the existence and uniqueness of solution to the posed problem. We also found the analytical solutions in implicit form [3].

In the current work we consider analogous problem for the same equation and propose an algorithm for numerical solving of this problem. Equations of this type were considered also in papers [1], [2], [4]–[6], where initial and characteristic problems have been studied.

Acknowledgments

The work was partially supported by the European Unions HORIZON EUROPE Grant Project GAIN.

References

[1] G. Baghaturia, Nonlinear versions of hyperbolic problems for one quasi-linear equation of mixed type. (Russian) *Sovrem. Mat. Prilozh.* no. 90 (2014); translation in *J. Math. Sci. (N.Y.)* **208** (2015), no. 6, 621–634.

[2] G. Baghaturia, J. Gvazava and M. Menteshashvili, Cauchy problem for a quasi-linear hyperbolic equation with closed support of data. (Russian) *Sovrem. Mat. Prilozh.* **80** (2012), Part 1; translation in *J. Math. Sci. (N.Y.)* **193** (2013), no. 3, 364–368.

[3] G. G. Bagaturia and M. Z. Menteshashvili, General integral of a quasilinear equation and its application for solving a nonlinear characteristic problem. (Russian) *Sibirsk. Mat. Zh.* **60** (2019), no. 6, 1209–1222; translation in *Sib. Math. J.* **60** (2019), no. 6, 940–951.

[4] G. G. Baghaturia and M. Z. Menteshashvili, On the numerical solution of the characteristic problems for one class of quasilinear equations. *Program. Comput. Softw.* **50** (2024), suppl. 1, S33–S38.

[5] A. V. Favorskaya and I. B. Petrov, Grid-characteristic method. *Innovations in wave processes modelling and decision making*, 117–160, Smart Innov. Syst. Technol., 90, *Springer, Cham*, 2018.

[5] M. Menteshashvili, The nonlinear Cauchy problem with solutions defined in domains with gaps. (Russian) *Sovrem. Mat. Prilozh.* no. 89 (2013); translation in *J. Math. Sci. (N.Y.)* **206** (2015), no. 4, 413–423.

Equivariant Embeddings of Trivial G-bundles into Complex G-Bundles Over G-CW Complexes

Malkhaz Bakuradze

Andrea Razmadze Mathematics Institute of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: malkhaz.bakuradze@tsu.ge

The talk is based on [1], where for a complex representation V of a compact Lie group G, sufficient conditions on the isotypic parts of a complex G-vector bundle E over a G-CW-complex are found so that E contains $X \times V$ as a G-subbundle.

Acknowledgments

The authors were supported by the Shota Rustaveli NSF grant # FR-23-779.

References

[1] M. Bakuradze and R. Meyer, Equivariant embeddings of trivial G-bundles into complex G-Bundles over G-CW complexes. (to appear).

On Khavin's Points of the Unit Circle

Mzevinar Bakuridze[1], **Vaja Tarieladze**[2]

[1] *Batumi Shota Rustaveli State University, Batumi, Georgia*
E-mail: bakuridzemzevinari@mail.ru

[2] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*
E-mail: v.tarieladze@gtu.ge

We denote by $\mathbb{S}$ *the unit circle*:

$$\mathbb{S} = \{\zeta \in \mathbb{C} : \ |\zeta| = 1\}$$

We plan to discuss the following results obtained in [1]:

Theorem *Let $z \in \mathbb{C} \setminus \mathbb{S}$.*

(a) *If $|z| < 1$, then z **is not** a Khavin's point for $\mathbb{S}$.*

(b) *If $|z| > 1$, then z **is** a Khavin's point for $\mathbb{S}$.*

References

[1] M. Bakuridze and V. Tarieladze, On Khavin's points of the unit circle, 2025 (to appear).

A Study on Zero Product Preserving Operators Defined on Banach Function Spaces

Öykü Bal, Ezgi Erdoğan

Department of Mathematics, Marmara University, İstanbul, Türkiye
E-mails: oyku.bal@marmara.edu.tr; ezgi.erdogan@marmara.edu.tr

In this talk, using the properties of norming products on normable Banach function spaces, we obtain several results of a general factorization theorem of a special class of multilinear operators called zero product preserving, defined on the topological product of Banach function spaces. Relying on a general factorization theorem, we explore how interpolation theory interacts with Banach function spaces in producing such factorizations. We analyze how this theorem applies to certain fundamental spaces, illustrating the consequences. Finally, we give some summability properties and geometric inequalities for orthogonally additive homogeneous polynomials.

References

[1] E. Erdoğan, Factorization of multilinear operators defined on products of function spaces. *Linear and Multilinear Algebra* **70** (2022), no. 2, 177–202.

[2] E. Erdoğan, E. A. Sánchez Pérez and Ö. Gök, Product factorability of integral bilinear operators on Banach function spaces. *Positivity* **23** (2019), no. 3, 671–696.

[3]] G. Ya. Lozanovskii, On some Banach lattices. *Siberian Math J.* **10** (1969), 419–431.

[4] A. R. Schep, Products and factors of Banach function spaces. *Positivity* **14** (2010), no. 2, 301–319.

Functional (Co)Homological Groups and their Applications

Vladimir Baladze, Fridon Dumbadze

Department of Mathematics, Batumi Shota Rustaveli State University, Batumi, Georgia
E-mails: vbaladze@gmail.com; fridon.dumbadze@bsu.edu.ge

Using the set of functionally open finite covers of completely regular spaces in the paper are constructed Čech type functional homology functor $\check{H}^F_q(-,-;G) : \boldsymbol{Top}^2_{\boldsymbol{cr}} \to \boldsymbol{Ab}$ and functional cohomology functor $\widehat{H}^q_F(-,-;G) : \boldsymbol{Top}^2_{\boldsymbol{cr}} \to \boldsymbol{Ab}$ from the category of pairs of completely regular spaces and their completely closed subspaces to the category of abelian groups, defined Bokstein–Nowak type functional coefficient of cyclicity $\eta^F_G : \boldsymbol{Top}_{\rm cr} \to \boldsymbol{N} \cup \{-1,\infty\}$ from the class of completely regular spaces to the set of integers $t \geq -1$ and proved the equalities

$$\check{H}^F_n(X,A;G) = \check{H}_n(\beta X,\beta A;G),$$
$$\widehat{H}^n_F(X,A;G) = \widehat{H}^n(\beta X,\beta A;G),$$
$$\eta^F_G(X) = \eta_G(\beta X),$$

where N, $\check{H}_n(\beta X,\beta A;G)$, $\widehat{H}^n(\beta X,\beta A;G)$ and $\eta_G(\beta X)$ are the set of natural numbers, Čech homology group, Čech cohomology group and Bokstein–Nowak coefficient of cyclisity of Stone-Čech compactifications of pair $(X,A) \in ob(\boldsymbol{Top}^2_{\boldsymbol{cr}})$ and space $X \in ob(\boldsymbol{Top}_{\boldsymbol{cr}})$, respectively.

The report also is devoted to investigation of cohomological dimensions of completely regular topological spaces and their Stone-Čech compactifications. Using the functionally cohomology groups of completelly regular spaces are defined large cohomological dimension $D_F(X;G)$, small cohomological dimension $d_F(X;G)$ and proved the following equalities

$$d_F(X;G) = d(\beta X;G),$$
$$D_F(X;G) = D(\beta X;G).$$

References

[1] V. Baladze, Characterization of precompact shape and homology properties of remainders. *Topology Appl.* **142** (2004), no. 1-3, 181–196.

[2] V. Baladze and L. Turmanidze, On homology and cohomology groups of remainders. *Georgian Math. J.* **11** (2004), no. 4, 613–633.

[3] V. Baladze, Intrinsic characterization of Alexander–Spanier cohomology groups of compactifications. *Topology Appl.* **156** (2009), no. 14, 2346–2356.

[4] V. Baladze, The coshape invariant and continuous extensions of functors. *Topology Appl.* **158** (2011), no. 12, 1396–1404.

[5] V. Baladze, On (co)homological properties of remainders of Stone–Čech compactifications. *Trans. A. Razmadze Math. Inst.* **173** (2019), no. 1, 1–10.

[6] V. Baladze and F. Dumbadze, An inverse system approach of map and its application in (co)homology theory. *J. Math. Sci.*, 2025; `https://doi.org/10.1007/s10958-024-07469-3`.

[7] V. Baladze and F. Dumbadze, On (co)homological properties of stone-cech compactifications of completely regular spaces. *Preprint* arXiv:1806.01566, 2018; `https://arxiv.org/abs/1806.01566`.

On Complex Interpolation Theorem in Variable Lebesgue Spaces with Mixed Norm

Rovshan Bandaliyev[1], **Konul Omarova**[2], **Sevda Akhundova**[1]

[1] *Higher Mathematics, Azerbaijan University of Architecture and Construction Baku, Azerbaijan*

[2] *Higher Mathematics, Baku Business University, Baku, Azerbaijan*

E-mails: rovshan.bandaliyev@azmiu.edu.az; omarovakonulk@gmail.com

We give a multiplicative interpolation inequality in variable Lebesgue spaces with mixed norm. By the method of complex interpolation we define an interpolation spaces between variable Lebesgue spaces with mixed norm. As application of main result we establish an analog of Riesz–Thorin interpolation theorem in variable Lebesgue spaces with mixed norm. We observe that the theory of interpolation of function spaces is a useful tool in the application of qualitative properties of PDE.

References

[1] R. A. Bandaliev, On an inequality in a Lebesgue space with mixed norm and variable summability exponent. (Russian) *Mat. Zametki* **84** (2008), no. 3, 323–333; translation in *Math. Notes* **84** (2008), no. 3-4, 303–313.

[2] R. A. Bandaliev and M. M. Abbasova, On an inequality and $p(x)$-mean continuity in the variable Lebesgue space with mixed norm. *Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.* **26** (2006), no. 7, Math. Mech., 47–56.

[3] R. A. Bandaliyev, A. Serbetci and S. G. Hasanov, On Hardy inequality in variable Lebesgue spaces with mixed norm. *Indian J. Pure Appl. Math.* **49** (2018), no. 4, 765–782.

[4] L. Diening, P. Harjulehto, P. Hästö and M. Růžička, *Lebesgue and Sobolev Spaces with variable exponents.* Lecture Notes in Mathematics, 2017. Springer, Heidelberg, 2011.

[5] V. Kokilashvili, A. Meskhi, H. Rafeiro and S. Samko, *Integral Operators in Non-Standard Function Spaces*, Vol. 1. *Variable Exponent Lebesgue and Amalgam Spaces.* Operator Theory: Advances and Applications, 248. Birkhäuser/Springer, [Cham], 2016.

Some Notes on Mazurkiewicz Type Sets

Lika Beraia

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: likaberaia444@gmail.com

We say that a subset of the Euclidean plane is a Mazurkiewicz type set with respect to the family of all straight lines if there exists a natural number $n > 1$ such that $card(X \cap \ell) = n$ for any straight line ℓ lying in the plane (see [1]).

We say that a subset of the Euclidean plane is a Mazurkiewicz type set with respect to the family of all circles if there exists a natural number $k > 2$ such that $card(X \cap C) = k$ for any circle C lying in the plane (see [1]).

In the talk propositions related to the existence and properties of the above-mentioned sets from measure-theoretic and topological viewpoints are discussed.

References

[1] L. Beraia and T. Tetunashvili, Some properties of Mazurkiewicz type sets. *Trans. A. Razmadze Math. Inst.* **177** (2023), no. 3, 491–493.

On Certain Properties of the Sierpiński–Zygmund Function with Respect to Various Point Sets

Mariam Beriashvili[1,2], **Marika Khachidze**[3]

[1] *Georgian Technical University, Tbilisi, Georgia*
[2] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: m.beriashvili@gtu.ge
[3] *Georgian National University SEU, Tbilisi, Georgia*
E-mail: m.khachidze1@seu.edu.ge

It is known, that there exists a Mazurkiewicz set which contains the graph of some Sierpinski–Zygmund function acting from R into R (see [1], [2]).

By assuming Martin's Axiom and applying some properties of so-called generalized Luzin subsets of R, it can be proved that there are additive absolutely nonmeasurable functions acting from R into R (see [1]).

On other hand, under $MA + \neg CH$:

- the set D from Blumberg's theorem can be ensured to be ω_1-dense;
- g is an arbitrary function acting from R into R, then, for each uncountable set $X \subset R$, there always exists an uncountable set $Y \subset X$ such that the restriction $g \mid Y$ is continuous.

In the presented talk, we discuss some properties of the Sierpinski–Zygmund function in the context of various point sets.

Acknowledgments

The work was supported by the Joint Rustaveli-DAAD-fellowship programme, 2025 (57812193), # 91956461.

References

[1] A. Kharazishvili, Almost measurable functions on probability spaces. *Georgian Math. J.* **31** (2024), no. 5, 813–818.

[2] W. Sierpiński and A. Zygmund, Sur une fonction qui est discontinue sur tout ensemble de puissance du continu. (French) *Fundamenta Math.* **4** (1923), 316–318.

On Some Problems about Isosceles Polyhedra

Shalva Beriashvili

Ilia State University, Tbilisi, Georgia
E-mail: Shalva.beriashvili@iliauni.edu.ge

It is known that there are substantially different notions of isosceles polyhedra in combinatorial and discrete geometry (see, e.g., [1], [2]). In this presentation, we will consider some versions of isosceles polyhedra and present several problems that are closely related to combinatorial properties of convex isosceles polyhedra in the three-dimensional Euclidean space. In particular, certain connections of such polyhedra with the classical Euler formula will be discussed.

References

[1] A. Kharazishvili, *Elements of Combinatorial Geometry*, Part II. The Publishing House of Georgian National Academy of Sciences, Tbilisi, 2020.

[2] A. Kharazishvili, *Introduction to Combinatorial Methods in Geometry*. CRC Press, Boca Raton, FL, 2024.

The Braid Group, Garside Structure and Burau Representations

Anzor Beridze[1,2]

[1]*Batumi Shota Rustaveli State University, Batumi, Georegia*
E-mail: a.beridze@bsu.edu.ge

[2]*Kutaisi International University, Kutaisi, Georgia*
E-mail: anzor.beridze@kiu.edu.ge

In this talk, we speak about the Burau representation faithfulness of the four-string braid group B_4, which is a well-known open problem. It is equivalent to the faithfulness of the Jones representation, which itself is related to the Jones hypothesis stating that the Jones polynomial detects the unknot. I will review the Garside and dual Garside structures on the braid group and their role in the Lawrence–Krammer–Bigelow representation. Finally, I will discuss the current progress on the Burau representation faithfulness problem in collaboration with my co-authors.

Solution of Bitsadze–Samarskii Boundary Value Problem for Elliptic Equations using Octo-Sweep Iterative Method

Vakhtang Beridze, **Marina Abashidze**, **Guladi Phartenadze**

Batumi Shota Rustavely State University, Batumi, Georgia

E-mails: v.beridze@bsu.edu.ge, vakhtangi@yahoo.com; marina.abashidze@bsu.edu.ge; guladi.phartenadze@bsu.edu.ge

The paper deals with the numerical solution of nonlocal boundary value problems for elliptic differential equations, which are of great theoretical and practical importance in physics, engineering and other applied sciences. Such problems often arise in modeling physical phenomena, including heat conduction, fluid dynamics and related applications, which highlights the relevance of developing efficient numerical methods in this area.

In [3], an algorithm is proposed for reducing a nonlocal problem to a classical Dirichlet problem, facilitating the use of numerical techniques for efficient computation. In [1], a numerical method for solving an optimal control problem is presented, implemented using the Mathcad software package. Additionally, [2] introduces a numerical algorithm based on the MEDG method to solve an optimal control problem for the Helmholtz equation with m-point nonlocal boundary conditions.

This study presents a comprehensive analysis of existing iterative schemes and, building on this analysis, proposes a novel method called Octo–Sweep [4]. The proposed approach is both theoretically formulated and computationally implemented using the Mathcad environment. Comparative numerical experiments demonstrate the high efficiency of the Octo–Sweep method, revealing a substantial reduction in computational time and iteration count while maintaining high solution accuracy.

References

[1] M. Abashidze and V. Beridze, Solution of an optimal control problem for Helmholtz equations with m-point nonlocal boundary conditions by means Mathcad. *Proceedings of 2018 IEEE East-West Design & Test Symposium (EWDTS)*, 2018, 1–5.

[2] D. Devadze and V. Beridze, Solution of the optimal control problem for Helmholtz equations with m-point nonlocal boundary conditions using MEDG method. *Proceedings of the 20th International Conference on Computer Systems and Technologie (CompSysTech '19)*, 2019. 131–136.

[3] D. G. Gordeziani, *On the Methods of Solution for One Class of Nonlocal Boundary Value Problems*. Tbilis. Gos. Univ., Tbilisi, 1981.

[4] M. Kamalrulzaman Md Akhir, M. Othman, M. Suleiman and J. Sulaiman, A new octo modified explicit group iterative method for the solution of 2D elliptic PDEs. *Appl. Math. Sci. (Ruse)* **6** (2012), no. 29-32, 1505–1524.

Enhancing Inductive Logic Programming with Neurosymbolic Inference and Probabilistic Reasoning for Relational Pattern Discovery

Merihan Hazem Anwar Labib Bishara

International Black Sea University, Tbilisi, Georgia
E-mail: merihan.hazem20@gmail.com

Inductive Logic Programming (ILP) serves as an effective approach for natural language processing. In this study, we will talk about a proposed enhanced framework called Propper [1], [2], which combines neurosymbolic reasoning with probabilistic ILP capabilities [3]. The framework utilizes program transformation through grammar-based operators to facilitate neurosymbolic inference. Additionally, it introduces a continuous evaluation metric for hypothesis selection and incorporates a flexible constraint system to ease the limitations typically placed on the hypothesis space. We assess the framework's performance by presenting a comparison by [4] of Propper with statistical machine learning models, such as Support Vector Machines (SVMs) and Graph Neural Networks (GNNs), on a practical task: showing the detection of relational patterns in satellite images using object predictions produced by a fallible deep learning model [4]. We will also discuss about a streamlined provenance mechanism by [5] that is also implemented to support real-world applicability.

References

[1] A. Cropper and R. Morel, Learning programs by learning from failures. *Mach. Learn.* **110** (2021), no. 4, 801–856.

[2] C. Hocquette, A. Niskanen, M. Järvisalo and A. Cropper, Learning MDL Logic Programs from Noisy Data. *Proceedings of the AAAI Conference on Artificial Intelligence* **38** (2024), no. 9, 10553–10561.

[3] A. d'Avila Garcez, M. Gori, L. C. Lamb, L. Serafini, M. Spranger and S. N. Tran, Neural-symbolic computing: an effective methodology for principled integration of machine learning and reasoning. *Preprint* arXiv:1905.06088, 2019; `https://arxiv.org/abs/1905.06088`.

[4] F. Hillerström and G. Burghouts, Towards probabilistic inductive logic programming with neurosymbolic inference and relaxation. *Theory Pract. Log. Program.* **24** (2024), no. 4, 628–643.

[5] A. Kimmig, G. Van den Broeck and L. De Raedt, Algebraic model counting. *J. Appl. Log.* **22** (2017), 46–62.

Understanding Ambiguity using Unranked Probabilistic Logic

Merium Hazem Anwar Labib Bishara

International Black Sea University, Tbilisi, Georgia
E-mail: merium.hazem20@gmail.com

Understanding ambiguity is a pervasive and difficult challenge in natural language processing (NLP), often arising when a sentence requires multiple semantic interpretations. Disambiguating such structures requires not only flexibility but also a principled way to model and reason about uncertainty. We prove such an issue through a use case example, "The man saw the dog with the telescope" which exhibits two plausible interpretations based on different parse trees. Capturing and comparing these derivations in a probabilistic framework is essential for meaningful disambiguation.

In this paper, we introduce a novel formalism that extends probabilistic first-order logic [1]–[3] with sequence variables and unranked function and predicate symbols [4]–[6], resulting in a highly expressive system that we call $LFOP_u$. This logic naturally accommodates variable-arity constructs and supports the concise representation of flexible grammatical rules and probabilistic dependencies. We formally define its syntax and semantics, establish soundness and completeness, and demonstrate its practical relevance through a detailed example of how this logic helps in understanding ambiguity. This showcases the potential of our formalism in bridging symbolic and probabilistic reasoning for advanced NLP.

References

[1] Z. Ognjanović and A. Ilić-Stepić, Logics with probability operators. *Probabilistic extensions of various logical systems*, 1–35, *Springer, Cham*, 2020.

[2] Z. Ognjanovic and M. Raškovic, Some first-order probability logics. *Theoret. Comput. Sci.* **247** (2000), no. 1-2, 191–212.

[3] Z. Ognjanović, M. Rašković and Z. Marković, *Probability Logics. Probability-Based Formalization of Uncertain Reasoning.* Springer, Cham, 2016.

[4] B. Dundua, L. Kurtanidze and M. Rukhaia, Unranked tableaux calculus for web related applications. 2017 IEEE First Ukraine Conference on Electrical and Computer Engineering (UKRCON), 2017, 1181–1184.

[5] L. Kurtanidze and M. Rukhaia, Skolemization in unranked logics. *Bull. TICMI* **22** (2018), no. 1, 3–10.

[6] T. Kutsia and B. Buchberger, Predicate Logic with Sequence Variables and Sequence Function Symbols. In: Asperti, A., Bancerek, G., Trybulec, A. (eds) *Mathematical Knowledge Management. MKM 2004*, pp. 205–219. Lecture Notes in Computer Science, vol. 3119. *Springer, Berlin, Heidelberg*, 2004.

Malcev Correspondence Between Nilpotent k-Groups and Nilpotent Lie k-Algebras

Tengiz Bokelavadze

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: tengiz.bokelavadze@atsu.edu.ge

In this paper, we study the famous Malcev correspondence between nilpotent k-groups G and nilpotent Lie k-algebras L over a field k of characteristic zero from the model-theoretic, algebro-geometric, and algorithmic view point. We prove that in this case the group G and the corresponding Lie algebra $L(G)$ are bi-interpretable by equations in each other. This gives a much more precise description of the correspondence, which implies that in addition to the classical categorical properties, the group G and the algebra $L(G)$ share many more algebraic, algorithmic, and model-theoretic properties.

References

[2] T. Bokelavadze, On some properties of W-power groups. *Bull. Georgian Acad. Sci.* **172** (2005), no. 2, 202–204.

[1] Ph. Hall, *The Edmonton Notes on Nilpotent Groups.* Queen Mary College Mathematics Notes. Queen Mary College, Mathematics Department, London, 1969.

[4] A. G. Myasnikov and V. N. Remeslennikov, Degree groups. I. Foundations of the theory and tensor completions. (Russian) *Sibirsk. Mat. Zh.* **35** (1994), no. 5, 1106–1118, translation in *Siberian Math. J.* **35** (1994), no. 5, 986–996.

[5] D. Quillen, Rational homotopy theory. *Ann. of Math. (2)* **90** (1969), 205–295.

[3] L. E. Sadovskiĭ, Projectivities and isomorphisms of nilpotent groups. (Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **29** (1965), 171–208.

An Isometry Theorem for Persistent Homology of Circle-Valued Functions

Nathan Broomhead, Mariam Pirashvili

School of Engineering, Computing and Mathematics, University of Plymouth Plymouth, UK

E-mails: nathan.broomhead@plymouth.ac.uk; mariam.pirashvili@plymouth.ac.uk

Circle-valued functions provide a natural extension of real-valued functions, where instead of measuring values along a linear scale the values lie on a circle. This opens up new possibilities for analysing data in settings where the underlying structure is periodic or has a direction associated to it.

There has been significant work on circle-valued maps in the context of persistent homology. Zig-zag persistence generalises to circle-valued functions, leading to persistence modules which are representations of a zig-zag cyclic quiver of type $\tilde{A}_n$. This approach was first introduced in the work of Burghelea and Dey, who classified the resulting indecomposable represenatitons of the $\tilde{A}_n$ quiver as barcodes and Jordan blocks and proposed an algorithm for computing these.

The stability of the numerical invariants of persistent homology with respect to the interleaving distance is the fundamental result in this area that gives this method its strong theoretical foundation. Over the years, this distance has been generalised to the zig-zag setting and to general poset representations, using tools from representation theory. Most notably, the involvement of the Auslander-Reiten translate in the definition of the interleaving distance has meant that the robust machinery of representation theory could be employed to derive algebraic stability theorems in more general settings.

Our main result is defining a stable interleaving distance on circle-valued persistence modules. Moreover, we propose a novel, computer-friendly way to encode the invariants of circle-valued functions via the so-called geometric model, a relatively new tool from representation theory.

We also propose a matching distance based on the geometric model, and show that this matching metric coincides with the interleaving distance.

Reference

[1] N. Broomhead and M. Pirashvili, An isometry theorem for persistent homology of circle-valued functions. *Preprint* arXiv:2506.02999, 2025; `https://arxiv.org/abs/2506.02999`.

A Fractional-Order MPC Framework for EEG-Based Seizure Prevention

Sadullah Bulut, **Muhammed Yiğider**

Department of Mathematics, University of Erzurum Technical, Erzurum, Turkey
E-mails: sadullah.bulut@erzurum.edu.tr; myigider@erzurum.edu.tr

This study presents a fractional-regular model predictive control (FO-MPC) approach based on EEG data for real-time suppression of epileptic seizures. Dynamic models that more accurately represent the memory effects of the neurological system were constructed using different fractional derivative definitions, such as Caputo, Riemann–Liouville, and Grunwald–Letnikov. Each derivative definition was evaluated in terms of model prediction performance and control performance; Caputo-derived models stood out in terms of computational efficiency and early seizure prediction. Simulations performed with real EEG data demonstrated that the FO-MPC method is effective in suppressing epileptic activity. This study contributes to the literature on the use of fractional derivative-based control structures in engineering applications and offers a new perspective for closed-loop neurostimulation systems.

References

[1] Z. Liang, Z. Luo, K. Liu, J. Qiu and Q. Liu, Online learning Koopman Operator for closed-loop electrical neurostimulation in epilepsy. *IEEE Journal of Biomedical and Health Informatics* **27** (2023), no. 1, 492–503.

[2] R. Magin, *Fractional Calculus in Bioengineering.* Begell House Publishers, 2006.

[3] I. Podlubny, *Fractional Differential Equations.* Academic Press, 1999.

[4] L.-H. Wang, Z.-N. Zhang, C.-X. Xie, H. Jiang, T. Yang, Q.-P. Ran, M.-H. Fan, I-C. Kuo, Z.-J. Lee and P. A. R. Abu, A novel real-time threshold algorithm for closed-loop epilepsy detection and stimulation system. *Sensors* **25** (2025), Paper no. 33, 16 pp.

Fractional-Order Model Predictive Control for Real-Time EEG-Based Epileptic Seizure Suppression

Sadullah Bulut, **Muhammed Yiğider**

Department of Mathematics, University of Erzurum Technical, Erzurum, Turkey
E-mails: sadullah.bulut@erzurum.edu.tr; myigider@erzurum.edu.tr

Epileptic seizures with excessive and abnormal activity present challenges for real-time prediction and suppression. This study proposes a novel EEG-based method for real-time suppression of epileptic seizures using the Fractional-Order Model Predictive Control (FO-MPC) framework. It reveals the advantages of fractional-order computation in capturing the internal memory of neural information and its genetic information in the system. It also combines the predictive capabilities of model predictive control. EEG playback is continuously monitored and processed over time, with adaptive intervention to detect perturbations at any time of day. A fractional-order dynamic model is used to represent neural behavior. Additionally, a predictive control program is designed to record optimal control inputs that mitigate the components of neural dysfunction. Simulation results using real EEG datasets demonstrate the effectiveness of the FO-MPC approach in suppressing storage onset while maintaining system stability and computational dispersion. This study demonstrated the potential of fractional order control results as a promising tool in closed-loop neurostimulation therapies for epilepsy.

References

[1] R. Magin, *Fractional Calculus in Bioengineering.* Begell House Publishers, 2006.

[2] I. Podlubny, *Fractional Differential Equations.* Academic Press, 1999.

[3] I. Ullah, M. Hussain, E.-ul-H. Qazi and H. Aboalsamh, An automated system for epilepsy detection using EEG brain signals based on deep learning approach. *Expert Systems with Applications* **107** (2018), no. 1, 61–71.

[4] Z. Wang, D. Liu and Q. Wei, Model predictive control of neural activity for seizure suppression using data-driven models. *IEEE Transactions On Neural Systems And Rehabilitation Engineering* **28** (2020), 402–412.

Regularity Problems for Evolution Anisotropic Equations

Kateryna Buryachenko[1,2]

[1] *Humboldt University, Berlin, Germany*
E-mail: buryachk@hu-berlin.de

[2] *Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine*
E-mail: k.buriachenko@donnu.edu.ua

The talk will be devoted to the derivation of potential estimates, study of asymptotic behavior, boundedness and solution regularity for elliptic and parabolic anisotropic equations (as well as their variational interpretation) of diffusion-absorption structure with non-standard growth conditions and external sources as well as related variational problems. Main attention will be given to the anisotropic porous medium equations

$$u_t = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(|u|^{m_i-1} u_{x_i}\right) + f,$$

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \cdots \leq m_n < \overline{m} + \frac{2}{n}, \ \ \overline{m} = \frac{1}{n}\sum_{i=1}^{n} m_i, \ \ f \in L^1(\Omega_T), \ \ \Omega_T = \Omega \times (0, T).$$

Such problems serve as mathematical models of many nonlinear processes in anisotropic and heterogeneous media. The main idea is to construct the analog of the regularity theory for anisotropic elliptic and parabolic equations with external sources via Riesz and Wolff potentials of right-hand side (see [4]), and to prove local boundedness and continuity of weak solutions. For some cases it is expected that the regularity theory for nonlinear equations can be linearized via Riesz potentials. Others cases will involve the nonlinear Wolff and parabolic potentials. Interesting observation arises: in all known related publications the cases of slow diffusion (or the degenerate case with $m_i > 1$) and fast diffusion (or the singular case with $m_i < 1$) were considered independently, and methods of proving depended on either fast or slow diffusion takes place [1]. The main novelty of this work ([2], [3]) is that we developed the method which allows to avoid former separation into degenerate and singular cases and proved the pointwise estimates for weak solutions of parabolic anisotropic equations via Riesz potential, and as the result, proved the local continuity of weak solutions for all cases.

References

[1] V. Bögelein, F. Duzaar and U. Gianazza, Sharp boundedness and continuity results for the singular porous medium equation. *Israel J. Math.* **214** (2016), no. 1, 259–314.

[2] K. O. Buryachenko and I. I. Skrypnik, Riesz potentials and pointwise estimates of solutions to anisotropic porous medium equation. *Nonlinear Anal.* **178** (2019), 56–85.

[3] K. O. Buryachenko and I. I. Skrypnik, Local continuity and Harnack's inequality for double-phase parabolic equations. *Potential Anal.* **56** (2022), no. 1, 137–164.

[4] F. Duzaar and G. Mingione, Gradient estimates via linear and nonlinear potentials. *J. Funct. Anal.* **259** (2010), no. 11, 2961–2998.

Relationship between the Beck–Chevalley and Kan Conditions in Simplicial Sets

Gaga Chakhvashvili

Department of Mathematics, Faculty of Exact and Natural Sciences,
Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: gaga.chakhvashvili4202@ens.tsu.edu.ge

Kan complexes play a central role in simplicial homotopy theory, being the fibrant objects with respect to the standard model structure.

A few years ago, Valentin Shehtman and Dmitry Skvortsov associated to each first-order theory certain simplicial set whose n-simplices are given by the n-types of that theory. For the purposes of his investigation, it is important that this simplicial set satisfies the so-called Beck–Chevalley condition: for any $i < j$ and any two simplices a, b such that the ith face of a coincides with the $(j-1)$st face of b, there exists a simplex x whose jth and ith faces are a and b, respectively.

It is easy to see that every Kan complex satisfies the Beck–Chevalley condition. However, the converse implication, namely whether a complex satisfying the Beck–Chevalley condition is necessarily a Kan complex, remains an open question. To address this, a counterexample is explicitly constructed: a simplicial complex that satisfies the Beck–Chevalley condition but is not a Kan complex.

We study the Beck–Chevalley condition for those simplicial sets that arise as the Duskin nerves of small 2-categories. In particular, we investigate what kind of conditions a monoid must satisfy in order for the Duskin nerves of some naturally defined sub-2-categories of the 2-category of endofunctors and their natural transformations of M viewed as a category with a single object to satisfy the Beck–Chevalley condition.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), grant # FR-24-9660.

References

[1] J. W. Duskin, Simplicial matrices and the nerves of weak n-categories. I. Nerves of bicategories. CT2000 Conference (Como). *Theory Appl. Categ.* **9** (2001/02), 198–308.

[2] P. G. Goerss and J. F. Jardine, *Simplicial Homotopy Theory.* Reprint of the 1999 edition [MR1711612]. Modern Birkhäuser Classics. Birkhäuser Verlag, Basel, 2009.

[3] J. Lurie, Kerodon; `https://kerodon.net`

[4] J. P. May, *Simplicial Objects in Algebraic Topology.* Reprint of the 1967 original. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1992.

[5] D. Pavlović, Categorical interpolation: descent and the Beck–Chevalley condition without direct images. *Category theory (Como, 1990)*, 306–325, Lecture Notes in Math., 1488, *Springer, Berlin*, 1991.

[6] R. Street, The algebra of oriented simplexes. *J. Pure Appl. Algebra* **49** (1987), no. 3, 283–335.

A Mathematical and Optimal Control Model for Rabies Transmission Dynamics Among Humans and Dogs with Environmental Effects

Mfano Charles[1], **Delfim F. M. Torres**[2], **G. A. Lyakurwa**[3], **Sayoki G. Mfinanga**[4], **Verdiana G. Masanja**[3]

[1]*Department of ICT and Mathematics, College of Business Education (CBE), P.O. BOX 1968, Dar es Salaam, Tanzania*
E-mail: mfanoc@nm-aist.ac.tz

[2]*Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal*
E-mail: delfim@ua.pt

[3]*School of Computational and Communication Science and Engineering, Nelson Mandela African Institution of Science and Technology (NM-AIST), P.O. Box 447, Arusha, Tanzania*
E-mail: verdiana.masanja@nm-aist.ac.tz

[4]*NIMR Chief Research Scientist Fellow, Mhimbili National Hospital, Dar-es-salaam, Tanzania*
E-mail: gsmfinanga@yahoo.com

Rabies, an infectious disease with significant global impact, remains a serious concern for both human and animal populations. This study presents a deterministic model to investigate rabies transmission dynamics, incorporating environmental effects and control strategies using optimal control theory. Qualitative and quantitative analyses reveal that the disease-free equilibrium is stable when the effective reproduction number $\mathscr{R}_e < 1$, and unstable when $\mathscr{R}_e > 1$. Mesh and contour plots illustrate an inverse relationship between $\mathscr{R}_e$ and control strategies, including dog vaccination, health promotion, and post-exposure treatment. Increased intervention reduces transmission, while higher contact rates among dogs raise $\mathscr{R}_e$. Numerical simulations with optimal control confirm the effectiveness of integrated strategies. Vaccination and treatment are identified as key interventions for achieving rabies elimination within five years.

Global Attractors of Non-autonomous Lattice Dynamical Systems

David Cheban, Andrei Sultan

State University of Moldova, Vladimir Andrunachievici Institute of Mathematics and Computer Science, Chişinău, Moldova

E-mails: david.ceban@usm.md; andrew15sultan@gmail.com

This work studies the long-term behavior of non-autonomous lattice dynamical systems in the Hilbert space ℓ_2, represented by the following infinite system of differential equations:

$$u_i' = \nu(u_{i-1} - 2u_i + u_{i+1}) - \lambda u_i + F(u_i) + f_i(t) \ \ (i \in \mathbb{Z}, \ \ \lambda > 0, \ \ \nu > 0)$$

where $u = (u_i)_{i\in\mathbb{Z}} \in \ell_2$, $F \in C(\mathbb{R}, \mathbb{R})$, and the non-autonomous forcing term $f = (f_i(t))_{i\in\mathbb{Z}}$ belongs to $C(\mathbb{R}, \ell_2)$. We establish the existence of a compact global attractor for this system under certain conditions on the nonlinear term F and the forcing term f. The proof methodology involves several key steps. First, we demonstrate that the system generates a continuous cocycle over a shift dynamical system on the hull of the function f. Subsequently, we prove the dissipativeness of this cocycle by establishing the existence of a bounded absorbing set. The core of the analysis lies in proving the asymptotic compactness of the cocycle, which is achieved by verifying the asymptotic tails property. The combination of dissipativeness and asymptotic compactness guarantees the existence of a compact global attractor.

Theorem 1 *Under the Conditions*

(**C1**) *f is translation-compact,*

(**C2**) *F is Lipschitz on bounded sets with $F(0) = 0$,*

(**C3**) *$sF(s) \leq -\alpha s^2$ for some $\alpha > 0$,*

the cocycle φ generated by the equation has a compact global attractor $\{I_g \mid \ g \in H(f)\}$, where $H(f)$ is the hull of f.

Lemma 1 *Under the Conditions* (**C1**)–(**C3**)*, there exists a closed ball $B[0, R] \subset \ell_2$ such that for any bounded subset $B \subset \ell_2$, there exists a time $L = L(B)$ such that all solutions starting in B enter and remain in $B[0, R]$. That is, $\varphi(t, B, H(f)) \subseteq B[0, R]$ for all $t \geq L(B)$.*

Definition A family $\{I_y \mid \ y \in Y\}$ of compact subsets I_y of a Banach space $\mathfrak{B}$ is a compact global attractor for a cocycle $\langle \mathfrak{B}, \varphi, (Y, \mathbb{R}, \sigma)\rangle$ if the set $\mathscr{I} := \cup\{I_y \mid \ y \in Y\}$ is precompact, the family is invariant (i.e., $\varphi(t, I_y, y) = I_{\sigma(t,y)}$), and it uniformly attracts all compact subsets of $\mathfrak{B}$.

Acknowledgments

This research was supported by the State Program of the Republic of Moldova *"Monotone Nonautonomous Dynamical Systems (24.80012.5007.20SE)"* and partially was supported by the Institutional Research Program 011303 "SATGED", Moldova State University.

A New Categorical Framework to Study Set Relations and Set Operators

Giampiero Chiaselotti

*Department of Mathematics and Computer Science, University of Calabria
Arcavacata di Rende, Italy*
E-mail: giampiero.chiaselotti@unical.it

In this talk we discuss a novel and comprehensive categorical framework to study maps arising between collections of mathematical structures axiomatized through set systems, binary set relations and set operators on some fixed ground set. Our approach is based on four fundamental new notions, namely: *class submaps* (i.e. specific pairs of $\mathrm{Obj}\,(\mathbf{Set})$-valued maps), *fsm-transfers* (that are mappings between class submaps), *fsm-cryptomorphisms* (i.e. categorical formalizations of the vague notion of cryptomorphism) and *displacements* (that are particular kinds of functorial extensions of fsm-transfers and of fsm-cryptomorphisms).

This point of view shifts the focus away from considering sets with given axioms on set systems, set relations and set operators to a more general context described by means of objects and arrows of a large category.

The talk begins by describing some recent studies conducted in [1]–[3] and, subsequently, outlines new research perspectives and related unpublished results.

References

[1] G. Chiaselotti and F. G. Infusino, Algebraic and order properties of maps and structures related to dependence relations arising in topology, algebra and rough set theory. *J. Algebra Appl.* **24** (2025), no. 4, Paper no. 2550102, 58 pp.

[2] G. Chiaselotti, T. Gentile and F. G. Infusino, On some categories of structured sets. *Eur. J. Math.* **10** (2024), no. 2, Paper no. 23, 41 pp.

[3] G. Chiaselotti and F. G. Infusino, Categorical interpretations of cryptomorphisms and maps arising from matroidal and combinatorial contexts. *J. Algebra* **665** (2025), 145–204.

Research of a Nonlinear Dynamic System Describing the Process of Interaction Between Georgian, Laz, Mingrelian and Svan Populations

Temur Chilachava, Gia Kvashilava, George Pochkhua

Department of Applied Mathematics, Sokhumi State University, Tbilisi, Georgia
E-mails: temo_chilachava@yahoo.com; gia.kvashilava@tsu.ge; g.pochkhua@sou.edu.ge

Previously, mathematical and computer models of three periods of transformation of the Proto-Kartvelian population were proposed and studied [1]–[4].

The presented paper addresses the fourth period of this process, during which the Colchian population splits into Laz and Mingrelian populations. The mathematical model is formalized as a four-dimensional nonlinear dynamical system, where each unknown function represents the population dynamics of one of the groups (Georgians, Laz, Mingrelians, Svans) over time.

First integrals for the nonlinear dynamical system have been found, provided certain conditions on the constant coefficients are satisfied. These first integrals allow the four-dimensional dynamical system to be reduced to a two-dimensional system. To study the two-dimensional dynamical system, the Poincaré–Bendixson theorem is applied. Several theorems have been established regarding the sign change of the divergence of a vector field within a simply connected domain located in the first quadrant of the phase plane. These results confirm the existence of closed integral trajectories, indicating that none of the groups undergo complete assimilation, and all four linguistic groups achieve stable coexistence within the same geographical space.

References

[1] T. Chilachava, G. Kvashilava and G. Pochkhua, Mathematical model for the Proto-Kartvelian population dynamics. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **37** (2023), 7–10.

[2] T. Chilachava, G. Kvashilava and G. Pochkhua, Mathematical model describing the transformation of the proto-Kartvelian population. *J. Math. Sci. (N.Y.)* **280** (2024), no. 3, 300–308.

[3] T. Chilachava, G. Kvashilava and G. Pochkhua, Research of a nonlinear dynamic system describing the process of interaction between Colchian, Georgian and Svan Populations. *Transactions of A. Razmadze Mathematical Institute* (to appear).

[4] T. Chilachava, G. Kvashilava and G. Pochkhua, Mathematical and computer models of the transformation of the Proto-Kartvelian populations into Svan and Georgian-Colchian populations. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **38** (2024), 23–26.

Numerical Solution of Two Nonlinear Parabolic Type Integro-Differential Equations Using Machine Learning

Teimuraz Chkhikvadze[1], **Mikheil Gagoshidze**[1,2], **Temur Jangveladze**[1,3]

[1] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2] *Central University of Europe, Kutaisi, Georgia*

[3] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*

E-mails: m.zarzma@gmail.com; mishagagoshidze@gmail.com; tjangv@yahoo.com

The initial-boundary value problems for the second and fourth-order nonlinear parabolic integro-differential equations are considered. The discussed type equations are based on the Maxwell system of nonlinear partial differential equations [7] and at first was proposed in [2]. The integro-differential models of such type are studied in many works (see, for example, [1]–[6] and references therein). Extensive references to the mentioned models are given in the monographs [3], [5]. Numerical experiments have been conducted using machine learning methods [8] for some of this type models in [4], [6]. Our aim is to continue the study of these models in this direction.

References

[1] T. A. Dzhangveladze, The first boundary value problem for a nonlinear equation of parabolic type. (Russian) *Dokl. Akad. Nauk SSSR* **269** (1983), no. 4, 839–842; translation in *Sov. Phys., Dokl.* **28** (1983), 323–324.

[2] D. G. Gordeziani, T. A. Dzhangveladze and T. K. Korshiya, Existence and uniqueness of the solution of a class of nonlinear parabolic problems. (Russian) *Differentsial'nye Uravneniya* **19** (1983), no. 7, 1197–1207; translation in *Differ. Equ.* **19**, (1983), no. 1, 887–895.

[3] T. Jangveladze, Investigation and numerical solution of nonlinear partial differential and integro-differential models based on system of Maxwell equations. *Mem. Differ. Equ. Math. Phys.* **76** (2019), 1–118.

[4] T. Jangveladze, Z. Kiguradze and T. Chkhikvadze, Application of Deep Neural Network for numerical approximation for averaged nonlinear integro-differential equation. *Bulletin of TICMI* **28** (2024), no. 1, 3-10.

[5] T. Jangveladze, Z. Kiguradze and B. Neta, *Numerical Solutions of Three Classes of Nonlinear Parabolic Integro-Differential Equations.* Elsevier/Academic Press, Amsterdam, 2016.

[6] T. Jangveladze, B. Tabatadze, Z. Kiguradze and T. Chkhikvadze, Approximate solution to one fourth-order nonlinear parabolic-type integro-differential equation. *Bull. Georgian Natl. Acad. Sci. (N.S.)*, 2025 (accepted).

[7] L. D. Landau and E. M. Lifschic, *Electrodynamics of Continuous Media.* (Russian) Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957.

[8] M. Raissi, P. Perdikaris and G. E. Karniadakis, Physics informed deep learning (Part I): data-driven solutions of nonlinear partial differential equations. *Preprint* arXiv 1711.10561, 2017; `https://arxiv.org/abs/1711.10561`.

Classification – Trichotomy of a Set of Text Fragments Borrowed from One Work to Another

Zurab Chkonia[1], **David Zarnadze**[2]

[1]*East European University, Tbilisi, Georgia*
E-mail: z.chkonia@yahoo.com

[2]*Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia*
E-mail: d.zarnadze@gtu.ge

This paper presents a classification of the set of text fragments borrowed from one work into another. It is established that this classification forms a *trichotomy*, meaning that the set of all borrowed fragments can be expressed as the union of three mutually exclusive subsets. A fragment may be only a quotation, only a paraphrase, or only a periphrase. Therefore, quotation, paraphrase, and periphrase are mutually exclusive concepts, corresponding to three non-intersection subsets: the set of quotations (the volume of the concept of quotation), the set of paraphrases (the volume of the concept of paraphrase), and the set of periphrases (the volume of the concept of periphrase). In terms of the logical operation of classification, this is expressed as follows: any text fragment borrowed from one work into another must be either only a quotation, or only a paraphrase, or only a periphrase [1]–[3]. The distinguishing features of these concepts are identified with the corresponding citation conditions for classification purposes.

References

[1] F. J. Pelletier and A. Hartline, Ternary exclusive or. *Log. J. IGPL* **16** (2008), no. 1, 75–83.

[2] D. Zarnadze, *Foundations of Logical-Analytical Thinking.* Mtsignobari, Tbilisi, 2017.

[3] D. Zarnadze, Z. Chkonia and S. Tsotniashvili, Some Logical and Legal Aspects of the Linguistic Definition of Quotation and Citation Rules. *Scientific Journal of Gori State University* **5** (2023), no. 1, 13–20.

PDE-Based Tumor Growth Modeling Via Artificial Neural Networks

Manana Chumburidze[1], **Miranda Mnatsakaniani**[2]

[1]*Department of Computer Technology, Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: manana.chumburidze@atsu.edu.ge

[2]*Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: miranda.mnatsakaniani@atsu.edu.ge

The multidisciplinary area of mathematical oncology uses theoretical, computational, and mathematical methods to tackle challenging issues in cancer biology and therapy. With the use of multiscale modeling, mathematical oncology has been instrumental in advancing scientific and clinical cancer research over the past 20 years. However, simulations typically do not incorporate computational fluids beyond simple diffusion, and the majority of cancer modeling still lacks a thorough depiction of cell and tissue mechanics. Boundary contact problems in tumor elasticity are an important area of study in mathematical oncology and biomechanics. These problems often involve modeling the interaction between a tumor and its surrounding tissue, considering factors such as elasticity, nutrient diffusion, and mechanical forces. This work is devoted to exploring the intersection of computational fluid dynamics, mechanics, and biological processes in the context of cancer. Mathematical modeling, simulation techniques and their applications have been developed to understand tumor growth, metastasis, and treatment strategies. This article will identify opportunities to build mathematical models of PDEs of computational fluids and mechanics into mathematical oncology. In order to simulate solid tumor growth, multiscale modeling of complicated tumor systems and boundary-contact challenges has been studied. These models describe how the tumor boundary evolves dynamically on the basis of biological and mechanical processes. This study focuses on the development of neural networks (NN) with basic answers for boundary contact problems of coupled thermo-diffusion models of tumor elasticity for 2-D isotropic inhomogeneous materials. In particular, the boundary contact problem is considered when the displacement vector, characteristic of rotation, temperature, and chemical potential are given on the surfaces of tumor cells in assumptions that the surface is sufficiently smooth. The approximate solution is constructed using the generalized Furrier neural network (FNN). This development predicts the outcome of cancer treatment. The Laplace Neural Platform, the potential method, the generalized FNN methodology, and singular integral equations are the foundations of the instruments used in this work. These models allow for simulations of tumor progression and response to different therapies, enabling more tailored treatment approaches based on individual tumor characteristics. This model requires future research to address essential aspects of machine learning optimality checks, representing an appropriate improvement for oncological medical prognoses and treatment.

References

[1] M. Chumburidze and V. Niminet, Efficient modelisation for complex geometries of tumor growth via thermo-elastic diffusion partial differential equations and artificial neural networks. *Broad Research in Artificial Intelligence and Neuroscience*, (2025), no. 16(1), 315–323.

On Various Aspects Related to the Generalized Monoid Rings

Elena Cojuhari[1], **Barry Gardner**[2]

[1]*Department of Mathematics, Technical University of Moldova*
Chişinău, Republic of Moldova
E-mail: elena.cojuhari@mate.utm.md

[2]*Hobart, Tasmania, Australia*
E-mail: barry.gardner1@mail.com

In [1], [2] it was introduced axiomatically a system, which later in [3] was called a D-structure, consisting of an associative ring A with identity 1, a multiplicative monoid G with identity e and a family of additive mappings $\sigma_{x,y} : A \to A$ for $x, y \in G$ satisfying the a long but fairly natural set of conditions. We connect the D-structure defined by means of the family σ with a monoid algebra $A\langle G\rangle$. The elements of $A\langle G\rangle$ are mappins $\alpha : G \to A$ such that $\alpha(x) = 0$ for almost all $x \in G$. In [2] it was shown that the free A-module with basis G, equipped with the multiplication defined via distributivity by the rule $(a \cdot x)(b \cdot y) = \sum\limits_{z\in G} a\sigma_{x,z}(b) \cdot zy$ $(a, b \in A;\ x, y \in G)$ becomes an associative ring. This construction of a generalized monoid ring $A\langle G, \sigma\rangle$ extends the notions of a group ring, a skew polynomial ring and other related ones. The n-th Weyl algebra over a field K is isomorphic to our construction with $A = K[t_1, \dots, t_n]$ and G the free commutative monoid of rank n. Furthermore, the generalized monoid ring introduced in [2] can be treated from more general aspect, see for instance, [6]. On other hand, it is important to investigate in our construction the notions of ideals, radicals and other from the structural theory. In [4] we characterized those σ for which $A\langle G, \sigma\rangle$ is a normalizing extension and gave examples of $A\langle G, \sigma\rangle$ which are subnormalizing, but not normalizing, extensions. In [4] we associated with a ring S which has A as a subring with the same identity, G as multiplicative submonoid which generates S as a free left A-module, a set of mappings $\sigma_{x,y} : A \to A$, $x, y \in G$, which satisfy all but (possibly) one of the defining conditions for a D-structure, and gave examples of monoids for which the “missing” condition is also satisfied. Further examples of such monoids were given in [5]. We now know that in fact the “missing” condition is satisfied for all monoids, and this is the main result of the talk:

Theorem 1 *Every generalized monoid ring arises from a D-structure.*

References

[1] E. P. Cojuhari, The property of universality for some monoid algebras over non-commutative rings. *Bul. Acad. Ştiinţe Repub. Mold. Mat.* **2006**, no. 2, 102–105.

[2] E. P. Cojuhari, Monoid algebras over non-commutative rings. *Int. Electron. J. Algebra* **2** (2007), 28–53.

[3] E. P. Cojuhari and B. J. Gardner, Generalized higher derivations. *Bull. Aust. Math. Soc.* **86** (2012), no. 2, 266–281.

[4] E. P. Cojuhari and B. J. Gardner, Skew ring extensions and generalized monoid rings. *Acta Math. Hungar.* **154** (2018), no. 2, 343–361.

[5] E. P. Cojuhari and B. J. Gardner, Subnormalizing extensions and D-structures. *ROMAI J.* **14** (2018), no. 2, 59–66.

[6] P. Nystedt, J. Öinert and J. Richter, Simplicity of Ore monoid rings. *J. Algebra* **530** (2019), 69–85.

A Study of Fractional Integral Inequalities and its Applications

Belete Debalkie

Wollo University, Dessie, Ethiopia
E-mail: beletedebalkie@gmail.com

Fractional integral inequalities are key tools in fractional calculus, providing important bounds and relationships for fractional integrals, which extend traditional integration to non-integer orders. These inequalities, such as those involving Hölder's, Hermite–Hadamard's, Ostrowski's, Hardy's, Grüss-type, Jensen's Grönwall's, and Minkowski's integral inequalities, as well as generalized fractional integral inequalities are used to analyze the properties of fractional operators and fractional differential equations. They have wide applications in various fields, including applied mathematics, physics, and engineering particularly in modeling systems. By utilizing fractional integral inequalities, researchers can derive approximations, existence, uniqueness, stability of solutions, and improve the accuracy of models, making them essential in the study of complex phenomena and special functions. During this presentation, I will discuss the integral inequalities with generalized fractional integral and differential operators, as well as their application in a Lyapunov type inequality to determine the necessary condition for the existence of non-trivial solutions for a boundary value problem with fractional differential equations.

The Role of Technology in the Process of Teaching Mathematics at the Primary Level

Manana Deisadze, Shalva Kirtadze

Faculty of Education, Akaki Tsereteli State University, Kutaisi, Georgia
E-mails: manana.deisadze@atsu.edu.ge; shalva.kirtadze@atsu.edu.ge

The rapid development of information technologies has introduced new opportunities into the educational process. In today's educational environment, the integration of technology is gaining special significance, especially at the primary level, where learning motivation and interest are crucial. The use of technology in the process of teaching mathematics provides opportunities for students to practice interactively, comprehend mathematical concepts visually and individually, as abstract thinking at the primary level is challenging.

Visual resources help students understand key mathematical concepts such as mathematical modeling, patterns, and logical reasoning. The paper offers specific examples of using various technological tools such as GeoGebra, Khan Academy, and Quizizz, which support the enhancement of motivation towards mathematics, consolidation of knowledge, and development of self-learning skills. The article reviews the effective use of technological resources at the primary level, including specific platforms and teaching methods.

Aggregate Model of Liquidity-Profit Dynamics

Michal Demetrian[1], **Rudolf Zimka**[2]

[1]*Faculty of Mathematics, Physics and Informatics, Comenius University Bratislava, Slovakia*
E-mail: I: michal.demetrian@uniba.sk

[2]*Faculty of Economics, Matej Bel University, Banská Bystrica, Slovakia*
E-mail: rudolf.zimka@umb.sk

We discuss the problem of limit cycles in an aggregate-type models of liquidity growth proposed and studied numerically for the first time by W. Semmler and M. Sieveking in [4]. Their model is of predator-prey type with cannibalism in predator (profit) and logistic bound in prey (liquidity). We modify the model to include the weak Allee effect in liquidity and study the local bifurcations of equilibria and limit cycles by means of the Hopf-Andronov theory. The main motivation for this study follows the economic theory of the so-called corridors of stability introduced by Leionhoofvud in [1].

Acknowledgments

This work was supported by the Slovak ministry of education under the VEGAprojects 1/0382/23 and 1/0084/23.

References

[1] A. Leionhoofvud, Effective Demand Failures. *The Swedish Journal of Economics* **22** (1973), no. 1, 27–48.

[2] R. Luís and H. Oliveira, An economical model with Allee effect. *J. Difference Equ. Appl.* **15** (2009), no. 8-9, 877–894.

[3] W. Semmler and L. Koçkesen, Liquidity, credit and output: a regime change model and empirical estimations. *University of Bielefeld, Department for Economics, Centre for Empirical Macroeconomics*, 2001, Working Paper no. 29.

[4] W. Semmler and M. Sieveking, Nonlinear liquidity-growth dynamics with corridor-stability. *Journal of Economic Behavior & Organization* **22** (1993), no. 2, 189–208.

B Meson's Double Photon Decays in 2HDM

Gela Devidze[1,2,3]

[1]*School of Science and Technology, The University of Georgia, Tbilisi, Georgia*
E-mail: gel.devidze@ug.edu.ge

[2]*High Energy Physics Institute, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: gela.devidze@tsu.ge

[3]*Georgian Aviation University, Tbilisi, Georgia*
E-mail: gela_devidze@ssu.edu.ge

The investigation of the B-meson's rare decays is of great interest in order to test the standard model (SM) and beyond SM (BSM) physics. In the SM the leading contribution to the two-photon decays of neutral B mesons $B_{s(d)} \to \gamma\gamma$ comes from one-loop diagrams, up-type quarks and W-bosons being particles exchanged in the loop [1].

Wave estimated additional to the SM contribution in the two Higgs doublet model (2HDM) type II [2], [3]. Real charged Higgs particle may contribute into $B \to \gamma\gamma$ decay in frame of 2HDM. For small value of charged Higgs particle $M_H \approx O$ (a few handred GeV) and large $\tan\beta$ the contribution of charged Higgs particle into branching ratio of the decay $B \to \gamma\gamma$ is significant large (on the same level as SM estimate). This region of parameters are practically excluded by current experimental and theoretical investigations. Though, allowed region of parameters $\tan\beta \approx 4$, $M_H \approx O$ (a few handred TeV) gives significant contribution (a few percent) into branching ratio of the decay $B_s \to \gamma\gamma$.

Our calculations are performed in the Feuynman't Hooft gauge and we use a dimensional regularization technique for divergent Feynman integrals (we have used useful properties of odd-like functions ($\int\limits_{-a}^{a} g(x)f(x)\,dx = 2k\int\limits_{0}^{a} g(x)\,dx$, here $f(x)$ is odd-like function $f(x)+f(-x)=k=const$, and g(x) is even function).

References

[1] S. W. Bosch and G. Buchalla, The double radiative decays $B \to$ in the heavy quark limit. *Journal of High Energy Physics* **2002**, 20 pp.

[2] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models. *Phys. Rep.* **516** (2012), no. 1–2, 1–102.

[3] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, The Higgs hunter's guide. *Front. Phys.* **80** (2000).

Invariants and Areas of Steiner 4-Chains

Ana Diakvnishvili, Gulnara Bibileishvili

Faculty of Business, Technology and Education, Ilia State University, Tbilisi, Georgia
E-mails: ana.diakvnishvili.1@iliauni.edu.ge; gulnara.bibileishvili.1@iliauni.edu.ge

We are concerned with the invariants of Steiner chains consisting of four circles. In particular, we compute the invariant moments of curvatures in Steiner 4-chains and give two applications of the obtained formulas. Specifically, we present an algorithmic feasibility criterion for Steiner 4-chains and identify the poristic Steiner chains having extremal areas, which yields a generalisation of the main results of a recent paper by K. Kiradjiev. The proofs are based on the invariants of Steiner chains described by R. Schwarz and S. Tabachnikov and on the relations between the radii of neighbouring poristic circles established by P. Yiu.

References

[1] M. Berger, *Geometry*, I. Springer-Verlag, Berlin, 2009.

[2] G. Bibileishvili and N. Sazandrishvili, Extremal problems for Steiner 3-chains. *Proc. I. Vekua Inst. Appl. Math.* **74** (2024), 3–13.

[3] R. R. Gazizov, On the maximum and minimum areas of the necklace. *Lobachevskii J. Math.* **44** (2023), no. 10, 4523–4530.

[4] M. N. Gurov and M. A. Volkov, Chains of tangent circles inscribed in curvilinear triangles. *Int. J. Geom.* **7** (2018), no. 1, 105–118.

[5] K. Kiradjiev, Exploring Steiner chains with Möbius transformations. *Math. Today (Southend-on-Sea)* **53** (2017), no. 6, 272–274.

[6] J. C. Lagarias, C. L. Mallows and A. R. Beyond the Descartes circle theorem. *Amer. Math. Monthly* **109** (2002), no. 4, 338–361.

[7] G. Lion, Variational aspects of Poncelet's theorem. *Geom. Dedicata* **52** (1994), no. 2, 105–118.

[8] D. Pedoe, *A Course of Geometry for Colleges and Universities.* Cambridge University Press, London-New York, 1970.

[9] M. Radić, An improved method for establishing Fuss' relations for bicentric polygons. *C. R. Math. Acad. Sci. Paris* **348** (2010), no. 7-8, 415–417.

[10] R. E. Schwartz and S. Tabachnikov, Descartes circle theorem, Steiner porism, and spherical designs. *Amer. Math. Monthly* **127** (2020), no. 3, 238–248.

[11] K. Stephenson, *Introduction to Circle Packing. The Theory of Discrete Analytic Functions.* Cambridge University Press, Cambridge, 2005.

[12] P. Yiu, Rational Steiner porism. *Forum Geom.* **11** (2011), 237–249.

Dynamics of Solutions of Inhomogeneous NLS System with a $\chi^{(2)}$ Nonlinearity

Van Duong Dinh[1], **Amin Esfahani**[2]

[1]*École normale supérieure de Lyon & CNRS, Lyon, France*
E-mail: contact@duongdinh.com

[2]*Nazarbayev University, Astana, Kazakhstan*
E-mail: amin.esfahani@nu.edu.kz

In this work, we investigate a system of inhomogeneous nonlinear Schrödinger equations that arise in optical media. Specifically, we consider the system

$$\begin{cases} \mathrm{i}\partial_t u + \dfrac{1}{2}\,\Delta u + |x|^{-\alpha}\overline{u}v = 0, \\ \mathrm{i}\partial_t v + \dfrac{\kappa}{2}\,\Delta v - \gamma v + \dfrac{1}{2}\,|x|^{-\alpha}u^2 = 0, \end{cases} \tag{1}$$

where the unknown functions $u, v : \mathbb{R}_t \times \mathbb{R}^d_x \to \mathbb{C}$ represent wave amplitudes. The parameters satisfy $d \geq 1$, $\alpha > 0$, $\kappa > 0$, and $\gamma \in \mathbb{R}$. For a detailed derivation of this model from the underlying physical principles, we refer to [2], [3]. By applying the abstract argument (see [1]), we show that system (1) is locally well-posed. More precisely, we have the following local well-posedness result. We also find the conditions under which the local solution is global. Once global solutions exist, we study the long time behavior of these solutions. We have the following energy scattering for global solutions in the mass-supercritical regime.

Theorem 1

(i) *Let $1 \leq d \leq 5$, $\kappa > 0$, $\gamma \in \mathbb{R}$, $0 < \alpha < \min\{2, d\}$, and $\alpha < \frac{6-d}{2}$ if $3 \leq d \leq 5$. For any $(u_0, v_0) \in \mathscr{H}^1$, there exists a unique maximal solution $(u, v) \in C((-T_*, T^*), \mathscr{H}^1) \cap C^1((-T_*, T^*), \mathscr{H}^{-1})$ to (1) with initial data $(u, v)\big|_{t=0} = (u_0, v_0)$, where $\mathscr{H}^{-1} := H^{-1}(\mathbb{R}^d) \times H^{-1}(\mathbb{R}^d)$ is the dual space of $\mathscr{H}^1$. The maximal time satisfies the blow-up alternative: if $T^* < \infty$ (resp. $T_* < \infty$), then $\lim\limits_{t \nearrow T^*} \|(u(t), v(t))\|_{\mathscr{H}^1} = \infty$ (resp. $\lim\limits_{t \searrow -T_*} \|(u(t), v(t))\|_{\mathscr{H}^1} = \infty$). In addition, there are conservation laws of mass and energy, i.e., $\mathbb{M}(u(t), v(t)) = \|u(t)\|^2_{L^2} + 2\|v(t)\|^2_{L^2} = \mathbb{M}(u_0, v_0)$, $\mathbb{E}(u(t), v(t)) = \frac{1}{2}\,\|\nabla u(t)\|^2_{L^2} + \frac{\kappa}{2}\,\|\nabla v(t)\|^2_{L^2} + \gamma\|v(t)\|^2_{L^2} - \Re\langle |x|^{-\alpha}u^2(t), v(t)\rangle = \mathbb{E}(u_0, v_0)$, for all $t \in (-T_*, T^*)$.*

(ii) *Let $3 \leq d \leq 5$, $\gamma = 0$ and $(u_0, v_0) \in \mathscr{H}^1$ satisfy $\mathbb{E}(u_0, v_0)(\mathbb{M}(u_0, v_0))^\sigma < \mathbb{E}(\varphi, \psi)(\mathbb{M}(\varphi, \psi))^\sigma$, $\mathbb{K}(u_0, v_0)(\mathbb{M}(u_0, v_0))^\sigma < \mathbb{K}(\varphi, \psi)(\mathbb{M}(\varphi, \psi))^\sigma$, $\sigma = \frac{6-d-2\alpha}{d+2\alpha-4}$. Then the corresponding solution to (1) scatters in $\mathscr{H}^1$ in both directions.*

Acknowledgments

Amin Esfahani is supported by Nazarbayev University under Faculty Development Competitive Research Grants Program for 2023-2025 (grant # 20122022FD4121).

References

[1] Van D. Dinh and A. Esfahani, A system of inhomogeneous NLS arising in optical media with a $\chi^{(2)}$ nonlinearity, Part I: Dynamics. *Commun. Pure Appl. Anal.* **24** (2025), no. 4, 584–625.

[23] T. S. Gill, Optical guiding of laser beam in nonuniform plasma. *Pramana – J. Phys.* **55** (2000), 835–842.

[3] Ch. Liu, A. Liu, J. Hu, V. Yuan and S. Halabi, Adjusting for misclassification in a stratified biomarker clinical trial. *Stat. Med.* **33** (2014), no. 18, 3100–3113.

Multi-Modal Epistemic Lukasiewicz Logics and Multi-Agent Systems

Antonio Di Nola[1], **Revaz Grigolia**[2,3], **Giacomo Lenzi**[1], **Ramaz Liparteliani**[2,3]

[1] *University of Salerno, Fisciano SA, Italy*
E-mails: adinola@unisa.it; glenzi@unisa.it

[2] *Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[3] *Georgian Technical University, Tbilisi, Georgia*
E-mails: revaz.grigolia@tsu.ge; r.liparteliani@yahoo.com

The new logics are introduced – multi-modal epistemic Lukasiewicz logic $EL(n)$, multi-modal epistemic Lukasiewicz $EL^{\Box}(n)$ obtained by adding global “knowledge operator” $\Box$. The knowledge operators model a community of ideal knowledge agents that possess the properties of veridical knowledge (everything they know is true), uncertain knowledge, positive introspection (they are aware of what they know), negative introspection (they are aware of what they do not know), and so on. The proposed logics allow the application of ideas and results based on Multi-modal epistemic logics to a broader class of Multi-agent Systems.

Limit State Zone at the Crack Tip in Composite Laminates

Roland Duduchava, Tatiana Kuzmina, Alexander Oleinikov

Viktor Kupradze Institute of Mathematics, The University of Georgia, Tbilisi, Georgia
E-mail: t.kuzmina@ug.edu.ge

The theory of asymptotic expansions of solutions to elliptic pseudodifferential equations on a manifold with boundary has turned out to be very useful in a broad range of engineering problems, especially problems in fracture mechanics.

In the report, a general approach to constructing the asymptotics of solutions of elliptic boundary value problems in singularly perturbed domains [1], [2] will be applied. We use asymptotic expansions here to study cracks with failure mechanics in composite laminates [3]–[5].

The main term in the asymptotic decomposition of stresses for cracks represents the stress state in the pre-fracture zone in which the fracture process begins. We distinguish two cases of crack location in a unidirectional composite, parallel and orthogonal to the fibers.

Based on the asymptotic solutions and the matrix and fibers failure criteria, the equations of the limit lines for cracks of mode I and II in the isotropic and orthotropic laminates are obtained. Particularly, the equations for mode I cracks:

$$\rho_I^{(||)}(\Theta) \equiv \frac{4S_{22}^2}{K_I^2}\, r(\Theta) = \frac{2}{\pi}\Big[\varphi_{22}(\Theta) + \Big(\frac{S_{22}}{kS_{21}}\Big)^2 \frac{\varphi_{12}^2(\Theta)}{\varphi_{22}(\Theta)}\Big]^2,$$

$$\rho_I^{\perp}(\Theta) \equiv \frac{4S_{11}^2}{K_I^2}\, r(\Theta)$$

$$= \frac{4}{\pi}\,\frac{[\varphi_{12}^2(\Theta) - \varphi_{11}(\Theta)\varphi_{22}(\Theta)]^2}{\varphi_{11}^2(\Theta) + \varphi_{22}^2(\Theta) + 2\varphi_{12}^2(\Theta) + [\varphi_{11}(\Theta) + \varphi_{22}(\Theta)]\sqrt{[\varphi_{11}(\Theta) - \varphi_{22}(\Theta)]^2 + 4\varphi_{12}^2(\Theta)}}\,,$$

where functions $\varphi_{ij}(\Theta)$ – coefficients of stresses σ_{ij} [5], and S_{11}, S_{22}, S_{21}, α – strength characteristics of monolayer [3].

These equations allow us to estimate, in the first approximation, the shape and size of the pre-fracture zone near cracks. If the doubled size of the limit zone in the direction of the crack axis does not exceed 1/5 of the crack length, then the criteria of its initiation and growth according to the theory of cracks-cuts within the framework of the linear theory of elasticity. These calculated sizes of pre-fracture zones in unidirectional T700S/2592 carbon-fiber reinforced polymer composite are presented.

References

[1] O. Chkadua and R. Duduchava, Pseudodifferential equations on manifolds with boundary: Fredholm property and asymptotic. *Math. Nachr.* **222** (2001), 79–139.

[2] O. Chkadua and R. Duduchava, Asymptotics of functions represented by potentials. *Russ. J. Math. Phys.* **7** (2000), no. 1, 15–47.

[3] A. Oleinikov, Fracture criteria for matrix and fiber in unidirectional polymeric composites. *J. Math. Sci., New York*, 2025; https://doi.org/10.1007/s10958-025-07733-0.

[4] A. Oleinikov and T. Kuzmina, Technical strength theory of elastic composite laminates. *Abstracts of the 12th Intern. Conf. on Composites Testing and Model Ident.* Riga, 2025, Abstract 116.

[5] G. C. Sih and H. Liebowitz, *Mathematical Theories of Brittle Fracture.* In: H. Liebowitz (Ed.). Fracture, Vol. 2, 67–190, Lecture Notes in Math., 1299, Academic Press, New York, 1968.

Long-Time Dynamics of PDEs: Nondegenerate vs Degenerate

Messoud Efendiyev

Helmholtz Center Munich, Institute of Computational Biology
Oberschleißheim, Neuherberg, Germany
E-mail: messoud.efendiyev@gmail.com

In my talk I consider a new class of degenerate parabolic PDEs arising in the modeling of life sciences. The long-time dynamics of solutions is studied in terms of their attractors. Some open problems will also be discussed.

Analysis of Effect of the Solar Magnetic Activity on the Climate Change Using AI Techniques

Khatuna Elbakidze[1], **Oleg Kharshiladze**[2], **Marika Tatishvili**[3]

[1]*Business and Technology University, Tbilisi, Georgia*
E-mail: khatuna.chargazia@gmail.com

[2]*Physics Department, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: oleg.kharshiladze@gmail.com

[3]*Institute of Hydrometeorology, Georgian Technical University, Tbilisi, Georgia*
E-mail: marika.tatishvili@yahoo.com

This study explores the intricate relationship between solar magnetic activity and climate change, utilizing advanced Artificial Intelligence (AI) techniques to uncover patterns, correlations, and potential causal links. Solar magnetic phenomena—such as sunspots, solar flares, and coronal mass ejections—play a significant role in modulating solar irradiance and cosmic ray influx, both of which can influence Earth's atmospheric dynamics and long-term climate patterns. Traditional models often struggle to capture the non-linear and multi-scale interactions between solar activity and terrestrial climate systems. To address this challenge, we employ AI methodologies including machine learning, deep learning, and time-series analysis to analyze large datasets comprising solar indices (e.g., sunspot number, solar flux, geomagnetic indices) and climate variables (e.g., global surface temperature, atmospheric pressure, and oceanic cycles). Our findings reveal statistically significant temporal correlations and lead-lag relationships, suggesting that solar variability, though not the dominant driver, may act as a modulating factor in climate variability over decadal scales. The application of AI provides enhanced predictive capability and model interpretability, offering new insights into the role of extraterrestrial forces in Earth's climate system. This interdisciplinary approach holds promise for improving climate forecasts and deepening our understanding of natural climate variability amidst anthropogenic influences.

References

[1] E. A. Barnes, R. J. Barnes and L. M. Polvani, Machine learning to understand climate variability and change. *Geophysical Research Letters* **46(16), 10830–10839** (2019), no. 4, 499–543.

[2] D. Rolnick, P. L. Donti, L. H. Kaack, K. Kochanski, A. Lacoste, K. Sankaran, A. S. Ross, N. Milojevic-Dupont, N. Jaques and A. Waldman-Brown, Tackling climate change with machine learning. *ACM Computing Surveys (CSUR)* **55** (2022), no. 2, 1–96.

A New Class of Lipschitz-Type Maps Defined on Banach Function Spaces

Ezgi Erdoğan

Department of Mathematics, Marmara University, Istanbul, Türkiye
E-mail: ezgi.erdogan@marmara.edu.tr

In this presentation, a new class of Lipschitz-type maps defined on Banach function spaces. These maps have a special inequality where the usual Lipschitz constant turns into a positive real function, meaning they follow a type of Lipschitz condition that influences the order of the lattice. After analyzing some general results for this class of operators, we give our attention to their representation given by pointwise composition with a strongly measurable function. We will introduce the neccessary and sufficient requirements for Lipschitz operators to be be representable by a strongly measurable function and we will finish the presentation with some applications.

This is a joint work with Roger Arnau, Jose M. Calabuig and Enrique A. Sánchez Pérez.

References

[1] R. Arnau, J. M. Calabuig, E. Erdoğan and E. A. Sánchez Pérez, Extension procedures for lattice Lipschitz operators on Euclidean spaces. *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM* **117** (2023), no. 2, Paper no. 76, 16 pp.

[2] R. Arnau, J. M. Calabuig and E. A. Sánchez-Pérez, Lattice Lipschitz operators on $C(K)$-spaces. *Ann. Funct. Anal.* **16** (2025), no. 3, Paper no. 32, 23 pp.

[3] W. V. Cavalcante, P. Rueda and E. A. Sánchez-Pérez, Extension of Lipschitz-type operators on Banach function spaces. *Topol. Methods Nonlinear Anal.* **57** (2021), no. 1, 343–364.

[4] Ş. Cobzaş, R. Miculescu and A. Nicolae, *Lipschitz Functions.* Lecture Notes in Mathematics, 2241. Springer, Cham, 2019.

Crystalline Measures, Fourier Quasicrystals, their Generalizations and some Applications

Sergii Favorov

V. N. Karazin Kharkiv National University, Kharkiv, Kharkiv Oblast, Ukraine
E-mail: sfavorov@gmail.com

We give definitions of crystalline measure and Fourier quasicrystal, discuss the relationship between these notions, and show some sufficient conditions for a crystalline measure to be a Fourier quasicrystal. We then present the theorem of Olevskii and Ulanovskii on the 1-1 correspondence between zeros of exponential polynomials and Fourier quasicrystals with unit masses. We show generalizations of this result to the zeros of absolutely convergence Dirichlet series, to slowly growing measures with countable spectrum, to the zeros of almost periodic functions in a strip. The last result yields the simple criterion of representability of almost periodic entire functions of exponential growth (in particular exponential polynomials) as a finite product of sines.

Rectangle Partitions Generalizing Integer Partitions

Krystian Gajdzica[1], **Maciej Zakarczemny**[2]

[1] *Theoretical Computer Science Department, Jagiellonian University, Kraków, Poland*
E-mail: krystian.gajdzica@uj.edu.pl

[2] *Department of Applied Mathematics, Cracow University of Technology, Kraków, Poland*
E-mail: mzakarczemny@pk.edu.pl

The story of integer partitions goes back to Euler, who, among other things, discovered the generating function for the partition function $p(n)$:

$$\sum_{n=0}^{\infty} p(n)q^n = \prod_{n=1}^{\infty} \frac{1}{1-q^n}\,.$$

There are a lot of possibilities to generalize the partition function. For instance, one can consider the k-colored partition function or higher-dimensional partitions introduced by MacMahon.

In this talk, we perform a "geometric" extension of the partition function, and consider the number $p(m, n)$ of ways to partition a rectangle of size $m \times n$ into rectangular blocks with integer sides, where two partitions of the rectangle are considered the same if they consist of the same collection of blocks (their geometric arrangement is neglected). We also present some basic properties of $p(m, n)$ and both the lower and the upper bounds for $p(2, n)$. Moreover, we investigate the asymptotic behavior of the number of restricted partitions of a rectangle, where only blocks of special sizes can occur as parts of the rectangle.

Acknowledgments

The first author was supported by the grant of the National Science Centre (Narodowe Centrum Nauki), Poland, # 2024/53/N/ST1/01538.

Safe Mutations of Communicating Automata

Natalia Garanina[1], **Igor Anureev**[1], **Dmitry Kondratyev**[1], **Sergei Gorlatch**[2]

[1] *Institute of Informatics Systems, Novosibirsk, Russia*
E-mail: natta.garanina@gmail.com

[2] *University of Muenster, Germany*
E-mail: gorlatch@uni-muenster.de

In this work we address the correctness problem for communication protocols which are used by automata interacting through channels [2]. Our motivation are numerous industrial applications, in particular in the banking sector. We are especially interested in the consequences of changes or mutations in the behavior of automata for the correctness of communication protocols with these automata as actors. The description of the behavior of actors in specialized protocol programming languages can change due to various reasons, such as changes in the content of messages, changes in the conditions or sequence of receiving and sending messages, changes in addresses, etc. In all these cases, it is necessary to ensure that the communication protocol with a changed actor remains correct. Formal verification of communication protocols is a resource-intensive procedure [1], [2], which in general is undecidable [2]. Therefore we aim at determining safe mutations in the behavior of actors that preserve the correctness of a particular protocol, as well as unsafe mutations that violate its correctness for sure. To the best of our knowledge, the mutability of communicating automata is still an open problem.

We formalize a communication protocol of automata as their asynchronous composition, and we propose a number of behavior mutations of the communicating automata that preserve the protocol correctness. We consider a protocol to be correct if each actor has completed its actions and each communication channel does not contain unreceived messages after the actors finished their work. We consider a class of actor mutations that assumes the inclusion of actor behaviors, i.e., the mutated actor performs only a part of the work of the original actor. Under this assumption, mutations in terms of communicating automata are characterized by the removal of some transitions. We identify several mutations for actor local actions, sending and receiving, and show that they are safe. In addition, we also identify complementary unsafe mutations. We illustrate our approach with an example of a protocol involving three actors.

Acknowledgments

The work at the University of Muenster is supported by the DFG-Project MDH-DL and LTD Case Studio.

References

[1] B. Boigelot and P. Godefroid, Symbolic verification of communication protocols with infinite state spaces using QDDs. *Formal Methods in System Design* **14** (1999), 237–255.

[2] D. Kuske and A. Muscholl, Communicating automata. *Handbook of automata theory. Vol. II. Automata in mathematics and selected applications*, 1147–1188, *EMS Press, Berlin*, 2021.

Mathematical Modeling of Some Thermohydrodynamic and Viscoelastic Processes

Giorgi Geladze, Archil Papukashvili, Meri Sharikadze, Manana Tevdoradze

*Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia*

E-mail: Giorgi.geladze@tsu.ge

In our group, based on the already developed mathematical models of the mesoscale boundary layer of the atmosphere and J. Boll's viscous-elastic beam, we will consider the following problems:

1. Breeze circulations taking into account humid processes (fog, clouds);
2. Aerosol distribution in the atmosphere and sea;
3. Genesis of smog in the atmosphere;
4. Calculation of J. Boll's beam in the functional dependence of viscosity on temperature and pressure;
5. J. Boll beam calculation using variable point heat sources in time and space (imitation of electric welding).

These issues can be used in connection with the Black Sea-related topic "Blue Development of the Black Sea" (seaside atmospheric processes; Anaklia Port; planned artificial island near Batumi; various information cables "laid" in the sea…).

Linguistic Diversity as a Resource for Teaching Numerical Notation and Number Names at the Elementary Level

Irma Ginturi

Public School no. 21, Tbilisi, Georgia
E-mail: ginturi.irma@teachers.gov.ge

Teaching numerical notation and number names at the elementary level is very important. In the Georgian language, the verbal naming of numbers (oral numeration) is based on a vigesimal (base-20) system, whereas the numerical notation is based on a decimal (base-10) system. This often causes confusion among elementary school students, especially second graders who are just beginning to learn numbers greater than 20. For example, when a student is asked to write "ormotsdatertmeti" ("fiftyone"), they may begin writing with the digit 2. Other ethnic minority students also face challenges, as the structure of number names/oral numeration varies across languages, making it difficult for them to comprehend number names in Georgian.

Instead of perceiving the diversity of Georgian and foreign languages as a problem, we can use it as an opportunity for effective teaching. If students compare number names in different languages and link them with their numerical notations, they will realize that differently expressed names are used to represent the same quantity.

Activities that help elementary students better understand the concept of numbers and the meaning of the decimal positional system will be presented at the conference. These activities will also assist teachers in turning multilingual environments into a resource, increasing inclusivity and students' confidence – especially among those for whom Georgian is not a native language.

References

All mathematics textbooks authorized by the Ministry of Education of Georgia, 1^{st} and 2^{nd} grades.

On Entropy and its Measure

Teimuraz Giorgadze, Mamuli Butchukhishvili

Faculty of Pedagogy, Akaki Tsereteli State University, Kutaisi, Georgia
E-mails: Teimuraz.giorgadze@atsu.edu.ge; mamuli.buchukhishvili@atsu.edu.ge

A person must live with uncertainty throughout life. This is primarily due to the fact that we cannot guarantee the occurrence of events influenced by randomness or by factors that, while possibly predictable, remain uncertain.

Assume that ω is a kind of experiment in which he elemental events ω_k $(k = 1, 2, \dots, n)$ have probabilities $P_1, P_1, \dots, P_n$, then ω is called entropy of an experiment

$$H(\omega) = -p_1 \log_2 P_1 - \cdots - P_n \log_2 P_n = -\sum_{k=1}^{n} P_k \log_2 P_k,$$

where it is implied that $-p_k \log_2 P_k = 0$ if for this k $P_k = 1$ $(0 \le P_k \le 1$ thus $H(\omega) \ge 0)$.

The unit of measurement for uncertainty is called a bit (abbreviated from the English words binary digit) i.e., $\log_2 n = 1$, when $n = 2$ Entropy is a Greek word (entropia) and in Georgian means turning, transformation.

Consider several examples of entropy usage: assume that we have 10 white and 4 red balls in a box. We randomly take two balls and observe the colors of the taken balls.

Obviously, we will have: both are white, both are red, or one is white and the other is red – no other case will occur – $\omega_1 = (w; w)$, $\omega_2 = (w; r)$, $\omega_3 = (r; r)$. Their corresponding probabilities are: $P_1 = \frac{45}{91}$, $P_2 = \frac{40}{91}$, $P_3 = \frac{6}{91}$. Therefore, the entropy will be

$$H(\omega) = -\frac{45}{91} \log_2 \frac{45}{91} - \frac{40}{91} \log_2 \frac{40}{91} - \frac{6}{91} \log_2 \frac{6}{91} \approx 1,2823 \text{ (bits)}.$$

Example Two different natural numbers not exceeding 100 are thought of. In how many questions can we guess this number if the questions are asked in such a way that only positive or negative answers (“yes” or “no”) can be given?

It is equally possible that any pair from numbers $C_{100}^2 = 4950$ could be thought of. consider their corresponding probabilities equal and we will have:

$$P_1 = P_2 = \cdots = P_{4950} = \frac{1}{4950}\,.$$

Therefore,

$$H(\omega) = \log_2 4950 \approx 12,273 \,(\text{bits}).$$

With each question asked $(k = 1, 2, \dots, m)$, only two equally expected answers are possible, i.e., with each question we can reduce uncertainty by at most $\log_2 2 = 1$ bits. Accordingly, solving the problem requires no fewer than $m = 13$ questions.

Sub-Gaussian Random Elements in Banach Spaces

George Giorgobiani, Vakhtang Kvaratskhelia, Vaja Tarieladze

Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia

E-mails: giorgobiani.g@gtu.ge; v.kvaratskhelia@gtu.ge; v.tarieladze@gtu.ge

We plan to discuss the further refinements of following results obtained in [1]:

1. If X is a Banach space and a weakly sub-Gaussian random element in X induces the 2-summing operator, then it is T*-sub-Gaussian* provided that X is *a reflexive type 2 space*.
2. A characterization of weakly sub-Gaussian random elements in a Hilbert space which are T-sub-Gaussian.

Acknowledgments

The work was supported by the European Union's HORIZON EUROPE Grant Project GAIN.

References

[1] G. Giorgobiani, V. Kvaratskhelia and V. Tarieladze, On sub-Gaussianity in Banach spaces. *Mathematical Problems of Computer Science* **62** (2024), 52–58.

Iteration Method for the Neumann Boundary Value Problem for the Laplace Equation

Giorgi Gochoshvili[1], **Maia Mrevlishvili**[2], **David Natroshvili**[1]

[1]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mails: gochoshvili5@gmail.ge; d.natroshvili@gtu.ge

[2]*School of Technology, Ilia State University, Tbilisi, Georgia*
E-mail: m_mrevlishvili@yahoo.com

We construct a convergent recurrence scheme for a solution of the Neumann boundary value problem for the Laplace equation. By a special approach, using the potential method we construct a uniquely solvable boundary integral equation containing a selfadjoint compact operator. The single layer potential constructed by the solution of the integral equation gives a particular solution of the Neumann problem if the corresponding necessary condition is satisfied. First, we construct a sequence of successive approximations which converges to the solution of the boundary integral equation in appropriate Bessel-potential spaces of functions defined on the boundary. Afterwards, using these approximations as densities of the single layer potential, we formulate another iteration which converges to a particular solution of the Neumann boundary value problem in the appropriate Sobolev–Slobodetskii spaces of functions defined in the three-dimensional domain under consideration. A general solution of the Neumann boundary value problem is obtained then by adding an arbitrary constant.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FR-23-267).

On Creative Collaboration with Artificial Intelligence in Mathematics Teaching and Learning

Guram Gogishvili[1], **Paata Gogishvili**[2]

[1]*Faculty of Business, Computing and Social Sciences, St. Andrew the First Called Georgian University, Tbilisi, Georgia*
E-mails: guramgog@gmail.com, g.gogishvili@sangu.edu.ge

[2]*Faculty of Business, Technology and Education, Ilia State University, Tbilisi, Georgia*
E-mails: paatagog@gmail.com, paata.gogishvili@iliauni.edu.ge

The role of artificial intelligence (AI) in mathematics education has become particularly significant in recent years and is expected to deepen and expand in the near future. The nature of collaboration with AI is distinct for both teachers/lecturers and students/pupils. In the academic and organizational activities of educators, AI can serve as a reliable and indispensable tool – this partnership continues to grow in scope. For students, AI has already become an effective assistant and an engaging resource. It is capable of performing computations, solving equations, outlining solution steps, and generating graphs with appropriate explanations.

However, alongside these developmental benefits, the use of AI also carries potential risks. Employing modern technologies requires a thoughtful and informed approach.

How should we guide this collaboration to ensure it yields the most fruitful outcomes both intellectually and ethically? Promoting the integration of AI into the learning process must also involve cultivating healthy modes of interaction with it, as well as fostering a critical attitude toward its outputs. Learners must understand that AI-generated advice can sometimes be inaccurate, and recognizing such inaccuracies may also involve turning to AI itself for correction and clarification.

Students/pupils must be made aware that frequent and uncritical reliance on AI can harm their academic integrity and undermine trust. Overdependence on AI – particularly when it leads to the neglect of one's own abilities – may eventually affect their employment prospects or opportunities to participate in high-responsibility projects.

The paper presents sample assignments specifically designed to not only permit but actively encourage the use of AI in a collaborative manner. This novel approach entirely eliminates the risk of plagiarism and enhances AI's educational and developmental role. The methodologies proposed here, when combined with thorough preparatory work by the teacher, facilitate the construction of personalized assignments as well as the accurate review and evaluation of students'/pupils' work.

In conclusion, this paper outlines not only theoretical approaches to integrating AI into the educational process but also introduces concrete forms of creative collaboration with it.

Selection of an Extreme Element in Problem Solving

Ivane Gokadze[1], **Vladimer Adeishvili**[2]

[1]*Andria Razmadze Kutaisi Physics and Mathematics Public School no. 41*
Kutaisi, Georgia
E-mail: Ivane.gokadze@atsu.edu.ge

[2]*Akaki Tsereteli State University, Kutaisi, Georgia*
E-mail: Vladimer.adeishvili@atsu.edu.ge

To successfully solve mathematical problems, it is essential to master a variety of mathematical methods and approaches, both general and specific. This paper explores one such general approach, known as the method of selecting an extreme element. This technique is particularly useful in tackling Olympiad-level and non-standard problems, where standard solution methods may not apply. The idea of identifying an extreme element often enables us to pinpoint components of a problem with distinctive properties, which can provide a critical breakthrough in reaching a solution. The method fosters the development of logical reasoning, analytical thinking, and the ability to approach problems from unconventional perspectives. It is especially effective when conventional methods are too complex or require excessive effort. In essence, the method involves choosing an object that possesses an extreme property. For instance, if a problem involves a set of numbers, the method suggests selecting the largest or smallest one; in the case of a convex polygon, one might choose the angle with the greatest or smallest measure; on a coordinate plane, one might identify the leftmost or rightmost point; or, among the sides of a polygon, select the longest or shortest. The paper also presents several example problems, and to further engage the reader, a set of exercises for independent practice is included at the end. We hope that by working through these problems, readers will both enjoy the process and gain a deeper understanding of the method presented.

On the Weighted Steklov Eigenvalue Problems in Outward Cuspidal Domains

Vladimir Gol'dshtein[1], **Prashanta Garain**[2], **Alexander Ukhlov**[1]

[1]*Department of Mathematics, Ben-Gurion University of the Negev, Beer Sheva, Israel*
E-mails: vladimir@math.bgu.ac.il; ukhlov@math.bgu.ac.il

[2]*Department of Mathematical Sciences, Indian Institute of Science Education and Research Berhampur Berhampur, Odisha, India*
E-mail: pgarain92@gmail.com

We investigate the weighted Steklov eigenvalue problem and the weighted Schrödinger–Steklov eigenvalue problem in outward cuspidal domains. We prove the solvability of these spectral problems in both linear and non-linear cases.

Acknowledgments

The authors thank Iosif Polterovich for very useful and valuable remarks.

The second author is supported by the seed grant from IISER Berhampur.

References

[1] A. Girouard and I. Polterovich, Spectral geometry of the Steklov problem (survey article). *J. Spectr. Theory* **7** (2017), no. 2, 321–359.

[2] V. Gol'dshtein and M. Ju. Vasiltchik, Embedding theorems and boundary-value problems for cusp domains. *Trans. Amer. Math. Soc.* **362** (2010), no. 4, 1963–1979.

[3] V. Gol'dshtein and A. Ukhlov, Weighted Sobolev spaces and embedding theorems. *Trans. Amer. Math. Soc.* **361** (2009), no. 7, 3829–3850.

[4] A. Ferrero and P. D. Lamberti, Spectral stability of the Steklov problem. *Nonlinear Anal.* **222** (2022), Paper no. 112989, 33 pp.

On Toeplitz–Hessenberg Determinants with Lichtenberg Number Entries

Taras Goy

Faculty of Mathematics and Computer Science,
Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine
E-mail: taras.goy@pnu.edu.ua

Lichtenberg numbers are named after the German scientist Georg Christoph Lichtenberg, who discussed them as early as 1769 in connection with the Chinese rings puzzle [3]. The Lichtenberg numbers, denoted by ℓ_n, are defined by the third-order recurrence relation $\ell_n = 2\ell_{n-1}+\ell_{n-2}-2\ell_{n-3}$ for $n \geq 3$, with initial conditions $\ell_0 = 0$, $\ell_1 = 1$, and $\ell_2 = 2$.

A Toeplitz–Hessenberg matrix is an $n \times n$ matrix of the form

$$M_n(a_0; a_1, \dots, a_n) = \begin{pmatrix} a_1 & a_0 & 0 & \cdots & 0 & 0 \\ a_2 & a_1 & a_0 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \ddots & \dots & \dots \\ a_{n-1} & a_{n-2} & a_{n-3} & \cdots & a_1 & a_0 \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_2 & a_1 \end{pmatrix},$$

where $a_0 \neq 0$ and $a_k \neq 0$ for at least one $k > 0$.

We investigate some families of Toeplitz–Hessenberg determinants with $a_0 = \pm 1$, where the remaining entries are Lichtenberg numbers.

Theorem 1 *Let $n \geq 1$, unless otherwise specified. Then*

$$\begin{aligned}
\det(1; \ell_2, \ell_3, \dots, \ell_{n+1}) &= 0, \quad n \geq 4; \\
\det(1; \ell_1, \ell_3, \dots, \ell_{2n-1}) &= n(-2)^{n-1}; \\
\det(-1; \ell_1, \ell_3, \dots, \ell_{2n-1}) &= \frac{1}{2}\sum_{k=0}^{n}\binom{n}{k}F_{3k}; \\
\det(1; \ell_3, \ell_5, \dots, \ell_{2n+1}) &= 0, \quad n \geq 3; \\
\det(-1; \ell_3, \ell_5, \dots, \ell_{2n+1}) &= \frac{\sqrt{17}}{68}\Big(\big(5+\sqrt{17}\big)^{n+1} - \big(5-\sqrt{17}\big)^{n+1}\Big); \\
\det(1; \ell_0, \ell_2, \dots, \ell_{2(n-1)}) &= 2\big((-3)^{n-1} - (-2)^{n-1}\big); \\
\det(-1; \ell_0, \ell_2, \dots, \ell_{2(n-1)}) &= \frac{2\sqrt{17}}{17}\bigg(\Big(\frac{5+\sqrt{17}}{2}\Big)^{n-1} - \Big(\frac{5-\sqrt{17}}{2}\Big)^{n-1}\bigg), \quad n \geq 2.
\end{aligned}$$

Here, F_n denotes the nth Fibonacci number.

Using the Trudi formula, these determinant expressions can be rewritten as identities involving sums of products of Lichtenberg numbers and multinomial coefficients (see [1], [2] for further details).

References

[1] T. Goy, Some families of identities for Padovan numbers. *Proc. Jangjeon Math. Soc.* **21** (2018), no. 3, 413–419.

[2] T. Goy and M. Shattuck, Determinant identities for Toeplitz-Hessenberg matrices with tribonacci number entries. *Trans. Comb.* **9** (2020), no. 2, 89–109.

[3] A. M. Hinz, The Lichtenberg sequence. *Fibonacci Quart.* **55** (2017), no. 1, 2–12.

An Axiomatic Theory of Infinitesimals and the Concept of Limit

Nikoloz Gunia

Mathematics Department, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail:nikolozgunia21@gmail.com

According to Cauchy's approach, the equality

$$\lim_{x\to x_0} y(x) = y_0, \tag{1}$$

where $x_0, y_0 \in \mathbb{C}$, and $y(x)$ is a complex-valued function of the variable x, means the following: if the difference $x - x_0$ (with $x \neq x_0$) is infinitesimal, then so is the difference $y(x) - y_0$. In Cauchy's original formulation, the notion of infinitesimal was intuitively understood.

Modern mathematical analysis relies on the well-known ε-δ definition of limit, formulated by K. Weierstrass. In contrast to Cauchy's method, this approach does not depend on the concept of infinitesimals; rather, an infinitesimal is defined as a function that tends to zero.

In our presentation, we define the equality (1) considering $y(x)$ as a function of a quantity x. The definition of the limit then formally replicates that of Cauchy.

Let the symbol "$\in$ Inf" denote that the corresponding expression is infinitesimal in the sense of our axiomatic theory. Then

$$x \to x_0 \iff_{\text{def}} (x - x_0)_{x\neq x_0} \in \text{Inf}.$$

Accordingly, equality (1) is defined as follows:

$$\lim_{x\to x_0} y(x) = y_0 \iff_{\text{def}} \Big[x \to x_0 \implies (y(x) - y_0)_{x\neq x_0} \in \text{Inf}\Big]. \tag{2}$$

At the foundation of our 'Inf-theory' lie the properties of infinitesimal quantities, formulated by Leibniz [1]. These properties, typically proven as theorems in standard analysis, are accepted here as axioms. To develop a rigorous theory, we have also introduced several additional axioms.

However, our theory remains within the framework of standard analysis, since the following theorem holds: if equality (1) is satisfied in the sense of definition (2), then it is also valid in the sense of the $\varepsilon - \delta$ definition of limit.

We rely on the axiomatic theory of real numbers $\mathbb{R}$, in which the axiom of completeness is formulated as the existence of the supremum for any set bounded above. The presentation demonstrates that this theory can be developed from the outset by building up the nonnegative rational numbers and gradually extending them to $\mathbb{R}$, and then, in the standard way, to $\mathbb{C}$.

References

[1] E. I. Gordon, A. G. Kusraev and S. S. Kutateladze, Excursus into the History of Calculus. In: *Infinitesimal Analysis. Mathematics and Its Applications*, vol. 544. *Springer, Dordrecht*, 2002.

On the Length of Jordan, Lie, and Maltsev Algebras

Alexander Guterman

Department of Mathematics, Bar-Ilan University, Israel
E-mail: alexander.guterman@gmail.com

The length of a finite system of generators for a finite-dimensional algebra over a field is the least positive integer k such that the products of length not exceeding k span this algebra as a vector space. The maximum length for the systems of generators of an algebra is called the length of this algebra. Length function is an important invariant widely used to study finite dimensional algebras since 1959. The length evaluation is a difficult problem even in associative case, where the length is always less than the dimension. However, in general, it is not true for non-associative algebras. Moreover, their length can have an exponential growth as a function of dimension.

In the talk we will discuss the length of Jordan, Malcev, or Leibnitz algebras. We prove that if A is an algebra from the above list and the characteristics $\operatorname{char}\mathbb{F} \neq 2$, then the length $l(A) \leq \dim(A)$. We also investigate the relationship between the length of an associative algebra and the length of its adjoint Jordan or Lie algebra. We show that the Jordan identity by itself without commutativity relation does not guarantee a linear bound on growth, although the length of a Leibnitz algebra is bounded by above by the dimension.

We also present some general properties that guarantee linear grows of the length.

The talk is based on the series of joint works with Dmitry Kudryavtsev and Svetlana Zhilina.

Adaptive Bias–Aware Query Optimization System

Eliza Gyulgyulyan, Hrachya Astsatryan

Institute for Informatics and Automation Problems, National Academy of Sciences
Yerevan, Armenia
E-mails: eliza.gyulgyulyan@gmail.com; hrach@sci.am

Data lakes manage large-scale, heterogeneous, evolving data collections across scientific computing, business intelligence, and open data platforms. Their schema-on-read flexibility supports exploratory analytics but introduces a significant challenge in data quality, most notably when dealing with inconsistent, missing, or biased data [1]. Analytical queries on such uncurated data risk returning unreliable, misleading, or unfair results, compromising scientific integrity and operational decision-making.

The authors proposed an alerting mechanism for quality anomaly detection during analytical processing in the data lake architecture to meet these requirements [2], [3]. It allowed analysts to detect data quality issues in real-time, leaving the user to interpret and respond to these alerts.

This paper brings the following natural next step: ABAQOS (Adaptive Bias-Aware Query Optimization), a novel mechanism that intercepts analyst queries, evaluates underlying data quality dimensions, and proactively optimizes query structure to reduce bias, resource waste, and analysis distortion. This is going to enhance the validity and precision of the alerts, making the informed decisions trustworthy. Through an enriched feedback loop and dynamic metrics model, ABAQOS provides real-time recommendations to analysts within exploratory workflows, enhancing reliability and decision impact.

Acknowledgments

The work is supported by the base program of the Institute for Informatics and Automation Problems funded by the Higher Education and Science Committee of the Ministry of Education, Science, Culture and Sports of the Republic of Armenia.

References

[1] M. Farid, A. Roatis, I. F. Ilyas, H.-F. Hoffmann and X. Chu, CLAMS: bringing quality to data lakes. *Proceedings of the 2016 International Conference on Management of Data (SIGMOD '16)*, 2016, 2089–2092.

[2] E. Gyulgyulyan, J. Aligon, F. Ravat and H. Astsatryan, Data quality alerting model for big data analytics. In: Welzer, T., et al. *New Trends in Databases and Information Systems. ADBIS 2019. Communications in Computer and Information Science*, vol. 1064, pp. 489–500, *Springer, Cham*, 2019.

[3] E. Gyulgyulyan, F. Ravat, H. Astsatryan and J. Aligon, Data quality impact in business inteligence. *2018 Ivannikov Memorial Workshop (IVMEM)*, IEEE Xplore, 2019, 47–51.

Lyapunov Functions for Stochastic Differential Equations: Theory and Computation

Sigurdur Hafstein

Department of Mathematics, University of Iceland, Reykjavik, Iceland
E-mail: shafstein@hi.is

Stability in probability of equilibria for stochastic differential equations (SDEs) can be characterized by the existence of Lyapunov functions, which are real-valued functions from the state-space that are decreasing along solution trajectories. The theory is not new, but until now applications have mainly concentrated on exponential mean-square stability as this is the easiest concept to verify. We will discuss the more general asymptotic stability in probability and how it can be ascertained for linear SDEs with constant coefficients using bilinear matrix inequalities. We show several examples from application, where global asymptotic stability in probability can be numerically proved, although the equilibrium is not exponentially mean square stable [1].

Acknowledgments

This work was partially supported by Te Icelandic Research Fund, Grant # 152429-051.

References

[1] G. Barrera, E. Bjarkason and S. Hafstein, The stability of the multivariate geometric Brownian motion as a bilinear matrix inequality problem. *SIAM J. Appl. Dyn. Syst.* **24** (2025), no. 2, 1251–1288.

Comparing Trivariate Copula with Machine Learning in Copper Recovery

H. Hernández[1], **E. Alberdi**[2], **A. Goti**[3]

[1] *Universidad Santo Tomás, Ejército Libertador 146, Santiago, Chile*
E-mail: hhernandez9@santotomas.cl

[2] *University of the Basque Country UPV/EHU, Paseo Rafael Moreno Pitxitxi 2 Bilbao, Spain*
E-mail: elisabete.alberdi@ehu.eus

[3] *University of Deusto, Avenida de las Universidades 24, Bilbao, Spain*
E-mail: aitor.goti@deusto.es

In this study, an innovative methodology using trivariate copula-based conditional quantile regression (CBQR) is proposed for estimating copper recovery. This approach is compared with machine learning regression methods.

An open access database representative of a porphyry copper deposit is used.

The results demonstrate that trivariate CBQR outperforms machine learning methods in accuracy but also in flexibility. It offers a practical and flexible alternative solution to model complex relationships between variables under limited data conditions. Unlike ML models, which require lengthy training and constant optimization of hyperparameters, copula allows direct implementation with minimal adjustments, facilitating its applicability in conditions of limited and heterogeneous data.

References

[1] F. Durante, G. Puccetti, M. Scherer and S. Vanduffel, My introduction to copulas: an interview with Roger Nelson. *Depend. Model.* **5** (2017), no. 1, 88–98.

[2] C. Genest and A.-C. Favre, Everything you always wanted to know about copula modeling but were afraid to ask. *Journal of Hydrologic Engineering* **12** (2007), no. 4, 347–368.

[3] H. Hernández, M. A. Díaz-Viera, E. Alberdi, A. Oyarbide-Zubillaga and A. Goti, Metallurgical Copper Recovery Prediction Using Conditional Quantile Regression Based on a Copula Model. *Minerals* **14** (2024), no. 7, 21 pp.

[4] P. K. Trivedi, D. M. Zimmer, Copula modeling: An introduction for practitioners. *Foundations and Trends®in Econometrics* **1** (2007) no. 1, 1–111.

On Gegenbauer Transformation on the Half-Line

Elman J. Ibrahimov

Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan, Baku, Azerbaijan

E-mail: elmanibrahimov@yahoo.com

In this abstract we construct Gegenbauer transformation and some similar properties to the Fourier transform is proved. An equation of Parseval–Plancherel type is obtained. The inversion theorem for the Gegenbauer transformation is proved.

References

[1] E. J. Ibrahimov, On Gegenbauer transformation on the half-line. *Georgian Math. J.* **18** (2011), no. 3, 497–515.

One-Sided Integral Operators in Variable Exponent Morrey Spaces

Giorgi Imerlishvili[1], **Alexander Meskhi**[1,2]

[1]*Kutaisi International University, Kutaisi, Georgia*

[2]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mails: imerlishvili18@gmail.com; alexander.meskhi@kiu.edu.ge

In the paper [1] the authors established the boundedness of one-sided maximal, fractional and singular integral operators in $L^{p(\cdot)}$ spaces under the conditions on exponents which are weaker than the log-Hölder continuity condition. Such conditions involve, for example, monotone exponents on finite intervals.

We established the boundedness of one-sided maximal, Riemann–Liouville and Weyl integral operators in variable exponent Morrey spaces $M^{p(\cdot),q(\cdot)}$ under certain conditions on exponents $p(\cdot)$ and $q(\cdot)$.

References

[1] D. E. Edmunds, V. Kokilashvili and A. Meskhi, One-sided operators in $L^{p(x)}$ spaces. *Math. Nachr.* **281** (2008), no. 11, 1525–1548.

About Using Internet Technologies in Education

Elza Imnadze, Nugzar Kereselidze

Faculty of Natural Sciences, Mathematics, Technology and Pharmacy,
Sokhumi State University, Tbilisi, Georgia
E-mails: elza.imnadze@iliauni.edu.ge; nkereselidze@sou.edu.ge

The article discusses the possibilities of using Internet technologies created on the client side in the education system. For example, one of several web browser data stores is selected – IndexedDB, in which the necessary information is recorded, searched, created and retrieved. IndexedDB is used for subject testing and for exams. The subject teacher has the ability to store test questions of several topics with probable answers and correct answers in IndexedDB. The proposed system – a comprehensive testing program (CTP) allows: each time to select different questions for each topic and display them in a different sequence; initialize a combined test from several topics for midterm and final exams, evaluate student testing results. The comprehensive testing program can work in places with problems with Internet access, allows an interested person to create and enter tests, use them for training or assessment. The comprehensive testing program is based on program code written in HTML [1], JavaScript [2] and JSON.

References

[1] http://kereseli.sou.edu.ge/CTP/damateba.pdf.

[2] http://kereseli.sou.edu.ge/CTP/gamotana.pdf.

On the Representability by Sums of Generalized Ridge Functions

Fidan M. Isgandarli

Baku State University, Baku, Azerbaijan
E-mail: fidanisgandarli100@gmail.com

A multivariate function $F : \mathbb{R}^n \to \mathbb{R}$ of the form

$$F(\mathbf{x}) = f(\mathbf{a}^1 \cdot \mathbf{x}, \dots, \mathbf{a}^d \cdot \mathbf{x})$$

is called a generalized ridge function, where $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$, $f : \mathbb{R}^d \to \mathbb{R}$ is a real-valued function of d variables ($1 \le d < n$) and $\mathbf{a}^j = (a_1^j, \dots, a_n^j) \in \mathbb{R}^n \setminus \{\mathbf{0}\}$, $j = 1, \dots, d$ are fixed vectors (directions). For $d = 1$ generalized ridge function called a ridge function.

Assume we are given a function $F : \mathbb{R}^n \to \mathbb{R}$ of the form

$$F(\mathbf{x}) = \sum_{k=1}^{m} f_k(\mathbf{a}^{k,1} \cdot \mathbf{x}, \dots, \mathbf{a}^{k,d} \cdot \mathbf{x}), \qquad (*)$$

where $\{\mathbf{a}^{1,1}, \dots, \mathbf{a}^{1,d}\}, \dots, \{\mathbf{a}^{m,1}, \dots, \mathbf{a}^{m,d}\}$ pairwise non-equivalent vector systems in $\mathbb{R}^n$, $1 \le d < n - 1$, $f_1, \dots, f_m$ are arbitrarily behaved real-valued functions of d variables.

Assume we are given a function $F \in C^s(\mathbb{R}^n)$ of the form (*). Is it true that there will always exist $g_k \in C^s(\mathbb{R}^d)$, $k = 1, \dots, m$ such that

$$F(\mathbf{x}) = \sum_{k=1}^{m} g_k(\mathbf{a}^{k,1} \cdot \mathbf{x}, \dots, \mathbf{a}^{k,d} \cdot \mathbf{x})?$$

In [2], this problem for ridge function representation was solved up to a multivariate polynomial: We give a partial solution posted roblem for generalized ridge function representation.

Theorem ([1]) *Assume a function $F \in C^m(\mathbb{R}^n)$ is of the form*

$$F(\boldsymbol{x}) = \sum_{k=1}^{m} f_k(\mathbf{a}^{k,1} \cdot \mathbf{x}, \dots, \mathbf{a}^{k,n-1} \cdot \mathbf{x}),$$

where $\{\mathbf{a}^{1,1}, \dots, \mathbf{a}^{1,n-1}\}, \dots, \{\mathbf{a}^{m,1}, \dots, \mathbf{a}^{m,n-1}\}$ pairwise non-equivalent vector systems in $\mathbb{R}^n$, $f_1, \dots, f_m$ are arbitrarily behaved real-valued functions of $n-1$ variables. Then there exist functions $g_k \in C^1(\mathbb{R}^{n-1})$, $k = 1, \dots, m$, such that

$$F(\mathbf{x}) = \sum_{k=1}^{m} g_k(\mathbf{a}^{k,1} \cdot \mathbf{x}, \dots, \mathbf{a}^{k,n-1} \cdot \mathbf{x}).$$

References

[1] A. A. Akbarov and F. M. Isgandarli, On the representability of a smooth function by sums of generalized ridge functions. *Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan* **51** (2025), no. 1, 42–50.

[2] R. A. Aliev and V. E. Ismailov, A representation problem for smooth sums of ridge functions. *J. Approx. Theory* **257** (2020), Article no. 105448, 13 pp.

Results from Combining Taylor Series and Block Pulse Functions

Yousef Jafarzadeh

Department of Mathematics, Ka.C., Islamic Azad University, Karaj, Iran
E-mail: Yousef.jafarzadeh@iau.ac.ir

In this paper, the new method that combines Taylor series and Block Pulse Functions is defined. The main results and theorems on the new method are investigated. The theory of solutions for higher-order linear Volterra–Fredholm Integro-Differential Equations (IDVFE) based on new method is developed in this work.

Acknowledgments

The work was supported by the Islamic Azad University, Karaj Branch.

References

[1] A. Akyüz-Daşcıoğlu and M. Sezer, Chebyshev polynomial solutions of systems of higher-order linear Fredholm–Volterra integro-differential equations. *J. Franklin Inst.* **342** (2005), no. 6, 688–701.

[2] Y. Jafarzadeh and B. Keramati, Convergence analysis of parabolic basis functions for solving systems of linear and nonlinear Fredholm integral equations. *Turkish J. Math.* **41** (2017), no. 4, 787–796.

[3] Y. Jafarzadeh and B. Keramati, Numerical method for a system of integro-differential equations and convergence analysis by Taylor collocation. *Ain Shams Engineering Journal* **9** (2018), no. 4, 1433–1438.

Some Properties of the Gradient Approach for Testing the Equality of Several Coefficients of Variation

Yousef Jafarzadeh

Department of Mathematics, Ka.C., Islamic Azad University, Karaj, Iran
E-mail: Yousef.jafarzadeh@iau.ac.ir

In this paper, we introduce the Gradient test and investigate its properties as a new approach for evaluating the equality of coefficients of variation across multiple datasets. Building on the foundations of the Wald and Score tests.

Acknowledgments

The work was supported by the Islamic Azad University, Karaj Branch.

References

[1] G. Alkan, F. Gökpınar and E. Gökpınar, A Wald test on the problem of homogeneity of variances. *Comm. Statist. Simulation Comput.* **54** (2025), no. 5, 1332–1345.

[2] A. G. Bedeian and K. W. Mossholder, On the use of the coefficient of variation as a measure of diversity. *Organizational Research Methods* **3** (2000), no. 3, 285–297.

[3] K. Krishnamoorthy and M. Lee, Improved tests for the equality of normal coefficients of variation. *Comput. Statist.* **29** (2014), no. 1-2, 215–232.

Solving Boundary Value Problem Including Second Order Multiplicative Differential Equation by Method of Green's Function

Mohammad Jahanshahi, **Ali Eyvazi**

Azarbaijan Shahid Madani University, Tabriz, Iran
E-mail: teach.eivazi42@gmail.com

Recently, multiplicative calculus have attracted the attention of mathematicians, physicists, and engineers due to its applications. After that, many authors develop and generalize many mathematical analysis methods to multiplicative analysis. In this article, first, multiplicative differential equations are introduced, then boundary value problems including these types of equations are presented. In the following, the method of constructing Green's function for such problems is presented. Finally, the solution of the boundary value problem including a second-order multiplicative differential equation is given by the Green's function method.

Review of the Classical Problem of Perfect Numbers and the Justification of the Uniqueness of Euclid's Formula

Temuri Jaoshvili

Tbilisi, Georgia
E-mail: jaoshvilitemuri@gmail.com

Perfect numbers are one of the oldest and most fascinating topics in number theory, studied since the era of Ancient Greece. This paper reviews well-known results related to perfect numbers and focuses on the Euclid–Euler formula, which defines all currently known even perfect numbers. The formula is as follows:

$$N = 2^{p-1}(2^{p-1} - 1), \tag{1}$$

where p is a prime number, and $2^p - 1$ is a Mersenne prime.

It is confirmed that all known perfect numbers are even and can be generated using Euclid's formula. The existence of odd perfect numbers remains one of the oldest unsolved problems in mathematics. The paper also reviews both historical and modern methods used in the search for Mersenne primes.

An Algebraic Approach to the Kuznetsov–Muravitsky Theorem

Mamuka Jibladze, Evgeny Kuznetsov

Department of Mathematical Logic, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia

E-mails: mamuka.jibladze@gmail.com; e.kuznetsov@freeuni.edu.ge

Around 1978, A. V. Kuznetsov introduced the now famous modal extension of the Intuitionistic Propositional Calculus IPC, later thoroughly investigated by him and his devoted collaborator A. Yu. Muravitsky in a series of papers. We will use for their system the by now well-established name KM.

In the posthumously published paper [1] (only available in Russian) Kuznetsov proved that, for any IPC-formulæ A, B,

$$\mathsf{KM} + A \vdash B \iff \mathsf{IPC} + A \vdash B. \tag{K}$$

His proof is essentially proof-theoretic, based on an inductive elimination of the modality from inferences.

On the other hand, from the very beginning Kuznetsov and Muravitsky studied their calculus in parallel with its algebraic semantics, via what they called Δ-*pseudoboolean algebras*. In this paper we will call them KM-algebras.

As established by Kuznetsov and Muravitsky, (K) is equivalent to the statement that every variety of Heyting algebras is generated by reducts of KM-algebras. In fact they also showed that (K) implies existence of an embedding of any Heyting algebra H into a KM-algebra generating the same variety of Heyting algebras as H. (See [2, p. 53] for these facts.)

As stressed by several people, including Muravitsky himself, it would be highly desirable to have a purely algebraic construction of such an embedding.

In this talk, an explicit embedding is addressed, manifestly remaining in the same variety.

Acknowledgments

This talk was supported by TSU Andrea Razmadze Mathematical Institute.

References

[1] A. V. Kuznetsov, The proof-intuitionistic propositional calculus. (Russian) *Dokl. Akad. Nauk SSSR* **283** (1985), no. 1, 27–30.

[2] A. Y. Muravitsky, The contribution of A. V. Kuznetsov to the theory of modal systems and structures. *Logic Log. Philos.* **17** (2008), no. 1-2, 41–58.

Performance Evaluation and Comparative Study of ECC, RSA and Kyber Cryptosystems in the Post-Quantum Era

Elza Jintcharadze, Tsitsino Sarajishvili

Department of Computer Sciences, Batumi Shota Rustaveli State University
Batumi, Georgia

E-mails: elza.jintcharadze@bsu.edu.ge; tsitsino.sarajishvili@bsu.edu.ge

The development of quantum computing presents a critical challenge for modern cryptographic algorithms. There is the need to create post-quantum alternatives. This article offers a performance evaluation and comparative analysis of different cryptographic algorithms: RSA, Elliptic Curve Cryptography (ECC) and Kyber.

RSA and ECC are widely used in modern systems because of their high security and efficiency. However, both are vulnerable to quantum attacks. Kyber is selected by NIST as a post-quantum cryptographic candidate and is designed to resist quantum threats. This research compares algorithms based on key size, encryption and decryption speed, memory usage, and security strength in both classical and quantum contexts.

The following results were obtained as a result of the research: ECC offers comparable security to RSA with significantly smaller key sizes. Kyber uses bigger keys, than ECC. This is because Kyber needs those larger keys to be strong enough to protect information from quantum computers, which ECC cannot do. At the same time, it is also Kyber's disadvantage, because you get stronger security, but the keys are a bit bigger. The comparative study reveals significant differences in the performance and security profiles of RSA, ECC, and Kyber. RSA generates large key sizes and slower performance, especially against quantum threat models. Performance evaluations indicate that Kyber is suitable for integration into modern cryptographic systems.

As quantum computing continues to develop, the use of secure and scalable cryptographic solutions such as Kyber will be crucial to ensure confidentiality and integrity of digital communications.

References

[1] G. Alagic, D. Apon, D. Cooper, Q. Dang, T. Dang, J. Kelsey, J. Lichtinger, Y.-K. Liu, C. Miller, D. Moody, R. Peralta, R. Perlner, A. Robinson and D. Smith-Tone, *Status Report on the Third Round of the NIST Post-Quantum Cryptography Standardization Process.* National Institute of Standards and Technology, 2022.

[2] E. Jintcharadze and M. Abashidze, Performance and comparative analysis of elliptic curve cryptography and RSA. *IEEE East-West Design & Test Symposium (EWDTS)*, 2023.

[3] R. Rivest, A. Shamir and L. Adleman, A method for obtaining digital signatures and public-key cryptosystems. *Communications of the ACM* **21** (21978), no. 2, 120–126.

[4] I. A. Shoukat, K. A. Bakar and S. Ibrahim, A Generic Hybrid Encryption System (HES). *Research Journal of Applied Sciences, Engineering and Technology* **5** (2013), no. 9, 2692–2700.

[5] Standards for Efficient Cryptography Group (SECG), *Elliptic Curve Cryptography*, 2000.

Recursive relations for the S-Matrix of Liouville Theory

George Jorjadze[1,2], **Lado Razmadze**[3], **Stefan Theisen**[4]

[1] *Free University of Tbilisi, Tbilisi, Georgia*
E-mail: g.jorjadze@freeuni.edu.ge

[2] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: george.jorjadze@tsu.ge

[3] *Forschungszentrum Jülich GmbH, Wilhelm-Johnen-Straße, Jülich, Germany*
E-mail: l.razmadze@fz-juelich.de

[4] *Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Golm, Germany*
E-mail: stefan.theisen@aei.mpg.de

We analyze the relation between the vertex operators of the in and out fields in Liouville theory. This is used to derive equations for the S-matrix, from which a recursive relation for the normal symbol of the S-matrix for discrete center-of-mass momenta is obtained. Its solution is expressed as multiple contour-integrals of a generalized Dotsenko–Fateev type. This agrees with the functional integral representation of the scattering matrix of Liouville theory which we had proposed previously.

Acknowledgments

This research has been supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) by the grant # FR-23-17899.

References

[1] G. Jorjadze and S. Theisen, On the S-matrix of Liouville theory. *J. High Energy Phys.* **2021**, no. 2, Paper no. 111, 24 pp.

[2] G. Jorjadze and S. Theisen, Generating functional for the S-matrix in Liouville theory. *Proceedings of Science* **394** (2021), 12 pp.

[3] D. Das, G. Jorjadze and L. Megrelidze, Functional Integral Representation for the S-Matrix in Integrable 2D Conformal Field Theories. *J. Contemp. Phys.* **60** (2025), no. 2 (in press).

[4] G. Jorjadze, L. Razmadze and S. Theisen, Recursive relations for the S-matrix of Liouville theory. *Preprint* arXiv:2507.11760, 2025; `https://arxiv.org/abs/2507.11760`.

Regularization of the Cauchy Problem for Elliptic Systems in Unbounded Domain

Davron Aslonqulovich Juraev[1,2], **Gulsum Allahyar Aghayeva**[3]

[1]*Scientific Research Center, Baku Engineering University, Baku, Azerbaijan*
[2]*Department of Mathematical Analysis and Differential Equations, Karshi State University Karshi, Uzbekistan*
E-mail: juraevdavron12@gmail.com

[3]*Department of Computational Mathematics, Baku State University, Baku, Azerbaijan*
E-mail: gulsumm_agayeva@mail.ru

This article addresses the construction of a regularized solution for the Cauchy problem associated with elliptic systems, particularly focusing on the matrix factorization of the Helmholtz equation in an unbounded three-dimensional domain. The proposed approach is grounded in the Carleman matrix method, which facilitates an integral representation of the solution. The ill-posed nature of the problem is tackled through careful regularization, wherein a strategically chosen parameter ensures the stability and convergence of the solution while maintaining a balance between noise suppression and accuracy. The authors derive a new class of integral formulas valid in unbounded domains, incorporating entire functions to refine the kernel behavior. Explicit estimates for the approximation error are provided, demonstrating the method's effectiveness and robustness. Additionally, the paper includes theoretical justifications for the regularization technique, including proofs of stability and convergence, and presents a numerical strategy for selecting the optimal regularization parameter based on a priori error bounds. The method is extended to handle boundary layers and allows uniform convergence away from singular perturbation zones. Overall, this work offers significant advancements in the treatment of ill-posed elliptic problems and contributes to the theoretical foundation and practical computation of stable solutions in mathematical physics and engineering applications.

In the development of the regularization method for the Cauchy problem in elliptic systems, the foundational concepts of well-posedness introduced by Hadamard [1] played a crucial role in defining the theoretical framework. Building upon this foundation, Juraev's earlier work [2] provided essential insights into matrix factorization techniques for the Helmholtz equation in unbounded domains. Furthermore, the approximation strategies for solving ill-posed problems, particularly in the context of first-order elliptic systems, were significantly advanced in the collaborative study by Juraev, Shokri, and Marian [3]. These contributions collectively inform the integral formulations and stability analyses presented in the current paper.

References

[1] Ž. Adamar, *The Cauchy Problem for Linear Partial Differential Equations of Hyperbolic Type.* (Russian) Translated from the French by F. V. Shugaev. Edited by D. M. Belocerkovskiĭ. "Nauka", Moscow, 1978.

[2] D. A. Zhuraev, The Cauchy problem for matrix factorizations of the Helmholtz equation in an unbounded domain. (Russian) *Sib. Èlektron. Mat. Izv.* **14** (2017), 752–764.

[3] D. A. Juraev, A. Shokri and D. Marian, On an approximate solution of the Cauchy problem for systems of equations of elliptic type of the first order. *Entropy* **24** (2022), no. 7, Paper no. 968, 18 pp.

Constrained Bayesian Method for Testing Composite Hypotheses Concerning Normal Distribution with Equal Parameters

Kartlos J. Kachiashvili[1,2,3], **Nitis Mukhopadhyay**[4], **Joseb K. Kachiashvili**[1,3]

[1]*Faculty of Informatics and Control Systems, Georgian Technical University, Tbilisi, Georgia*

[2]*Ilia Vekua Institute of Applied Mathematics of Tbilisi State University Tbilisi, Georgia*

[3]*Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia*

E-mails: k.kachiashvili@gtu.ge, kkachiashvili@gmail.com; s.kachiashvili@gtu.ge

[4]*Department of Statistics, University of Connecticut, 215 Glenbrook Road, CLAS Building, U-4120, Storrs, CT 06269-4120, U.S.A.*

E-mail: nitis.mukhopdahyay@uconn.edu

The problem of testing composite hypotheses with respect to the equal parameters of a normal distribution using the constrained Bayesian method is discussed. Hypotheses are tested using maximum likelihood and Stein's methods. The optimality of our decision rule is shown by the criteria: the mixed directional false discovery rate; the false discovery rate; the Type I and Type II errors, under the conditions of providing a desired level of constraint. The algorithms for implementing the proposed methods and the computational tools for their application are included. Simulation results show validity of the theoretical results along with their superiority over classical Bayesian method.

Acknowledgments

K. Kachiashvili's and J. Kachiashvili's researches were partially supported by European Commission HORIZON EUROPE WIDERA-2021-ACCESS-03 Grant Project GAIN (grant agreement # 101078950).

References

[1] K. J. Kachiashvili, *Constrained Bayesian Methods of Hypotheses Testing: A New Philosophy of Hypotheses Testing in Parallel and Sequential Experiments.* Nova Science Publishers, New York, 2018.

[2] N. Mukhopadhyay and G. Cicconetti, Applications of sequentially estimating the mean in a normal distribution having equal mean and variance. *Sequential Anal.* **23** (2004), no. 4, 625–665.

Modeling and Handwriting Examination of Sensorimotor Intelligence

A Shell to Support Sensorimotor Development of Intelligence

Boris Karapetyan, Edward Pogossian

Academy of Sciences of Armenia, Institute for Informatics and Automation Problems
Yerevan, Armenia

E-mail: boris.karapetyan@iiap.sci.am; epogossi@aua.am

We present a digital tool and methods designed to support child, thus AI, sensorimotor development to advance in adequate constructive modeling of stabilized and developmental modes of cognization [3], [4]. We are based on descriptive psychological models of human cognizers by outstanding psychologist Jean Piaget [1], who revealed and experimentally argued that human cognizing is *develops through the acquisition and identification of mental structures, their organization and balancing, transitions from the sensorimotor to the operational stages, and then to the abstract, which can develop indefinitely.*

Sensorimotor development is a crucial stage in a child's ontogenesis, forming the foundation for the development of higher mental functions such as attention, memory, speech, thinking, and self-regulation [1], [2]. The sensitive period for the formation of motor and sensory skills falls on the age of up to 7 years, and during this period, the development of fine motor skills is most effective [2]. It contributes to the formation of speech and writing, as well as adaptation to educational activities. Sensorimotor development is a crucial stage in a child's ontogenesis, forming the foundation for the development of attention, memory, speech, thinking, and self-regulation [1], [2]. The sensitive period for the formation of motor and sensory skills occurs before the age of 7, and it is during this time that the targeted intervention is most effective [2]. The development of fine motor skills contributes to the formation of speech and writing, as well as to the adaptation to academic activities. The choice of handwriting as the starting stage is based on several reasons [2]. Children learn not only handwriting, but also perseverance and patience. Repetitive writing of letters in workbooks ensures memorization and faster acquisition of reading skills, expands their vocabulary.

Planning to integrate this tool with one based on cognizers of combinatorial games [4], we adapt it now for graphics tablets without requiring an internet connection. Displaying a PDF worksheet as a background on-screen eliminates the need for printed workbooks. To evaluate the learning process, we plan to add features to measure the execution speed of tasks, track changes, and suggest repetitions. We have prepared XML and SVG files for English, Russian, Georgian, and even Arabic, and we seek opportunities for collaborative development. Animation and test implementation can be viewed at `http://karapret.am`.

References

[1] J. H. Flavell, The Developmental Psychology of Jean Piaget. D Van Nostrand, 1963; `https://doi.org/10.1037/11449-000`.

[2] P. Ya. Galperin, *Methods of Teaching and Mental Development of the Child.* MSU Publishing House, Moscow, 1985.

[3] E. M. Poghossian, *Constructing Models of Being by Cognizing.* Publishing House of the Institute for Informatics and Automation Problems of the National Academy of Sciences of Armenia, Armenia, 2020.

[5] E. Pogossian, *Adaptation of Combinatorial Algorithms.* Academy of Sciences of Armenia, Yerevan, 1983.

A Study on Generalized Laplace Transform: Iteration and Parseval-Type Identities

Hilal Başak Karataş, Faruk Uçar

Department of Mathematics, Marmara University, Istanbul, Türkiye

E-mails: hilal.karatas@marmara.edu.tr; fucar@marmara.edu.tr

Generalized integral transforms extend classical transforms by allowing the kernel function to depend on additional parameters. These transforms are useful in solving a wide range of problems in mathematics and applied sciences. In this study, a type of integral transform called the generalized Laplace transform is considered. It is shown that the iteration of this transform and the Fourier sine and Fourier cosine transforms naturally leads to what is known as the generalized Glasser transform. Some identities involving these transforms are also obtained, including their relations with the Mellin transform and the generalized Stieltjes transform.

References

[1] N. M. Temme, Incomplete Laplace integrals: uniform asymptotic expansion with application to the incomplete beta function. *SIAM J. Math. Anal.* **18** (1987), no. 6, 1638–1663.

[2] O. Yürekli, A Parseval-type theorem applied to certain integral transforms. *IMA J. Appl. Math.* **42** (1989), no. 3, 241–249.

[3] F. Uçar, Some identities for the Glasser transform and their applications. *Turkish J. Math.* **39** (2015), no. 4, 538–546.

[4] A. Erdélyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, *Tables of Integral Transforms*. Vol. I. Based, in part, on notes left by Harry Bateman. McGraw-Hill Book Co., Inc., New York–Toronto–London, 1954.

[5] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. Translated from the Russian. Translation edited and with a preface by Alan Jeffrey and Daniel Zwillinger. With one CD-ROM (Windows, Macintosh and UNIX). Seventh edition. Elsevier/Academic Press, Amsterdam, 2007.

The Maximal Operator on Variable $L^{p(\cdot)}$ Spaces Over Spaces of Homogeneous Type

Oleksiy Karlovych[1], **Alina Shalukhina**[2]

[1]*Departamento de Matemática, NOVA University Lisbon, Caparica, Portugal*
E-mail: oyk@fct.unl.pt

[2]*FCT NOVA, NOVA University Lisbon, Caparica, Portugal*
E-mail: a.shalukhina@campus.fct.unl.pt

In the Euclidean setting, it has long been known that if the Hardy–Littlewood maximal operator M is bounded on a variable Lebesgue space $L^{p(\cdot)}(\mathbb{R}^n)$, then necessarily $p_-(\mathbb{R}^n) > 1$, where

$$p_-(X) := \operatorname*{ess\,inf}_{x\in X} p(x)$$

denotes the essential lower bound of the exponent function $p(\cdot) : X \to [1,\infty]$. Surprisingly, nothing like this was known in literature for the setting of spaces of homogeneous type (X,d,μ), which generalize $\mathbb{R}^n$.

We prove that the condition $p_-(X) > 1$ is necessary for the boundedness of the maximal operator M on the variable Lebesgue space $L^{p(\cdot)}(X,d,\mu)$ over a *reverse doubling* space of homogeneous type (X,d,μ) with a *Borel-semiregular measure* μ. The talk is based on the results of [1].

Acknowledgments

This work is funded by national funds through the FCT – Fundação para a Ciência e a Tecnologia, I. P., under the scope of the projects # UIDB/00297/2020 and # UIDP/00297/2020 (Center for Mathematics and Applications – NOVA MATH) and the PhD scholarship # UI/BD/154284/2022.

References

[1] O. Karlovych and A. Shalukhina, A necessary condition for the boundedness of the maximal operator on $L^{p(\cdot)}$ over reverse doubling spaces of homogeneous type. *Anal. Math.* **51** (2025), no. 1, 241–248.

On Some Combinatorial Properties of Discrete Point-Sets in Euclidean Spaces

Tamar Kasrashvili[1,2]

[1] *Georgian Technical University, Tbilisi, Georgia*
[2] *Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: tamarkasrashvili@yahoo.com

Discrete point-systems can be met in various felds of pure and applied mathematics. We may indicate several such directions in contemporary mathematics, for instance, discrete and computational geometry, classical number theory, combinatorics (finite or infinite), the theory of convex sets, etc. The investigation of the combinatorial structure of various discrete and finite point-systems in Euclidean spaces is a rather attractive and important topic. Properties of various discrete point systems are considered in many works (see, for example, [1]–[6]).

A system X of points in the Euclidean space $\mathbf{R}^d$ $(d \geq 1)$ is called discrete if every ball in $\mathbf{R}^d$ contains only finitely many points from X.

In general, a set X is discrete in a topological space E if every point$x \in X$ has an eighborhood U such that $X \cap U = x$.

The present report is devoted to some aspects highlighting profound connections between the theory discrete point-sets with some geometric properties of intersections of finite families each member of which is a hyperboloid in the Euclidean space $\mathbf{R}^d$. The main fact here is that the intersection of suficiently many algebraic surfaces, which are is general position, always yields the empty set.

References

[1] P. Erdös, Integral distances. *Bull. Amer. Math. Soc.* **51** (1945), 996.

[2] H. Hadwiger, *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie.* (German) Springer-Verlag, Berlin–Göttingen–Heidelberg, 1957.

[3] H. Hadwiger and H. Debrunner, *Kombinatorische Geometrie in der Ebene.* (German) Monographies de L'Enseignement Mathématique [Monographs of L'Enseignement Mathématique], no. 2. Université de Genève, Institut de Mathématiques, Geneva, 1960.

[4] A. B. Kharazishvili, *Introduction to Combinatorial Geometry.* (Russian) Tbilis. Gos. Univ., Tbilisi, 1985.

[5] A. Kirtadze and T. Kasrashvili, On some properties of certain discrete point-sets in Euclidean spaces. *Proc. A. Razmadze Math. Inst.* **159** (2012), 155–157.

[6] W. Sierpiński, Sur les ensembles de points aux distances rationnelles situés sur un cercle. (French) *Elem. Math.* **14** (1959), 25–27.

Numerical Method for Solving the System of Timoshenko Equations

Nikoloz Kachakhidze[1], **Jemal Peradze**[2]

[1] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mail: n.kachakhidze@gtu.ge

[2] *Department of Mathematics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: jemal.peradze@tsu.ge

A boundary value problem is considered for the system of Timoshenko equations describing the shell in the static regime. The given system of ordinary differential equations with respect to the displacement functions u, w and ψ is reduced to an integro-differential equation with respect to the function w using Green's functions. This equation, together with the Dirichlet boundary condition, forms a boundary value problem. In order to find an approximate solution to this problem, we use a method consisting of the Galerkin method and Jacobi–Cardano iteration. The obtained approximation for the function w and the error estimate are used to build the approximations of the functions u and ψ and estimate their errors. The results of a numerical experiment are presented.

Exploration of the Solar Activity Indices Data with the Near Periodicities of its Synodical Rotation Period

Vitali Kavlashvili

Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: Kavlashvilivitali@gmail.com

Discrete Fourier transforms were conducted for the solar activity indices such as solar daily sunspot numbers and daily measurements of the integrated emission from the solar disc at 2800 MHz (10.7 cm wavelength) at the rising phase of solar activity in deferent epochs. Spectral analyses were carried out at the periods near the periodicity associated with the solar synodic rotation period ≈ 27 days.

The solar activity data were filtered against low and high frequencies to reveal more clearly the real periodicities in the researching intervals.

The special approach was developed to estimate the reality of the revealed frequencies In the power spectrum of the solar activity indices. The adjusted to the power of the solar data random numbers were used to estimate frequency peaks in the power spectrum for the validation of the real physical processes in the solar activities and 99 percent confidence level was calculated for these frequencies.

Full Transitivity of a Cotorsion Hull and the Group of Rational Points of an Elliptic Curve

Tariel Kemoklidze

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: tariel.kemoklidze@atsu.edu.ge

The full transitivity of the cotorsion hull of a separable primary abelian group T and the related issues of describing the lattice of fully invariant subgroups of the cotorsion hull of the group T for various type of T are discussed. The additive abelian group of rational points of an elliptic curve and the problems raised here are also discussed.

References

[1] T. Kemoklidze, On the full transitivity of a cotorsion hull of a separable primary abelian group. *Comm. Algebra* **52** (2024), no. 5, 2079–2085.

[2] J. H. Silverman And J. T. Tate, *Rational Points On Elliptic Curves*. Second edition. Undergraduate Texts in Mathematics. Springer, Cham, 2015.

Modified Model for Effective Fight Against Disinformation

Nugzar Kereselidze

Sokhumi State University, Tbilisi Georgia
E-mail: nkereselidze@sou.edu.ge

A new, modified mathematical and computer model of effective counteraction to disinformation is presented. The model that was presented in 2024 [1] has been updated. That model described the fight against disinformation only by means of counter-information flows. Since then, legislative restrictions have been introduced to combat disinformation. Taking into account these legislative restrictions, the existing model has been modified. Thus, the model for combating disinformation takes into account both counter-information flows and possible sanctions. Keywords. Mathematical model, computer model, disinformation, information, flow, sanction.

References

[1] N. Kereselidze, Mathematical and Computer Modeling of a Dynamic System for Effectively Combating Disinformation. *WSEAS Transactions on Systems* **23** (2024), 66–72.

Some Current Issues of Quantum Mechanics

Tamaz Kereselidze

Faculty of the Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia

E-mail: tamaz.kereselidze@tsu.ge

In 1929, Morse and Stueckelberg published an article in which they addressed the so-called two-Coulomb center problem [2]. Their solution was based on the intuitive idea that the number of nodes in the wave functions remain unchanged as the distance between the Coulomb centers varies. In the article, Morse stated that he would prove this "node conservation law" in a subsequent work. However, his next publication focused entirely on introducing the famous Morse potential. Despite the absence of formal proof, the idea of node conservation proved so effective that it was widely adopted in the study of diatomic molecules by subsequent researchers. We have examined this problem and, using quantum mechanical principles, demonstrated that the number of nodes in the one-dimensional wave functions remains unchanged as the distance between the Coulomb centers varies [4]. The creation of hydrogen atoms in the early, so-called recombination stage of the Universe's evolution is a key issue in cosmology. In 2019-2022, we proposed and studied a non-standard quasi-molecular mechanism of recombination [3], [6], [7]. According to this mechanism, an electron interacts with two nearest neighboring protons and forms a hydrogen molecular ion in the excited state, which then cascades to lower energy levels or dissociates into a hydrogen atom and a proton. According to a paper published by Flower in 2023, the rate of three-particle recombination is 12 orders of magnitude lower than that of two-particle recombination [1]. Consequently, the author concludes that the contribution of quasi-molecular mechanisms to the formation of hydrogen atoms in the early Universe is negligible and can therefore be disregarded. Flower's estimate does not take into account that recombination occurs via highly excited states. We have shown that including this factor increases the rate of three-particle recombination by seven orders of magnitude [5]. Consequently, the contribution of the quasi-molecular mechanism to the neutralization process is significant and should be considered in studies of hydrogen atom formation. The next problem focuses on the study of electron–proton radiative recombination in the solar wind. Previous investigations of electron–proton recombination relied on the key simplifying assumption that the electron collides with a stationary proton – an approximation that does not accurately reflect solar wind conditions. In our analysis, we considered radiative recombination under conditions where both the electron and proton are in motion. The results will be presented at the conference.

References

[1] D. R. Flower, *Mon. Not. R. Astron. Soc.* **523** (2023), L1–L3.

[2] Ph. M. Morse and E. C. G. Stueckelberg, Diatomic molecules according to the wave mechanics, I. Electronic levels of the hydrogen molecular ion. *Physical Review (2)* **33** (1929), 932–947.

[3] T. Kereselidze and I. Noselidze, Formation of hydrogen in the early universe: quasi-molecular mechanism of recombination. *Mon. Not. R. Astron. Soc.* **488** (2019), no. 2, 2093–2098.

[4] T. Kereselidze and I. Noselidze, Conservation of the number of nodes in the wavefunctions of one-electron diatomic quasimolecules. *Eur. Phys. J. D* **78** (2024), Article no. 134.

[5] T. Kereselidze and I. Noselidze, Primordial three-body recombination in the early Universe involving an electron and two protons. *Eur. Phys. J. D* **79** (2025), Article no. 51.

[6] T. Kereselidze,I. Noselidze and J. F. Ogilvie, Influence of a quasi-molecular mechanism of recombination on the formation of hydrogen in the early Universe *Mon. Not. R. Astron. Soc.* **501** (2021), no. 1, 1160–1167.

[7] T. Kereselidze, I. Noselidze and J. F. Ogilvie, Non-standard mechanism of recombination in the early universe. *Mon. Not. R. Astron. Soc.* **509** (2022), no. 2, 1755–1763.

Generalized-Phase Boas Transforms: Extensions and Applications in Wavelets

Nikhil Khanna

Department of Mathematics, College of Science, Sultan Qaboos University, Muscat, Oman

E-mail: n.khanna@squ.edu.om

In this work, we explore vanishing moments in wavelets through the lens of generalized-phase Boas transforms. Building upon the classical Boas transform framework, we introduce a generalized-phase kernel incorporating polynomial phase components. We derive sufficient conditions for the existence of vanishing moments in generalized wavelets, establish convolution and correlation theorems for continuous wavelet transforms, and investigate their link to higher-order vanishing moments.

Joint work with Stanzin Dorjai, Ahmad Al-Salman, and S. K. Gandhi.

References

[1] R. P. Boas, Jr., Some theorems on Fourier transforms and conjugate trigonometric integrals. *Trans. Amer. Math. Soc.* **40** (1936), no. 2, 287–308.

[2] R. R. Goldberg, An integral transform related to the Hilbert transform. *J. London Math. Soc.* **35** (1960), 200–204.

[3] P. Heywood, On a transform discussed by Goldberg. *J. London Math. Soc.* **38** (1963), 162–168.

[4] N. Khanna and L. Kathuria, On convolution of Boas transform of wavelets. *Poincare J. Anal. Appl.* **2019**, no. 1, 53–65.

[5] N. Khanna, S. K. Kaushik and A. M. Jarrah, Some remarks on Boas transforms of wavelets. *Integral Transforms Spec. Funct.* **31** (2020), no. 2, 107–117.

The Mathematical Model of Bones Growth and Pathologies

Nino Khatiashvili[1], David Kaladze[1], Irene Khatiashvili[2]

[1] *Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mails: ninakhatia@gmail.com; datokala@yahoo.com

[2] *Tbilisi Medical University, Tbilisi, Georgia*

Mobility in the environment is a main characteristic of mammals. For the human body, the foundation of movement is the skeleton, which is composed of about 206 bones connected by joints. The skeleton constitutes approximately 0.1 of total body mass in men and 0.08 in women [1], [6], [8]. Bone is a permanently remodeled mineralized connective tissue. The skeleton contains about 0.99 of total body calcium (Ca). The remaining Ca contains body fluids, muscles, and other tissue. Ca is consumed from the body fluids and its consumption is regulated by calciotropic hormones [1], [4], [6]–[8]. Bones begin developing at the early stage of life and continue until adulthood, around age 30. This process is genetically determined and depends on adequate nutrient intake. From age 30 to 50, total bones mass is stabilized. In older age, tissue recovery ability diminishes [1], [6], [8].

We propose two models of bones growth:

(1) The discrete model: Let V_t be the bones mass at the time t, V_{t+1} be the bones mass after 24 ours, β is the Ca consumption per unit mass, the following formula is valid $V_{t+1} = \alpha\beta V_t$. This formula means, that V_{t+1} is proportional to the Ca consumption by the previous volume V_t, α is the coefficient, which can be determined experimentally. $\alpha\beta > 1$ from the early stage of life until age 30, $\alpha\beta = 1$ between ages $30 - 50$, $\alpha\beta < 1$ after age 50.

(2) The continuous model: $\frac{dV_t}{dt} = \alpha\beta_t V_t - V_d$, where V_d is the mass of the death cells within 24 ours.

Inadequate Ca intake leads abnormalities in bones structure and causes diseases, such as osteoarthritis, osteoporosis, etc. Osteophytosis may develop following bones and joint injury or the tissue damage and in need of repair. This process we describe by the nonlinear reaction-diffusion equation [1], [3]–[5]. The solution of this equation is obtained.

References

[1] V. Akhobadze, D. Gamrekeli, N. Khatiashvili and K. Pirumova, On some pathologies connected with the crystalization process at the human body (gallstones formation). *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **29** (2015), 1–4.

[2] D. G. Blankenbaker and K. W. Davis , *ExpertDDx: Musculoskeletal E-Book.* Elsevier, 2017.

[3] N. Khatiashvili, O. Komurjishvili and Z. Kutchava, On the numerical treatment of the axisymmetric turbulent-diffusion equation. *Appl. Math. Inform. Mech.* **18** (2013), no. 1, 55–60.

[4] N. Khatiashvili, O. Komurjishvili, Z. Kutchava and K. Pirumova, On numerical solution of axisymmetric reaction-diffusion equation and some of its applications to biophysics. *Appl. Math. Inform. Mech.* **15** (2011), no. 1, 36–42.

[5] N. Khatiashvili, O. Komurjishvili, A. Papukashvili, R. Shanidze, V. Akhobadze, T. Makatsaria and M. Tevdoradze, On some mathematical models of growth of solid crystals and nanowires. *Bull. TICMI* **17** (2013), no. 1, 28–48.

[6] N. Khatiashvili, Chr. Pirumova and V. Akhobadze, On the mathematical model of vascular endothelial growth connected with tumor proliferation. *World Academy of Science, Engineering and Technology, Paris* **79** (2011), 545–548.

[7] N. Khatiashvili, K. Pirumova, I. Khatiashvili and V. Akhobadze, On the influence of the cancer proteins on the blood flow. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **29** (2015), 60–63.

[8] E. N. Marieb, *Essentials of Human Anatomy and Physiology.* 10th Ed., Benjamin, 2012.

Electron-Electron Interaction in Semiconductor Quantum Nanostructures

Zakaria Kherkeladze, Tamar Tchelidze

The Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia

E-mails: zakaria.kherkeladze374@ens.tsu.edu.ge; tchelidze@tsu.ge

This thesis explores the intricate role of space confinement in controlling electron-electron interactions within semiconductor quantum nanostructures, focusing specifically on $(Al_xGa_{1-x})_2O_3/G_2O_3$ quantum dots. These structures, known for their tunable electronic properties and wide bandgap characteristics, serve as an ideal platform for investigating confined quantum systems. A central component of this work is the adoption of the Pöschl–Teller potential, a realistic model that effectively captures the spatial confinement of single-particle states within quantum dots. The Pöschl–Teller potential is particularly advantageous due to its analytical tractability and capacity to model smooth confinement profiles more accurately than idealized square or parabolic wells.

To analyze electron-electron interactions, the Hartree self-consistent field (SCF) method is employed. This approach enables the iterative solution of the coupled Poisson and Schrödinger equations, allowing for a realistic description of the mutual Coulomb repulsion between electrons in confined geometries.

A key focus of the study is the investigation of how variations in quantum dot size influence electron-electron interactions. By systematically altering the spatial dimensions of the quantum dot, the thesis examines the resulting changes in confinement strength and interaction energy. These size-dependent effects are critical for the design of quantum devices, where precise control over electronic behavior is essential. Additionally, the study extends to include scenarios with varying potential well depths, providing further insight into how the depth of confinement influences the electronic structure and interaction dynamics.

Through this comprehensive analysis, the thesis contributes to a deeper understanding of electron correlation effects in oxide-based quantum dot systems and lays the groundwork for future applications in quantum computing and nanoelectronic device engineering.

Central Derivations, Centroids and Skew-Symmetric Bi-Derivations of Leibniz (n-)Algebras

Emzar Khmaladze[1,2]

[1] *The University of Georgia, Tbilisi, Georgia*
E-mail: e.khmaladze@ug.edu.ge

[2] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: e.khmal@gmail.com

In this talk, we present the conditions under which the well-known forgetful and so-called Daletskii–Takhtajan's functors between the categories of Leibniz and Leibniz n-algebras preserve central derivations and centroids. On the other hand, we introduce and study skew-symmetric bi-derivations and commuting linear maps of Leibniz algebras, and under appropriate conditions, we relate them to centroids. The results of this research are presented in the papers [J. M. Casas, E. Khmaladze and M. Ladra, Central derivations, centroids and skew-symmetric bi-derivations of Leibniz (n-)algebras. *Bull. Braz. Math. Soc. (N.S.)* **56** (2025), no. 2, Paper no. 29, 21 pp.].

Analysis of Different Intensity Fire Development in a Road Tunnel Using Numerical Modeling

Zaza Khokerashvili, Nino Ratiani

Department of Occupational Safety and Emergency Management, Georgian Technical University, Tbilisi, Georgia

E-mail: zaza196822@gmail.com

It is known that in road tunnels equipped with longitudinal ventilation system, backlayering frequently occurs during the initiation and development of fires. This means that the ventilation airflow does not exert sufficient impact on the buoyant forces generated by the fire. As a result, a tunnel is intensively filled with smoke and other toxic combustion products, creating an environment incompatible with human survival and hazardous for human life. An organized and immediate self-evacuation is considered to be the only effective means to rescue people under such conditions. The time period for an effective self-evacuation is significantly dependent on the fire development in enclosed spaces, and the propagation dynamics of dangerous factors.

In order to determine this, we have conducted a numerical modeling of 10, 30, 50 MW heat release intensity fire within the FDS program. The Finite Volume Method was applied. A 500-m-long tunnel was selected, with the cross-sectional area of 40-42 m^2. The minimum size of the finite volume outline – $0,25\times 0,25\times 0,25\,m$, maximum – $0,5\times 0,5\times 0,5\,m$, modeling duration – 190 $sec.$, the fire source, with an area of 10–15 m^2, is located at the center of the tunnel. The modeling was carried out using four types of fuel: petrol, diesel, kerosene, and log wood. The thesis underlines that the dynamic pressure induced by a severe fire significantly exceeds the static pressure of the tunnel jet fans. Hence, after the algebraic summation of the positively directed ventilation flows and the negatively directed flows caused by the fire, intense backlayering occurs, which must be taken into account when implementing emergency ventilation projects.

The numerical modeling revealed the dynamics of the spread of hazardous factors features, which significantly impact the evacuation duration and the structural stability of tunnels.

The dynamics of the spread of the critical temperature of $60°C$, as one of the hazardous factors, were determined, along with its geometric parameters of propagation in fires of corresponding intensity. Additionally, the propagation dynamics of carbon monoxide at a hazardous concentration of 200 mg/m^3 in the tunnel's vertical plane were established for fires of appropriate intensity.

The obtained results are instrumental in formulating ventilation strategies and emergency management protocols in operational tunnels when fires occur, serving as essential guidance for both personnel and rescue teams.

Acknowledgments

This work was funded by the Shota Rustaveli National Science Foundation of Georgia (Grant # FR-22-12 949 – *Study of Critical Velocity and Fire-Induced Backlayering Life-saving Strategies in Road Tunnels.*

Almost Disjoin Family of Sets and Their Applications

Aleks Kirtadze

Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: kirtadze2@yahoo.com

Let E be an infinite set and let $\{X_i :\ i \in I\}$ be a family of subsets of E. We say that this family is almost disjoint if the following conditions hold:

(a) $card(X_i) = card(E)$ for arbitrary $i \in I$;

(b) $card(X_i \cap X_j) < card(E))$ $(i \in I,\, j \in I,\, i \neq j)$.

The notion of an almost disjoint family of sets was first introduced and investigated by W. Sierpinski (see [2]).

Lemma *Let E be an infinite set and let $\{Y_k : \in K\}$ be a family of subsets of E such that:*

(1) $card(K) \leq card(E)$;

(2) $card(Y_k) = card(E)$.

Then there exists an almost disjoint family $\{X_i :\ i \in I\}$ of subsets of E, satisfying the relations:

(3) $card(I) > card(E)$;

(4) $card(X_i \cap Y_k) = card(E))$;

(5) $\cup\{X_i :\ i \in I\} = E$.

The proof of Lemma see [1].

According to the lemma we can formulate the following result.

Theorem *In infinite dimensional linear Polish spaces there exist a family of σ-invariant mutually singular measures $\{\mu_i :\ i \in I\}$ such that:*

$$card(I) > \mathbf{c},$$

where $\mathbf{c}$ is cardinality of the continuum.

References

[1] A. B. Kharazishvili, *Topological Aspects of Measure Theory.* (Russian) “Naukova Dumka”, Kiev, 1984.

[2] W. Sierpiński, *Cardinal and Ordinal Numbers.* Polska Akademia Nauk. Monografie Matematyczne, Tom 34. Państwowe Wydawnictwo Naukowe, Warsaw, 1958.

Cotriple Cohomologies of Lie Super p-Algebras and Abstract Kernels

Bakur Kotchlamazashvili[1], **Nino Tophuridze**[2]

[1] *Ilia State University, Tbilisi, Georgia*
E-mail: bkochlamazashvili@gmail.com

[2] *Sokhumi State University, Tbilisi, Georgia*
E-mail: ntophuridze@sou.edu.ge

If Lie p-algebra Γ is defined over a field, their lower cohomology groups, defined by projective resolutions, can be interpreted using extensions and abstract kernels. But if Γ is defined over a commutative ring R the Hochschild [1] and Pareigis [2] cohomologies does not lead to the desired interpretation. This is true for Lie p-superalgebras too. For this reason cohomology groups of Lie p-algebras over rings of positive characteristics were constructed using a cotriple method and the second cotriple cohomology group were interpreted as equivalence classes of two-term extensions [5]. We construct similarly cotriple cohomologies of Lie super p-algebras over rings of positive characteristics and interpret the second cotriple cohomology group as the equivalence classes of two-term extensions (Theorem 1) and abstract kernels (Theorem 2). In addition, a connection between abstract kernels and two term extensions is proved what of course was expected. Isomorphism of the first cotriple cohomologies groups of Lie super p-algebras and equivalence classes of short exact sequences were proved earlier in [3], [4].

Let G_i, $i = 1, 2$, are cotriples induced by construction of free Lie super p-algebra over a set (over an R-module). Let M be an Abelian Lie (p-)superalgebra (i.e., in M the multiplication is zero, but the triviality of the p-mapping is not assumed), on which Γ acts. So we construct the cohomologies groups $H^n_{G_i}(\Gamma, M), i = 1, 2$ of the functor of derivations $Der(-, M)$.

Definition A two-term extension of Γ by M is an exact sequence

$$0 \longrightarrow M \xrightarrow{\varphi_2} X_1 \xrightarrow{\varphi_1} X_0 \xrightarrow{\varphi_0} \Gamma \longrightarrow 0\,,$$

which satisfies the conditions from [5] (here $u, v \in X_1$, $x \in X_0$): $\varphi_2(\varphi_0(x)m) = x\varphi_2(m)$, $[u, v] = \varphi_1(u)v$, $[x, \varphi_1(u)] = \varphi_1(xu)$.

Let $Ext^2_i(\Gamma, M)$ be the Abelian group of equivalence classes of two-term extensions of Γ by M (when $i = 2$, the extensions are assumed to be R-split). Let $Ker_i(\Gamma, M)$ be the Abelian group of equivalence classes of abstract kernels (when $i = 2$, the kernels are assumed to be R-split).

Theorem 1 *There exist isomorphisms of Abelian groups* $H^2_{G_i}(\Gamma, M) \simeq Ext^2_i(\Gamma, M)$, $i = 1, 2$.

Theorem 2 *There exist isomorphisms of Abelian groups* $H^2_{G_i}(\Gamma, M) \simeq Ker_i(\Gamma, M)$, $i = 1, 2$. *Any kernel is induced by some two-term extension. The class of extendable kernels vanishes.*

References

[1] G. Hochschild, Cohomology of restricted Lie algebras. *Amer. J. Math.* **76** (1954), 555–580.

[2] B. Pareigis, Kohomologie von p-Lie-Algebren. (German) *Math. Z.* **104** (1968), 281–336.

[3] G. G. Rakviashvili, On products in the algebraic K-theory of crossed enveloping superalgebras. (Russian) *Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR* **91** (1988), 67–75.

[4] G. Rakviashvili, On cohomologies and algebraic K-theory of Lie p-superalgebras. *Tbilisi Math. J.* **13** (2020), no. 2, 217–224.

[5] N. Shimada, H. Uehara, F. Brenneman and A. Iwai, Triple cohomology of algebras and two term extensions. *Publ. Res. Inst. Math. Sci.* **5** (1969), 267–285.

Semi-Smooth Newton Method for Contact and Dynamic Problems

Victor A. Kovtunenko[1,2]

[1]*Department of Mathematics and Scientific Computing,*
Karl-Franzens University of Graz NAWI Graz, Graz, Austria
[2]*Lavrentyev Institute of Hydrodynamics, Siberian Branch of the Russian Academy of Sciences*
Novosibirsk, Russia
E-mail: victor.kovtunenko@uni-graz.at

Motivated by inequality constraints appearing in convex and non-convex optimization context, we study non-smooth problems with respect to its properties of generalized differentiability. The use of Lagrange multipliers and merit functions leads to a semi-smooth Newton method for solution of linear and nonlinear complementarity problems and equivalent primal-dual active-set numerical algorithms. For application in mechanics, problems describing cohesive obstacle, two-body contact, non-penetrating cracks and fluid-driven fractures are considered. In the semi-smooth framework, we investigate contact and impact dynamics, in particular, the dynamic contact problem between beam and foundation under moving load stemming from railway applications.

Acknowledgments

The author thanks the Georgian Mathematical Union for the long collaboration.

References

[1] E. Bauer, V. A. Kovtunenko, P. Krejčí, G. A. Monteiro, L. Paoli and A. Petrov, Modeling the railway track ballast behavior with hypoplasticity. *J. Math. Sci.*, 2025 (to appear).

[2] M. Hintermüller, V. A. Kovtunenko and K. Kunisch, Generalized Newton methods for crack problems with nonpenetration condition. *Numer. Methods Partial Differ. Equ.* **21** (2005), no. 3, 586–610.

[3] V. A. Kovtunenko and Y. Renard, Convergence analysis of semi-smooth Newton method for mixed FEM approximations of dynamic two-body contact and crack problems. *J. Comput. Appl. Math.*, 2025 (submitted).

[4] A. M. Khludnev and V. A. Kovtunenko, *Analysis of Cracks in Solids.* WIT-Press, Southampton, Boston, 2000.

On Some Generalized Class of Yule Distribution

C. Satheesh Kumar

Department of Statistics, University of Kerala, Trivandrum, India
E-mails: profcskumar@gmail.com, csk@keralauniversity.ac.in

The Yule distribution (also known as Yule–Simon distribution) is an important class of discrete distributions, which can be viewed as a discrete analogue of the well-known Pareto distribution. It was originally developed as a beta mixture of the geometric distribution and it has been found extensive applications in several areas of scientific research. In the present talk, we provide an outline of the Yule distribution and discuss some important aspects of certain recently developed classes of generalized Yule distributions and their bivariate versions.

On Some Properties and Mixtures of BGHFM Distribution

C. Satheesh Kumar[1], **S. Nikhil Chandra**[2]

[1]*Department of Statistics, University of Kerala, Trivandrum, India*
E-mail: csk@keralauniversity.ac.in

[2]*Department of Mathematics & Statistics, Mar Ivanios College, Trivandrum, India*
E-mail: nikhilchandra.s@mic.ac.in

The bivariate generalized hypergeometric factorial moment distribution (BGHFMD), introduced by Kumar (2008), provides a unifying framework for numerous classical bivariate discrete distributions. The BGHFMD encapsulates a wide family of models, including the bivariate versions of Bernoulli, binomial, geometric, negative binomial, and Poisson distributions as special cases. Each marginal corresponds to the generalized hypergeometric factorial moment distribution (GHFMD) developed by Kemp and Kemp (1969). Through this presentation we discuss some important properties of the distribution.

References

[1] C. D. Kemp and A. W. Kemp, Some distributions arising from an inventory decision problem. *Bulletin of the International Statistical Institute* **43** (1969), 336–338.

[2] C. S. Kumar, A unified approach to bivariate discrete distributions. *Metrika* **67** (2008), no. 1, 113–123.

Completeness and Decidability in Infinitary Probability Logics

Lia Kurtanidze

Georgian National University SEU, Tbilisi, Georgia
E-mail: L.kurtanidze@seu.edu.ge

The intersection of formal logic and probability theory has emerged as a powerful framework for reasoning under uncertainty, particularly in Artificial Intelligence and complex systems. In this talk, we explore infinitary probability logics, a class of logical systems that extend classical propositional and first-order logics by incorporating probability operators and allowing infinitary inference rules. This presentation aims to showcase how infinitary methods can restore completeness and enable meaningful decision procedures in wide logical systems that handle probabilistic reasoning.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under the project # FR-22-4254.

References

[1] Z. Ognjanović (Ed.), *Probabilistic Extensions of Various Logical Systems.* Springer Cham, 2020.

[2] Z. Ognjanović, M. Rašković and Z. Marković, *Probability Logics: Probability-Based Formalization of Uncertain Reasoning.* Springer, 2016.

Multivariate Membership Function for Analysis and Control of Hierarchical System

Elguja Kurtskhalia, Merab Akhobadze

Department of Interdisciplinary Informatics, Georgian Technical University
Tbilisi, Georgia
E-mails: e.kurtskhalia@gtu.ge; m.akhobadze@gtu.ge

The decision-making process mainly includes three stages: initial, evaluation formation of the existing situation, problem; determination of possible ways to solve the problem; forecasting the time and finances required to solve the problem.

At the initial stage, each project is discussed (evaluated) at all levels using the so-called linguistic variables, for example: "acceptable", "requires correction", etc. This is due to the fact that each stage of decision-making initially contains a large uncertainty (fuzziness), the project is assessed by experts as a whole and subjectively.

As is known, at present, the theory of fuzzy sets and fuzzy technologies are used to model and manage such uncertain systems [1]. To use the theory of fuzzy sets for optimal project evaluation, it is necessary to introduce the concept of a multidimensional membership function.

The paper discusses the principles of constructing a multidimensional membership function and an algorithm for its formalization.

References

[1] L. A. Zadeh, Fuzzy sets. *Information and Control* **8** (1965), 338–353.

The Problem of Covering a Curved Area with Circles

Ketevan Kutkhashvili

The University of Georgia, Tbilisi, Georgia
E-mail: kkutkhashvili@yahoo.com

The problem of covering any flat figure with circles can be interesting not only from the point of view of mathematics, but also from the point of view of application in many different fields. Solving such problems is often necessary also to solve various issues in the field of economics, technology and services, such as the placement of antennas or other devices and the regulation of their power, the placement of various service centers and the management of their effective functioning, the solution of the problem of optimal packing in a container, etc. In particular, a multi-parameter dynamic problem of covering a region bounded by a curved line with circular objects with a varying radius is considered. Optimization of the characteristic values of objects is considered as an optimality criterion. The coverage radius, power and spatial parameters of each object are selected as parameters. A mathematical model and an algorithm of polynomial complexity have been developed for this problem.

References

[1] Dynamic Programming Algorithms. California State University. `https://home.csulb.edu>teaching>lectures,PDF.2024`.

[2] P. D. Lebedev and A. L. Kazakov, Construction of optimal coverings of convex planes figures with circles of various radius. *Institute of Mathematics and Mechanics UB RAS* **25** (2019), no. 2.

[3] K. Kutkhashvili, Algorithm for one-dimensional cutting and containerization problem. *Georgian Academy of Sciences A. Eliashvili Institute of Control Systems Proceedings*, Tbilisi, 2002, no. 6.

[3] K. Kutkhashvili, On a single dynamic problem of discrete optimization. *Georgian Academy of Sciences A. Eliashvili Institute of Control Systems Proceedings*, Tbilisi, 2023, no. 27.

Estimation of a Regression Curve Using Observations with Chain Dependence

Zurab Kvatadze[1], **Beqnu Pharjiani**[1], **Tsiala Kvatadze**[2]

[1]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mails: zurakvatadze@yahoo.com; beqnufarjiani@yahoo.com

[2]*International Black Sea University, Tbilisi, Georgia*
E-mail: ttkvatadze@gmail.com

On the probability space (Ω, F, P), a two-component stationary (in the narrow sense) sequence $\{(X_i, Y_i)\}_{i\geq 1}$ is considered. The control sequence $\{X_l\}_{l\geq 1}$ $(X_l : \Omega \to \{s_1, s_2, \dots, s_r\})$ is a finite, homogeneous regular Markov chain, $\eta = (X, Y)$ $(\eta : \Omega \to R^2)$ is a two-dimensional random variable with density $f(x, y)$. $g(x)$ is the density of the marginal distribution of the X. The regression function $R(x) = \mathbb{E}(Y \mid X = x)$ if $E|Y| < \infty$ is discussed. When the conditional variances $\sigma^2(a) = D(Y_1 \mid \xi_1 = s_a)$, $a = \overline{1, r}$, are finite, then using observations with a chain dependence $\{\eta_i\}_{\geq 1} = \{X_i, Y_i\}_{\geq 1}$, by the value $\eta = (X, Y)$ a nonparametric estimate of this function of the E. Nadaraya type is constructed:

$$\widehat{R}_n(x) = \begin{cases} \dfrac{\sum\limits_{i=1}^{n} Y_i K(\frac{x - X_i}{a_n})}{\sum\limits_{i=1}^{n} K(\frac{x - X_i}{a_n})}, & \text{if } \sum\limits_{i=1}^{n} K\Big(\dfrac{x - X_i}{a_n}\Big) \neq 0, \\ 0, & \text{if } \sum\limits_{i=1}^{n} K(a_n(x - X_i)) = 0. \end{cases}$$

$K(x)$ is some density that satisfies certain conditions, and $a_n = o(n)$.
$\widehat{R}_n(x)$ is a consistent estimate of $R(x)$.

References

[1] J. G. Kemeny and J. L. Snell, *Finite Markov Chains.* Reprinting of the 1960 original. Undergraduate Texts in Mathematics. Springer-Verlag, New York–Heidelberg, 1976.

[2] Z. A. Kvatadze and T. L. Shervashidze, On limit theorems for conditionally independent random variables controlled by a finite Markov chain. *Probability theory and mathematical statistics (Kyoto, 1986)*, 250–258, Lecture Notes in Math., 1299, *Springer, Berlin*, 1988.

[3] L. G. Loitsansky, *Mechanic of a fluid and gas.* (Russian), Nauka, Moscow, 1987.

[4] È. A. Nadaraja, On a regression estimate. (Russian) *Teor. Verojatnost. i Primenen.* **9** (1964), 157–159.

Perturbation Theory for High Precision Studies of the Bound State Characteristics

Alexander N. Kvinikhidze

Andrea Razmadze Mathematical Institute
of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: sasha_kvinikhidze@hotmail.com

High precision of contemporary experimental meusurements open the possibility of the high precision study of characteristic features of the bound states' structure, such as their size, shape, etc, if the precision of the corresponding theoretical approaches match the mentioned experimental precision. For this purpose a Perturbation Theory (PT) is presented for the Lorentz boost operator in the space of few-nucleon wave functions. The latter is expressed in terms of the nucleon-nucleon (NN) potentials, developed so far in great detail for their use in the chiral PT for NN scattering studies. The PT is designed to take into account the boost relativistic corrections in a systematic way which contribute to the bound state motion in particular. As such, it is the only missing part in the approaches developed up to now in the low-energy effective field theories which allows ab initio high precision studies of the bound state structure.

Integrated Modeling of Fire Safety in Road Tunnels with Longitudinal Ventilation

Omar Lanchava, **Nino Ratiani**, **Nino Arudashvili**

Department of Occupational Safety and Emergency Management, Georgian Technical University, Tbilisi, Georgia

E-mail: o.lanchava@yahoo.com

To ensure comprehensive life safety and emergency management during road tunnel fire scenarios, a fully integrated multifactorial modeling approach is essential. Traditional single-factor models, such as those relying solely on critical velocity or static Froude number values, fail to reflect the variable scenarios that can occur under real-world circumstances, providing only a superficial representation. This results from a wide spread application of fixed (peak) numerical values of critical parameters, which fail to reflect the variable dynamics of these parameters during actual fire scenarios, and thus are not aligned with the optimal safety strategy. The implementation of analogous integrated models encounters difficulties, primarily because of the differing temporal scales between fire progression and evacuation processes. This problem can be addressed by applying the dual synchronization method. Evacuation requirements (particularly those related to the temporal scale) must be reflected in fire progression and ventilation priorities; in order for the ventilation to ensure a safe movement, it is crucial to coordinate the rapidly evolvng processes of high temperatures, toxic compounds and smoke spread with evacuation capacity and pace. Such models are essential during both preliminary planning phase and real-time response.

Only such a complex model has the capacity to define peak values in advance, as well as the temporal changes of important factors – such as the pace of development of critical situations and how efficient the existing mitigating measures are (for instance, operation of ventilation fans, activation of smoke extraction tunnels, deployment of various barriers, and use of other auxiliary measures).

Thus, such an integrated fire safety model must encompass:

1. Dynamics of pressure variation in the ventilation flow – assessment of thermal and mechanical effects on air movement considering time and space.
2. Heat release rate progression: reflecting realistic combustion phases based on fuel loads and tunnel geometry.
3. Evacuation simulations: tracking human movement under variable visibility, temperature, and toxicity conditions.
4. Thermal field mapping: monitoring how hot gas layers evolve and influence survivability zones.
5. CFD-based analysis: combining airflow simulations with heat transfer and pollutant dispersion.
6. Structural geometry integration: considering tunnel slope gradient, cross-sectional dimensions, and obstruction effects.

The model synchronization with eevacuation processes – acknowledging human movement pace, toxic thresholds and temperature effects – represents the basis of an effective safety strategy.

Acknowledgments

This work was funded by the Shota Rustaveli National Science Foundation of Georgia (Grant # FR-22-12 949 – *Study of Critical Velocity and Fire-Induced Backlayering Life-saving Strategies in Road Tunnels*.

Unital C_∞-Algebras and the Real Homotopy Type of $(r-1)$-Connected Compact Manifolds of Dimension $\leq \ell(r-1)+2$

Hong Van Le

Institute of Mathematics of the Czech Academy of Sciences, Prague, Czech Republic
E-mail: hvle@math.cas.cz

Let M be an n-dimensional $(r-1)$-connected compact manifold, where $r \geq 2$. In my talk I shall explain how to use a Hodge homotopy transfer for encoding the real homotopy type of M into a minimal unital C_∞-structure on $H^*(M,\mathbb{R})$. I shall show how to apply this method for proving several vanishing theorems, in particular, for proving a conjecture by Zhou, stating that if $n \leq \ell(r-1)+4$ and $b_r(M)=1$ then the multiplication μ_k of the minimal unital C_∞-structure on $H^*(M,\mathbb{R})$ for all $k \geq \ell-1$ vanishes. Zhou's conjecture implies two formality results by Cavalcanti in 2006. My talk is based on my joint work with Fiorenza, Kawai, and Schwachhöfer in 2021 and with my joint work with Fiorenza in 2025.

"Projective" Theory of Linear Ordinary Differential Equations

Vakhtang Lomadze[1,2]

[1]*Mathematics Department, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[2]*Andrea Razmadze Mathematical Institute*
of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: vakhtang.lomadze@tsu.ge

We consider linear ordinary differential equations (ODEs) with constant coefficients that depend on a parameter. When this parameter tends to zero, the solution set typically converges to that of the limiting differential equation – provided that the leading coefficient remains non-zero. However, the behavior becomes significantly more delicate in the singular case, where the leading coefficient vanishes in the limit. In such cases, the solution set may even disappear entirely. In this talk, we develop a "projective" framework in which the dependence of the solution set on the equation's coefficients is always continuous.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Project # FR-24-8249.

Discrete Boundary Value Problem on the Integer Lattice for a Linear Difference Equation

Alexander Lyapin

School of Mathematics and Computer Science, Siberian Federal University
Krasnoyarsk, Russia
E-mail: aplyapin@sfu-kras.ru

We study a boundary value problem for a two-dimensional linear difference equation with constant coefficients. The problem is formulated in a rectangular subset of the integer lattice and involves specifying the solution on a set of ”boundary” points structured in vertical and horizontal directions. We analyze the solvability conditions of the discrete equation by constructing the associated finite linear system and examining the determinant of its coefficient matrix, which has a block tridiagonal form. A necessary and sufficient condition for the existence and uniqueness of the solution is established in terms of the non-vanishing of this determinant.

Let us define a finite subset A of the integer lattice $\mathbb{Z}^2$ as follows: $A = \big\{(x_1,x_2) \in \mathbb{Z}^2 : \ 0 \leqslant x_1 \leqslant 2,\ 0 \leqslant x_2 \leqslant 2\big\}$. For integers $p_1, q_1 \geqslant 2$, we define the subset

$$X_{p_1}^{q_1} = \big\{(x_1,x_2) \in \mathbb{Z}^2 : \ \ (0,t),\ (p_1,t),\ (t,0),\ (t,q_1),\ t = 0,1,2,\dots\big\},$$

on which we define a function $\varphi : X_{p_1}^{q_1} \to \mathbb{C}$. Consider a homogeneous two-dimensional difference equation with constant coefficients:

$$\sum_{(\alpha_1,\alpha_2)\in A} c_{\alpha_1,\alpha_2} f(x_1+\alpha_1, x_2+\alpha_2) = 0, \tag{1}$$

where $c_{2,2} \neq 0$.

We formulate the problem: find a function $f(x)$ satisfying the difference equation (1) and coinciding with the function $\varphi(x)$ on the set $X_{p_1}^{q_1}$, namely

$$f(x) = \varphi(x), x \in X_{p_1}^{q_1}. \tag{2}$$

We will call problem (1) ,(2) a *boundary value problem* for the two-dimensional difference equation with constant coefficients (1), and we will refer to function (2) as the *boundary data function.*

The work contributes to the ongoing development of algorithmic methods for discrete boundary value problems and has potential applications in combinatorics and digital signal processing. This is joint research with Anna Leinartene.

Acknowledgements

This research was supported by the Russian Science Foundation and Krasnoyarsk Regional Science Foundation, project # 25-21-20076.

References

[1] E. K. Leĭnartas, Multiple Laurent series and difference equations. (Russian) *Sibirsk. Mat. Zh.* **45** (2004), no. 2, 387–393; translation in *Siberian Math. J.* **45** (2004), no. 2, 321–326.

[2] E. K. Leinartas and A. P. Lyapin, On the rationality of multidimentional recusive series. *J. Sib. Fed. Univ. Math. Phys.* **2** (2009), no. 4, 449–455.

[3] A. P. Lyapin and S. S. Akhtamova, Recurrence relations for the sections of the generating series of the solution to the multidimensional difference equation. (Russian) *Vestn. Udmurt. Univ. Mat. Mekh. Komp'yut. Nauki* **31** (2021), no. 3, 414–423.

A Non-Perturbative Causal Model for an Effective Charge in QCD

Badri Magradze

Department of Theoretical Physics, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: badri.maghradze@tsu.ge

We propose a semi-phenomenological non-perturbative causal (analytic) model for the QCD effective charge corresponding to the static quark-antiquark potential. We employ a modified version of Analytic Perturbation Theory [1], [2]. The spectral density $\rho(s)$ of the Källen–Lehmann representation of the effective charge is appropriately approximated. Hence, the effective charge is naturally represented as a sum of the "analytically improved" perturbative and non-perturbative components. The non-perturbative component corresponds to the linearly rising quark-antiquark static potential. To reduce the number of independent parameters, we impose physically motivated conditions on the asymptotic behavior of the charge at large momenta. The only parameter of the model is the QCD scale. Within this approach, we suggest a natural definition of the infrared boundary for perturbative QCD. In numerical and analytic calculations, we demonstrate the usefulness of the explicit solutions for the running coupling determined in terms of the Lambert-W function [3], [4]. We examine the stability of predictions of the model charge with respect to higher loop corrections up to the three loop order. To set the values of the parameters, we use the phenomenological value of the string tension parameter $\sigma_{st} = (0.420, GeV)^2$. With the three-loop order perturbative component, calculated in the $\overline{\mathrm{MS}}$ scheme for $n_f = 3$ quark flavors, the model-based analysis leads to the reasonable values for the QCD scale parameter and the infrared boundary of QCD: $\Lambda_{\overline{MS}} = 0.282\,GeV$, $\mu_{\mathrm{B}} = 0.894\,GeV$.

References

[1] D. V. Shirkov and I. L. Solovtsov, Analytic model for the QCD running coupling with universal $\overline{\alpha}_s(0)$ value. *Phys. Rev. Lett.* **79** (1997), 1209–1212.

[2] G. Cvetič, C. Valenzuela and I. Schmidt, A modification of minimal analytic QCD at low energies. *Nucl. Phys., B, Proc. Suppl.* **164** (2007), 308–311.

[3] B. A. Magradze, The gluon propagator in analytic perturbation theory. *Conference Proceedings* **C 980518** (1999), 158–170; Contribution to: *10th International Seminar on High-Energy Physics (Quarks 98).*

[4] E. Gardi, G. Grunberg and M. Karliner, Can the QCD running coupling have a causal analyticity structure? *Journal of High Energy Physics* **1998**, 25 pp.

Classical vs. Fuzzy Logic in Artificial Intelligence: Foundations and Applications

Dali Magrakvelidze

Department of Computational Mathematics, Georgian Technical University
Tbilisi, Georgia
E-mail: d.magrakvelidze@gtu.ge

Receiving the benefits of the alternative is determined by the unintended consequences that really arise in the future. the connection between the benefit and the Artificial Intelligence (AI) systems increasingly demand reasoning approaches capable of handling both precision and ambiguity. Classical logic, based on binary true/false values, offers clear and consistent reasoning but struggles with the uncertainty inherent in real-world data. Fuzzy logic extends this framework by introducing degrees of truth, enabling AI systems to model imprecise concepts and human-like reasoning more effectively. This paper explores the core differences between classical and fuzzy logic, with a focus on their roles in AI. We discuss their theoretical foundations, implementation challenges, and use cases in intelligent systems such as expert systems, natural language processing, and adaptive control. The comparison highlights how combining both approaches can enhance the flexibility and interpretability of AI solutions.

References

[1] L. A. Zadeh, The foundational paper introducing fuzzy logic. *Fuzzy Systems, IEEE Transactions on Fuzzy Systems* **4** (1965), 338–353.

[2] S. Russell and P. Norvig, *A Standard AI Textbook that Discusses Logic-Based Reasoning and Fuzzy Systems.* Artificial Intelligence: A Modern Approach (4th ed.), 2021.

[3] H. J. Zimmermann and P. Norvig, *A comprehensive resource on fuzzy logic applications, especially in intelligent systems.* Fuzzy Set Theory – and Its Applications (4th ed.), Springer, 2010.

[4] M. Negnevitsky, *Covers classical logic, fuzzy logic, and their application in AI.* Artificial Intelligence: A Guide to Intelligent Systems (3rd ed.). Pearson, 2011.

Multilinear Extrapolation in Grand Lebesgue Spaces

Dali Makharadze[1], **Alexander Meskhi**[2,3], **Tsira Tsanava**[4]

[1]*Department of Mathematics, Batumi Shota Rustaveli State University, Batumi, Georgia*
E-mail: dali.makharadze@bsu.edu.ge

[2]*School of Mathematics, Kutaisi International University, Kutaisi, Georgia*
[3]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: alexander.meskhi@kiu.edu.ge

[4]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mail: ts.tsanava@gtu.ge

Rubio de Francía's extrapolation theorem plays a central role in the weight theory of integral operators (see, e.g., [1]). Our aim is to present multilinear Rubio de Francía's extrapolation theorem in grand Lebesgue spaces. Mapping properties of some operators of harmonic analysis will also be discussed. We will formulate Marcinkiewicz–Zygmund inequalities for multilinear Calderón–Zygmund operators and weighted norm inequalities for the bilinear rough singular integral operators in the frame of grand Lebesgue spaces. We refer, e.g., to [2], [3] for related topics. The talk is based on the note [4].

References

[1] D. V. Cruz-Uribe, J. M. Martell and C. Pérez, *Weights, Extrapolation and the Theory of Rubio de Francia.* Operator Theory: Advances and Applications, 215. Birkhäuser/Springer Basel AG, Basel, 2011.

[2] V. Kokilashvili, M. Mastyło and A. Meskhi, Multilinear integral operators in weighted grand Lebesgue spaces. *Fract. Calc. Appl. Anal.* **19** (2016), no. 3, 696–724.

[3] K. Li, J. M. Martell and Sh. Ombrosi, Extrapolation for multilinear Muckenhoupt classes and applications. *Adv. Math.* **373** (2020), Article no. 107286, 43 pp.

[4] D. Makharadze, A. Meskhi and Ts. Tsanava, Multilinear extrapolation in grand Lebesgue spaces, *Trans. A. Razmadze Math. Inst.* **179** (2025), no. 2, 309–314.

Applications of Mechanical Equilibrium Principles in Mathematics

Vardan Manukyan[1,2], **Gagik Nikoghosyan**[2]

[1]*Research Institute of Physics, Yerevan State University, Yerevan, Armenia*

[2]*Faculty of Natural Science and Mathematics, State University of Shirak Gyumri, Armenia*

E-mails: v.f.manukyan@gmail.com; gnikoghosyan@gmail.com

The use of equilibrium conditions to prove mathematical—particularly geometric—statements has been explored by pedagogical mathematicians over the past century (see [1] and references therein). While this approach demands additional effort from students and requires a deeper understanding of physics and mathematics, pedagogical experience suggests that it enhances the learning process by making the justification of geometric statements more engaging and meaningful. Integrating mechanics knowledge and skills into mathematics education can serve as a powerful research tool and foster students' creative thinking.

With the global rise of STEM education, which emphasizes integrated and holistic learning, there has been renewed interest in applying physical principles to prove mathematical statements. Given the extensive theoretical groundwork already established, the foundations needed to fully incorporate this approach into the educational process are readily available.

In this context, the report presents various problem-solving strategies in school mathematics based on the postulate that perpetual motion is impossible, along with the application of equilibrium conditions. It explores geometric problems traditionally solved using the intersecting chord theorem and the tangent–secant theorem, offering alternative solutions based on the principle of moments. Additionally, the report presents a novel proof of the generalized Pythagorean theorem by analyzing the equilibrium conditions of a gas-filled cylinder. It also introduces a method for computing certain finite algebraic sums using equilibrium principles. A comparison is made between solutions based on mechanics and those derived using traditional mathematical methods.

Acknowledgments

This research was carried out with the financial support of M. Nalbandyan State University of Shirak within the framework of the scientific project # SUSh-PRIOR N01-SCI-2024.

References

[1] M. Levi, *The Mathematical Mechanic: Using Physical Reasoning to Solve Problems.* Princeton University Press, Princeton, NJ, 2009.

The Third Cohomology Group of a Monoid and Admissible Abstract Kernels II

Nelson Martins-Ferreira[1], **Andrea Montoli**[2], **Alex Patchkoria**[3], **Manuela Sobral**[4]

[1] *Polytechnic Institute of Leiria, Leiria, Portugal*
E-mail: martins.ferreira@ipleiria.pt

[2] *University of Milan, Milan, Italy*
E-mail: andrea.montoli@unimi.it

[3] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: alex.patchkoria@tsu.ge

[4] *University of Coimbra, Coimbra, Portugal*
E-mail: sobral@mat.uc.pt

We will define the product of admissible abstract kernels of the form $\Phi\colon M \to \frac{End(A)}{Inn(A)}$, where M and A are monoids, and Φ is a monoid homomorphism. Identifying C-equivalent abstract kernels, where C is the subgroup of invertible elements of the center of A, we will obtain that the set $\overline{\mathscr{M}}(M,C)$ of C-equivalence classes of admissible abstract kernels inducing the same action of M on C is a commutative monoid. Considering the submonoid $\overline{\mathscr{L}}(M,C)$ of abstract kernels that are induced by Schreier extensions, we will prove that the factor monoid $\overline{\mathscr{A}}(M,C) = \frac{\overline{\mathscr{M}}(M,C)}{\overline{\mathscr{L}}(M,C)}$ is an abelian group. Moreover, we will show that this abelian group is isomorphic to the third cohomology group $H^3(M,C)$. This generalizes the results of [1], where A was a group instead of a monoid.

Acknowledgments

The second and third authors were supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) through grant # FR-24-9660.

References

[1] N. Martins-Ferreira, A. Montoli, A. Patchkoria and M. Sobral, The third cohomology group of a monoid and admissible abstract kernels. *Internat. J. Algebra Comput.* **32** (2022), no. 5, 1009–1041.

Weighted Multilinear Rellich and Hardy Inequalities

Alexander Meskhi[1,2]

[1] *Kutaisi International University, Kutaisi, Georgia*
[2] *Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: alexander.meskhi@kiu.edu.ge

In [2] the authors established weighted multilinear Rellich differential inequalities in the one-dimensional case. As far as we know, such inequalities and Hardy differential inequalities are not known in the higher dimensional case. Our aim is to present two-weight multilinear Rellich and Hardy differential inequalities, generally speaking, on stratified nilpotent Lie groups $\mathbb{G}$ (homogeneous groups). In particular we established the inequalities:

$$\bigg(\int\limits_G \nu(x)\Big|\prod_{k=1}^m u_k(x)\Big|^q\,dx\bigg)^{\frac{1}{q}} \le C\prod_{k=1}^m \bigg(\int\limits_G w_k(x)|\Delta_G u_k|^{p_k}\,dx\bigg)^{\frac{1}{p_k}}$$

and

$$\bigg(\int\limits_G \nu(x)\Big|\prod_{k=1}^m u_k(x)\Big|^q\,dx\bigg)^{\frac{1}{q}} \le C\prod_{k=1}^m \bigg(\int\limits_G w_k(x)|\nabla_G u_k|^{p_k}\,dx\bigg)^{\frac{1}{p_k}},$$

where ν is a general weight, w_k are power-type weights. Here ∇_G is the horizontal gradient on G, and $\Delta_G := \nabla_G \cdot \nabla_G$ is the sub-Laplacian on G.

From these estimates it follows appropriate multilinear weighted inequalities with power weights.

The results are new even for the Euclidean case. These results were announced in [1].

References

[1] D. E. Edmunds and A. Meskhi, Weighted multilinear Rellich and Hardy inequalities on homogeneous groups. *Bull. Georgian National Acad. Sci.* **19(193)** (2025), no. 2, 7–11.

[2] D. E. Edmunds and A. Meskhi, A multilinear Rellich inequality. *Math. Inequal. Appl.* **24** (2021), no. 1, 265–274.

Criteria for Multilinear Sobolev Inequality with Non-doubling Measure in Lorentz Spaces

Alexander Meskhi[1,2], **Lazare Natelashvili**[3]

[1] *Department of Mathematical Analysis, Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

[2] *School of Mathematics, Kutaisi International University, Kutaisi, Georgia*

E-mail: alexander.meskhi@tsu.ge

[3] *Department of Mathematics, Faculty of Informatics and Control Systems, Georgian Technical University, Tbilisi, Georgia*

E-mail: lazarenatelashvili@gmail.com

Necessary and sufficient conditions on a measure μ guaranteeing the boundedness of the multilinear fractional integral operator $T^{(m)}_{\gamma,\mu}$ (defined with respect to a measure μ) from the product of Lorentz spaces $\prod\limits_{k=1}^{m} L^{r_k,s_k}\mu$ to the Lorentz space $L^{p,q}_{\mu}(X)$ are established. These results are new even for the linear fractional integral operator $T_{\gamma,\mu}$ (i.e., for $m=1$).

From the general results, we derive a criterion for the validity of a Sobolev-type inequality in Lorentz spaces defined with respect to non-doubling measures. Finally, we investigate the same problem for Morrey–Lorentz spaces.

Similar problem in the Lebesgue spaces were studied [1], [2].

References

[1] V. Kokilashvili, M. Mastyło and A. Meskhi, On the boundedness of multilinear fractional integral operators. *J. Geom. Anal.* **30** (2020), no. 1, 667–679.

[2] V. Kokilashvili and A. Meskhi, Fractional integrals on measure spaces. *Fract. Calc. Appl. Anal.* **4** (2001), no. 1, 1–24.

On Teaching Problems of Extremum

Rusudan Meskhia

Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: rusudan.meskhia@tsu.ge

The question of solving extremum problems without using the derivative of a function is considered, as the derivative in the school course of Mathematics is not taught. The solution of the oldest extremum problems is provided both geometrically and analytically. Besides, well-known inequalities, propertes of elementary functions, elements of vector Algebra and linear programming method are applied in solving extremum problems.

The problems considered are mainly of practical nature and will help motivate more students to study Mathematics.

Classification of C^*-Dynamical Systems in the Bootstrap Class for Actions of a Finite Commutative Square-Free Groups

Ralf Meyer[1], **George Nadareishvili**[2]

[1] *Mathematisches Institut, Universitat Göttingen, Göttingen, Germany*
E-mail: ralf.meyer@mathematik.uni-goettingen.de

[2] *Kutaisi International University, Kutaisi, Georgia*
E-mail: Giorgi.Nadareishvili@kiu.edu.ge

In this talk, we will demonstrate how to extend the results of Manuel Köhler [1] on classification of C^*-dynamical systems with the action of finite cyclic groups of prime order to finite commutative square-free groups. We will explain how by [2], the classification will apply to all such actions on Type I C^*-algebras.

References

[1] M. Köhler, Universal coefficient theorems in equivariant KK-theory. Ph.D. Thesis, *Georg-August-Universität Göttingen*, 2010; `http://hdl.handle.net/11858/00-1735-0000-0006-B6A9-9`.

[2] R. Meyer and G. Nadareishvili, A universal coefficient theorem for actions of finite groups on C^*-algebras. *Preprint*, arXiv:2406.11787, 2024; `https://arxiv.org/abs/2406.11787`.

AB Theory

Oleg Mikeladze

Physics Department, Shota Rustaveli State University, Batumi, Georgia
E-mail: mikeladze.olegi68@gmail.com

The paper aims to present the AB theory in a manner that is accessible to a broad audience and to demonstrate its compatibility with recent discoveries from the James Webb Space Telescope, asserting that the theory predates the telescope's launch. The author contends that the AB theory can explain modern astronomical achievements, unlike the Big Bang theory:

1. Visible matter is the result of the interaction of dark matter and dark energy.

2. The total volume of the vacuum obtained during the origin of the visible universe does not change and is a significant law of the universe's stability.

References

[1] E. Hubble, *The Observational Approach to Cosmology.* Clarendon Press, Oxford, 1937.

[2] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha and R. P. Kirshner, Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal* **116** (1998), no. 3, 1009–1038.

[3] E. Rutherford, The scattering of α and β particles by matter and the structure of the atom. *Philiosophical Magazine* **21** (1911), 669–688.

[4] F. Zwicky, On the masses of nebulae and of clusters of nebulae. *Astrophys. J.* **86** (1937), 217–224.

Some Aspects of Bilinear Hardy Type Inequalities

Saujanya Mohanty

Department of Mathematics, South Asian University, Rajpur Road, Maidan Garhi
New Delhi, India
E-mail: mohantysaujanya29@gmail.com

We study weighted bilinear Hardy-type inequalities by reducing them to the classical Hardy inequalities. Moreover, we study the inequality

$$\bigg(\int\limits_0^\infty (H_2(f,g)(x))^q u(x)\,dx\bigg)^{1/q} \le C\bigg(\int\limits_0^\infty f^{p_1}(x)v_1(x)\,dx\bigg)^{1/p_1}\bigg(\int\limits_0^\infty f^{p_2}(x)v_2(x)\,dx\bigg)^{1/p_2}$$

for all non-negative f, g on (a,b) and $1<p_1,p_2,q<\infty$. The same technique is applied to various further problems, in particular those involving multilinear mixed integral operators of Hardy type and obtained the result.

References

[1] M. Křepela, Iterating bilinear Hardy inequalities. *Proc. Edinb. Math. Soc. (2)* **60** (2017), no. 4, 955–971.

[2] A. Kufner, L.-E. Persson and N. Samko, *Weighted Inequalities of Hardy Type.* Second edition. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017.

[3] S. Mohanty, S. Jain and P. Jain, On weighted bilinear inequalities with mixed Hardy operators. (submitted).

[4] V. D. Stepanov, Integral operators on the cone of monotone functions. *J. London Math. Soc. (2)* **48** (1993), no. 3, 465–487.

Some Linear Maps on Nest Algebras

Amir Hossein Mokhtari

Technical Faculty of Ferdows, University of Birjand, Bijand, Iran
E-mail: a.mokhtari@birjand.ac.ir

This paper investigates the structure of proper linear and Lie-type mappings on generalized matrix algebrasand we present a unified framework that encompasses commuting mappings, Lie centralizers, and Lie triple centralizers. We derive general forms for these maps and establish sufficient conditions for their properness. Our approach highlights how results for triangular and nest algebras are immediate corollaries. The theoretical findings are complemented by explicit characterizations and examples that demonstrate the map decomposition into derivation components and central-valued components. These insights both generalize classical results and offer a concise, algebraic method for analyzing map behavior in structured ring settings.

References

[1] W.-S. Cheung, Lie derivations of triangular algebras. *Linear Multilinear Algebra* **51** (2003), no. 3, 299–310.

[2] K. R. Davidson, *Nest Algebras.* Triangular forms for operator algebras on Hilbert space. Pitman Research Notes in Mathematics Series, 191. Longman Scientific & Technical, Harlow; copublished in the United States with John Wiley & Sons, Inc., New York, 1988.

[3] C. R. Miers, Derived ring isomorphisms of von Neumann algebras. *Canadian J. Math.* **25** (1973), 1254–1268.

[4] M. Gazor and F. Mokhtari, Normal forms of Hopf-zero singularity. *Nonlinearity* **28** (2015), no. 2, 311–330.

Analysis of Boundary-Domain Integral Equations for Variable-Coefficient Helmholtz Equation with Interior Mixed BVP on n-dimensional Lipschitz Domain

Shumatir Y. Monijore[1], **Tsegaye G. Ayele**[2], **Mulugeta A. Dagnachew**[3]

[1]*Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia*
E-mail: yshumatir@gmail.com

[2]*Department of Mathematics, Addis Ababa University, Addis Ababa, Ethiopia*
E-mail: tsegaye.ayele@aau.edu.et

[3]*Department of Mathematics, Injibara University, Injibara, Ethiopia*
E-mail: malemayehu3@gmail.com

In this study, together with parametrix-based boundary domain integral equations (BDIEs), the Helmholtz equation with a non-smooth Hölder continuous variable coefficient was examined in the Lipschitz domain using interior mixed (Dirichlet-Neumann) boundary value problems. When neither classical nor weak canonical co-normal derivatives are well defined, the right-hand side of the PDE belongs to the Sobolev (Bessel potential) space $H^{s-2}(\Omega)$ or $\widetilde{H}^{s-2}(\Omega)$; $\frac{1}{2} < s < \frac{3}{2}$. For the Helmholtz equation with varying wave numbers, using the methodology established in [2], we build BDIE systems by extending [1]. In suitable Sobolev spaces, the equivalence of BDIEs with the original mixed BVP, BDIE solvability, solution uniqueness, the Fredholm property, and the invertibility of the BDIE operators are examined.

Acknowledgments

The work was supported by the Arba Minch University.

References

[1] O. Chkadua, S. E. Mikhailov and D. Natroshvili, Analysis of direct boundary-domain integral equations for a mixed BVP with variable coefficient. I. Equivalence and invertibility. *J. Integral Equations Appl.* **21** (2009), no. 4, 499–543.

[2] S. E. Mikhailov, Analysis of segregated boundary-domain integral equations for BVPs with non-smooth coefficients on Lipschitz domains. *Bound. Value Probl.* **2018**, Article no. 87, 52 pp.

Reproducing Kernels for Eigenspaces Attached to Fock–Darwin Levels on $\mathbb{C}^n$

Zouhair Mouayn

University Sultan Moulay Slimane, Beni-Mellal, Morocco
E-mail: mouayn@gmail.com

Modern semiconductor technology has made it possible to fabricate ultrasmall structures that confine electrons on scales comparable to their de Broglie wavelength. Structures that restrict the motion of electrons in all directions are known as quantum dots. The *Fock–Darwin levels*, which describe the energy quantization of particles in such a system, have been a critical point of study due to their implications for the behavior of quantum dots. The number of electrons, N, in a quantum dot can range from zero to several thousand. A parabolic confinement potential, $V(x,y)=\frac{1}{2}\,\omega_0^2(x^2+y^2)$, is often used as a realistic and at the same time computationally convenient approximation. The corresponding Hamiltonian is of the form

$$H_N=\sum_{j=1}^{N}H_1^{(j)}+\sum_{1\le i,j\le N}W(\mathbf{r}_i-\mathbf{r}_j)$$

$\mathbf{r}_\ell\in\mathbb{R}^2$, $\ell=1,\dots,N$ and $H_1^{(j)}=1\otimes\cdots\otimes H_1\otimes\cdots\otimes 1$ (H_1 in the j-th place) with one body Hamiltonian $H_1=H_1^{orb}+\gamma_0\mathscr{S}_z.B+\gamma_1$,

$$H_1^{orb}=\frac{1}{2m_*}\Big(-i\hbar\nabla-\frac{e}{c}\,\mathbf{A}\Big)^2+V(\mathbf{r}),$$

where $\mathbf{r}=(x,y)$ ($i=\sqrt{-1}$, $m_*=e=\hbar=c=1$). Here, $\mathbf{A}$ is the vector potential of a homogeneous magnetic field $\mathbf{B}$ in the z-direction, V is the confining potential, $\mathscr{S}_z$ the spin operator in the z-direction and γ_0, γ_1 are physical constants. The potential W represents the Coulomb repulsion between electrons. In this talk, we will discuss some spectral properties of an analog form of H_1^{orb}, given by operators

$$\Delta_{B,\omega}=-\frac{1}{4}\sum_{j=1}^{n}\frac{\partial^2}{\partial z_j\partial\overline{z}_j}+\lambda_-\sum_{j=1}^{n}z_j\,\frac{\partial}{\partial z_j}+\lambda_+\sum_{j=1}^{n}\overline{z}_j\,\frac{\partial}{\partial\overline{z}_j}\,,\quad \lambda_\pm:=\frac{1}{2}\big(\sqrt{B^2+4\omega^2}\pm B\big),$$

called Fock–Drawin Laplacians and will be acting on $L^2(\mathbb{C}^n,e^{-\sqrt{B^2+4\omega^2}\,|z|^2}\,d\mu)$, $d\mu$ being the Lebesgue measure on $\mathbb{C}^n$. We obtain the reproducing kernels of the corresponding L^2 eigenspaces.

An Alternative Potential Method for Pseudo-Oscillation Equations of Elasticity Theory

Maia Mrevlishvili[1], **David Natroshvili**[2]

[1]*School of Technology, Ilia State University, Tbilisi, Georgia*
E-mail: m_mrevlishvili@yahoo.com

[2]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mail: natrosh@hotmail.com

We consider an alternative approach to investigate three-dimensional interior and exterior mixed boundary value problems (BVP) for the pseudo-oscillation equations of the elasticity theory for isotropic bodies. The domain occupied by an elastic body, $\Omega \subset \mathbb{R}^3$, has a compact boundary surface $S = \partial\Omega$, which is divided into two disjoint parts, the Dirichlet part S_D and the Neumann part S_N, where the displacement vector (the Dirichlet type condition) and the stress vector (the Neumann type condition) are prescribed, respectively.

Our new approach is based on the classical potential method and has several essential advantages compared with the existing approaches. We look for a solution to the mixed boundary value problem in the form of a linear combination of the single layer and double layer potentials with densities supported on the Dirichlet and Neumann parts of the boundary, respectively. This approach reduces the mixed BVP under consideration to a system of boundary integral equations, which contain neither extensions of the Dirichlet or Neumann data nor the Steklov-Poincaré type operator involving the inverse of a special boundary integral operator, which is not available explicitly for an arbitrary boundary surface. Moreover, the right-hand sides of the resulting boundary integral equations system are vector-functions coinciding with the given Dirichlet and Neumann data of the problem in question. We show that the corresponding matrix integral operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies unconditional unique solvability of the mixed BVP in the class of vector-functions belonging to the Sobolev space $[W_2^1(\Omega)]^3$. We also show that the obtained matrix boundary integral operator is invertible in the L_p-based Besov spaces and prove that under appropriate boundary data a solution to the mixed BVP possesses C^α-Hölder continuity property in the closed domain $\overline{\Omega}$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FR-23-267).

On Projective Class Group of Crossed Products II

Vitali Muladze[1], **Giorgi Rakviashvili**[1], **Lela Turmanidze**[2]

[1]*Ilia State University, Tbilisi, Georgia*

E-mails: vitali.muladze.1@iliauni.edu.ge; giorgi.rakviashvili@iliauni.edu.ge

[2]*Batumi State University, Batumi, Georgia*

E-mail: lela.turmanidze@bsu.edu.ge

Let R be a Dedekind ring, K its quotient field and $R[\pi,\rho]$, $K[\pi,\rho]$ crossed products [2]. We will consider those finitely generated projective $R[\pi,\rho]$-modules such that $K\otimes P$ is $K[\pi,\rho]$-free. Let $\mathbf{P}_0$ be the category of such projectives and let $P_0(R[\pi,\rho])$ be the Grothendieck group of $\mathbf{P}_0$. Let $P_0^{'}(R[\pi,\rho])$ be the subgroup of $P_0(R[\pi,\rho])$ generated by all $[F]$ with F free over $R[\pi,\rho]$.

Definition 1 Let R be a Dedekind ring, K its quotient field. Let $\underline{P}_0(R[\pi,\rho])$ be the category of finitely generated projective $R[\pi,\rho]$-modules P such that $K\otimes_R P$ are free over $K[\pi,\rho]$. We denote the corresponding Grothendieck group as $K_0'(R[\pi,\rho])=K(\underline{P}_0(R[\pi,\rho]))$.

Definition 2 The special projective class group [4] is defined to be

$$C_0(R[\pi,\rho])=P_0(R[\pi,\rho])/P_0^{'}(R[\pi,\rho]).$$

The special projective class group coincides with the usual projective class group $C(R[\pi,\rho])$ as defined in [3].

Definition 3 Let $G^{\#}(R[\pi,\rho])$ denote a subgroup of $G(R[\pi,\rho])$ generated by all $[A]$ where A is an R-free $R[\pi,\rho]$-module.

Theorem 1 *Suppose char(R) does not divide $(\pi:1)$. Then there is a unique map μ making the following diagram commutative:*

$$\begin{array}{ccc} G^{\#}(R[\pi])\otimes K'{}_0(R[\pi,\rho]) & \longrightarrow & K'{}_0(R[\pi,\rho]) \\ & {\scriptstyle j_*\otimes 1}\searrow \quad \nearrow{\scriptstyle \mu} & \\ & G(K[\pi])\otimes K'{}_0(R[\pi,\rho]) & \end{array}.$$

Definition 4 If M is a class of subgroups, define $C_0^M(R[\pi,\rho])$ as the sum of the images of the maps $i_*:C_0(R[\pi',\rho])\to C_0(R[\pi,\rho])$ where $i:\pi'\subset\pi$, $\pi'\in M$.

Definition 5 Let k be an integer. We say [4] that $G_M(K[\pi,\rho])$ has exponent k in $G(K[\pi,\rho])$ if $kG(K[\pi,\rho])\subset G_M(K[\pi,\rho])$. It is not required that k is the smallest such integer.

Theorem 2 *Suppose char(R) is prime to $(\pi:1)$. Let k be the exponent of $G_M(K[\pi,\rho])$ in $G(K[\pi,\rho])$. Let $x\in C_0(R[\pi,\rho])$ be such that $i^*(x)=0\in C_0(R[\pi',\rho]$ for all $i:\pi'\subset\pi$, $\pi'\in M$. Then $kx=0$.*

References

[1] V. Muladze and G. Rakviashvili, Projective class group of crossed product. *Adv. Stud. Euro-Tbil. Math. J.* **17** (2024), no. 3, 123–136.

[2] G. Rakviashvili, On algebraic K-functors of crossed group rings and its applications. *Tbilisi Math. J.* **11** (2018), no. 2, 1–15.

[3] D. S. Rim, Modules over finite groups. *Ann. of Math. (2)* **69** (1959), 700–712.

[4] R. G. Swan, Induced representations and projective modules. *Ann. of Math. (2)* **71** (1960), 552–578.

Constructive Martingale Representation of Non-Smooth Brownian Functionals

Ekaterine Namgalauri[1], **Omar Purtukhia**[1,2]

[1]*Department of Mathematics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[2]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mails: ekanamgalauri96@gmail.com; o.purtukhia@gmail.com

The martingale representation theorem stands as a cornerstone of stochastic calculus. It plays a crucial role in a wide range of applications, from pricing financial derivatives and managing risk to analyzing the structure of martingales and their connection to Brownian motion. Additionally, it is instrumental in addressing challenges in control theory and signal filtering. At its heart, the theorem asserts that any square-integrable martingale driven by Brownian motion can be represented as a stochastic integral. This insight provides a powerful and unified framework for modeling randomness, making the theorem relevant not just in finance, but also in physics, engineering, economics, and other fields that deal with uncertainty.

A key result in this area is the Clark–Ocone formula, introduced by Ocone in 1984, which offers a way to explicitly construct such integral representations – though it is limited to functionals that are Malliavin-differentiable. Recognizing this limitation, Professor Purtukhia, in collaboration with the late Professor Glonti [1], extended the formula to apply even when the functional itself lacks Malliavin smoothness, as long as its conditional expectation remains Malliavin-differentiable. Here, we introduce a novel constructive method that generalizes stochastic integral representations to encompass both Malliavin-smooth and nonsmooth functionals.

Definition The stochastic derivative (derivative in the Malliavin sense) of a smooth random variable $F=f(B_{t_1},B_{t_2},\dots,B_{t_n})$ ($f\in C_p^\infty(R^n)$, $t_i\in[0,T]$) is defined as a random process D_tF defined by the relation $D_tF=\sum\limits_{i=1}^n\frac{\partial f}{\partial x_i}(B_{t_1},B_{t_2},dots,B_{t_n})I_{[0,t_i]}(t)$ and $D_{1,2}$ is equal to the closure of the class of smooth random variables in the norm $\|F\|_{1,2}:=\big\{E[F^2]+E[\|DF\|^2_{L_2([0,T])}]\big\}^{1/2}$.

Let $h_s(\omega)=h_s(B_s(\omega))$ be an integrable process adapted to the flow of σ-algebras $\Im_s^B$. Let's consider the functional $F(a,b)=\exp\big\{\int_a^b h_s(\omega)\,ds\big\}$, where $0\le a\le b\le T$.

Theorem 1 *If the function $V(t,x)=E[F(t,T)|B_t=x]$ satisfies the requirements of the classical Itô formula (i.e. $V(\,\cdot\,,\,\cdot\,)\in C^{1,2}([0,T]\times R)$), then the following stochastic integral representation is fulfilled*

$$F(0,T)=EF(0,T)+\int_0^T F(0,t)\cdot V_x'(t,B_t)\,dB_t\ \ (P\text{-a.s.}).\tag{1}$$

Theorem 2 *Let $F(0,t)\in D_{1,2}$ for almost all t. Then the Clark–Ocone representation formula for the functional $F(0,T)$ follows from the representation* (1).

References

[1] O. A. Glonti and O. G. Purtukhiya, On an integral representation of a Brownian functional. (Russian) *Teor. Veroyatn. Primen.* **61** (2016), no. 1, 158–164; translation in *Theory Probab. Appl.* **61** (2017), no. 1, 133–139.

[2] D. Ocone, Malliavin's calculus and stochastic integral representations of functionals of diffusion processes. *Stochastics* **12** (1984), no. 3-4, 161–185.

Iteration Method for the Dirichlet and Neumann Type Boundary Value Problems of the Elasticity Theory

David Natroshvili[1], **Maia Mrevlishvili**[2]

[1]*Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mail: d.natroshvili@gtu.ge

[2]*School of Technology, Ilia State University, Tbilisi, Georgia*
E-mail: m_mrevlishvili@yahoo.com

Based on the potential method, we construct a convergent recurrence scheme for the solution of the three-dimensional Dirichlet and Neumann boundary value problems of the elasticity theory.

By the single layer potential, the Dirichlet problem is reduced to the uniquely solvable Fredholm integral equation of the first kind with a symmetric weakly singular boundary integral operator. First, we construct a sequence of successive approximations which converges to the solution of the boundary integral equation in the appropriate Bessel-potential spaces of functions defined on the boundary. Afterwards, using these approximations, we construct another iteration which converges to the solution of the Dirichlet boundary value problem in the appropriate Sobolev spaces of functions defined in the region occupied by the elastic body. We assume that the boundary surface belongs to a Lipschitz class.

Similar approach is developed for the Neumann boundary value problem with some nontrivial modification. The case is that, on the one hand, the boundary integral operator corresponding to the Neumann problem and generated by the single layer potential is neither symmetric nor weakly singular and needs symmetrization by a compact operator. On the other hand, the null-space of the operator is not trivial.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FR-23-267).

Maximal Operators and Differentiation with Respect to Collections of Shifted Convex Bodies

Giorgi Oniani

Kutaisi International University, Kutaisi, Georgia
E-mail: giorgi.oniani@kiu.edu.ge

Let Ω be a collection of shifted balls in $\mathbb{R}^n$. The term "shifted ball" here means that the ball does not contain the origin. Maximal operators and differentiation of integrals with respect to the collection Ω, i.e. with respect to the basis $\mathbf{B}_\Omega$ for which $\mathbf{B}_\Omega(x) = \{B + x : \ B \in \Omega\}$ $(x \in \mathbb{R}^n)$, have been studied by Nagel and Stein, Stein, Aversa and Preiss, Csörnyei, and Hagelstein and Parissis. One of the motivations for the research was the intimate connection with the boundary behaviour of Poisson integrals along regions more general than cones. Moonens and Rosenblatt examined the case of collections of shifted two-dimensional intervals, while Laba and Pramanik studied the topic for dilation invariant collections of sparse one-dimensional sets.

We will discuss some new results related with maximal operators and differentiation of integrals with respect to collections of shifted convex bodies.

The talk is based on a joint research with Emma D'Aniello and Laurent Moonens.

Geometric Aspects of Orlicz Sequence Spaces with the s-Norm

Serap Öztop[1], **Badik Hüseyin Uysal**[1,2]

[1]*Department of Mathematics, Faculty of Science, İstanbul University, İstanbul, Türkiye*
[2]*Institute of Graduate Studies in Sciences, İstanbul University, İstanbul, Türkiye*
E-mails: oztops@istanbul.edu.tr; huseyinuysal@istanbul.edu.tr

Let Φ be an Orlicz function, and let $l^{\Phi}(\mathbb{N}, \mathscr{P}(\mathbb{N}), \mu)$ denote the corresponding Orlicz sequence space over the counting measure space $(\mathbb{N}, \mathscr{P}(\mathbb{N}), \mu)$. Our main interest is to relate the geometric properties of $l^{\Phi}(\mathbb{N}, \mathscr{P}(\mathbb{N}), \mu)$ to various geometric conditions centered around the weak behavior of the sequences as the norm of the Orlicz sequence spaces. Moreover, we examine the behavior of weakly convergent sequences in Orlicz sequence spaces equipped with the s-norm. Our study generalizes and unifies the results that have been obtained for the Orlicz norm, the Luxemburg norm, and the p-Amemiya norms where $1 \le p \le \infty$, respectively.

Acknowledgments

This study was funded by Scientific Research Projects Coordination Unit of Istanbul University, Project # 41350.

Effects of Macroalgal Toxicity and Elevated Sea Surface Temperature on Coral Reef Dynamics

Samares Pal

Department of Mathematics, University of Kalyani, Kalyani, India
E-mails: samaresp@yahoo.co.in, samaresp@gmail.com

Coral reef ecosystems are most vulnerable to changes in sea surface temperature (SST), a key environmental factor critical to reef-building growth. Elevated SST reduces the ability of corals to produce their calcium carbonate skeletons. Prolonged high SST results in coral bleaching owing to the uncoupling of symbiosis among corals and microalgae. Corals have narrow temperature tolerances. The skeletal growth rate of corals falls sharply to zero even at a slight increase of SST above its temperature tolerance level. Corals are also vulnerable to macroalgal toxicity. Several benthic macroalgae species are known to bring about allelopathic chemical compounds that are very harmful to corals. The toxic-macroalgae produce allelochemicals for which the survivability and settlement of coral larvae are highly affected. Toxic macroalgae species damage coral tissues when in contact by transferring hydrophobic allelochemicals present on macroalgal surfaces, leading to a reduction of corals and even coral mortality. The abundance of toxic macroalgae changes the community structure towards a macroalgae-dominated reef ecosystem.

On one Nonlocal Problem for Differential Equations with a Deviating Argument

Nino Partsvania

Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
E-mail: nino.partsvania@tsu.ge

On the interval $[a,b]$, we consider the differential equation

$$u'(t)=f(t,u(\tau(t))) \tag{1}$$

with the nonlocal condition

$$\sum_{i=1}^{m}\ell_i u(t_i)=c_0, \tag{2}$$

where $f:[a,b]\times[0,+\infty[\,\to\mathbb{R}$, $\tau:[a,b]\to[a,b]$ are continuous functions, m is a natural number, c_0 is a positive constant, $\ell_i>0$, $a\le t_i\le b$ $(i=1,\dots,m)$, $t_i<t_k$ for $i<k$.

Sufficient conditions for the solvability and unique solvability of problem (1), (2) are established. Namely, the following theorem is proved.

Theorem *Let*

$$f(t,0)=0,\ \ f(t,x)\le 0\ \ \text{for}\ \ a\le t\le b,\ \ x\ge 0,$$

$t_1=a$*, and let there exist* $b_0\in\,]a,b[$ *such that*

$$\tau(t)>t\ \ \text{for}\ \ a\le t<b_0,\ \ \tau(t)=b\ \ \text{for}\ \ b_0\le t\le b.$$

Then problem (1), (2) *has at least one positive solution. If the function* f *is nonincreasing in the phase variable, then this solution is unique.*

On the Farrell–Tate K-Theory of $Out(F_n)$

Irakli Patchkoria

Department of Mathematics, University of Aberdeen, Aberdeen, UK
E-mail: irakli.patchkoria@abdn.ac.uk

This is joint work with Naomi Andrew. The classical Farrell–Tate cohomology measures the failure of duality in group (co)homology. Brown in 70s gave a general method for computing the p-local part of the Farrell–Tate cohomology. Using Brown's methods Farrell-Tate cohomology has been computed for various arithmetic groups, mapping class groups and $Out(F_n)$-s, outer automorphism groups of Free groups. Later Klein introduced generalised Farrell-Tate cohomology with coefficients in an arbitrary spectrum. In this project we investigate the Farrell-Tate K-theory of $Out(F_n)$. We will show that for any discrete group with finite classifying space for proper actions, the p-adic Farrell–Tate K-theory is rational. Then using Lück's Chern character, we will give a general formula for the p-adic Farrell–Tate K-theory in terms of centralisers. In particular, we apply this formula to $Out(F_{p+1})$ which has curious p-torsion behaviour: It has exactly one conjugacy class of a p-torsion element which does not come from $Aut(F_{p+1})$. Computing the rational cohomology of the centraliser of this element allows us to fully compute the p-adic Farrell–Tate K-theory of $Out(F_{p+1})$. As a consequence we show for example that the 11-adic Farrell–Tate K-theory of $Out(F_{12})$ is non-trivial, thus detecting a non-trivial class in odd K-theory of $Out(F_{12})$ without using any computer calculations.

Acknowledgments

The work is supported by the EPSRC grant # EP/X038424/1 “Classifying spaces, proper actions and stable homotopy theory”.

References

[1] N. Andrew and I. Patchkoria, On the Farrell–Tate K-theory of $Out(F_n)$. *Preprint* arXiv:2505.21803, 2025; `https://arxiv.org/abs/2505.21803`.

Hardy Inequalities for Monotone Functions using Sawyer Duality Principle

Aditi Pathak

Department of Mathematics, South Asian University, Rajpur Road, New Delhi, India
E-mail: aditipathak675@gmail.com

The classical Sawyer principle provides the necessary and sufficient conditions for the boundedness of linear integral operators in various function spaces. This principle is well known for weighted Lebesgue as well as for Orlicz spaces, providing both lower and upper bounds for the following expressions

$$I(g) := \sup_{0\le f\downarrow} \frac{\int\limits_0^\infty f(x)g(x)\,dx}{\big(\int\limits_0^\infty f^p v\big)^{1/p}}$$

and

$$J(g) := \sup_{0\le f\downarrow} \frac{\int\limits_0^\infty f(x)g(x)\,dx}{\|f\|_{P(v)}}\,.$$

While discussing this principle, we shall also present the cases when f is replaced by its integral average $\frac{1}{x}\int\limits_0^x f(t)\,dt$ and also by its adjoint $\int\limits_x^\infty \frac{f(t)}{t}$.

References

[1] H. P. Heinig and A. Kufner, Hardy operators of monotone functions and sequences in Orlicz spaces. *J. London Math. Soc. (2)* **53** (1996), no. 2, 256–270.

[2] A. Kufner, L.-E. Persson and N. Samko, *Weighted Inequalities of Hardy Type.* Second edition. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017.

[3] A. Kufner, O. John and S. Fučík, *Function Spaces.* Monographs and Textbooks on Mechanics of Solids and Fluids, Mechanics: Analysis. Noordhoff International Publishing, Leiden; Academia, Prague, 1977.

[4] V. D. Stepanov, Integral operators on the cone of monotone functions. *J. London Math. Soc. (2)* **48** (1993), no. 3, 465–487.

Minimality of the Inner Automorphism Group

Dekui Peng[1], **Menachem Shlossberg**[2]

[1] *Institute of Mathematics, Nanjing Normal University, Nanjing, Jiangsu, Gulou, China*
E-mail: pengdk10@lzu.edu.cn

[2] *School of Computer Science, Reichman University, Herzliya, Israel*
E-mail: menachem.shlossberg@post.runi.ac.il

This talk is based on the paper [4].

By [1], a minimal group G is called *z-minimal* if $G/Z(G)$ is minimal. In this paper, we present the *z-Minimality Criterion* for dense subgroups. For a locally compact group G, let $\mathrm{Inn}(G)$ be the group of all inner automorphisms of G, endowed with the Birkhoff topology. Using a theorem by Goto [2], we obtain our main result which asserts that if G is a connected Lie group and $H \in \{G/Z(G), \mathrm{Inn}(G)\}$, then H is minimal if and only if H is centre-free and topologically isomorphic to $\mathrm{Inn}(G/Z(G))$. In particular, if G is a connected Lie group with discrete centre, then $\mathrm{Inn}(G)$ is minimal. We prove that a connected locally compact nilpotent group is z-minimal if and only if it is compact abelian. In contrast, we show that there exists a connected metabelian z-minimal Lie group that is neither compact nor abelian. As in the papers [3], [5], some applications to Number Theory are provided.

References

[1] D. Dikranjan, W. He, D. Peng, W. Xi and Z. Xiao, Products of locally minimal groups. *Topology Appl.* **329** (2023), Paper no. 108368, 28 pp.

[2] M. Goto, Absolutely closed Lie groups. *Math. Ann.* **204** (1973), 337–341.

[3] M. Megrelishvili and M. Shlossberg, Minimality of topological matrix groups and Fermat primes. *Topology Appl.* **322** (2022), Paper no. 108272, 21 pp.

[4] D. Peng and M. Shlossberg, Minimality of the inner automorphism group. *Topology Appl.* **370** (2025), Paper no. 109425, 17 pp.

[5] M. Shlossberg, Minimality conditions equivalent to the finitude of fermat and mersenne primes. *Axioms* **12** (2023), no. 6, Paper no. 540, 8 pp.

GRACYASK: Graphs, Cycles, and Skyrmions

Sinuhe Perea

Max Planck Institute of Colloids and Interfaces (MPIKG), Postdam, Germany
E-mail: sinuhe.perea@mpikg.mpg.de

In recent years, the boundaries between mathematics and natural sciences have increasingly blurred out due to the parallel advances in fabrication techniques and lab characterisation of nanomaterials. By extending initial simple re-writing rules from Cayley diagrams, it is possible to construct hypergraph sets that elucidate topological features across a wide range of phenomena in soft-matter dynamics. These include fiber bundles, self-assembling cholesteric liquid crystal templates, near-field chiral nanomagnetism, or the formation of mother-of-pearl. Classifying these topological defects using Toulouse–Kleman parameters fa cilitates the definition of a generalised framework for encompassing skyrmion-like structures in discretised inelastic lattices, liquid crystals, chiral electromagnetism, and biomineralisation, respectively. This frame work reveals important nuances regarding general principles of discontinuity cancellation in continuous media and stabilization mechanisms via topological protection, which are crucial for technological applications like toroidal metasurfaces characterisation, racetrack memory spintronics or bottom-up approaches to structural colour.

Acknowledgments

The work was supported by the SALTO (CNRS/MPG).

The Main Aim of Research Institute for Cultural Protection and Technological Development of Georgian State Languages at the Georgian Technical University and the Georgian-Abkhazian "smart" Internet Network Technologically Completely Supported with Georgian and Abkhazian Computer "Brains"

Konstantine Pkhakadze[1], **Aleksandre Maskharashvili**[1,2]

[1] *Research Institute for Cultural Protection and Technological Development of Georgian State Languages, Georgian Technical University, Tbilisi, Georgia*
[2] *Department of Linguistics, University of Illinois, Urbana, USA*
E-mail: gllc.ge@gmail.com

In the presentation, we will review the main aim of the "Research Institute for the Cultural Protection and Technological Development of Georgian State Languages", launched at the Georgian Technical University on May 15, 2025, and this aim is to develop a Georgian-Abkhazian "smart" Inter-Net, technologically fully supported with technological alphabets of the Georgian and Abkhazian languages, in other words, with Georgian and Abkhazian computer "brains". In addition, based on Georgian and European research, we will argue that on the verge of the era of perfect artificial intelligence, which is already approaching:

1. Due to weak technological support, the Georgian language, along with other weakly supported European languages, is under threat of digital extinction;
2. Due to its extremely weak technological support, the Abkhazian language, along with other very weakly supported European languages, is at high risk of digital extinction.

References

[1] K. Pkhakadze, A. Maskharashvili and L. Abzianidze, Program Financing Plan for 2024-2028 of the Research Institute for Cultural Protection and Technological Development of Georgian State Languages at the Georgian Technical University. *Journal "Logic, Language, Artificial Intelligence"*, 2023-2024, no. 1(15), 62–134; https://drive.google.com/file/d/1RbH-m7EN68HNkkeo0l9ajpwVxcXjuVML/view?usp=sharing

[2] K. Pkhakadze, M. Chikvinidze, G. Chichua, Sh. Malidze, D. Kurtskhalia, C. Demurchev and N. Okroshiashvili, In the European Union with Georgian and Abkhazian Languages – Aims and Problems of Complete Technology Support of Georgian and Abkhazian Languages. *Bull. Georgian Natl. Acad. Sci. (N.S.)* **14** (2020), no. 3, 36–42; http://science.org.ge/bnas/t14-n3/06_Pkhakadze_Informatics.pdf.

[3] K. Pkhakadze, M. Chikvinidze, G. Chichua, D. Kurtskhalia, I. Beriashvili and Sh. Malidze, Georgian Intellectual Web Corpus: Goals, Methods, Recommendations. Georgian Technical University, 2017; https://geoanbani.com/other/pdfs/aboutus/2.pdf.

[4] G. Rehm and A. Way (Ed.), *European Language Equality: A Strategic Agenda for Digital Language Equality.* Cognitive Technologies. Springer, 2023.

For the Trial-Applied Version of the Hybrid Syntactic-Logical Generator and Analyzer of the Core part of the Georgian Language

Konstantine Pkhakadze[1], **Aleksandre Maskharashvili**[1,2], **Luka Pkhakadze**[1]

[1] *Research Institute for Cultural Protection and Technological Development of Georgian State Languages, Georgian Technical University, Tbilisi, Georgia*
[2] *Department of Linguistics, University of Illinois, Urbana, USA*
E-mail: gllc.ge@gmail.com

In our presentation, based on the new insights and methods developed by K. Pkhakadze's Logical Grammar of the Georgian Language, we will briefly review the goals and methods of developing a hybrid syntactic-logical generator and analyzer of the core part of the Georgian language, and also present the first trial-applied version of such a system.

References

[1] A. Maskharashvili, Bachelor's thesis: "Mathematical analysis of some Georgian complex sentences." *TSU, Thesis Supervisor Konstantine Pkhakadze*, 2008, 1–30.

[2] K. Pkhakadze, On the issue of linguistic relations and logical declination in Georgian. *Journal "Georgian Language and Logic"*, (2005), no. 1, 19–77; `https://geoanbani.com/other/gllc.ge/publications/issues/Jurnali_1_2005.pdf`.

[3] K. Pkhakadze, M. Chikvinidze, G. Chichua, D. Kurtskhalia, I. Beriashvili and Sh. Malidze, *Georgian Intellectual Web Corpus: Goals, Methods, Recommendations.* GTU Press, Tbilisi; `https://geoanbani.com/other/pdfs/aboutus/2.pdf`.

Homotopy in the Geometry of Monoids

Ilia Pirashvili

Department of Computer Science, Baden-Württemberg Cooperative State University Lörrach, Germany

E-mails: ilia_p@ymail.com, pirashvili@dhbw-loerrach.de

The geometry of monoids is a term used to describe new types of algebraic geometry where monoids play a central role. Examples include F_1 geometry, tropical geometry, toric varieties and many others. Much like (ring-)schemes, monoid schemes play a crucial part in all of these.

We can obtain classical varieties from monoid schemes via realisations. There are two particularly noteworthy such processes: Locally, one is based on monoid rings $K[M]$, the other is based on $\mathsf{Hom}(M,K)$. Relevant to our talk is the latter, which is denoted by $K\mathsf{Spec}(M)$ or KX for a monoid scheme X. This construction generalises toric varieties.

We will focus on the cases $K=\mathbb{C}$ and $\mathbb{R}$. For $\mathbb{R}$, Prof. Brenner and I developed an explicit and algorithmic way of calculating its fundamental groupoid. Much of it can also be generalised to $\mathbb{C}$ and if time permits, we will mention this example as well. We will also talk about their homologies.

Acknowledgments

Parts of the results discussed in this talk were developed in a joint work with Professor Brenner from the University of Osnabrück.

For the Georgian-Abkhazian Semi Self-developing Terminology Portal

Konstantine Pkhakadze[1], **Aleksandre Maskharashvili**[1,2], **Luka Pkhakadze**[1]

[1]*Research Institute for Cultural Protection and Technological Development of Georgian State Languages, Georgian Technical University, Tbilisi, Georgia*

[2]*Department of Linguistics, University of Illinois, Urbana, USA*

E-mail: gllc.ge@gmail.com

In this presentation, we will mainly review the goals and methods of developing a Georgian-Abkhazian semi self-developing terminology portal and, also, we will briefly overview the first trial-experimental version of such a portal.

References

[1] K. Pkhakadze, In the European Union with the Georgian and Abkhazian languages – Aims, Problems and Action Plan for Establishing a Three-Level Educational Program in “Digital Humanities and Computational Linguistics” at the Georgian Technical University. *Journal “Logic, Language, Artificial Intelligence”*, 2023-2024, no. 1(15), 4–61; https://drive.google.com/file/d/1RbH-m7EN68HNkkeo0l9ajpwVxcXjuVML/view?usp=sharing.

[2] K. Pkhakadze, A. Maskharashvili and L. Abzianidze, Program Financing Plan for 2024-2028 of the Research Institute for Cultural Protection and Technological Development of Georgian State Languages at the Georgian Technical University. *Journal “Logic, Language, Artificial Intelligence”*, 2023-2024, no. 1(15), 62–134; https://drive.google.com/file/d/1RbH-m7EN68HNkkeo0l9ajpwVxcXjuVML/view?usp=sharing.

[3] Georg Rehm, Andy Way (Ed.). *European Language Equality: A Strategic Agenda for Digital Language Equality. Cognitive Technologies.* Springer, 2023.

[4] K. Pkhakadze, M. Chikvinidze, G. Chichua, Sh. Malidze, D. Kurtskhalia, C. Demurchev, N. Okroshiashvili and B. Mikaberidze. In The European Union With The Georgian And Abkhazian Languages – Aims, Problems, Results, And Recommendations Of The Complete Technological Support Of The Georgian And Abkhazian Languages. *AMIM* **25** (2020), no. 2, 147–173.

[5] K. Pkhakadze, M. Chikvinidze, G. Chichua, Sh. Malidze, D. Kurtskhalia, C. Demurchev and N. Okroshiashvili, In the European Union with Georgian and Abkhazian Languages – Aims and Problems of Complete Technology Support of Georgian and Abkhazian Languages. *Bull. Georgian Natl. Acad. Sci. (N.S.)* **14** (2020), no. 3, 36–42; http://science.org.ge/bnas/t14-n3/06_Pkhakadze_Informatics.pdf.

[6] K. Pkhakadze, M. Chikvinidze, G. Chichua, D. Kurtskhalia, I. Beriashvili and Sh. Malidze, Georgian Intellectual Web Corpus: Goals, Methods, Recommendations. Georgian Technical University, Tbilisi, 2017; https://geoanbani.com/other/pdfs/aboutus/2.pdf.

On the Design, Investigation, and Numerical Implementation of a Semi-Discrete Scheme for a Kirchhoff-Type Equation with Time-Dependent Coefficients

Jemal Rogava[1,2], **Zurab Vashakidze**[3,4]

[1] *Department of Partial Differential Equations, Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

[2] *Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: jemal.rogava@tsu.ge

[3] *Viktor Kupradze Institute of Mathematics, The University of Georgia, Tbilisi, Georgia*

[4] *Department of Numerical Mathematics and Modelling, Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: z.vashakidze@ug.edu.ge

In this talk, we examine an initial-boundary value problem for a Kirchhoff-type nonlinear dynamic string equation with time-dependent coefficients (see [1]). Our principal objective is to construct a time discretization scheme capable of approximating the solution of this problem. To this end, we employ a symmetric three-layer semi-discrete scheme according to the temporal variable. The nonlinear term is evaluated at the temporal midpoint node, ensuring balanced treatment of the nonlinearity. This formulation reduces the problem at each time step to the inversion of a linear operator, yielding a system of second-order linear ordinary differential equations. We establish local convergence of the scheme and prove that the method exhibits second-order (quadratic) accuracy with respect to the time step. Finally, we present numerical experiments to validate the proposed algorithm across several benchmark problems. The computational results are in strong agreement with the theoretical consequences.

References

[1] J. Rogava and Z. Vashakidze, On convergence of a three-layer semi-discrete scheme for the non-linear dynamic string equation of Kirchhoff-type with time-dependent coefficients. *ZAMM Z. Angew. Math. Mech.* **104** (2024), no. 4, Paper no. e202300608, 36 pp.

On the Way to Adequate Theory of Cognizing

A Shell to Support Sensorimotor Development of Intelligence

Edward Pogossian

Academy of Sciences of Armenia, Institute for Informatics and Automation Problems
Yerevan, Armenia
E-mail: epogossi@aua.am

Cognizing is essentially ensuing being of humans, nevertheless, the ultimate frontiers of human ways of cognizing the Universe U so far stay to be challenged. Thus, human cognizing inevitably entails the necessity of reliable prognostication of solutions of problems on

1. the most effective human cognizers and conditions to meet them constructively;
2. origination of cognizing and conditions necessary for it;
3. the limits on human cognizing the Universe and ways to overcome them;
4. the frontiers of human-AI relationships.

Such prognostication, expectedly, can provide **the theory of cognizing** so far **stating** that

1. *Human cognizers hcogs of U can be modeled by descriptive pcogs Piaget's ones [2].*
2. *Constructive models cogs of hcogs can adequately model pcogs [4], [5].*
3. *Humans cognizing of U can be modeled by cognizing of combinatorial games, particularly, of rg reproducible subsets of such games by corresponding rgcogs models of cogs [1], [4], [5].*
4. *Progressing of cognizers to the most effective ones in given classes can be measured algorithmically [4], [5].*
5. *The theory challenges the questioning of the power and limits of cognizing the entire U^* and observable U universes.*
6. *Cogs and means of formation cogs are decomposable to atomic 1/2 place classifiers [5], [6].*
7. *There are premises of origination of atomic* 1/2 *place classifiers in nature and their development to generalized cognizers cogs [5], [6].*

The theory acknowledging that information is inseparable from classifying and measured equally with one of neg entropic provide premises of origination of information and neg entropic in nature. The theory along with enriching current AI applications, if successful - will support to shed light on the fundamental question ***of the origin of cellular,*** and thus, ***humans.***

References

[1] R. B. Banerji, *Theory of Problem Solving. An Approach to Artificial Intelligence.* Modern Analytic and Computational Methods in Science and Mathematics, no. 17. American Elsevier Publishing Co., Inc., New York, 1969.

[2] J. H. Flavell, The Developmental Psychology of Jean Piaget. D Van Nostrand, 1963; `https://doi.org/10.1037/11449-000`.

[3] S. Grigoryan, E. Pogossian and T. Shahinyan, *On The Way to Learning Expert Meaning Processing.* European University of Armenia, Armenia, 2024.

[4] E. Pogossian, *Adaptation of Combinatorial Algorithms.* Academy of Sciences of Armenia, Armenia, 1983.

[5] E. Pogossian, *Constructing Models of Being by Cognizing.* Academy of Sciences of Armenia, Armenia, 2020.

[6] E. Pogossian, Promoting Origination of Non-Cellular Cognizers. *Pattern Recognition and Image Analysis* **34** (2024), 158–168.

The Ladybug Chronicles: Predation, Defense Coalitions & Cannibalism

Florian Rupp[1], **Anano Tamarashvili**

[1]*Kutaisi International University, Kutaisi, Georgia*
E-mail: florian.rupp@kiu.edu.ge

The dynamics of the ladybug, aphid, and ant interactions are driven by ladybug-aphid predation, ladybug-ant inter-species competition (aphid-ant symbiosis/ mutualism in defense to ladybugs), and ladybug-ladybug intra-species competition (cannibalism). With this, the ladybug, aphid, and ant dynamics are far richer than typical predator-prey models; though also less studied. Here, we are presenting a two-fold approach to modeling and analyzing the interaction dynamics further.

First, we give a Discrete Age-Structured Model of the situation capturing the maturation stages of ladybugs and aphids. The inclusion of 'age', although eminent for biological systems, is often discarded for mathematical simplicity. Simulations enable us to illustrate biologically observed scenarios, like pest control by ladybugs.

Second, we give a two-dimensional accumulated Ordinary Differential Equation Model for the ladybug-aphid interactions (where ants occur as a function of the number of aphids) with saturation terms. The mathematical analysis of this model exhibits a transcritical bifurcation such that for certain values of the parameters an asymptotically stable mixed-species equilibrium exists with aphid numbers well below those without ladybug presence. Moreover, the non-existence of periodic orbits is shown analytically.

Quantization of the Theory of Topological Insulators

Armen Sergeev

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia
E-mail: sergeev@mi-ras.ru

Topological insulators are the solid bodies having a broad energy gap stable under small deformations. It motivates the usage of topological methods for their investigation.

A key role in the theory of solid is played by their symmetry groups. It was Kitaev who has pointed out the relation between the symmetries of solid bodies and Clifford algebras. Following this idea the quantization of topological insulators should reduce to the theory of irreducible representations of Clifford algebras.

The next important step was done by Kennedy and Zirnbauer who introduced the notion of pseudosymmetries. While the algebra of observables of topological insulators is formed by the Hamiltonians, satisfying the commutation relations with symmetry operators, the quantum observables are given by complex structures on the Nambu space, satisfying the anticommutation relations with pseudosymmetries. This correspondence determines the quantization of topological insulators.

Acknowledgements

The study is supported financially by Russian Scientific Foundation (grant # 24-11-00196).

Dynamic Contact Problem for Volterra Viscoelastic Model

Nugzar Shavlakadze[1,2]

[1]*Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

[2]*Georgian Technical University, Tbilisi, Georgia*

E-mail: nusha1961@yahoo.com

In the condition of Volterra viscoelastic model the dynamic boundary value contact problem for half-space is considered. The half-space is reinforced by elastic inclusion with variable rigidity, along the contact surface the tangential stress has unknown jump and displacement is continuous. The problem is reduced to the Carleman type problem of the theory of analytic functions, the effective solution of this boundary value problem is constructed. Based on contact condition the integro-differential equation with respect of jump of the tangential stress is obtained. Using the method of orthogonal polynomials this equation reduced to an infinite system of linear algebraic equations. The quasi-regularity of the equivalent infinite system is proved and the convergence rate of the approximate solution to exact solution is determined.

The approximate solutions of the system can be constructed by the reduction method with any accuracy. The problem can be solved under assumptions when the inclusion rigidity changes according to a different law, including when it is constant.

G-Riesz Basis and Q-Dual of G-Frames in Quaternionic Hilbert Spaces

Sumit Kumar Sharma

Department of Mathematics, Kirori Mal College, University of Delhi, Delhi, India
E-mail: sumitkumarsharma@gmail.com

In this work, we define and study the concepts of g-Riesz bases and g-orthonormal bases for $\mathbb{H}^R(\mathfrak{Q})$ with respect to $\{\mathbb{H}_i^R(\mathfrak{Q})\}_{i\in\mathbb{N}}$, and provide necessary and sufficient conditions for them using sequences induced by sequences of right-bounded linear operators. Furthermore, a characterization of the dual g-frame via the induced sequence is presented. It is shown that a g-Riesz basis for $\mathbb{H}^R(\mathfrak{Q})$ with respect to $\{\mathbb{H}_i^R(\mathfrak{Q})\}_{i\in\mathbb{N}}$ is the image of a g-orthonormal basis under a bounded invertible linear operator. Additionally, we define alternate dual g-frames of a g-frame and give a characterization of dual g-frames. Finally, we introduce the notion of approximate dual g-frames and provide several characterizations of such frames.

Joint work with Nikhil Khanna and S. K. Kaushik.

Parameter Estimate by the Method of Least Squares of Relative Errors Under Non-Classical Assumptions and its Recurrent Form of Representation

Alexander Slabospitsky

Department of Applied Statistics, Taras Shevchenko National University of Kyiv
Kyiv, Ukraine
E-mail: alexsl@knu.ua

The problem of unknown parameter estimation for systems is considered in the case when the classical assumption that guarantees the uniqueness of parameter estimate may be violated.

To solve this parameter estimation problem, it is proposed to use the method of least squares of relative errors. This method is widely used in practice in economics, finance, stock market, physics, chemistry, computer vision and image processing, environmental and Earth sciences. Concentration of attention on minimization of relative error is often the main goal when building models for these areas.

The purpose of the research is to extend the explicit form of the presentation of the desired estimate for the selected class of systems to the situation when the classical assumption that guarantees the uniqueness of the parameter estimate can be violated. When obtaining it, it becomes necessary to use the Moore–Penrose matrix pseudo-inversion operator. And it was also necessary to propose a recurrent algorithm for recalculating the corresponding weighted sum of squares of the residuals.

Let us consider the above-mentioned problem of estimating the vector of unknown parameters α for such a regression model

$$y(k) = x^T(k)\alpha + e(k), \ \ k = \overline{1, N}, \ \ N \in \mathbb{N},$$

where $y(k)$, $e(k)$ – scalar observations of the dependent variable and model errors, respectively, $x(k)$ – known regressor vectors, $\alpha \in \mathbb{R}^p$.

Suppose that $y(k) \neq 0$, $k \in \mathbb{N}$. Then the set of all estimates by the method of least squares of relative errors for this regression model under non-classical assumption when this estimate may be not unique is defined as

$$\operatorname{Arg}\min_{\alpha} Q(\alpha, N),$$

where

$$Q(\alpha, N) = \sum_{k=1}^{N} \Big(\frac{e(k)}{y(k)}\Big)^2.$$

Using the Moore–Penrose matrix pseudo-inversion operator, for this class of objects, the explicit form of representing the estimate of the method of least squares of relative errors was transferred to the case when the classical assumption guaranteeing its uniqueness may be violated. The estimate $\widehat{\alpha}(N)$ with the lowest norm can be used as a unique estimate on the set of all these estimates. It is for this estimate $\widehat{\alpha}(N)$ that a recurrent form of representation was proposed. The advantage of the proposed recurrent estimation algorithm is the absence of the need to use either the Moore–Penrose matrix pseudo-inversion operation or even the usual matrix inversion operation. Recurrent form of representation for recalculating the corresponding weighted sum of squared residuals $q(N) = Q(\widehat{\alpha}(N), N)$ is also derived.

Lévy Processes on Quantum Groups

Adam Skalski[1], **Ami Viselter**[2]

[1]*Institute of Mathematics of the Polish Academy of Sciences, Warszawa, Poland*
E-mail: a.skalski@impan.pl

[2]*Department of Mathematics, University of Haifa, Haifa, Israel*
E-mail: aviselter@univ.haifa.ac.il

Lévy processes, i.e. continuous-time random walks on groups, are important probabilistic objects with applications that go as far as option pricing in mathematical finance. In this talk we discuss a natural extension of Lévy processes to quantum groups. These have been studied in depth in the compact setting, while we focus on the general, locally compact case. Applications include novel characterizations of approximation properties and insights into generating functionals.

References

[1] J. Kustermans and S. Vaes, Locally compact quantum groups. *Ann. Sci. École Norm. Sup. (4)* **33** (2000), no. 6, 837–934.

[2] A. Skalski and A. Viselter, Convolution semigroups on locally compact quantum groups and noncommutative Dirichlet forms. *J. Math. Pures Appl. (9)* **124** (2019), 59–105.

[3] A. Skalski and A. Viselter, Generating functionals for locally compact quantum groups. *Int. Math. Res. Not. IMRN* **2021**, no. 14, 10981–11009.

[4] A. Skalski and A. Viselter, Convolution semigroups on Rieffel deformations of locally compact quantum groups. *Lett. Math. Phys.* **114** (2024), no. 2, Paper no. 52, 39 pp.

Kneser Type Theorems on a Structure of Sets of Solutions of the Weighted Cauchy Problem for Nonlinear Singular Delayed Differential Equations

Zaza Sokhadze, **Irakli Nanava**

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mails: zaza.sokhadze@atsu.edu.ge; irakli.nanava@atsu.edu.ge

The weighted Cauchy problem is considered for nonlinear n-th order delayed differential equations having nonintegrable singularities with respect to a time variable. The conditions are found under which the set of solutions is compact and connected in the topology of the space C^{n-1}.

Refereces

[1] L. E. Bobisud and D. O'Regan, Existence of solutions to some singular initial value problems. *J. Math. Anal. Appl.* **133** (1988), no. 1, 214–230.

[2] Ph. Hartman, *Ordinary Differential Equations.* John Wiley & Sons, Inc., New York–London–Sydney, 1964.

[3] J. Kalas, Nonuniqueness theorem for a singular Cauchy problem. *Georgian Math. J.* **7** (2000), no. 2, 317–327.

[4] I. T. Kiguradze, On the Cauchy problem for ordinary differential equations with a singularity. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **37** (1965), 19–24.

[5] I. T. Kiguradze, On the question of variability of solutions of nonlinear differential equations. (Russian) *Differencial'nye Uravnenija* **1** (1965), 995–1006; translation in *Differ. Equations* **1** (1965), 995–1011.

[6] I. T. Kiguradze, On a singular problem of Cauchy–Nicoletti. *Ann. Mat. Pura Appl. (4)* **104** (1975), 151–175.

[7] I. T. Kiguradze, On the modified problem of Cauchy–Nicoletti. *Ann. Mat. Pura Appl. (4)* **104** (1975), 177–186.

[8] I. Kiguradze, *Some Singular Boundary Value Problems for Ordinary Differential Equations.* (Russian) Izdat. Tbilis. Univ., Tbilisi, 1975.

[9] I. Kiguradze, *The Initial Value Problem and Boundary Value Problems for Systems of Ordinary Differential Equations*, Vol. I. *Linear Theory.* (Russian) "Metsniereba", Tbilisi, 1997.

[10] I. Kiguradze and Z. Sokhadze, On the structure of the set of solutions of the weighted Cauchy problem for evolution singular functional-differential equations. *Fasc. Math.* no. 28 (1998), 71–92.

[1] T. I. Kiguradze, Estimates for the Cauchy function of linear singular differential equations and some of their applications. (Russian) *Differ. Uravn.* **46** (2010), no. 1, 29–46; translation in *Differ. Equ.* **46** (2010), no. 1, 30–47.

[12] Z. Sokhadze, On the structure of the set of solutions of the weighted Cauchy problem for high order evolution singular functional differential equations. *Mem. Differential Equations Math. Phys.* **25** (2002), 153–155.

[13] Z. Sokhadze, The weighted Cauchy problem for linear functional differential equations with strong singularities. *Georgian Math. J.* **18** (2011), no. 3, 577–586.

[14] Z. Sokhadze and B. Půža, Weighted Cauchy problem for nonlinear singular differential equations with deviating arguments. (Russian) *Differ. Uravn.* **49** (2013), no. 1, 33–45; translation in *Differ. Equ.* **49** (2013), no. 1, 32–44.

Application of Poisson's Equation in Image Processing Problems

Zaza Tabagari, Zaza Sanikidze, George Giorgobiani

Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia

E-mails: z.tabagari@gtu.ge; z.sanikidze@gtu.ge; giorgobiani.g@gtu.ge

Currently, image processing and analysis are primarily carried out using artificial intelligence technologies. Among these, Deep Learning, OCR, NLP and Computer Vision can be highlighted. Methods and algorithms have also been developed that perform informational or reconstructive analysis of images, aiming for their restoration or quality improvement, based on certain approaches such as geometric transformations or correction/filtration. In some cases, combined image processing algorithms, which represent a combination of these various technologies, can also yield significant effects.

The authors discuss one such mathematical approach, which is currently less studied, but a deep analysis of methodologies built upon it may lead to noticeable results. Specifically, this report reviews issues concerning the possibility of applying Poisson's equation in image processing, in conjunction with artificial intelligence technologies. The structural connections of Poisson's equation with individual image components are shown, as well as the influence of the solvability of this equation on characteristics of the visual object under study, such as texture, colors, and so on. Specific examples are provided for illustration.

Acknowledgments

The work was supported by the European Union s Grant Project GAIN (grant agreement # 101078950).

The Global Representation Formulas of Solutions for the Nonlinear Controlled Functional-Differential Equations with Several Delays and Continuous Initial Condition

Tamaz Tadumadze[1,2], **Nika Gorgodze**[3]

[1]*Department of Mathematics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[2]*Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: tamaz.tadumadze@tsu.ge

[3]*Department of Mathematics, Akaki Tsereteli Kutaisi State University, Kutaisi, Georgia*
E-mail: nika.gorgodze@atsu.edu.ge

The global analytic representation formulas of solutions are proved for the nonlinear controlled functional-differential equations with several delays in the phase coordinates and controls. In the formulas the effects of perturbations of the initial moment, the initial function, the control function, delays parameters containing in the phase coordinates as well as the effect of the continuous initial condition are revealed. The representation formula of solution is used in the investigation of optimization problems, in finding of an approximate solution of the perturbed functional-differential equation and to carry out a sensitivity analysis of mathematical models.

Uniqueness in Some Inverse Problems of Potential Theory

Zurab Tediashvili

Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
E-mail: z.tediashvili@gtu.ge

Let $K(x,y)$ be the fundamental solution to the Laplace operator in R^n $(n=2,3)$ and $V_\Omega(\mu) = \int\limits_\Omega K(x,y)\mu(y)\,dy$ be a Newtonian volume potential with a density μ.

Uniqueness problem

Let Ω_1 and Ω_2 be two bounded domains of R^n and μ be some density function defined in $\Omega_1 \cup \Omega_2$. Moreover, let $V_{\Omega_1}(\mu) = V_{\Omega_2}(\mu)$ for all $x \in R^n \setminus (\overline{\Omega_1 \cup \Omega_2})$. By this condition we have to define the location of the domains Ω_1 and Ω_2 with respect to each other.

In the general case, this problem is not uniquely solvable. Under some additional restrictions, we prove the following uniqueness theorems.

Uniqueness theorems

Let $\Omega_\infty^{(1,2)}$ be an unbounded connected component of $R^n \setminus (\overline{\Omega_1 \cup \Omega_2})$ and $\Omega_0 = R^n \setminus \overline{\Omega_\infty^{(1,2)}}$.

Theorem 1 *Let Ω_1 and Ω_2 be two bounded piecewise smooth domains of R^2 and let there exist a point $x_0 \in \partial\Omega_\infty^{(1,2)}$ and a number $r > 0$ such that $\sigma_0^r \cap \overline{\Omega_1} = \varnothing$, where $\sigma_0^r = \{x : \ |x - x_0| < r\} \cap \partial\Omega_\infty^{(1,2)}$. Assume that $\sigma_0^r = L \cup \{x_0\} \cup L'$ where L and L' are line segments of some different lines. If $\mu \in C^1(\overline{\Omega_1 \cup \Omega_2})$, $\frac{\partial\mu}{\partial L} = 0$ and $\mu(x_0) \neq 0$, then the potentials $V_{\Omega_1}(\mu)$ and $V_{\Omega_2}(\mu)$ do not coincide on $\Omega_\infty^{(1,2)}$.*

Theorem 2 *Let Ω_1 and Ω_2 be two bounded piecewise smooth domains of R^3 and let there exist a point $x_0 \in \partial\Omega_\infty^{(1,2)}$ and a number $r > 0$ such that $\sigma_0^r \cap \overline{\Omega_1} = \varnothing$, where $\sigma_0^r = \{x : \ |x - x_0| < r\} \cap \partial\Omega_\infty^{(1,2)}$. Assume that $\sigma_0^r = P \cup \{x_0\} \cup P'$ where P and P' are parts of some different planes. If $\mu \in C^1(\overline{\Omega_1 \cup \Omega_2})$, $\frac{\partial\mu}{\partial L} = 0$ for a line L parallel to P and $\mu(x_0) \neq 0$, then the potentials $V_{\Omega_1}(\mu)$ and $V_{\Omega_2}(\mu)$ do not coincide on $\Omega_\infty^{(1,2)}$.*

References

[1] A. I. Prilepko, Inverse problems of potential theory (elliptic, parabolic, hyperbolic equation and transport equations). (Russian) *Mat. Zametki* **14** (1973), 755–767.

[2] Z. Tediashvili, Uniqueness questions in the inverse problems of the potential theory. *Bull. Georgian Acad. Sci.* **161** (2000), no. 3, 391–394.

Sharp Strong Convergence Result of the Two-Dimensional Walsh–Fourier Series in Martingale Hardy Spaces

George Tephnadze

Viktor Kupradze Institute of Mathematics, The University of Georgia
Tbilisi, Georgia
E-mail: g.tephnadze@ug.edu.ge

Unlike the classical theory of Fourier series which deals with decomposition of a function into sinusoidal waves the Walsh functions are rectangular waves. Some important steps in early development can be found in the book by F. Schipp, W. R. Wade, P. Simon and J. Pál [1] from 1990. The research continued intensively also after this. Some of the most important steps in these developments are presented in the recent book by L. E. Persson, G. Tephnadze and F. Weisz [2] from 2022.

In [4] Weisz investigated strong convergence of partial sums $S_{m,n}$ of the two-dimensional Walsh–Fourier series in the martingale Hardy spaces, but under the condition when $2^{-\alpha} < m/n \leq 2^{\alpha}$. This talk is devoted to investigate strong convergence of the two-dimensional Walsh–Fourier series in the martingale Hardy spaces $H_p^{\square}(G^2)$ for $0 < p < 1$, without any restriction on the indices (for details see [3]). Moreover, sharpness of this result is also showed.

References

[1] L.-E. Persson, L.-E. Persson, G. Tephnadze and F. Weisz, *Martingale Hardy Spaces and Summability of One-Dimensional Vilenkin–Fourier Series.* Birkhäuser/Springer, Cham, 2022.

[2] F. Schipp, W. R. Wade, P. Simon and J. Pál, *Walsh series. An Introduction to Dyadic Harmonic Analysis.* Adam Hilger., Bristol etc., 1990.

[3] G. Tephnadze, Sharp strong convergence result of the two-dimensional Walsh–Fourier series in martingale Hardy spaces. *Anal. Math. Phys.* **15** (2025), no. 3, Paper no. 78, 23 pp.

[4] F. Weisz, Strong convergence theorems for two-parameter Walsh–Fourier and trigonometric-Fourier series. *Studia Math.* **117** (1996), no. 2, 173–194.

On Absolute Convergence of Blaschke–Djerbashyan Canonical Product on the Boundary of the Unit Disk

Giorgi Tetvadze, Lili Tetvadze, Iuri Tvalodze, Lamara Tsibadze

Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: giorgi.tetvadze@atsu.edu.ge

Blaschke–Djerbashyan canonical product has the following form:

$$\mathscr{B}_{p+1}(z,(a_n)) = z^{\lambda}\prod_{n=1}^{\infty}\Big(1-\frac{1-|a_n|^2}{1-\overline{a_n}z}\Big)\exp\Big(\sum_{k=1}^{p}\frac{1}{k}\Big(\frac{1-|a_n|^2}{1-\overline{a_n}z}\Big)^k\Big),$$

where $\lambda+1$ and p are natural numbers $0<|a_n|\le|a_{n+1}|<1$, $\lim\limits_{n\to\infty}|a_n|=1$,

$$\sum_{n=1}^{+\infty}(1-|a_n|)^{p+1}<+\infty \quad [1].$$

Theorem 1 *In order for Blaschke–Djerbashyan canonical product to be absolutely convergent at a point $e^{i\theta}$, it is necessary and sufficient that*

$$\sum_{n=1}^{+\infty}\Big(\frac{1-|a_n|}{|e^{i\theta}-a_n|}\Big)^{p+1}<+\infty.$$

Theorem 2 *If Blaschke–Djerbashyan canonical product is absolutely convergent at a point $z=e^{i\theta}$, then there is an angular limit at this point and the following equality holds*

$$\lim_{z\to e^{i\theta}}\mathscr{B}_{p+1}(z,(a_n)) = \mathscr{B}_{p+1}(e^{i\theta},(a_n)) \neq 0;+\infty.$$

References

[1] G. Tetvadze, L. Tetvadze and L. Tsibadze, On boundary properties of the boundary values of Blaschke-Djerbashian Canonical product. *Moambe*, 2021, 195–201.

A Probabilistic Extension of τSR Logic: Reasoning with Uncertainty and Tau Terms

Lali Tibua

Ilia Vekua Institute of Applied Mathematics of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia

E-mail: ltibua@gmail.com

The τSR logic extends classical logical frameworks with epsilon terms, enabling reasoning about existence without explicit witnesses and supporting selective quantification. However, in many real-world applications, particularly in artificial intelligence, uncertain databases, and knowledge representation, deterministic reasoning is insufficient. In this presentation, we propose a probabilistic extension of τSR logic that integrates probabilistic semantics with epsilon-based term selection. By interpreting epsilon terms as probabilistic selectors and incorporating probability measures over the domain, we enrich the expressive power of τSR to handle quantified statements under uncertainty. This extension preserves essential logical properties while allowing fine-grained reasoning about likelihood and belief. We outline the formal semantics, illustrate its applicability through examples, and discuss connections with probabilistic first-order logic and potential completeness theorems. The resulting framework bridges the gap between logical abstraction and probabilistic reasoning, offering a principled foundation for uncertain inference.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia under the project # FR-22-4254.

Analysis of Selected Problems Used in the Georgian National Exams

Davit Tsamalashvili, **Lamara Kurchishvili**

Faculty of Psychology and Educational Sciences, Institute of Teacher Education, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia

E-mails: davit.tsamalashvili@tsu.ge; lkurchishvili@hotmail.com

National university entrance exams have been held in Georgia since 2005. From that same year, mathematics has been included in the list of elective subjects. It is chosen by those applicants for whom mathematics is a core subject required for admission to their desired faculty or program. Over the years, the structure of the mathematics test has undergone several modifications. Likewise, the mathematics syllabus for the National Exams has also changed multiple times. However, test questions from different years are often similar in structure.

The principle of task distribution is as follows:

- One-point questions (accompanied by four or five answer choices, for which the applicant is required to select only one specific answer).
- Questions where applicants must provide a full written solution in the answer sheet. These are graded with either 0 or a full positive integer score, depending on the quality of the applicant's work.

Over time, a large bank of questions has accumulated, with many being similar to each other. Some stand out due to their originality. In online mathematics groups, certain problems often become subjects of discussion and debate, with group members even competing in coming up with elegant solutions.

Some one-point questions are often critically evaluated – especially those whose solutions clearly exceed the expected difficulty level of such questions. These may require several non-trivial steps, and a single mistake in the final step can result in the loss of the entire point. Therefore, the psychological impact of the test design on applicants must be taken into account.

To perform well, applicants must not only possess factual mathematical knowledge but also appropriate logical reasoning skills. It is also important that a certain number of questions be solvable using only formal knowledge.

Given this, our aim is to analyze the questions used in the mathematics sections of the Georgian National Exams and the common mistakes made by applicants.

References

[1] Unified National Exams. *Mathematics Tests*, 2005–2025.

A Method of Solution of an Initial and Time Dependent Boundary Values Problem for a Dynamic Beam Kirchhoff Type Nonlinear Differential Equation

Zviad Tsiklauri

Georgian Technical University, Tbilisi, Georgia
E-mail: zviad_tsiklauri@yahoo.com

An initial boundary value problem for Kirchhoff type beam nonlinear integro-differential equation is considered in case of inhomogeneous boundary conditions. The problem is reduced to a solution of some more complex equation than the original one, but when the homogeneous boundary conditions hold and the type of initial conditions remain the same. To find an approximate solution of the received problem, a numerical method that is a combination of the Galerkin method, Crank–Nicolson difference scheme and Jacobi–Cardano iteration process is used. Outlines how to obtain an approximate solution of the original problem using the solution of the auxiliary problem.

Complete Semigroups of a Binary Relations Defined by Semilattices Type R

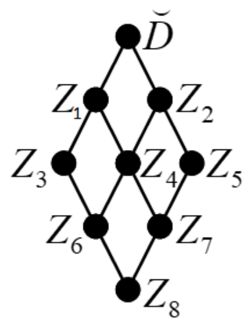
Nino Tsinaridze, Omar Givradze, Giuli Tavdgiridze, Guladi Partenadze

Department of Mathematics, Batumi Shota Rustaveli State University
Batumi, Georgia
E-mails: n.tsinaridze@bsu.edu.ge; omar.givradze@bsu.edu.ge; g.tavdgiridze@bsu.edu.ge; guladi.phartenadze@bsu.edu.ge

We say that the finite semilattes are type R, if the number completeness sources is equal or great to the number of a basic sources.

We study the right units, the idempotent and Their calculation of the complete semigroup of binary relations defined by X-semilattice unions of the class type R.

Let $D = \{\breve{D}, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8\}$ be some X-semilattice of unions and $C(D) = \{P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$ be the family of sets of pairwise nonintersecting subsets of the set. If φ is a mapping of the semilattice D on the family of sets $C(D)$ which satisfies the condition



$$\varphi = \begin{pmatrix} \breve{D} & Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 & Z_7 & Z_8 \\ P_0 & P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 \end{pmatrix},$$

where P_0, P_4, P_6, P_7, P_8 are the completeness sources and P_1, P_2, P_3, P_5 is basic sources $(5 > 4)$, i.e. D is finite semilattes type R.

If D be the given semilattice of unions, then the semigroup $B_X(D)$ always has right unit.

References

[1] Ya. Diasamidze and Sh. Makharadze, *Complete Semigroups of Binary Relations.* Kriter, Istambul Turkey, 2013.

[2] G. Tavdgiridze, Ya. Diasamidze and O. Givradze. Idempotent elements of the semigroups $B_X(D)$ defined by semilattices of the class $\Sigma_3(X, 8)$, when $Z_7 \neq \varnothing$. *Applied Mathematics* **7** (2016), 193–218.

Question-Answering System Based on AI and NLP Models

Sergo Tsiramua[1], **Hamlet Meladze**[2], **Tinatin Davitashvili**[3], **Davit Bitmalkishev**[1], **Tatia Elbakidze**[1]

[1]*IT Department, School of Science and Technologies, Institute of Informatics, University of Georgia, Tbilisi, Georgia*

E-mails: s.tsiramua@ug.edu.ge; davit.bitmalkishev@ug.edu.ge; tatia.elbakidze@ug.edu.ge

[2]*Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*

E-mail: h.meladze@gtu.ge

[3]*The Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*

E-mail: tinatin.davitashvili@tsu.ge

This paper discusses the technologies of creating an interactive question-answering system using Artificial Intelligence (AI) and Natural Language Processing (NLP) models. The NLP system, presented as a web application, allows the system administrator to upload PDF files in advance and create a database of textual documents on various topics. Subsequently, any user will be able to ask questions to the application based on the content of the documents and receive relevant answers both in text and voice format. To achieve this, the uploaded documents need to be analyzed, transformed into vectors, processed, and used to generate relevant answers. This requires the integration of several components: creating a Flask web application, ensuring the processing of PDF documents, generating embeddings and storing them in a database, and developing the question-answering module. The application will use the LangChain framework, Ollama language models, and ChromaDB for efficient information retrieval. It will also incorporate a Word2Vec model, cosine similarity algorithm, and the generative GPT-4o model. The NLP system will support voice-based interactions — users will be able to ask questions and receive answers by voice (including in Georgian), using Google and Microsoft AI services.

The goal of developing this NLP technology is to simplify information retrieval from large documents and ensure fast, accurate, and context-based response generation. Moreover, it aims to be adaptable to different needs and technological requirements. As a result, it offers a user-assisted tool that enables rapid access to the information needed from large collections of documents. This system can be applied in various fields with extensive textual databases, including education, scientific research, law, and more.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia. Grant # SC-24-753.

Machine Learning Methods for Adaptive Reconfigurable Systems Based on Multifunctional Elements

Sergo Tsiramua[1], **Hamlet Meladze**[2], **Tinatin Davitashvili**[3], **George Tsiramua**[4]

[1] *IT Department, School of Science and Technologies, Institute of Informatics, University of Georgia, Tbilisi, Georgia*
E-mail: s.tsiramua@ug.edu.ge

[2] *Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University Tbilisi, Georgia*
E-mail: h.meladze@gtu.ge

[3] *The Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: tinatin.davitashvili@tsu.ge

[4] *Bussiness Technology Department of Toyota Caucasus, Tbilisi, Georgia*
E-mail: gtsiramua@bilikiapp.com

Adaptive reconfigurable systems composed of multifunctional elements represent a new class of intelligent architectures capable of dynamic structural and functional transformation in response to changing tasks, environments, or fault conditions. This paper explores the application of modern machine learning methods to enhance adaptability, reliability, and control efficiency in such systems. We investigate a range of learning approaches, including supervised learning for fault classification, reinforcement learning for dynamic reconfiguration, and sequence modeling techniques such as Hidden Markov Models (HMM) and Long Short-Term Memory (LSTM) networks for operational state prediction. Emphasis is placed on how these methods support intelligent decision-making under uncertainty, optimize system performance, and enable real-time adaptation. Case studies on reconfigurable robotic platforms and modular control systems demonstrate the effectiveness of the proposed models. The results highlight the potential of machine learning to drive autonomy and resilience in next-generation multifunctional reconfigurable systems.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia. Grant # SC-24-753.

Handling Standard and Non-Standard Mathematical Problems: A Comparative Analysis on GPT and Gemini Models

Soso Tsotniashvili[1], **Andria Kelekhsaevi**[2]

[1]*Department of Mathematics, Gori State University, Gori, Georgia*
E-mail: stsotniashvili@gmail.com

[2]*Student of Caucasus School of Technology, Caucasus University, Tbilisi, Georgia*
E-mail: andria11kelekhsaev@gmail.com

This study examines how large language models, specifically GPT and Gemini, handle standard versus non-standard mathematical tasks. We reviewed several simple examples, observing that both systems resolve routine calculations and elementary problems reliably. However, when presented with ill-posed integrals, unconventional equations, or ambiguous logical constraints, GPT demonstrates a more robust capacity to identify inconsistencies and appropriately qualify or reject invalid tasks. This suggests a notable divergence in the reasoning patterns of the models: GPT's chain-of-thought reflects stronger internal consistency checks, while Gemini is more prone to offer plausible yet flawed solutions when confronted with non-standard cases.

References

[1] D. Lay, S. Lay and J. McDonald, *Linear Algebra and Its Applications.* Pearson, 2021.

[2] R. Larson, *Elementary Linear Algebra* (8th edition). Cengage Learning, 2016.

[3] S. S. Epp, *Discrete Mathematics with Applications* (Metric version). Cengage Learning, 2021.

[4] K. H. Rosen and Dr. K. Krithivasan, *Discrete Mathematics and its Applications (SIE)* (8th Edition). McGraw-Hill, 2021.

[5] G. Thomas, M. Weir, J. Hass and C. Heil, *Thomas' Calculus* (15th edition in SI units). Pearson Education, 2024.

[6] R. Barnett, M. Ziegler and K. Byleen, *Calculus for Business, Economics, Life Sciences, and Social Sciences* (14th edition). Pearson, Prentice Hall, 2019.

Magnetohydrodynamic Flow of Fluid in a Circular Pipe with Account of the Volume Sources and Sink Mass

Varden Tsutskiridze[1], **Mariam Tsutskiridze**[2,3]

[1] *Department of Mathematics, Georgian Technical University, Tbilisi, Georgia*
E-mail: btsutskirid@yahoo.com

[2] *MS of Caucasus University, Tbilisi, Georgia*
[3] *MS of Grenoble University, Grenoble, France*
E-mail: Tsutskiridzegrenobleem@gmail.com

The subject of this work is the study of the stationary motion of an electrically conductive viscous incompressible fluid in a round pipe with stationary volumetric mass sources placed in an external magnetic field perpendicular to the axis of the pipe. Is accepted that the motion is created by a constant longitudinal pressure drop applied at the initial moment of time, although it is not difficult to generalize the problem both to the case of the presence of an initial distribution of velocities and to the case of moving walls.

Bounded Linear Functionals on Banach Spaces

Giorgi Tutberidze[1], **Vakhtang Tsagareishvili**[2]

[1] *Viktor Kupradze Institute of Mathematics, The University of Georgia, Tbilisi, Georgia*
E-mail: g.tutberidze@ug.edu.ge

[2] *Department of Mathematics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mail: cagare@ymail.com

S. Banach [1] demonstrated that strong differentiability conditions are not necessarily sufficient to guarantee almost everywhere (a.e.) convergence of Fourier series with respect to arbitrary orthonormal systems (ONS). On the other hand, it is well established that the Menshov-Rademacher Theorem provides a sufficient criterion for the a.e. convergence of orthonormal series.

The main aim of this talk is to explore the convergence of the general Fourier series in the $Lip\alpha$ class, where $0 < \alpha < 1$. Specifically, we investigate the conditions on the functions φn of an orthonormal system (φn) under which the variational Fourier series (with respect to general orthonormal systems) of functions from the BV class converges almost everywhere on $[0, 1]$. Additionally, we establish specific conditions for the functions of the orthonormal system that ensure the convergence of Fourier series for functions in the $Lip\alpha$ class. It is shown that these conditions are the best possible in a certain sense. These results not only deepen our understanding of the convergence behavior of Fourier series but also provide valuable insights into the interplay between various function classes and orthonormal systems.

References

[1] S. Banach, Sur la divergence des séries orthogonales. (French) *Studia Math.* **9** (1940), 139–155.

[2] L.-E. Persson, V. Tsagareishvili and G. Tutberidze, Properties of sequence of linear functionals on BV with applications. *Nonlinear Stud.* **30** (2023), no. 4, 1319–1328.

[3] V. Tsagareishvili and G. Tutberidze, Some problems of convergence of general Fourier series. (Russian) *Izv. Nats. Akad. Nauk Armenii Mat.* **57** (2022), no. 6, 70–80; translation in *J. Contemp. Math. Anal.* **57** (2022), no. 6, 369–379.

[4] V. Tsagareishvili, G. Tutberidze and G. Cagareishvili, Unconditional convergence of general Fourier series. *Publicationes Mathematicae Debrecen* (to appear).

Restriction of Fourier Multipliers on Orlicz Spaces

Rüya Üster

Department of Mathematics, Faculty of Science, İstanbul University, İstanbul, Türkiye
E-mail: ruya.uster@istanbul.edu.tr

In this talk we shall deal with Orlicz spaces on general locally compact abelian groups. Our interest is to deal with transference and restriction for bilinear multipliers acting on Orlicz spaces defined on different locally compact abelian groups such as $\mathbb{R} \times \mathbb{R}$, $\mathbf{D} \times \mathbf{D}$, $\mathbb{Z} \times \mathbb{Z}$ and $\mathbb{T} \times \mathbb{T}$.

Joint work with Oscar Blasco from University of Valencia.

Acknowledgments

The work was supported by the The Scientific and Technological Research Council of Turkey" TÜBİTAK-BİDEB.

References

[1] O. Blasco, Notes in transference of bilinear multipliers. *Advanced courses of mathematical analysis III*, 28–38, *World Sci. Publ., Hackensack, NJ*, 2008.

[2] O. Blasco and R. Üster, Transference and restriction of Fourier multipliers on Orlicz spaces. *Math. Nachr.* **296** (2023), no. 12, 5400–5425.

[3] M. M. Rao and Z. D. Ren, *Theory of Orlicz Spaces.* Monographs and Textbooks in Pure and Applied Mathematics, 146. Marcel Dekker, Inc., New York, 1991.

On the Construction of a Refined Theory for Nonlinear Models of Solid Mechanics in the Case of Thin-Walled Structures

Tamaz Vashakmadze

Ilia Vekua Institute of Applied Mathematics
of Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mail: tamazvashakmadze@gmail.com

We have constructed a class of "refined theories" of continuum power for plates. In the case of bending, such an amount was determined, on the one hand, by the introduction of a large number of simplifying hypotheses of a geometric and physical nature, and, on the other hand, by the methodology developed in [3], when the class of models is also represented by a parameter dependent on two variables, the selection of which also yields known theories. In the case when the elastic thin-walled structure represents the shell, the number of simplifying hypotheses is rather scarce (see [1, 4]). Such a smallness of theories is due to the fact that the bending and expansion-compression problems for shells are non- splitting (as is the case for anisotropic plates of a certain subclass) and the difficulty of defining simplifying hypotheses. In this report, using the method presented in [3] and some results from [2, 4], a class of parameter-dependent two-dimensional models will be presented not only for elastic shells.

In the second part of the report, refined theories for thin-walled structures will be presented, when the initial system of differential equations and boundary conditions are replaced by the essentially nonlinear Piola–Trousdell–Noll model.

References

[1] Ph. G. Ciarlet, *Mathematical Elasticity.* Vol. III. *Theory of Shells.* Studies in Mathematics and its Applications, 29. North-Holland Publishing Co., Amsterdam, 2000.

[2] S. Lukasiewicz, *Local Loads in Plates and Shells.* Springer, Dordrecht, 1979.

[3] T. S. Vashakmadze, *The Theory of Anisotropic Elastic Plates.* Mathematics and its Applications, 476. Kluwer Academic Publishers, Dordrecht, 1999.

[4] I. N. Vekua, *Shell Theory: General Methods of Construction.* Monographs, Advanced Texts and Surveys in Pure and Applied Mathematics, 25. Pitman (Advanced Publishing Program), Boston, MA; distributed by John Wiley & Sons, Inc., New York, 1985.

On the Differentiation of Integrals by a Product of Density Bases

Sandro Vatsadze

Kutaisi International University, School of Mathematics, Kutaisi, Georgia
E-mail: vatsadze.sandro@kiu.edu.ge

Properties of density differentiation bases have been studied by various authors. Significant work on the topic has been done by, among others, Busemann and Feller [1], Oniani [5], [6], Hagelstein and Parissis [2], Hagelstein and Stokolos [3], and Japaridze [4].

We show that the product of density differentiation bases $B_1, \dots, B_k$ is a density basis as well, provided the factor bases B_j are measurable in the sense that the values of the truncated maximal operators associated with them are measurable for characteristic functions of finite unions of intervals.

The talk is bases on a joint research with Giorgi Oniani.

References

[1] H. Busemann, Areas in affine spaces. III. The integral geometry of affine area. *Rend. Circ. Mat. Palermo (2)* **9** (1960), 226–242.

[2] P. Hagelstein and I. Parissis, Tauberian constants associated to centered translation invariant density bases. *Fund. Math.* **243** (2018), no. 2, 169–177.

[3] P. Hagelstein and A. Stokolos, Tauberian conditions for geometric maximal operators. *Trans. Amer. Math. Soc.* **361** (2009), no. 6, 3031–3040.

[4] I. Japaridze, Density property for a product of translation invariant density differentiation bases. *Trans. A. Razmadze Math. Inst.* **177** (2023), no. 1, 53–57.

[5] G. G. Oniani, Some statements connected with the theory of differentiation of integrals. (Russian) *Soobshch. Akad. Nauk Gruzii* **152** (1995), no. 1, 49–53 (1996).

[6] G. Oniani, Approximation and transfer of properties between net-type and translation invariant convex differentiation bases. *Colloq. Math.* **176** (2024), no. 2, 159–170.

On the Different Definitions of Some Mathematical Notions

Teimuraz Vepkhvadze

Department of Mathematics, Ivane Javakhisvili Tbilisi State University, Tbilisi, Georgia
E-mail: t-vepkhvadze@ hotmail.com

The comparative analysis of some mathematical notions are given. The advantage of using one definition over other definitions in secondary school is substantiated.

The Method of Probabilistic Solution for 3D Dirichlet Generalized and Classical Harmonic Problems in Some Finite Bodies with Cavities

Mamuli Zakradze[1], **Zaza Tabagari**[1], **Nana Koblishvili**[1], **Hamlet Meladze**[1], **Tinatin Davitashvili**[2]

[1] *Department of Computational Methods, Muskhelishvili Institute of the Computational Mathematics of Georgian Technical University, Tbilisi, Georgia*
E-mails: m.zakradze@gtu.ge; z.tabagari@gtu.ge; n.koblishvili@gtu.ge; h.meladze@gtu.ge

[2] *Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University Tbilisi, Georgia*
E-mail: tinatin.davitashvili@tsu.ge

In this paper, we investigate the application of the method of probabilistic solutions(MPS) to numerical solving of Dirichlet generalized and classical harmonic problems in some finite bodies with cavities. The term "generalized" indicates that a boundary function has a finite number of first kind discontinuity curves. The suggested algorithm for numerical solution of boundary problems consists of the following stages:

(a) application of the MPS, which in its turn is based on a computer modeling of the Wiener process;

(b) finding the intersection point of the trajectory of the simulated Wiener process and the surface of the problem domain;

(c) developing a code for numerical implementation and verifying the accuracy of the results;

(d) finding of the probabilistic solution of generalized problems at any fixed points of the considered domains. The algorithm does not require approximation of a boundary function. To illustrate the effectiveness and simplicity of the proposed method five examples are considered. Numerical results are presented and discussed.

On Replacing the Hilbert Space with a Strict Fréchet–Hilbert Space in the Mathematical Model of Quantum Mechanics

David Zarnadze[1], **Soso Tsotniashvili**[2]

[1]*Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia*
E-mail: d.zarnadze@gtu.ge

[2]*Department of Mathematics, Gori State University, Gori, Georgia*
E-mail: stsotniashvili@gmail.com

In this work, we propose a modification to the mathematical model of quantum mechanics by replacing the traditional Hilbert space $L^2(\mathbb{R})$ with the strict Fréchet–Hilbert space $L^2_{\mathrm{loc}}(\mathbb{R})$, obtained by weakening the topology. This replacement addresses a known limitation noted in [1, Section 3.4, p. 52], namely that the conventional quantum Hilbert space does not contain polynomials and other important elementary functions.

We present the strict Fréchet–Hilbert space as the projective limit of an increasing sequence of Hilbert subspaces, and its strong dual space as the inductive limit of the same sequence [5, Section 4.2.4]. Using these representations, we extend the main operators in quantum mechanics – including those for energy, position, momentum, creation, and annihilation – to act on the strict Fréchet–Hilbert space.

We prove that the canonical commutation relations between the position and momentum operators, as well as between creation and annihilation operators, remain valid in the $L^2_{\mathrm{loc}}(\mathbb{R})$ setting [2], [4]. The theory of self-adjoint operators is also generalized to Fréchet–Hilbert spaces, and the topological and geometric properties of the strict Fréchet–Hilbert space are investigated [3], see also [5, Section 2.4.2]. We identify a broad class of subspaces that admit topological complements and prove the existence of interpolating splines for non-adaptive information of cardinality 1.

References

[1] B. C. Hall, *Quantum Theory for Mathematicians.* Graduate Texts in Mathematics, 267. Springer, New York, 2013.

[2] S. Tsotniashvili, On the extension of selfadjoint operators from Hilbert spaces to Fréchet–Hilbert spaces. *Bull. Georgian Acad. Sci.* **153** (1996), 338–341.

[3] D. N. Zarnadze, Some topological and geometric properties of Fréchet–Hilbert spaces. (Russian) *Izv. Ross. Akad. Nauk Ser. Mat.* **56** (1992), no. 5, 1001–1020; translation in *Russian Acad. Sci. Izv. Math.* **41** (1993), no. 2, 273–288.

[4] D. Zarnadze and S. A. Tsotniashvili, *A Generalization of the Canonical Commutative Relation in the Quantum Fréchet–Hilbert Space.* Book of Abstracts XI International Conference of the Georgian Mathematical Union (August 23-28, 2021, Batumi), p. 164; `https://gmu.gtu.ge/Batumi2021/Conference_Batumi_2021+.pdf`.

[5] D. Zarnadze and D. Ugulava, *Central Spline Algorithms in the Hilbert and Fréchet Spaces of Orbits.* SRNSFG, 2024;
`https://rustaveli.org.ge/geo/tsignebi/Central-Spline-Algorithms-in-the-Hilbert-and-Frechet-Spaces-of-Orbits`.

Consistent Estimators and Consistent Criteria for Parameters of Linear Regression Models

Zurab Zerakidze, **Lela Aleksidze**, **Laura Eliauri**

Gori State Teaching University, Gori, Georgia

E-mails: zura.zerakidze@mail.ru; lelaaleksidze@gmail.com; lauraeliauri@gmail.com

Most econometric models are described by regression equations. We consider statistical relationships, in particular, we consider the linear regression model $y = \alpha + \beta x + u$, where y is represented by non-random component $\alpha + \beta x$ and a random component u. In addition, u is normally distributed with a zero mean and unknown variance σ_u^2. In this paper, we consider Gaussian statistical structures to estimate the unknown parameters $\alpha, \beta, \sigma_u^2$. We consider two types of statistical structure: 1) the Gaussian statistical structures with a zero mean and unknown variance σ_u^2 ($\sigma_u^2 \in \Theta_+$, where Θ_+ is a set of nonnegative rational numbers); 2) the Gaussian statistical structures with known variance σ_u^2 and unknown mathematical expectations α, β. For strongly separable Gaussian statistical structures, consistent estimates and a consistent criterion are constructed for σ_u^2, $\sigma_u^2 \in \Theta_+$. The necessary and sufficient conditions for the existence of consistent estimators and consistent criterion of parameters σ_u^2 are given for Gaussian statistical structures.

Suppose that the linear regression model $y = \alpha + \beta x + u$, the values $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ of the variables x and y for the sample size n are known, and we have selected a consistent estimate and consistent criterion of σ_u^2.

Theorem *Let* $\sum\limits_{i=1}^{n} (x_i - \overline{x})^2 \neq 0$, *then there is an estimate of the unknown parameter* β*:*

$$b = \frac{cov(x,y)}{Var\, x}$$

and an estimate of the unknown parameter α*:*

$$a = \overline{y} - b\overline{x}.$$

The estimates a *and* b *of the parameters* α *and* β are an unbiased, weakly consistent and

$$a \sim N(\alpha, \sigma_a^2), \ \ b \sim N(\beta, \sigma_b^2), \ \ \sigma_a^2 = \frac{\sigma_u^2}{n} \left(1 + \frac{\overline{x}^2}{Var\, x}\right), \ \ \sigma_b^2 = \sigma_u^2 \left(\frac{1}{n\, var\, x}\right).$$

References

[1] L. Aleksidze and L. Eliauri, The "Z-CRITERIA" of hypothesis testing in Hilbert space of measures for Haar statistical structure. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **37** (2024), 1–7.

[2] T. Chkonia and M. Tkebuchava, The "Z-CRITERIA" of hypothesis testing for exponential statistical structure in Banach space of measures. *Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math.* **37** (2024), 8–14.

[3] Z. Zerakidze, Generalization criteria of Neyman–Pearson. *Proceedings of the International Scientific Conference "Information Technologies"*, Georgian Technical University, Tbilisi, 2008, 22–24.

Describing Real-Life Situations Through Mathematical Models at the Primary Level

Manana Zivzivadze-Nikoleisvili

Faculty of Education, Akaki Tsereteli State University, Kutaisi, Georgia
E-mail: manana.zivzivadze@atsu.edu.ge

One of the key objectives in teaching mathematics at the primary level is to help students understand that mathematics is not merely an abstract system of symbols, but a powerful tool for representing and analyzing real-life situations and events. Students achieve this through mathematical modeling—when they describe real problems, drawn from personal experience or presented scenarios, using mathematical language and structures.

This paper discusses what constitutes a mathematical model at the primary level, and how real-life situations can be translated into mathematical problem-solving models.

The work emphasizes and supports with examples that through mathematical modeling, students learn to identify problems and convert them into mathematical structures; they comprehend the connection between mathematics and everyday life; they develop skills in reasoning, drawing conclusions, and mathematical argumentation; and they cultivate creative and analytical thinking.

F-Topology on Integers

Fahed Zulfeqarr

Self-employed
E-mail: fahedzul@gmail.com

The present piece of work introduces a new topology on integers, referred as F-topology. In contrast to Furstenberg topology on integers, the F-topology is hyperconnected, compact and does not even satisfy T_0 axiom whereas the former one is totally disconnected, non-compact and satisfies T_3 axiom. Integers are studied under this F-topology and examined for their properties. Furthermore a topological characterization of infinitude of primes is also presented.

Mini-Symposium: Applied Data Science

Abstracts of Talks

A Rule-Based Text-to-Speech (TTS) System Adapted for the Georgian Language

Maia Archuadze, **Manana Khachidze, Nika Iakobashvili, Mariam Luarsabishvili, Nino Kasabishvili, Nikoloz Tsiskadze**

Computer Science Department, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia

E-mails: ail: maia.archuadze@tsu.ge; manana.khachidze@tsu.ge
mariam.luarsabishvili494@ens.tsu.edu.ge; nino.kasabishvili994@ens.tsu.edu.ge;
nikoloz.tsiskadze308@ens.tsu.edu.ge

Text-to-speech (TTS) technology has become a vital component of modern human-computer interaction, enabling the automated transformation of written text into spoken language. While substantial advancements have been made in developing TTS systems for widely spoken and well-resourced languages, under-resourced languages such as Georgian remain significantly underrepresented. The Georgian language presents unique phonetic, morphological, and syntactic characteristics that pose specific challenges, necessitating the development of language-tailored solutions [1].

Currently, there is no open and fully-fledged text-to-speech system for the Georgian language in Georgia. Existing systems are either closed source and functionally limited, or do not reflect the full range of phonetic and prosodic features of the Georgian language. The project involves creating a database of Georgian phonemes using the necessary linguistic rules, which ensures the accuracy and naturalness of the audio produced by our system.

This paper presents a prototype of a rule-based, syllable-based TTS system designed to address the linguistic complexities of Georgian. A database containing 241 of the most frequently used syllables was created and used in the system, successfully covering approximately 80% of texts on various topics. The text processing and normalization module developed as part of this work effectively handled acronyms, abbreviations, numbers, and irrelevant symbols – resulting in a complete grapheme-to-phoneme conversion pipeline.

In recent years, most TTS systems have been based on neural models such as Tacotron and WaveNet [2], which achieve high-quality prosody in languages with abundant digital resources. In contrast, the system described here relies on a rule-based algorithm that prioritizes accurate phonetic representation and natural prosody through phoneme concatenation. To evaluate the system, a Mean Opinion Score (MOS) test was conducted with six listeners (three male, three female) who had no prior exposure to the system. Each participant listened to five phonetically diverse test audio samples and rated them on a five-point scale, where $1 =$ "poor"$and 5 =$ "excellent". The prototype received an average MOS of 2.33 ± 0.41. Based on these results, several areas for improvement were identified to enhance the system's performance and efficiency.

References

[1] Th. Dutoit, High-quality text-to-speech synthesis: an overview. *Journal of Electrical Electronics Engineering, Australia* **17** (1999), 25–37.

[2] A. van den Oord, S. Dieleman, H. Zen, K. Simonyan, O. Vinyals, A. Graves, N. Kalchbrenner, A. Senior and K. Kavukcuoglu, WaveNet: a generative model for raw audio. *Preprint* arXiv:1609.03499, 2016; https://arxiv.org/abs/1609.03499.

Algorithms for Building Multi-Level Forecasts

Natela Archvadze[1], **Irina Khutsishvili**[1], **Merab Pkhovelishvili**[2], **Lia Shetsiruli**[3]

[1]*Department of Computer Scieces, Ivane Javakhishvili Tbilisi State University*
Tbilisi, Georgia
E-mails: natela.archvadze@tsu.ge; irina.khutsishvili@tsu.ge

[2]*Faculty of Informatics and Control Systems, Georgian Technical University*
Tbilisi, Georgia
E-mail: m.pkhovelishvili@gtu.ge

[3]*Batumi Navigation University, Batumi, Georgia*
E-mail: lika77u@gmail.com

Predicting natural disasters is one of the most complex challenges in forecasting asynchronous events. The inherent complexity and multi-level nature of such phenomena demand forecasting models that not only evaluate individual processes, but also analyze their interconnections, resulting effects, and cascading impacts.

Multi-level forecasting models are structured approaches in which predictions are generated sequentially across several distinct levels or stages. Each stage may involve different data types, models, or analytical methods.

This structure can be effectively represented using extended semantic networks, where each node corresponds to the optimal model for a specific cataclysm (or the intersection of predictions from the best-performing models [1]). The edges between nodes represent the approximate probabilities (in percentage) of transitions from one stage to another – effectively, the likelihood of one cataclysm triggering another. To calculate these probabilities reliably, a large volume of statistical data is required. In cases where such data is lacking, expert judgment is incorporated into the forecasting process.

Notably, the semantic network in question is extended, as each node can itself represent an independent semantic subnetwork.

This structure consists of three principal levels:

- Initial (first-level) events (M): foundational factors that initiate the process (e.g., natural disasters or socio-political events)
- Intermediate (second-level) events (K): transitional developments that emerge as a result of first-level events
- Final (third-level) events (N): ultimate outcomes arising from both direct and indirect influences.

In this network architecture, the vertices represent events, while the edges encode causal logic, quantified through probabilities derived either from historical data or expert evaluations.

Had this risk-based forecasting algorithm been applied, it is plausible that the tragic event of August 3, 2023, in the Bubisskali gorge could have been predicted in advance using a multi-level forecasting approach.

References

[1] A. Prangishvili, Z. Gasitashvili, M. Pkhovelishvili and N. Archvadze, Predicting Events by Analyzing the Results of the Work of Predictive Models. In: Tuzikov, A. V., Belotserkovsky, A. M., Lukashevich, M. M. (eds) *Pattern Recognition and Information Processing. PRIP 2021*, Communications in Computer and Information Science, vol. 1562. *Springer, Cham*, 2022.

Detect Cyber Threats and Assess Vulnerabilities by Creating and Using Scripts

Elisabed Asabashvili

Department of Informatics of Science and Technologies School, University of Georgia, Tbilisi, Georgia

E-mail: z.asabashvili@ug.edu.ge

With the growth of technological development, cybersecurity has become a serious global issue. Although the Internet has never been a completely safe space, the sharp rise in digitized data has led to a significant increase in cyberattacks, making them far more dangerous. Cyber defense plays a critical role in countering such threats to systems, networks, and applications. This is why the need for powerful, reliable, and secure programming languages and tools has never been more relevant. According to data from 2025, the top 10 most popular programming languages in the field of cybersecurity are: Python, JavaScript, C and C++, Java, Bash/Shell, SQL, PHP, PowerShell, Ruby, and Assembly. Each of these has its own strengths, and they are used accordingly in various areas of cybersecurity. This report focuses on one of these ten languages – the versatile and powerful Ruby programming language – and its use in cybersecurity. The focus on Ruby is due to its readability and concise syntax, which make it an ideal language. In addition, Ruby is cross-platform, meaning it can be used across different operating systems, which is critical for cybersecurity. It also boasts a rich ecosystem of libraries, including those for network programming and cryptography, and gems such as OpenSSL, net-ssh, and net-ping, which simplify working with secure connections, encryption, and network scanning. Finally, Ruby's metaprogramming capabilities allow for the creation of dynamic scripts and frameworks. The report examines one of the most common cybersecurity activities – network scanning. Network scanning helps assess the security of a network, identify potential vulnerabilities, and ensure its operability. The article focuses on the use of Ruby gems. A script was written to scan a range of IP addresses and use ping to check which devices are online. The results were analyzed in the report, necessary and sufficient conditions for ensuring security were considered, and existing challenges were discussed.

References

[1] E. Asabashvili, Challenges of Media Digitization in Georgia. *Conference materials*, 2023.

[2] E. Asabashvili, Comparative analysis of ruby language libraries in the field of data science. *Conference materials*, 2024.

[3] J. Evans, *Polished Ruby Programming: Build Better Software with More Intuitive, Maintainable, Scalable, and High-Performance Ruby Cod.* Packt Publishing, 2021.

[4] N. Rappin and D. Thomas, *Programming Ruby 3.3.* (5th Edition) *Pragmatic Bookshelf*, 2024; https://pragprog.com/titles/ruby5/programming-ruby-3-3-5th-edition/.

Crowdsourcing for the Validation of Lexical Data on Georgian Verbs

Archil Elizbarashvili[1], **David Gross-Amblard**[2], **Magda Tsintsadze**[1], **Mireille Ducassé**[2], **Manana Khachidze**[1]

[1] *Computer Science, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
E-mails: archil.elizbarashvili@tsu.ge; magda.tsintsadze@tsu.ge manana.khachidze@tsu.ge

[2] *Université de Rennes, IRISA, France*
E-mails: david.gross-amblard@irisa.fr; mireille.ducasse@irisa.fr

Georgian, a morphologically rich and syntactically complex language, poses persistent challenges for both human learners and natural language processing systems. The irregularity and depth of its verbal inflection system make it difficult to connect inflected forms to their dictionary base forms (lemmas), which reduces the effectiveness of traditional dictionaries. To address this, we present Kartuverbs – a comprehensive Semantic Web-based lexical resource featuring over five million verb forms extracted from more than 16,000 Georgian verbs. To enhance and verify the dataset, we applied machine learning techniques – specifically decision tree algorithm – to infer missing verbal nouns. Recognizing the need for high-precision linguistic data, we introduced Headwork, a targeted crowdsourcing platform designed to validate and refine the predicted forms through expert and informed user participation. Ongoing validation campaigns help eliminate outdated or incorrect entries while improving the overall coverage and quality of the dataset. Together, these efforts contribute to building a more accurate, scalable, and linguistically sound infrastructure to support Georgian language technology, lexicography, and educational applications.

A Comparative Perspective: Strategic Decision-Making with AHP, TOPSIS, and ELECTRE

Julieta Gagloshvili

Computer Science Department, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia

E-mail: julieta.gagloshvili@tsu.ge

Multi-Criteria Decision-Making (MCDM) methods are key tools for solving complex problems involving multiple, often conflicting, criteria. This paper offers a comparative analysis of three widely used MCDM techniques: Analytic Hierarchy Process (AHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and ELimination and Choice Expressing REality (ELECTRE). Building on prior work such as Gagloshvili et al. (2016), which applied TOPSIS in expert evaluations [2], this study extends the comparison to include AHP and ELECTRE in both theoretical and applied settings.

Incorporating insights from Goyal and Durai (2020) on algorithmic valuation [3] and Brown and Duguid (2001) on organizational knowledge systems [1], the paper examines the methods' structure, transparency, and performance. A case-based comparison highlights how each method operates under varying levels of complexity and stakeholder involvement.

Findings suggest that AHP ensures consistency in hierarchical judgments, TOPSIS stands out for its simplicity and clarity, and ELECTRE is effective for handling incomparability and outranking scenarios. The analysis underscores that method selection should consider both technical and organizational factors. The paper concludes with practical recommendations for integrating these tools into real-world decision processes.

References

[1] J. S. Brown and P. Duguid, Knowledge and organization: A social-practice perspective. *Organization Science* **12** (2001), no. 2, 198–213.

[2] J. Gagloshvili, Z. Gasitashvili and S. Khutsishvili, Innovative idea multicriteria expert evaluation system based on TOPSIS method. *GESJ: Computer Science and Telecommunications*, 2016, no. 1(47).

[3] N. Goyal and T. S. K. Durai, The role of artificial intelligence in idea management systems. *International Journal of Innovation Management* **24** (2020)), Article no. 2050012, no. 22.

Shade-Aware Routing

Levan Gogorikidze, **Giorgi Svanadze**, **Magda Tsintsadze**

Department of Computer Sciences, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia

E-mails: levan.gogorikidze874@ens.tsu.edu.ge; giorgi.svanadze070@ens.tsu.edu.ge; Magda.tsintsadze@tsu.ge

Walking is the most sustainable form of transportation. On average, 15% of the population walks for at least 30 minutes a day, often in high temperatures and under the sun. Such conditions can cause discomfort for pedestrians and increase the risk of oveheating, skinburns and melanoma. In urban environments, buildings might cast enough shade to protect against sun radiation and provide relief from extreme heat. We present a JavaScript library to easily maximize shade. We model the city and use approximated shade coverage to plan comfortable walkable routes in the city. The program uses openly accessible and continually updated building geometry and road network data from OpenStreetMap and simulates shade coverage in a fast and convenient way. The existence of this library can facilitate the creation of applications for sustainable urban planning and help develop better ways to maximize pedestrian comfort in modern cities.

Random Number Generator

Levan Khorava, Dimitri Kvateladze, Saba Bezhashvili

Computer Science Department, Faculty of Exact and Natural Sciences,
Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia
E-mails: levan.khorava550@ens.tsu.edu.ge; dimitri.kvateladze344@ens.tsu.edu.ge;
saba.bezhashvili820@ens.tsu.edu.ge

The work concerns a desktop application for generating random numbers, which implements a random number generator based on several high-fidelity algorithms [1]–[4]. It will simplify the process of generating random values and, in general, the conduct of research experiments that contain stochastic components. The functional capabilities offered by this application will solve a number of existing problems, such as: limited functionality of existing applications, lack of ability to check the suitability of random numbers, pseudo-randomness of generated values, etc. The application generates random numbers distributed in the interval $(0;1)$, from which the user can optionally use them in their "raw" form, as well as obtain specific continuous (standard normal distribution, normal distribution, Erlang distribution, etc.) [1], [5] and discrete random variates (Bernoulli, binomial, Poisson, etc.). The application includes a set of empirical-statistical tests [4], which will allow the user to test for (uniformity, independence, degree of correlation) the generated random numbers. To solve the problem of pseudo-randomness of the generated numbers, as a kind of experiment, we developed the so-called "true" random number generator. In our true random number generator, we will consider some noise as the source of randomness, from which our application will generate random numbers. To create the basic structure and logic of the application, we used the Python programming language, OOP and functional programming principles and Qt Designer for developing the graphical interface.

Acknowledgments

The work was supported by the our project heads professor Manana Khachidze and associate professor Maia Archuadze.

References

[1] A. M. Law, *Simulation Modeling and Analysis* (5th edition)). McGraw-Hill Series in Industrial Engineering and Management Science. McGraw-Hill Book Co., New York, 2013.

[2] P. L'ecuyer, Good parameters and implementations for combined multiple recursive random number generators. *Operations Research* **47** (1999), no. 1, 1–173.

[3] P. L'ecuyer, Software for uniform random number generation: distinguishing the good and the bad. In: *Proceeding of the 2001 Winter Simulation Conference (09-12 December, 2001), IEEE Xplore* **1** (2001), 95–105.

[4] D. E. Knuth, *The Art of Computer Programming*, Vol. 2. *Seminumerical Algorithms* (3 ed.). Addison-Wesley Professional, 1997.

[5] G. Sirbiladze, *System Modeling and Simulation.* Tbilisi, 2023.

Investment Projects Selection in a Picture Fuzzy Environment

Irina Khutsishvili, **Natela Archvadze**

Department of Computer Sciences, Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia

E-mails: irina.khutsishvili@tsu.ge; natela.archvadze@tsu.ge

In the present research, a decision support methodology for the multi-attribute group decision-making (MAGDM) problem is developed, namely for the selection of investment projects. The objective of the investment project selection is to choose the best project among a set of projects, or to rank all projects in descending order.

A set of potential alternatives (projects seeking investment) is typically involved in this kind of problem, and they are assessed based on some weighted criteria. The alternative that best meets each criterion will be selected for investment.

To evaluate the criteria, our approach employs experts' assessments. In the proposed methodology, the values of the criteria are expressed in picture fuzzy numbers, determined by the group of experts. The case when the information on the criteria weights is completely unknown is considered. The attribute weights are identified based on the concept of De Luca and Termini's information entropy, determined in the context of picture fuzzy sets.

The decisions are made using the extended TOPSIS method under a picture fuzzy environment. Hence, a methodology is based on a picture-valued fuzzy TOPSIS decision-making model with entropy weights.

The ranking of alternatives is performed by the proximity of their distances to both the fuzzy positive-ideal solution (FPIS) and the fuzzy negative-ideal solution (FNIS). For this purpose, the picture-weighted Hamming distance is used.

An example of investment decision-making is shown that clearly explains the procedure of the proposed methodology.

Bayesian Approach to Identification of Function and Taxonomy of DNA Sequences

Alexandre Lomsadze, Mark Borodovsky

School of Computational Science and Engineering, Wallace H. Coulter Department of Biomedical Engineering, Georgia institute of Technology Atlanta, GA, USA
E-mail: borodovsky@gatech.edu

Papillomaviruses pose a significant threat to human health due to their ease of transmission and well-established carcinogenic potential. Their evolutionary history spans over 300 million years. The papillomavirus genome, consisting of approximately 8,000 nucleotides of double-stranded DNA, evolves slowly because replication is carried out by the host's high-fidelity DNA polymerase. Despite this slow evolutionary rate, human papillomaviruses (HPVs) – a relatively young clade – have diversified into more than 400 distinct types. This diversification is driven by the interplay of mutation, natural selection, and human population migration. These HPV types form distinct genomic clusters, further subdivided into lineages and sublineages. These classifications rely on thresholds on divergence in average nucleotide identity (ANI): 1.0-10.0% for lineages and 0.5-1.0% for sublineages (1). Importantly, the oncogenic potential of a given HPV type can vary across its lineages and sublineages. Therefore, determining the taxonomic origin of a newly sequenced HPV genomic fragment – down to its lineage and sublineage – has significant clinical relevance. Previously, we developed an algorithm to identify HPV types directly from raw sequencing data (2). Here, we present a computational method and software tool for assigning sequences of a given HPV type to their specific lineages and sublineages. The new algorithm employs statistical models of sequences of each known sublineage. The training sets are selected through:

- (i) building MSA for all sequences of a given type;
- (ii) construction of a phylogenetic tree;
- (iii) identification of genomes that belong to sublineage-specific branches.

A probabilistic model of each sublineage is then built using position-specific nucleotide frequencies derived from the corresponding MSA segment. To classify a query DNA sequence, the algorithm computes posterior probabilities for all candidate sublineages and selects the one with the highest probability. In our evaluation across nine medically significant (vaccine-targeted) HPV types, the method achieved high accuracy in assigning correct sublineages for genomic fragments of at least 2,000 nucleotides.

Acknowledgments

The work has been supported in part by the US Centers for Disease Control.

References

[1] R. D. Burk, A. Harari and Z. Chen, Human papillomavirus genome variants. *Virology* **445** (2013), no. 1-2, 232–243.

[2] A. Lomsadze, T. Li, M. S. Rajeevan, E. R. Unger and M. Borodovsky, Bioinformatics pipeline for human papillomavirus short read genomic sequences classification using support vector machine. *Viruses* **12** (2020), no. 1, Article no. 710, 12 pp.

Proof in Terms of Divisibility by n

Merab Pkhovelishvili[1], **Kakha Gogichaishvili**[2]

[1] *Faculty of Informatics and Control Systems, Georgian Technical University, Tbilisi, Georgia*
E-mail: m.pkhovelishvili@gtu.ge

[2] *Tbilisi, Georgia*
E-mail: kakhagogichaishvili6@gmail.com

Claim: We want to prove that the expression

$$n \mid [x^n + (-x)]$$

logically implies the association $x^n \to (-x)$ in terms of divisibility by n. (Here, the symbol $\to$ denotes logical association.)

Proof Indeed, we have already established that

$$n \mid [x^n + (-x)]$$

means that the expression x^n is logically associated with $-x$ with respect to divisibility by n.

Now observe: both expressions x^n and $-x$ contain the variable x. Therefore, we can say that within this expression, there exists a logical association $x^n \to (-x)$, which is valid in the context of divisibility by n.

Since we have already stated that x^n is logically associated with $-x$ in terms of divisibility by n, it follows that

$$x^n \to (-x) \text{ (in terms of divisibility by } n\text{)},$$

which completes the proof.

References

[1] M. Pkhovelishvili, N. Archvadze, K. Gogichaishvili and Z. Zandarashvili, Modern confirmation theory. Book of Abstracts XIV International Conference of the Georgian Mathematical Union dedicated to the 100-th Anniversary of the Georgian Mathematical Union (September 2-7, 2024, Batumi), p. 165; https://gmu.gtu.ge/conferences/wp-content/uploads/2024/08/Conference_GMU_2024_22.08.pdf.

Index