საქართველოს
მათემატიკოსთა კავშირი
**Georgian
Mathematical Union**

აჭარის ავტონომიური რესპუბლიკის
განათლებისა და სპორტის
სამინისტრო
**Ministry of Education and Sport
of Adjara Autonomous Republic**

ბათუმის შოთა რუსთაველის
სახელმწიფო უნივერსიტეტი
**Batumi Shota Rustaveli
State University**

საქართველოს მათემატიკოსთა კავშირის XVI საერთაშორისო კონფერენცია

თეზისების კრებული

XVI International Conference of the Georgian Mathematical Union

BOOK OF ABSTRACTS

ბათუმი, 31 აგვისტო – 5 სექტემბერი
Batumi, August 31 – September 5

2026

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რედაქტორები: გურამ გოგიშვილი, მაია ჯაფოშვილი

Editors: Guram Gogishvili, Maia Japoshvili

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Professor Vladimer (Lado) Baladze (1952–2026)

The Georgian mathematical community has suffered a profound loss with the passing of Professor Vladimer (Lado) Baladze, an internationally recognized specialist in topology, a devoted teacher, and one of the leading representatives of the Georgian school of topology. His scientific, pedagogical, and organizational work played a decisive role in the development of geometric and algebraic topology in Georgia and, in particular, in establishing Batumi as an important center of topological research.

Vladimer Baladze was born on August 9, 1952, in the Keda district of Georgia. After completing secondary school, he entered the Faculty of Mechanics and Mathematics of Ivane Javakhishvili Tbilisi State University, graduating in 1974. He subsequently pursued postgraduate studies at the Andrea Razmadze Mathematical Institute. In 1980 he defended his Candidate of Sciences dissertation, *"On Dimension-like Functions Based on Distinguished Classes of Spaces and Complexes"*. His Doctor of Sciences dissertation, *"Resolutions of Topological Spaces and Continuous Maps and Their Applications to Spectral Homotopy Theory"*, was defended at Tbilisi State University in 1993.

His mathematical outlook was formed at the Chair of Geometry and Topology of Tbilisi State University and the Department of Geometry and Topology of the Andrea Razmadze Mathematical Institute. At that time, prominent Georgian topologists and algebraists worked in these institutions under the intellectual leadership of Academician George Choghoshvili, the founder of the Georgian school of topology. Professor Baladze continued and substantially developed this tradition. His research interests encompassed general topology, dimension theory, transformation groups, homology and homotopy theory, retract theory, shape and coshape theory, and category theory.

The first major cycle of Professor Baladze's work concerns dimension theory modulo distinguished classes of spaces. He investigated small inductive, large inductive, and covering modular dimensions, established their fundamental properties and their relations to classical inductive and Lebesgue dimensions, and studied their connections with the dimension and multiplicity of continuous maps. Among the consequences were an axiomatic characterization of large inductive

dimension, extensions of Morita-type theorems, and approximation and factorization results for spaces and maps.

A second central direction of his research was shape theory. He introduced and studied proper resolutions of locally compact spaces and used them to develop proper shape theory. This led to shape classifications different from the classical approaches of Borsuk, Mardešić, and Morita, as well as to continuous, proper-shape-invariant extensions of functors, a theory of proper shape dimension, and analogues of the Hurewicz and Whitehead theorems in the proper shape category.

Professor Baladze made especially important contributions to fiber shape theory. From the viewpoint of retract theory, he developed a geometric theory of continuous maps, introduced absolute (neighborhood) retracts and absolute (neighborhood) extensors for classes of maps, and established their basic properties. These results were essential for proving the existence of fiber resolutions and polyhedral approximations of continuous maps. Their applications include fiber-shape classifications, fiber-shape-invariant extensions of functors, exact sequences associated with maps, and characterizations of spectral and projective homology and cohomology without recourse to relative groups. He later advanced strong shape theory for continuous maps and fiber strong shape theory.

Another substantial part of his work was devoted to coshape theory. He studied factor categories of direct systems, established conditions for the existence of abstract coshape categories, and analyzed epimorphisms, monomorphisms, and isomorphisms in these categories. On this basis he constructed the topological coshape category, obtained coshape-invariant continuous extensions of algebraic functors, and proved Hurewicz-type and long exact sequence results. These investigations contributed to the general problem of extending algebraic functors from subcategories of well-behaved spaces to broader topological categories.

Beginning in 2000, Professor Baladze and his students in Batumi systematically investigated Alexander–Spanier cohomology. Using normal coverings, he constructed Alexander–Spanier normal cohomology for P-embedded closed pairs of arbitrary topological spaces and proved its equivalence with Alexandrov–Čech normal cohomology. This opened the way to the construction and axiomatic characterization of Alexander–Spanier normal exact homology. His later work also addressed uniform and border (co)homology, the (co)homological properties of compactifications and their remainders, and inverse-system methods in the (co)homology theory of maps. His research remained active until the final years of his life.

Professor Baladze's scientific work was inseparable from his teaching. From 1976 onward, his academic career was closely connected with Tbilisi State University, where he progressed from assistant to professor and for many years taught the principal course in topology, as well as advanced courses in general, algebraic, and geometric topology. At the insistence of Academician George Choghoshvili, he was entrusted with continuing the foundational topology course that Choghoshvili himself had led for decades. Professor Baladze regarded this responsibility as one of the greatest honors of his career.

From 1991, during one of the most difficult periods in Georgia's recent history, he taught the same subjects in parallel at Batumi Shota Rustaveli State University. His pedagogical experience culminated in the publication of the first comprehensive Georgian-language textbook on topology, intended for undergraduate, graduate, and doctoral students. He also authored works on fiber shape theory and geometric topology. Through his lectures, books, seminar, and supervision of doctoral students, he ensured the continuity of the Georgian topological tradition and formed a new generation of researchers.

His principal institutional achievement was the development of geometric topology in Batumi and the creation of the Batumi school of topology. He led the research seminar on the shape and coshape classifications of topological spaces and continuous maps and their algebraic and dimension-like invariants. The seminar's participants and his students developed these ideas in

several directions, including homology and cohomology theories, uniform and fiber shape theory, and invariants of continuous maps. Professor Baladze authored about one hundred scientific papers and participated in more than forty international mathematical forums.

Professor Baladze also rendered exceptional service to university life and the wider mathematical community. At Batumi Shota Rustaveli State University he served as Deputy Rector in 2005–2006 and Rector in 2006–2007; from 2008 he headed the Department of Mathematics. He chaired university dissertation councils, served as a referee for *Mathematical Reviews* and zbMATH, and was a member of the editorial board of the *Tbilisi Mathematical Journal*. He was Vice-President of the Georgian Mathematical Union in 2008–2013 and later a member of its Council. His organizational efforts were instrumental in establishing Batumi as the principal venue for the Union's annual international conferences and for numerous schools and meetings in geometry, topology, algebra, and analysis.

His contributions were recognized beyond academia. He was named an Honorary Citizen of Keda Municipality in 2014 and an Honorary Citizen of Batumi in 2024, and in 2022 he was awarded the Order of Tbel Abuseridze. These distinctions reflected not only his scholarly achievements but also his service to education, his region, and his country.

Professor Vladimer Baladze belonged to the generation that inherited the traditions of the Georgian school of topology and carried them forward into new fields. As a researcher, he created original theories and methods; as a teacher, he educated generations of mathematicians; and as an organizer, he built lasting institutions and scholarly communities. His passing is an irreparable loss to the Georgian Mathematical Union, Batumi Shota Rustaveli State University, Tbilisi State University, and the international topological community. His scientific legacy, his school, and the gratitude of his colleagues and students will preserve his memory.

Georgian Mathematical Union

Abstracts of Plenary Talks

A Stein–Weiss Inequality with Nonstandard Power Weights and Applications

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In this talk, we discuss a generalization of the Stein–Weiss inequality for the fractional integral operator in variable Lebesgue spaces involving nonstandard power weights. As applications, we also show how Hardy–Sobolev inequalities and other related inequalities with such weights can be derived.

This is based on joint work with D. Cruz-Uribe (Alabama, USA) and H. Rafeiro (Al Ain, UAE).

A Bound-Preserving and Conservative Enriched Galerkin Method for Elliptic Problems

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We propose a locally conservative enriched Galerkin scheme that preserves the physical bounds for an elliptic problem. To this end, we use a substantial over-penalization of the discrete solution's jumps to obtain optimal convergence. To avoid the ill-conditioning issues that arise in over-penalized schemes, we introduce an involved splitting approach that separates the system of equations for the discontinuous solution part from the system of equations for the continuous solution part, yielding well-behaved subproblems. We prove the existence of discrete solutions and optimal error estimates, which are validated numerically.

Acknowledgments

The first author has been supported by the Leverhulme Trust through the Research Project grant RPG-2021-238. The third author has been supported by the Research Council of Finland's decision 350101, 354489, 359633, 358944 and by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 577175348.

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Model Theory of Chevalley Groups and their Extensions

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In this talk I will discuss model-theoretic properties of Chevalley groups over rings. The main result is that adjoint Chevalley groups over arbitrary commutative rings are regularly bi-interpretable with their coefficient rings. There are several consequences of this result, including the elementary definability of natural classes of Chevalley groups, and the finite axiomatizability of the corresponding classes. I will also present related results for non-adjoint Chevalley groups and for natural extensions of Chevalley groups.

On the Söhngen Problem for the Finite Hilbert Transform

Guillermo P. Curbera

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We address the problem of inversion of the finite Hilbert transform on $L^2(-1,1)$, first noted by H. Söhngen in 1954.

The work is joint with Javier Carrillo-Alanís from Sevilla.

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Intersection Type Galore

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The first part of the talk reviews, in full generality, intersection type theories and the filter models of the λ-calculus generated by them.

The second part gives an overview of how various authors have exploited intersection types in different theoretical developments and practical applications.

Finally, some new results concerning the interpretation of unsolvable λ-terms in filter models will be discussed.

Riemann–Roch Bases in Kummer Extensions

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The explicit computation of Riemann–Roch spaces is a classical problem in algebraic geometry and computational algebra. Starting from the classical work of Brill and Noether, several algorithmic approaches to the Riemann–Roch problem have been developed.

Since the introduction of algebraic geometric codes by Goppa, explicit bases of Riemann–Roch spaces have found applications in coding theory and cryptography, including McEliece-type public key system.

In this paper we work with the Kummer function field $K = \mathsf{k}(x, y)$, $y^n = f(x)$, where $\mathrm{char}\ \mathsf{k} \nmid n$, $f(x) \in \mathsf{k}[x]$ is separable of degree d, and $\gcd(n, d) = 1$. The infinite place of $\mathsf{k}(x)$ is then totally ramified in K. We write Ω for the unique place above infinity. Its Weierstrass semigroup is $H(\Omega) = \langle n, d\rangle$, whose Frobenius number coincides with $2g - 1$ for the genus g of K.

We study Riemann–Roch spaces of the form $\mathscr{L}(\Delta + m\Omega)$, where $\Delta = P_1 + \cdots + P_t - t\Omega$ is a degree-zero divisor supported at rational places of K. When the places P_i lie above pairwise distinct rational places of $\mathsf{k}(x)$, we write $P_i = (a_i, b_i)$, (with $b_i^n = f(a_i)$), and associate with them the interpolation data

$$u(x) = \prod_{i=1}^{t}(x - a_i), \quad v(x) : \deg(v) < t, \quad v(a_i) = b_i.$$

This is the Kummer analogue of the Mumford representation familiar from the hyperelliptic case.

The basic auxiliary functions used in the paper are obtained from subsets of the support of Δ. If $J \subseteq \{1, \ldots, t\}$, let $u_J(x)$ be the product of the corresponding factors $x - a_i$, and let $v_J(x)$ be the interpolation polynomial attached to the selected places. We define

$$\Psi_J(x, y) = \frac{y^n - v_J(x)^n}{(y - v_J(x))u_J(x)}\,.$$

The valuation of Ψ_J at Ω depends only on $|J|$ and on $\deg v_J$. These values form a combinatorial object, the *interpolation array*, which records the thresholds at which the functions Ψ_J enter the spaces $\mathscr{L}(\Delta + m\Omega)$.

The computation is organized around the Frobenius level $2g-1$, which is the Frobenius number of the numerical semigroup $\langle n, d\rangle$. The spaces below this level are obtained by Gaussian elimination on the leading coefficients at Ω. Thus the interpolation array supplies the candidate functions, while the valuation filtration at Ω supplies the linear-algebraic reduction.

Twisted Localization in Representation Theory

Vyacheslav Futorny

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We will discuss the role of twisted localization techniques in the study of smooth representations of Affine Kac-Moody algebras, their geometric realization and relevance to quantum field theory.

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Two Views on Unification: Terms as Strategies

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In [1], the authors have shown that *linear application* in Geometry of Interaction (GoI) models of λ-calculus amounts to *resolution* between *principal types* of linear λ-terms. This analogy also works in the reverse direction. Indeed, an alternative definition of *unification* between algebraic terms can be given by viewing the terms to be unified as *strategies*, i.e. sets of pairs of occurrences of the same variable, and verifying the termination of the GoI interaction obtained by playing the two strategies. In this paper we prove that such a criterion of unification is equivalent to the standard one. It can be viewed as a local, bottom-up, definition of unification. Dually, it can be understood as the GoI interpretation of unification.
The proof requires generalizing earlier work to arbitrary algebraic constructors and allowing for multiple occurrences of the same variable in terms. In particular, we show that two terms σ and τ unify if and only if $\mathscr{R}(\sigma) \mathrel{\widehat{\subseteq}} \mathscr{R}(\tau)\mathbin{\widehat{;}}(\mathscr{R}(\sigma)\mathbin{\widehat{;}}\mathscr{R}(\tau))^*$ and $\mathscr{R}(\tau) \mathrel{\widehat{\subseteq}} \mathscr{R}(\sigma)\mathbin{\widehat{;}}(\mathscr{R}(\tau)\mathbin{\widehat{;}}\mathscr{R}(\sigma))^*$, where $\mathscr{R}(\sigma)$ denotes the set of pairs of paths leading to the same variable in the term σ, $\mathrel{\widehat{\subseteq}}$ denotes "inclusion up to substitution" and $\mathbin{\widehat{;}}$ denotes "composition up to substitution".

Acknowledgments

Work supported by project PNRR SERICS PE00000014 M4C2I1.3 – SECCO – "SecCo-OC – Secure Containers Open Call" – CUP D33C22001300002, and by project PNRR SERICS M4C2I1.3 – COVERT OpenCall In searCh Of eVidence of stEalth cybeR Threats – CUP G23C24000790006.

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About Banach–Mazur's Problem from the 1930s. Classical and New Results

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One of the famous unsolved problems in modern functional analysis is the question (the Banach–Mazur problem (1933)) of whether every infinite-dimensional Banach space E can be mapped by a continuous linear operator onto an infinite-dimensional separable Banach space (equivalently, the space E has an infinite-dimensional separable quotient).

This problem is known as the Separatable Quotient Problem. For many specific classes of Banach spaces, the answer to this question is positive, for example, for reflexive and even WCG Banach spaces. Argyros, Dodos, and Kanellopoulos proved that every dual Banach space has an infinite-dimensional separable quotient. It turns out that the separable quotient problem is equivalent to the problem whether the dual of every infinite-dimensional Banach space in its weak*-topology has a basic sequence. We will present these relations.

On the other hand, Rosenthal showed that all infinite-dimensional Banach spaces $C(X)$ of continuous (real-valued) functions have a quotient isomorphic to c_0 or ℓ_2. The same problem (still open) will also be discussed for the space $C_p(X)$ with pointwise topology. The connection of this problem with the long-standing open problem of the existence of compact Efimov spaces will also be discussed.

The Bergstra–Tucker Problem: a c.e. Algebra with no Finitely Presented Expansion

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A classical result of Bergstra and Tucker states that every finitely generated computable algebra admits a finite presentation after expanding the signature with finitely many auxiliary operations. This led Bergstra and Tucker, and independently Goncharov, to ask in the late 1970s whether every finitely generated computably enumerable (c.e.) algebra admits a finite presentation by equations or quasi-equations in an expansion without extending the domain. The equational case was fully resolved in the 1990s–2010s.

This paper addresses the remaining quasi-equational case. Unlike equations, quasi-equations are not preserved under homomorphisms, rendering the methods for the equational case inapplicable. We develop a new approach and construct a finitely generated c.e. algebra that admits no finite presentation by quasi-equations in any expansion. The proof combines universal algebra (almost free algebras), model theory (quantifier elimination), and computability theory (a finite injury priority argument) to build a c.e. algebra whose computable operations are essentially term operations. We then show that no expansion of such an algebra can be finitely presented by quasi-equations. This completes the solution to the Bergstra–Tucker problem.

Integral Identities and Inequalities for the Equations of the Electromagnetism in a Nonvariational Setting

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We plan to show those basic integral representation theorems, equalities and inequalities that are necessary to analyze the equations of the electromagnetism if we look for Hölder continuous solutions that may have an infinite Dirichlet integral. Thus for solutions that do not belong to the classical variational setting.

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Maximal Regularity for Abstract Cauchy Problems

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We will present selected results on maximal regularity estimates for abstract Cauchy problems, with particular emphasis on the L^1-theory for sectorial operators and on L^p-maximal regularity for $p \in (1, \infty)$. We will discuss extensions of the Da Prato–Grisvard approach, weighted estimates and their relation to the unweighted theory, as well as global-in-time estimates. We will also comment on the role of abstract interpolation methods in this setting. The presentation will focus on the main ideas and the general framework.

The talk is based on joint work with S. Król and J. Sarnowski.

Diophantine Problem in Groups and Rings

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Algebraic equations play an important part in mathematics. Recent breakthroughs in the solution of the Diophantine problem (Hilbert's 10th problem) in rings of algebraic integers have had a tremendous impact on the Diophantine problem in groups, rings, and algebras. In this talk, I will discuss some recent results and new methods that allow one to reduce various algorithmic questions on algebraic equations in non-commutative structures to Diophantine problems in commutative rings.

Hyperidentites in Action

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This survey talk illustrates many important current trends and perspectives for the field and its applications. Main areas cover questions of modern algebra, mathematical logic and discrete mathematics, including classification of hyperidentities, characterization of algebras with hyperidentities, structure results for varieties of lattices and bilattices. Also the functional representations of finitely-generated free algebras of various varieties are presented via generalized Boolean functions (De Morgan functions, quasi-De Morgan functions, super-Boolean functions, super-De Morgan functions, ets), and some categorical questions.

Equations in Products of Free Groups and 3-Manifold Groups

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Perelman's proof of the Poincare conjecture shows that every simply connected closed 3-manifold is homeomorphic to the 3-sphere. The fundamental groups of 3-manifolds attract lots of interest from mathematicians of different fields. As it was stated in a famous survey of Allen Hatcher "The classification of 3-manifolds", one would want to know exactly which groups occur as fundamental groups of these manifolds.

The Stallings–Jaco–Hempel reformulation of the Poincare conjecture inspired several connections between low-dimensional topology, equations over free groups, and combinatorial group theory. The reformulation reduces the problem to study epimorphisms from the fundamental group of a closed orientable surface onto the direct product of two free groups (they correspond to Heegaard splittings of 3-manifolds and were named splitting homomorphisms). The Poincare conjecture is equivalent to standardness of splitting epimorphisms. Olshankii (1989) constructed (in non-explicit form) first non-trivial examples of such splitting epimorphisms and verified the standardness of some of them.

We construct up to equivalence all the splitting coordinate-surjective homomorphisms (among them, the genuine splitting epimorphisms are exactly those for which our constructed associated group balanced presentation is trivial). We give generators and relations of the corresponding balanced presentation (so all closed orientable 3-manifold groups) that can be studied by algebraic methods. We analyse by algebraic methods a big class of such homomorphisms/presentations (including all Olshanskii's epimorphisms) and show that splitting epimorphisms are very rare, in this case the corresponding balanced presentation of the trivial group can be reduced to the standard one by Andrews–Curtis transformations and the epimorphisms are standard.

Convexity and Continuous Forms of Inequalities as a Source of Inspiration for New Research

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First I present some new ideas and results from the 2025 books [4] and [5]. After that I present some remarkable examples how these ideas already influenced the development in Interpolation Theory, Hardy-Type Inequalities (see [2]), modern Fourier Analysis (see [3]) and even in Composites Engineering. Finally, I present my latest thoughts how such ideas can contribute to the further developments of various kinds both in Mathematics and Engineering Sciences with special focus on a requested second Edition of the book [1].

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Skein Modules of 3-Manifolds: the Personal Perspective

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Skein modules, from their humble beginnings in April 1987, grew into a mainstream branch of mathematics (even according to MathSciNet). I will outline the path skein modules took, from my personal historical perspective. I will start briefly with the ancient Greek surgeon Heraklas and the medieval philosopher Llull, via Leibniz, Kuhn, Euler, Vandermonde, and Gauss (and his student Listing) to the Scottish school: Maxwell, Kelvin, and Tait. It was Tait who first played with a skein relation (not published). Our history really starts with Alexander and his polynomial in 1928, and even before, when Alexander outlined his research to Veblen in a letter in 1919. I will mention the revolution in knot theory by Vaughan Jones and describe how a comparison of the Alexander polynomial to the Jones polynomial led to the discovery of the HOMFLYPT polynomial in 1984 (T is for Pawe l Traczyk). In April 1987, I had the enlightenment that one can generalize the Jones polynomial (and much more) to arbitrary 3-manifolds. These skein modules are the main characters of today's talk. I would call this part of mathematics "Algebra Situs" (mixing Arabic and Latin was on purpose). However, when I suggested it to Jones, he was not too impressed. "Good try, Jozef!" he said. So the longer name "Algebraic Topology based on Knots" is in use. My talk will mix pure history with related anecdotes. The story about Burstin and Mayer, and the job found for them by Einstein, will lead us to nonassociative structures (the entropic condition connects the HOMFLYPT polynomial with nonassociative structures). I will end by advertising the determinant of the multiplication table to study quasigroups, e.g. Bol–Moufang quasigroups (following Dedekind's idea for finite groups).

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Evolution PDEs with Strong Singularities

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In this talk we will discuss the notion of very weak solutions suited to deal with PDEs with strong singularities.

The Tangent Lie Algebras of Automorphism Groups of Free Algebras

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We introduce the notion of tangent Lie algebras for certain automorphism groups of free algebras as a subalgebra of the algebra of derivations. We show that for many classical varieties of algebras the tangent Lie algebra is a subalgebra of the Lie algebra of all derivations with constant divergence. We also introduce the notions of approximately tame and absolutely wild automorphisms of free algebras of any variety of algebras and use tangent Lie algebras for their study. It is shown that almost all known examples of wild automorphisms of free algebras are absolutely wild except Nagata and Anick automorphisms. We show that the Bergman automorphism of free matrix algebras of order two is absolutely wild. It is also shown that free algebras of any variety of polynilpotent Lie algebras, except abelian and metabelian varieties, have absolutely wild automorphisms.

It is a joint work with Ualbai Umirbaev.

On the Current State of Gau–Wang–Wu Conjecture

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The *numerical range* (a.k.a. Hausdorff set) $W(A)$ of an n-by-n matrix A is the image of the unit sphere of $\mathbb{C}^n$ under the mapping $x \mapsto x^*Ax$. This set is the convex hull of some algebraic curve $C(A)$ of class n, sometimes called the Kippenhahn curve of A. So, for $n = 2$ the numerical range is an elliptical disk (for normal A degenerating into a line segment); the complete classification is also known for $n = 3$ and, to some extent, $n = 4$, but not in higher dimensions.

Recall also that A is a *partial isometry* if it acts as an isometry on the orthogonal complement to its kernel. It was conjectured in [1] that if the numerical range of a partial isometry is a circular disk, then this disk has to be centered at the origin.

This conjecture was justified in [1] for $n \leq 4$, implying, in particular, that it holds for any n if the rank of A does not exceed two. In this talk, we will describe further progress. Namely, abbreviating the name of the conjecture to GWW for brevity:

In [2], GWW was proved for $n = 5$. The case of arbitrary n when the kernel of A is one-dimensional was covered in [3]. Finally, in [4] we have shown that GWW holds for arbitrarily sized partial isometries of rank three. So, the simplest situation in which GWW is still open is rank four 6-by-6 matrices.

Time permitting, we will also discuss the obtained in [4] classification of rank three 6-by-6 partial isometries A for which $C(A)$ contains at least one circular component. In particular, all such matrices with circular numerical ranges will be described,

Acknowledgments

The work was partially supported by the Faculty Research funding from the Division of Science and Mathematics, New York University Abu Dhabi.

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Revisiting Fourier Inequalities

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We review classical and recent results on the Fourier estimate

$$\|\widehat{f}\|_X \lesssim \|f\|_Y$$

for various couples (X, Y). We are particularly interested in the case where X is a weighted Lebesgue space and Y is either an integrability space or a smoothness space.

Orlicz Convolutions in QHA with Applications to Orlicz Schatten Properties for Toeplitz Operators

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For any (Weyl symbol) $a \in \mathscr{S}(\mathbf{R}^{2d})$, the Weyl operator $\operatorname{Op}^w(a)$ is the pseudo-differential operator, defined by

$$\operatorname{Op}^w(a)f(x) = (2\pi)^{-d} \iint_{\mathbf{R}^{2d}} a\Big(\frac{1}{2}\,(x+y),\xi\Big) f(y) e^{i\langle x-y,\xi\rangle}\, dy\, d\xi,$$

when $f \in \mathscr{S}(\mathbf{R}^d)$. The definition extends to any $a \in \mathscr{S}'(\mathbf{R}^{2d})$, and then $\operatorname{Op}^w(a)$ is continuous from $\mathscr{S}(\mathbf{R}^d)$ to $\mathscr{S}'(\mathbf{R}^d)$.

Let s_p^w be the set of all $a \in \mathscr{S}'(\mathbf{R}^{2d})$ such that $\operatorname{Op}^w(a)$ belongs $\mathscr{I}_p$, the set of all all Schatten-p operators on $L^2(\mathbf{R}^d)$, $p \in (0,\infty]$. Then essential convolutions were established by R. Werner during the 80th. In the language of Weyl symbols, these convolution results can be formulated as

$$s_p^w * s_q^w \subseteq L^r \ \text{ and } \ s_p^w * L^q \subseteq s_r^w, \quad \frac{1}{p}+\frac{1}{q} = 1+\frac{1}{r}\,, \ \ p,q,r \in [1,\infty].$$

Here $*$ denotes the usual convolution.

In the talk we first extend these convolution properties to Orlicz situations. Thereafter we apply these results to deduce Orlicz Schatten properties for Toeplitz operators.

The project behind the talk is based on collaborations with Wolfram Bauer and Robert Fulsche.

Sharp Lee–Ni Estimates for the Semilinear Parabolic Problems

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We discuss Lee–Ni-type lifespan estimates for blow-up solutions of higher-order and fractional-order semilinear parabolic equations. By employing suitable families of test functions together with the corresponding backward heat kernels, we derive upper bounds for the lifespan of solutions. For several classes of equations, we also establish matching or complementary lower lifespan estimates.

The main results presented here are based on joint works with N. Tobakhanov [1, 2] and with M. Majdoub [3].

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The Jacobi Last Multiplier and the SIR Model

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We investigate the Jacobi last multiplier method as a tool for deriving Lagrangians associated with systems of ordinary differential equations arising in mathematical biology. The theoretical foundations of the method are presented, in particular the relationship between the Jacobi last multiplier and the Lagrangian of a second-order ordinary differential equation or systems of two first-order equations. Our contribution is a Lagrangian formulation for an epidemiological SIR model.

Acknowledgments

This work is supported by FCT via CIDMA under grant UID/04106/2025 (`https://doi.org/10.54499/UID/04106/2025`).

Parameterized Analysis of Optimization Problems

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Parameterized analysis for an intractable optimization problem allows a deeper view into the problem and more efficient and practical solutions. A Fixed Parameter algorithm for such a problem carries out a brute-force enumeration for a subproblem, restricted by the corresponding fixed parameters. Then the remaining objects can be arranged in a polynomial time. Thus for the fixed parameters, the fixed parameter algorithm runs in polynomial time. A Variable Parameter algorithm applied to an intractable optimization problem employes a special kind of a subroutine that determines a core subproblem that unavoidably yields an exponential-time enumeration. The core subinstance is arranged by an exponential-time subroutine while the remaining part of the problem is solved in polynomial time. Since the core substance is typically essentially smaller and can be drastically smaller than the original problem instance, a VP-algorithm is much more efficient compared to the traditional solution methods that apply an implicit enumeration to the whole problem instance. We plan to give a brief overview of these two parameterized approaches and some related results.

Trotter Product Formula for Nonlinear Affine Semigroups on Hilbert Space

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Similarly to *linear* strongly continuous (C_0)-semigroups $\{T(t)\}_{t\geq 0}$ on a vector space $\mathfrak{X}$ the *nonlinear* C_0-semigroups $\{S(t,x)\}_{t\geq 0,\, x\in D}$ (for some $D\subset\mathfrak{X}$) have their origine in *nonlinear* abstract Cauchy problem (ACP).

Nonlinearity modifies and complicates analysis of semigroup $\{S(t,x)\}_{t\geq 0,\, x\in D}$ starting on the problem of description of domain D of its definition. Besides the nonlinearity also complicates the problem of the semigroup approximations, in particular by the Trotter product formula even in the *strong* operator topology.

In my lecture I present a result about the Trotter product approximation in the *operator-norm* topology for *nonlinear affine* semigroups on Hilbert space $\mathfrak{X}=\mathfrak{H}$. The last step, which ensures the operator-norm convergence, is based on the key Dzh.Rogava theorem about operator-norm convergence of the Trotter product formula for self-adjoint linear C_0-semigroups on a Hilbert space $\mathfrak{H}$.

Growth and Complexity Functions

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I will discuss history and recent results concerning (i) growth functions of groups, algebras, monoids and languages, (ii) complexity functions of infinite sequences.

Abstracts of Sectional Talks

Development of a Block Method for Solving Multiple Order ODEs

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In this work, a convergent hybrid block method (CHBM) with two off-grid points for direct integration of first, second, and third-order initial value problems (IVPs) is proposed. The development of a block method for the solution of IVPs has been considered overwhelmingly in the literature. However, using a block method to directly solve multi-order IVPs has not been so common. Thus, the formulation of a single numerical algorithm for the direct numerical integration of first, second and third-order IVPs is our focus. The method is formulated from a continuous scheme derived using collocation and interpolation techniques and implemented in a block-by-block manner as a numerical integrator for IVPs. To assess the method's applicability, efficiency, and accuracy, the convergence analysis has been investigated, and six test problems are considered

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On Sawyer Duality Principle in Function Spaces

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The Sawyer duality principle is well known which provides both lower and upper bounds for the following expressions

$$I(g) := \sup_{0\le f\downarrow} \frac{\int_0^\infty f(x)g(x)\,dx}{\left(\int_0^\infty f^p v\right)^{1/p}}.$$

A similar study is also known for higher dimensional setting. In this presentation we will consider the above expression when f is replaced by its integral average $(\frac{1}{x}\int_0^x f(t)\,dt)$ and by its adjoint $(\int_x^\infty \frac{f(t)}{t})$ in the denominator. With these modification we shall discuss the two sided estimates of the corresponding expressions in the framework of Orlicz spaces as well as in the framework of higher dimensional Lebesgue spaces.

Optimal Algorithm for Designing and Reconstructing Urban Systems

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A city, an urban system, is a living organism: it is "born", grows, develops, "gets sick", "recovers", and so on. Each city has its own individual characteristics, a "genetic" code. If improperly managed, it is quite possible for it to lose its "individuality", and unpredictable, chaotic processes begin to develop. The modern concept of urban management – the "smart city" concept – views the city as a single dynamic system, where any action causes a "disturbance" to the entire system. Consequently, an urban planner managing a city must use a compromise solution – the Pareto principle (it is impossible to simultaneously ensure the well-being of all segments of the population) – when making specific decisions.

In the modern urban management concept of "smart city", city modeling and management (decision-making) are fully realized using mathematical methods and information and communication technologies. In our work, the mathematical model of the city represents a set L of relationships between objects and processes of various categories and objectives operating in the city – that is, it represents a geometric structure, for the analysis and management of which we use the q-analysis method. An algorithm is presented for the sustainable reorganization (reconstruction) of existing infrastructure based on a systems approach. Two necessary conditions for an optimal solution are presented, and a corresponding example is given.The theory of percolation explores the features and properties of both isolated and interconnected territories and landscapes. Therefore, it is a very valuable and applicable theory for the planning and management of urban systems, as well as for determining the placement of buildings, blocks, green areas and their optimal density. To create sustainable systems, it is very important to achieve a certain compromise between urbanization, nature conservation, agricultural expansion and other areas, because buildings and road infrastructure are the anthropogenic concerns of the landscape. Indeed, among them, distances, topology determine the environmental, social, economic and other problems of the urban system.

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A Modified Trend in Matrix Analysis and Applications

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In interval analysis, we write a new approach which can be written as:

$$[A, B] = \big\{x_\alpha = (1-\alpha)A + \alpha B : \ \alpha \in [0,1]\big\}.$$

Consequently, we may adopt interval operations by applying the scalar operation point-wise to the corresponding interval points.

With the usual restriction $0 \notin J$ if $\cdot = \dot{-}$.

These operations are associative:

$$I + (J + K) = (I + J) + K,$$
$$I * (J * K) = (I * J) * K.$$

These two properties, which are missing in the usual interval operations, will enable the extension of the usual linear system concepts to the interval setting in a seamless manner. The arithmetic introduced here avoids such vague terms as "interval extension", "inclusion function", determinants which we encounter in the engineering literature that deal with interval linear systems.

On the other hand, these definitions were motivated by our attempt to arrive at a definition of interval random variables and investigate the corresponding statistical properties. We feel that they are the natural ones to handle interval systems.

We will enable the extension of many results from usual state space models to interval state space models.

This feeling is reassured by the numerical results we obtained in a simulation examples.

Keywords: Matrix analysis, interval computations, convex analysis.

A Unified Difference-Distribution Framework for Cyclotomic Difference Sets and Almost Difference Sets

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Cyclotomic classes provide a classical source of difference sets and related combinatorial structures. In this work, we study the additive difference distributions of unions of cyclotomic classes in prime fields. Let $p = ef + 1$ be an odd prime and let

$$D_I = \bigcup_{i \in I} C_i^{(e,p)}$$

be a union of cyclotomic classes of order e. For a nonzero element $a \in C_h^{(e,p)}$, the number of ordered representations of a as a difference of two elements of D_I is expressed in terms of cyclotomic numbers by

$$N_{D_I}(a) = \sum_{i,j \in I} (j - h, i - h)_e.$$

Hence, the difference frequency depends only on the cyclotomic class containing a. This gives finite criteria for determining whether D_I is a difference set or a two-level almost difference set. The theoretical framework is complemented by exact computational verification based on modular difference distributions. The method is illustrated through Paley-type quadratic residue constructions, quartic and octic examples, and Hall-type sextic difference sets. The approach provides a unified way to organize cyclotomic difference sets and almost difference sets in prime fields.

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The author was supported by the Scientific Research Projects Coordination Unit of Recep Tayyip Erdoğan University (BAP Project # 2030).

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Some Special Structures on Walker-4 Manifolds

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This paper presents a new class of geometric manifolds, referred to as golden-Walker manifolds, obtained by endowing a four-dimensional Walker manifold with a golden structure. The golden structure is represented by a tensor field ψ satisfying the polynomial identity $\psi^2 - \psi - I = 0$, where I denotes the identity endomorphism. We first establish the basic geometric framework of these manifolds and then investigate how the presence of the golden structure influences their curvature properties. Several necessary and sufficient conditions characterizing distinguished curvature features are derived. Finally, we focus on the existence of geometric solitons on golden-Walker manifolds, examining Ricci solitons together with more general soliton equations and identifying the conditions under which such structures occur.

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Topological Invariants of Random Polynomials

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We will discuss topological invariants of random polynomials. There are considered the results on the expected value of topological degree and Euler characteristic of the fibers of certain random polynomial mappings. We will touch only random mappings defined by Gaussian random polynomials with certain special distributions of coefficients. There are also presents some results on a closely related topic concerned with the estimation of topological invariants of random polygonal knots.

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Minimality of Root Functions of Sturm–Liouville Problems with Linear Eigenparameter Boundary Conditions

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Consider the following spectral problem

$$-y'' + q(x)y = \lambda y, \quad 0 < x < 1, \tag{1}$$

$$y(0)\cos\beta = y'(0)\sin\beta, \quad 0 \le \beta < \pi, \tag{2}$$

$$(a\lambda + b)y(1) = (c\lambda + d)y'(1), \tag{3}$$

where a, b, c, d are real constants, $ad - bc < 0$, λ is the spectral parameter, and $q(x)$ is a real valued and continuous function over the interval $[0, 1]$.

Theorem 1 *In this case, where all the eigenvalues of* (1)–(3) *are real, and except one triple eigenvalue* λ_k*, all are simple, the systems*

$$\{y_n\} \ \ (n = 0, 1, \dots; \ \ n \neq k + 2),$$
$$\{y_n\} \ \ (n = 0, 1, \dots; \ \ n \neq l),$$

where $l \neq k$, $k + 1$, $k + 2$ *is a non-negative integer, are minimal in space* $L_2(0, 1)$.

Here y_{k+2} is an associated function of the second order corresponding to the eigenfunction y_k and the first associated function y_{k+1} [1].

Acknowledgments

This work was financially supported by ADA University and Baku State University.

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On Generalization of Mitev's Inequalities

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This work solves an optimization problem concerning the ratio of differences between classical symmetric means. Specifically, we determine the global extrema for the ratio of the arithmetic-harmonic difference to the quadratic-harmonic difference for n variables:

$$\frac{A_n - H_n}{Q_n - H_n}.$$

We prove that the maximum and minimum values of this ratio are $\frac{\sqrt{n-1}}{\sqrt{n}}$ and $\frac{1}{\sqrt{n}}$, respectively, effectively generalizing prior results established for specific cases like $n = 3$ and $n = 4$ etc. Our methodology reduces the multivariate problem to a single-variable optimization task. To ensure analytical rigor on a compact set, we resolve a removable discontinuity at the singular point where all variables coincide. The proof leverages a L'Hôpital-type monotonicity lemma, and the final results offer meaningful interpretations within probability and mathematical statistics.

Lemma 1 *If $x > 0$, then*

$$\lim_{(x_1,x_2,\ldots,x_n)\to(x,x,\ldots,x)} \frac{A_n - H_n}{Q_n - H_n} = \frac{2}{3}.$$

Lemma 2 *Let x, y, and z be positive real numbers such that, $x + y + z = \beta$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \alpha$. Without loss of generality, we can sort the variables in such a way that, $x \le y \le z$. Then the sum $x^2 + y^2 + z^2$ is decreasing when y is increasing staying between x and z.*

Acknowledgments

The work was supported by the ADA University Faculty Research and Development Fund and Baku State University.

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Free Temporal Gödel Algebras Generated by Constant Elements

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Modalized Heyting calculus **mHC**, introduced by Leo Esakia, is the augmentation of the intuitionistic logic **Int** by a modal operator $\Box$. The temporal Heyting calculus **tHC** was defined (by Esakia) on the basis of **mHC** with additional axioms for the "adjoint" modality $\Diamond$.

Algebraic models of **mHC** are **fHA**-algebras (frontal Heyting algebras). In [2] Castiglioni, Sagastume and San Martin have extended Heyting duality to the category **fHA**. The variety of temporal Heyting algebras **tHA**, which represent algebraic models of temporal Heyting calculus **tHC** has been investigated with various results [3, 4].

We investigate Temporal Gödel algebras **tGA**, which are **tHA**-algebras with pre-linearity condition: $(x \to y) \vee (y \to x) = 1$. We describe 0-generated free algebras in the variety of **tGA**.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number # FR-25-19092].

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Norm and Almost Everywhere Convergence of Subsequences of Vilenkin–Nörlund Means with Non-Increasing Sequences

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The convergence and summability of Fourier series with respect to Walsh and Vilenkin systems constitute an important part of modern dyadic harmonic analysis. A systematic presentation of martingale Hardy spaces, Vilenkin–Fourier series, and the corresponding summability methods can be found in the monograph by Persson, Tephnadze, and Weisz [1]. In particular, Fejér means play a fundamental role in this theory. Recently, Areshidze, Persson, and Tephnadze [2] investigated almost everywhere convergence of partial sums and Fejér means of Vilenkin–Fourier series and proved that, for every set of Haar measure zero, there exists an integrable function whose Vilenkin–Fejér means diverge at every point of that set.

In this talk, we investigate subsequences of Vilenkin–Nörlund means generated by non-increasing sequences (see [3]). We establish a new approximation estimate for the subsequence indexed by the generalized numbers M_n. This estimate is essentially stronger than previously known bounds and, in particular, yields the norm convergence

$$\|t_{M_n} f - f\|_p \to 0 \ \text{ as } \ n \to \infty$$

for every $f \in L^p(G_m)$, where $1 \leq p < \infty$. We also prove that the corresponding restricted maximal operator is of weak type $(1,1)$. Consequently, the subsequence $t_{M_n} f$ converges almost everywhere to f for every $f \in L^1(G_m)$. Finally, we show that this result is sharp: for every subset $E \subset G_m$ of Haar measure zero, there exists a concrete function $f \in L^1(G_m)$ such that its M_n-th Vilenkin–Nörlund means diverge at every point of E.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under Grant # FR-24-698.

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Low Light Image Enhancement Based on Coefficient Bounds Low of Sakaguchi Type Functions Associated with Gregory Polynomials

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Low-light image enhancement is a fundamental challenge in image processing as image captured under insufficient illumination often suffer from low contrast, poor visibility and loss of structural details. In this article, we propose a novel Mathematical framework for enhancing low-light images from the concept of geometric function theory (GFT). We employ a class of Sakaguchi-type analytic function associate with the Gregory polynomials. Sharp bound of the functions belonging to this class are obtained and later incorporated into compact directional convolution kernel to achieve controlled and stable image enhancement. The proposed method is then validated through comparative testing with the existing frontier methods, over key matrices like PSNR (Peak Signal-to-Noise Ratio) and SSIM (Structural Similarity Index), as well as via visual comparisons. The results obtained demonstrate that this method produces visually improved images and performs competitively with existing enhancement techniques, enhancing brightness and contrast while preserving essential image structures and suppressing noise amplification.

Keywords: Coefficient bounds, Sakaguchi-type, low-light image enhancement, Gregory polynomial image processing.

On Subset Sum Problem

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In this talk we plan to discuss the *Subset Sum Problem (SSP)* and a case when it has an exact solution.

The talk is based mainly on [1].

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A Characteristic Finite-Difference Method to Solve a Nonlinear Goursat Problem

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This article proposes a numerical approach based on the method of characteristics for solving a nonlinear characteristic Goursat-type problem for a class of second-order quasi-linear partial differential equations of non-strictly hyperbolic type [1]. The equation contains squares of the first derivatives of the unknown function, causing both the characteristic curves and the domain of parabolic degeneracy to depend on the solution itself, which makes the direct use of classical linear numerical methods impossible. A finite-difference algorithm based on discrete analogues of the characteristic differential relations, consistent with the nonlinear structure of the characteristics, is developed. The proposed scheme enables the computation of both the approximate solution and its first derivatives directly on characteristic curves belonging to different characteristic families. The uniqueness of solvability of the difference schemes is proved, and their uniform first-order convergence with respect to the grid step is established. It is rigorously shown that the nonlinear characteristic problem can be reduced to a finite-difference analogue of the classical Goursat problem. Numerical experiments confirm the stability and efficiency of the proposed method.

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On the Analytical Solution of the Pursuit Problem for a Moving Target

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This talk addresses a generalization of the classic Apollonius pursuit-evasion problem in a two-medium plane involving an amphibious pursuer and a moving target. Using the method of isochrones, we construct a geometric description of reachability boundaries and establish optimal refraction rules governed by Snell's law. We partition the motion space into distinct domains via a critical separatrix line, determining whether direct or refracted paths minimize interception time. Furthermore, explicit analytical formulas and optimization algorithms are derived for both stationary and moving target scenarios. The presented approach provides a geometric characterization of optimal trajectories and interception times, offering a framework applicable to broader differential games.

On Garsia's Conjecture

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In our talk, which is dedicated to *Adriano Mario Garsia* (1928–2024), we plan to discuss a conjecture posed in [1] about finite orthonormal sequences from $H = L_2[0,1]$ and its consequences.

The talk is based mainly on [2] and [3].

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On Boundedness and Compactness of the Positive Linear Integral Operators in Weighted Lebesgue Spaces

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Classical Hardy's inequalities focus on the Hardy operator and its adjoint, the Bellman operator, while fractional Hausdorff operators naturally generalize fractional versions of these two operators. In this abstract, we give a sufficient condition for the compactness property of the Hausdorff operator on weighted Lebesgue spaces. Also, the boundedness of the discrete Hausdorff operator on Lebesgue spaces is given. Moreover, the authors establish strong type two-weight inequalities for positive linear integral operators on weighted Lebesgue spaces.

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Norm, Pointwise, and Almost Everywhere Convergence of T Means of Vilenkin–Fourier Series

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The convergence and summability of Fourier series with respect to Walsh and Vilenkin systems form an important part of modern dyadic harmonic analysis. A systematic treatment of Vilenkin–Fourier series, martingale Hardy spaces, and various summability methods can be found in the monograph by Persson, Tephnadze, and Weisz [1]. Among these summability methods, T means constitute a broad class that includes several classical procedures, such as Fejér means, Riesz logarithmic means, and inverse Cesàro-type means.

In this talk, we investigate norm, pointwise, and almost everywhere convergence of T means of Vilenkin–Fourier series generated by monotone sequences of nonnegative coefficients (see [2] and [3]). For non-increasing coefficients, we prove convergence in $L^p(G_m)$, where $1 \leq p < \infty$, as well as pointwise convergence at continuity points and at Vilenkin–Lebesgue points of integrable functions. Analogous results are established for non-decreasing coefficients satisfying the condition

$$\frac{q_{n-1}}{Q_n} = O\Big(\frac{1}{n}\Big), \quad n \to \infty.$$

For arbitrary non-decreasing coefficients, we also obtain norm and pointwise convergence for the subsequence indexed by the generalized numbers M_n, including convergence at Lebesgue points. Furthermore, we establish a general criterion for the weak-type $(1,1)$ boundedness of the maximal operator associated with T means. Applying this criterion, we prove almost everywhere convergence for regular T means generated by non-increasing coefficients and for those generated by non-decreasing coefficients satisfying the condition above. We also show the almost everywhere convergence of the subsequence $T_{M_n} f$ for regular T means generated by arbitrary non-decreasing coefficients. As consequences, convergence results for several classical summability methods are recovered and extended.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under Grant # PHDF-25-2055.

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Local Regression on Path Spaces with Signature Metrics

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We study nonparametric regression and classification for path-valued data $(X, Y) \in \mathscr{X} \times \mathscr{Y}$, where $mathcal X$ is an (infinite-dimensional) space of paths $x : [0, T] \to \mathbb{R}^d$ and $\mathscr{Y}$ a finite-dimensional Euclidean space. The object of interest is the regression function $F(x) = \mathbb{E}[Y \mid X = x]$, which we estimate by the functional Nadaraya–Watson estimator [3] $\widehat{F}(x) = \frac{\sum_{i=1}^{M} Y^{(i)} K(h^{-1}\varrho(x, X^{(i)}))}{\sum_{j=1}^{M} K(h^{-1}\varrho(x, X^{(j)}))}$, built from i.i.d. samples $(X^{(i)}, Y^{(i)})$, a bandwidth $h > 0$, an asymmetric kernel K, and a semi-metric ϱ on $\mathscr{X}$. Our central proposal is to let ϱ be induced by the *signature* | $transform$ [1] of rough path theory, $\varrho^{\mathrm{Sig}}(x, y) = \|\mathrm{Sig}\,(x) - \mathrm{Sig}\,(y)\|$, where $\mathrm{Sig}\,(x)$ collects the iterated integrals of x and $\|\cdot\|$ is the Hilbert–Schmidt norm on the extended tensor algebra $\mathscr{T} = \prod_{k \geq 0} (\mathbb{R}^d)^{\otimes k}$; in practice one truncates the signature at a level N. The signature encodes the geometry of a path, is invariant under time reparametrization, and lets sequences of different lengths be compared directly, while avoiding the large Gram-matrix inversions inherent to signature-kernel methods. Assuming only that F is H¨older with respect to ϱ, that is $|F(x) - F(x')| \leq L\varrho(x, x')^{\beta}$ for some $\beta \in (0, 1]$, we prove a finite-sample deviation bound whose bias-variance trade-off is governed by β and by the small-ball behaviour of X, measured through its concentration function $\phi_x^{\varrho}(h) = \mathbb{P}[\varrho(X, x) \leq h]$. For the truncated signature distance this concentration function decays only *polynomially*, with exponent equal to the homogeneous dimension $\nu(N)$ of the free nilpotent Lie group carrying the signature; this yields Euclidean-type nonparametric rates $M^{-\beta/(2\beta+\nu(N))}$ in an infinite-dimensional setting, in sharp contrast to the merely logarithmic rates that classical L^p or Hölder metrics produce there [5]. In this respect the signature metric is never worse than the 1-variation distance. We further derive rates for learning the Itô–Lyons solution map of rough differential equations [4], and—to counter the instability caused by unbounded signature entries-employ the robust (tensor-normalized) signature [2]. Experiments on SDE learning and on real-world time-series classification confirm competitive accuracy with substantial computational savings.

Acknowledgments

Christian Bayer and Luca Pelizzari gratefully acknowledge funding by the Deutsche Forschungsgemeinschaft (DFG) through CRC/TRR 388 *"Rough Analysis, Stochastic Dynamics and Related Fields"* (Project # ID 516748464), Project # B03. The work of Davit Gogolashvili was supported by the LMBayes project (Linguistic Meaning and Bayesian Modelling).

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Some Remarks on the Intersections of Mazurkiewicz Type Sets with Plane Algebraic Curves

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It is known that a Mazurkiewicz type set with respect to the family of straight lines has only finite intersections with every line in the plane. This naturally leads to the question of whether analogous properties hold for irreducible plane algebraic curves of degree at least two. Motivated by this question, and by Larman's well-known result, it is natural to investigate the relationship between irreducible plane algebraic curves and Mazurkiewicz type sets. (see, e.g., [1]).

Theorem *Let $C \subset \mathbb{R}^2$ be any irreducible algebraic curve of degree at least two and let n be an arbitrary fixed natural number such that $n \geq 2$. Then there exist Mazurkiewicz n-type sets: $M_1^k \subset \mathbb{R}^2$, $M_2^k \subset \mathbb{R}^2$, $M_3^k \subset \mathbb{R}^2$, $M_4^k \subset \mathbb{R}^2$ each of which is with respect to the family of all straight lines in $\mathbb{R}^2$ and possesses the following properties:*

- *The intersection of M_1^k with any nondegenerate simple arc contained in C is of cardinality continuum, has Lebesgue linear measure zero, and is of the Baire first category;*
- *The intersection of M_2^k with any nondegenerate simple arc contained in C is of cardinality continuum, has Lebesgue linear measure zero, and is of the Baire second category;*
- *The intersection of M_3^k with any nondegenerate simple arc contained in C is of cardinality continuum, is non-measurable with respect to the Lebesgue linear measure (moreover, is thick with respect to this measure), and is of the Baire first category;*
- *The intersection of M_4^k with any nondegenerate simple arc contained in C is of cardinality continuum, is non-measurable with respect to the Lebesgue linear measure (moreover, is thick with respect to this measure), and is of the Baire second category (see, e.g., [2]).*

An analogous question assertion is valid for Mazurkiewicz type sets with respect to the family of circles.

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Sierpiński Topological Spaces and Hamel Bases

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It is known, that there exists a Mazurkiewicz set which contains the graph of some Sierpinski–Zygmund function acting from R into R. A Sierpiński space E is characterized by the absence of Luzin subspaces of the same cardinality. Under the assumption of the Continuum Hypothesis (CH) or Martin's Axiom (MA), we demonstrate the simultaneous existence of a subset of the real line that functions concurrently as a Sierpiński set and a Hamel basis. see [1, 2]).

Given that any classical or generalized Sierpiński set constitutes a Sierpiński topological space, these constructions verify that algebraic structures can embody the precise measure-theoretic pathologies that define such spaces. The existence of these dual-natured sets carries significant mathematical weight, particularly in the study of Lebesgue nonmeasurable sets and absolutely nonmeasurable functions, as a Hamel basis fundamentally guarantees the presence of nonmeasurable subsets within R. Ultimately, this research establishes that the topological "largeness" inherent to Sierpiński spaces is entirely compatible with the algebraic property of linear independence over the field of rational numbers Q.

In the presented talk, we discuss some characterization of Sierpiński topological spaces by analyzing and integrating various algebraic, topological, and set-theoretic objects and axioms.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) # FR-25-19092 – *"Komori type epistemic MV-algebras with constants"*.

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Vertex Enumeration and Structural Properties of k-Isosceles Simplexes in Euclidean Space

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This research explores the geometric and combinatorial properties of k-isosceles simplexes within m-dimensional Euclidean space (R^m), highlighting their role in combinatorial geometry. We analyze the definition of k-isosceles simplexes based on the congruence of their k-dimensional faces and the recursive properties of their sub-simplexes.

A primary focus of this study is the relationship between face geometry and the vertex bounds of convex polyhedra. We demonstrate that in a strictly convex polyhedron where all faces are congruent equilateral triangles (a deltahedron), the sum of face angles around a vertex restricts the degree of each vertex to a maximum of five. Utilizing Euler's formula, we confirm that such polyhedra are constrained to a maximum of 12 vertices.

Crucially, the research identifies a significant structural transition: by slightly modifying the geometric constraints-specifically by transitioning from equilateral faces to congruent isosceles triangles – the upper bound on the number of vertices is entirely removed, allowing for an unbounded total of vertices.

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On the Homotopical Invariants of the Braid Groups Modulo the Centre

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Recent developments have shown that studying stable homotopy theoretic invariants of infinite discrete groups is interesting and useful in geometry, number theory and group cohomology (see [1–4]). The paper [2] introduces a general duality theory for infinite groups that generalizes Bieri–Eckmann duality and produces new type of Euler characteristics. In particular, the paper [2] studies the question when does a group have a duality with coefficients in a generalized (co)homology. The failure of this generalized duality is witnessed by the generalized Farrell–Tate cohomology. The paper [1] examined the Farrell–Tate cohomology with coefficients in topological K-theory and produced a general formula for computing it. The paper [1] then applied this formula to compute the p-adic Farrell–Tate K-theory for $Out(F_n)$ for certain n.

In this talk we will discuss a joint work with Anzor Beridze where we study the Farrell–Tate K-theory of the braid groups module the centre. Using the main formula from [1], we give a full calculation of these groups. This relies on the Garside structure and knowledge of periodic elements and their centralizers in braid groups. In particular, we compute the rational cohomology of these centralizers modulo Δ^2. The final answer involves a concrete number theoretic function which computes the dimension of the p-adic Farrell–Tate K-theory groups as $\mathbb{Q}_p$-vectors spaces. At the end we also compute higher chromatic invariants as studied in [2] and [3] and examine the congruences discussed in [4].

Acknowledgments

Both authors were supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) through the grant # FR-25-10267. The second author was supported by the EPSRC grant EP/X038424/1 *"Classifying spaces, proper actions and stable homotopy theory"*.

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Burau Representation for $n = 4$: the Problem of Faithfulness – Algebraic vs Topological Approach

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The problem of the faithfulness of the Burau representation for $n = 4$ appears to have been solved affirmatively by Joan Birman and her coauthors.

We briefly describe the topological approach that led to this remarkable result, using some elementary homotopy theory.

We also describe our own attempts to solve the same problem by treating it as a purely linear algebraic question. This reduces to asking whether certain matrices A and B, whose entries are Laurent polynomials, generate a free group of rank two. This is, in turn, a classical problem in mathematics, and we hope that it admits a solution based solely on linear algebra, independent of any topological considerations.

Acknowledgments

First author were supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) through the grant # FR-25-10267.

Symplectic Modules Over Geometric Subrings and Monoid Algebras

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Let R be a commutative Noetherian ring of dimension d and N a commutative partially-cancellative torsion-free seminormal monoid. Let A be a geometric subring of polynomial ring $R[t]$. We prove that any symplectic module over A or $R[N]$ of *rank* d is cancellative. We also study the splitting problem for symplectic modules over such algebras.

An Extension of Narayana Difference Sequence Spaces

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In recent years, numerous studies have been devoted to sequence spaces and their applications in summability theory, functional analysis, and matrix transformations. Among these, Narayana sequence spaces and their difference counterparts have recently been introduced and investigated from various perspectives. In this paper, we extend this framework to the classical space ℓ_p, where $1 \leq p \leq \infty$, by introducing the sequence space $\ell_p(\mathscr{N}_\Delta)$, defined as the matrix domain of the Narayana difference matrix $\mathscr{N}_\Delta$.

We investigate the topological and algebraic properties of the newly defined space and prove that it is a BK-space under a suitable norm. Furthermore, we establish a linear isomorphism between $\ell_p(\mathscr{N}_\Delta)$ and ℓ_p. In addition, we determine a Schauder basis for $\ell_p(\mathscr{N}_\Delta)$ and characterize its α-, β-, and γ-duals. These results are subsequently employed to characterize infinite matrix transformations from $\ell_p(\mathscr{N}_\Delta)$ into the classical sequence spaces ℓ_∞, c, c_0, and ℓ_p. The proposed construction provides a natural extension of Narayana difference sequence spaces and offers a broader framework for the study of matrix domains generated by recursive sequence matrices.

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Sufficient Optimality Conditions for a Control Problem Driven by McKean–Vlasov FBDSDEs

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In this work, we study a control problem for systems of forward-backward doubly SDEs of McKean–Vlasov type, (MV-FBDSDEs) in which the coefficients depend on the states of the solution processes as well as their distribution. Moreover the cost functional is also of McKean-Vlasov type. In particular, we establish sufficient optimality conditions for a stochastic control problem governed by MV-FBDSDEs in infinite-dimensional separable real Hilbert spaces.

Theorem (Sufficient conditions for optimality) *Let U be convex. Assume that conditions* (A_1)–(A_3) *hold. Given $\widehat{u}_\cdot \in \mathscr{U}_{ad}$, let $V_t^{\widehat{u}_\cdot} \equiv (y_t^{\widehat{u}_\cdot}, Y_t^{\widehat{u}_\cdot}, z_t^{\widehat{u}_\cdot}, Z_t^{\widehat{u}_\cdot})$ and $(p^{\widehat{u}}, P^{\widehat{u}}, q^{\widehat{u}}, Q^{\widehat{u}})$ be the corresponding solutions of MV-FBDSDEs (1) and (4), respectively. Assume the following:*

(i) *φ and ψ are convex.*

(ii) *For all $t \in [0,T]$, P-a.s., the function $\mathscr{H}(t,\cdot,\cdot,\cdot,\cdot,\cdot,\cdot,p^{\widehat{u}},P^{\widehat{u}},q^{\widehat{u}},Q^{\widehat{u}})$ is concave.*

(iii) $\mathscr{H}(t,V_t^{\widehat{u}_\cdot},\widehat{u}_t,\chi_t^{\widehat{u}_\cdot},\mathbb{P}_{V_t^{\widehat{u}_\cdot}}) = \max\limits_{u\in U}\mathscr{H}(t,V_t^{\widehat{u}_\cdot},v,\chi_t^{\widehat{u}_\cdot},\mathbb{P}_{V_t^{\widehat{u}_\cdot}})$, *a.e. t, $\mathbb{P}$ – a.s.*

Then $(y^{\widehat{u}_\cdot}, Y^{\widehat{u}_\cdot}, z^{\widehat{u}_\cdot}, Z^{\widehat{u}_\cdot}, \widehat{u}_\cdot)$ is an optimal solution of our control problem.

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On $H = W$ in Banach function spaces

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In this talk, the proof of "$H = W$" in the context of a Banach function space $X(\Omega)$ is considered. Let Ω be a subset of $\mathbb{R}^n$ and denote by $W^1_X(\Omega)$ the collection of all those $f \in X(\Omega)$ whose distributional derivatives $\partial_j f$ are contained in $X(\Omega)$. Our main result provides a small collection of "universal" hypotheses on $X(\Omega)$ that ensure $W^1_X(\Omega)$ is equal to $H^1_X(\Omega)$, the formal closure of $\mathrm{Lip}(\Omega) \cap W^1_X(\Omega)$ with respect to the norm

$$\|f\|_{W^1_X(\Omega)} = \|f\|_{X(\Omega)} + \|\nabla f\|_{X(\Omega)}.$$

The required hypothesess are summarized below. These draw upon recent papers by Lorist and Nieraeth [2] and by Nieraeth [4] on Banach function spaces.

Hypothesis *Given a domain Ω and a Banach function space $X(\Omega)$, we assume that it has the following properties:*

- *The Fatou property holds;*
- *The norm on $X(\Omega)$ is absolutely continuous;*
- *The strong Muckenhoupt condition holds.*

Acknowledgments

The first author is supported by the TUBITAK 2219 – International Postdoctoral Research Fellowship Program for Turkish Citizens. The second author is partially supported by a Simons Foundation Travel Support for Mathematicians Grant and by NSF Grant DMS-2349550. The third author is partially supported by an NSERC Development Grant and an NSERC Discovery Grant, Canada.

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A Partial Differential Model Describing the Dynamics of the Kartvelian – Speaking Population

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Previously, we proposed and developed continuous mathematical models describing the transformation of the Proto-Kartvelian population into Georgian, Laz, Mingrelian, and Svan populations. These models were represented by systems consisting of two, three, and four ordinary differential equations [1–4].

The paper presents a new mathematical partial differential model that describes the dynamics of the Kartvelian-speaking population size. The linear partial differential equation of parabolic type contains diffusion and demographic factor terms, as well as a term that determines the greater protection of the population with increasing elevation above sea level.

As a result of an annihilating transformation of the boundary conditions of the mixed problem, a boundary value problem is obtained for a second-order ordinary differential equation with variable coefficients. Its solution can be expressed in terms of Airy functions of the first and second kinds.Fourier's method is used to solve the linear homogeneous partial differential equation of parabolic type. For the coefficient depending on the geometric variable, a Sturm–Liouville boundary value problem is obtained, which is solved analytically in a particular case.

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On the Asymptotic Solution on the Explosion Problem of a Triaxial Rotating Homogeneous Self-Gravitating Gaseous Ellipsoid

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The paper considers a non-self-similar problem of a central explosion in a homogeneous triaxial gaseous ellipsoid that exists within its own gravitational field and rotates about the z-axis with a constant angular velocity as a rigid body. The initial conditions are based on the exact solution of the stationary rigid-body rotation problem for a homogeneous triaxial Jacobi gaseous ellipsoid.

It is assumed that, at the initial moment of time, a central explosion occurs, releasing a finite amount of energy. As a result, a propagating detonation shock wave (a first-kind discontinuity surface of the unknown functions) is generated at the center of the gaseous ellipsoid.

To solve the problem, the asymptotic thin shock-layer method previously proposed by the authors is employed [1–5]. Analytical solutions in quadratures are obtained for the zeroth- and first-order approximations of the singular asymptotic expansions describing the law of motion and velocity behind the detonation shock wave, as well as the thermodynamic characteristics of the self-gravitating gas.

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Development of the Localized Boundary-Domain Integral Equations Method for the Dynamical Dirichlet Problem of the Theory of Inhomogeneous Generalized Thermo-Electro-Magnetic Elastic Solids

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The paper deals with the three-dimensional Dirichlet dynamical problem of the genera-lized thermo-electro-magneto-elasticity for anisotropic inhomogeneous solids and develops the potential method based on the localized parametrix method. Using Laplace trans-form Green's integral representation formula and properties of the localized layer and vo-lume potentials we reduce the Dirichlet dynamical problem to the localized boundary-do-main integral equations (LBDIE) system. The equivalence between the Dirichlet BVP and the corresponding LBDIE system is studied. We establish that the obtained localized boun-dary-domain integral operator belongs to the Boutet de Monvel algebra and with the help of the Wiener-Hopf factorization method we investigate corresponding Fredholm properties and prove invertibility of the localized operator in appropriate function spaces.

This is joint work with D. Natroshvili.

Acknowledgments

This research was supported by the S. Rustaveli National Science Foundation (SRNSF) Grant # FR-23-267.

Numerical Solution of a Fourth Order Nonlinear Parabolic Type Integro-Differential Equations Using Machine Learning

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The initial-boundary value problems for the fourth-order nonlinear parabolic integro-differential equations are considered. The integro-differential models of such type are studied in many works (see, for example, [1, 2] and references therein). Numerical experiments have been conducted using machine learning methods [3]. Our aim is to continue the study of these models in this direction.

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Matrix Representations of D-Structures and Generalized Monoid Rings

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The generalized monoid rings proposed in [1] naturally encompass various classes of group rings, differential polynomial rings, especially skew and q-skew polynomial rings, and other related rings.

We have recently shown [2] that every generalized monoid ring is determined by a family of additive maps, called a D-structure, satisfying a collection of compatibility conditions that encode both the multiplicative structure of the coefficient ring and the underlying monoid [1]. This description provides a unified framework for studying numerous classes of ring extensions.

The purpose of this talk is to present matrix representations associated with D-structures and to demonstrate that they provide an effective tool for studying generalized monoid rings. Given a ring A, a monoid G, and a D-structure $\sigma = (\sigma_{g,h})$ $g, h \in G$, we associate with each element $a \in A$ the matrix $\alpha(a) = (\sigma_{g,h}(a))_{g,h\in G}$. The defining axioms of a D-structure imply that α is an injective ring homomorphism from A into a matrix ring over A. When G is infinite, the finite-support condition ensures that the image consists of row-finite matrices. Consequently, every D-structure admits a faithful matrix representation in which the algebraic information carried by the family of maps $\sigma_{g,h}$ is encoded in a single matrix. Some important classes of generalized monoid rings are examined from this perspective. For D-structures arising from monoid actions by endomorphisms, the associated matrices are diagonal and recover the familiar representations of skew monoid and skew group rings. D-structures determined by derivations produce lower triangular matrices whose entries involve iterated derivations and binomial coefficients, yielding the classical skew polynomial rings of derivation type [2]. Explicit matrix representations are also presented for more general D-structures, including examples defined over upper triangular matrix algebras [3]. The matrix approach extends naturally beyond D-structures. In particular, every ring equipped with a derivation admits a faithful representation by 2×2 lower triangular matrices thereby providing an alternative realization of derivation rings as triangular matrix rings and establishing a connection with classical results on triangular matrix extensions due to Chase [4]. These constructions demonstrate that matrix representations offer a unified and computationally convenient framework for studying generalized monoid rings and related skew ring extensions. They also provide new insight into the interplay between endomorphisms, derivations, and the structural properties of D-structures, suggesting further applications to radicals, skew polynomial extensions, and other classes of noncommutative ring extensions [5, 6].

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On q-Central Idempotents and Radical Classes of Rings

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The talk is based on results published in [1–4], partially presented on XII International Conference of the Georgian Mathematical Union in 2022 [5].

We examine connections between ring radicals and corners, namely subrings of the form eAe, where e is an idempotent of a ring A. Our principal focus is on idempotents which are q-central in the sense of T. Y. Lam [6]. Such idempotents are defined in terms of the Peirce decomposition, but an idempotent e of a ring A is q-central if and only if eAe is is an endomorphic image of A via the map which sends each element a to eae. All radical classes are therefore hereditary for corners defined by q-central idempotents, though the analogous statement for idempotents in general is false. We explore conditions under which the radical of a ring contains all corners (or all those defined by certain types of idempotents) belonging to a given radical class, and investigate radical properties characterized by the presence or absence of idempotents of various kinds.

Acknowledgments

The first author is partially supported by the Institutional Research Program of the Moldova State University for 2024–2027 subprogramme SATGED 011303.

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Some Results Around Feed-Forward Networks Definable in o-Minimal Expansions of $\mathbb{R}$

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We introduce the classical model theoretic concept of an o-minimal expansion of $\mathbb{R}$ in which one may enjoy a "tame" geometry that generalizes familiar real analytic geometry. We give some familiar examples to convince the audience that most realistic feed-forward architectures naturally live in an o-minimal setting as definable families of functions. We will discuss some previous results on learnability and trainability of networks definable in o-minimal settings, and finish some recent work on learnability of architectures definable in an o-minimal settings that is joint work with Anastatsis Kratsios, Simone Brugiapaglia, Bum Jun Kim, and Haitz Sáez de Ocáriz Borde.

Equations and their Solution for the S-Matrix of Integrable 2d CFT's

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We investigate the S-matrix of coset WZW theories such as: Liouville theory, SL(2,R)/U(1) Black Hole model, SL(3) Toda theory, etc. Based on integrability of these theories, we derive equations for the S-matrix functional. We construct some exact solutions, which describe Fock space transition amplitudes. We then show that the general solution allows a special type functional integral representation, which can be used for practical calculations.

Acknowledgments

This research has been supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) by the grant # FR-23-17899.

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Flight Delay Prediction using a Multilayer Neural Network

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This paper discusses the prediction of air transport flight delays using Multi-Layer Perceptron. The aim of the study is to create a mathematical model that will comprehensively analyze both time and operational characteristics, as well as climatic risks. The indicators of departure and destination points are taken into account. At the data pre-processing stage, Min-Max normalization and linear transformation of categorical variables (One-Hot Encoding) methods were used, which ensured the formation of a 22-dimensional input vector. A controlled matrix of one-way factor analysis (Sensitivity Analysis) was used to validate the model.

Empirical testing showed that the neural network stably identifies aviation seasonality, airport infrastructure load, and the logistics domino effect. The results obtained are characterized by high accuracy and confirm the effectiveness of machine learning algorithms in the process of optimizing aviation management.

The results obtained confirm that the integration of two-way climate risks (departure/destination) with temporal parameters provides a reliable tool for aviation risk management and forms the basis for automated decision support systems.

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On One Nonstationary Problem in the Theory of Elasticity with Periodic Initial Conditions

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The report considers a nonstationary problem describing the vibration of an infinite strip under the action of periodic external loads in the presence of periodic initial conditions.

The work is based on the method of regularization of difference schemes proposed by A.Samarskii for the case of a positively defined spatial operator. This method provides broad possibilities for constructing efficient (economical) difference schemes with prescribed properties for specific problems, provided that the approximation rule for the spatial operator is known.

For the considered problem, an absolutely stable factorized difference scheme is constructed using the regularization method. A distinctive feature of this problem is that the spatial operator A is required to be only nonnegative, and therefore the inverse operator A^{-1} does not exist.

The absolute stability of the proposed scheme is proven, and an a-priori estimate is obtained. Based on this estimate, the convergence of the factorized scheme with second-order accuracy with respect to both the time step t and the spatial step $|h|$ is demonstrated.

Acknowledgments

The work is supported by the EU Horizon Europe project FORGE-AI.

The Role of Puzzle Problems in the Development of Mathematical Thinking

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This article discusses the importance of puzzle-based problems in the process of teaching mathematics and their impact on the development of students' cognitive, logical, and creative thinking skills. It demonstrates that puzzle problems are an effective educational resource that promotes the development of problem-solving strategies, analytical and reasoning skills, a deeper understanding of mathematical concepts, and increased interest in learning.

The article reviews the main types of puzzle problems, the principles for their selection, and the methodological features of their implementation in mathematics instruction. It also presents practical recommendations and examples of classroom activities that support teachers in the purposeful integration of puzzle problems into mathematics lessons.

The systematic use of puzzle problems enhances students' motivation, develops critical and logical thinking, strengthens problem-solving abilities, and contributes to the effective development of mathematical competencies.

On Hopf Bifurcation in Predator-Prey Models with Prey's Self-Defence

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In present work we investigate the dynamics of predator-prey type models of the form

$$\dot{x} = \mathscr{A}_L(x) - xy, \quad \dot{y} = -\gamma y + xy - \alpha x^2 y,$$

where $\mathscr{A}_L : \mathbb{R}_0^+ \to \mathbb{R}$ is continuous function which describes evolution of isolated population of preys mainly affected by the Allee effect. Strong suppression of the population of the predator by the term $-\alpha x^2 y$ in the second equation is then balanced by the destabilisation of the system by means of the Allee effect in preys. Dynamics of such the models is quite rich since there may exist two distinct coexistence equilibria besides Allee's threshold unstable equilibrium and, if applied, logistic bound equilibrium which stability varies with the parameters of the system. As an example, we summarize one of our results.

Theorem *In the case*

$$\mathscr{A}_L(x) = (x - L)\Big(1 - \frac{x}{K}\Big) \text{ and } \gamma = \frac{p}{4\alpha}, \ 0 < p < 1,$$

and $L = qK$, $0 < q < 1$, *our system undergoes soft Hopf bifurcation at the coexistence equilibrium with smaller values of* x *at the point*

$$\alpha_0 = \frac{1 - \sqrt{1-p}}{2\sqrt{q}\,K}$$

with stable limit cycle for values of α *(slightly) exceeding the bifurcation value* α_0. *Further, all the trajectories of our system are bounded.*

Acknowledgments

The work was supported by the by the grant schemes VEGA 1/0084/23 and VEGA 1/0346/25 of the Ministry of Education, Research, and Sport of the Slovak Republic.

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Optimal Control Problem for a Contact Problem with Nonlocal Boundary Conditions

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Contact problems for equations of mathematical physics play an important role in the theory of boundary value problems and have numerous applications in the mechanics of deformable solids, elasticity theory and other applied fields. Recent developments include nonlocal contact problems with multipoint contact conditions for elliptic equations, as well as numerical approaches to optimal control problems.

In this work, we study an optimal control problem for a two-dimensional contact problem with nonlocal boundary conditions. A Helmholtz equation is considered in a rectangular domain divided by an interior line into two subdomains. The corresponding boundary value problem is formulated with nonlocal contact conditions. The objective is to derive optimality conditions for the minimization of a linear functional depending on the state and control variables. The method of variations is employed to obtain an integral representation of the increment of the functional. An adjoint problem is constructed, which allows the optimality conditions to be formulated in the form of a Pontryagin-type maximum principle.

For the adjoint problem, an iterative method based on the Green's function is developed. Convergence estimates for the iterative process are established, and the existence and uniqueness of a generalized solution of the adjoint problem are proved. The main contribution is the derivation of necessary and sufficient optimality conditions in the form of an integral relation for the optimal control.

The obtained results provide a theoretical basis for the development of numerical algorithms for optimal control problems associated with nonlocal contact models.

Acknowledgments

This work was supported by the targeted scientific-research project of Batumi Shota Rustaveli State University (Resolution # 06-01/33).

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Boundary Value Problems for the Generic Laplace Equation

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On a Lie group $G \subset \mathbb{R}^n$ with a generic Fourier transformation $mathcal F_G$ and the Haar measure $d_G x$ we define the generic Bessel potential space $\mathbb{GH}^s_p(G, d_G x)$, endowed with the norm

$$\big\|\psi \mid \mathbb{GH}^s_p(G, d_G x)\big\| := \big\|W^0_{G,\langle\xi\rangle^s}\psi \mid \mathbb{L}_p(G, d_G x)\big\| = \big\|\mathscr{F}^{-1}_G\langle\xi\rangle^s\mathscr{F}_G\psi \mid \mathbb{L}_p(G, d_G x)\big\|, \quad \langle\xi\rangle^s := (1+|\xi|^2)^s, \quad s \in \mathbb{R}, \quad (1)$$

where $1 < p < \infty$, $s \in \mathbb{R}$. For an integer $s = m = 1, 2, \dots$ the space $\mathbb{GH}^m_p(G, d_G x)$ is isomorphic to the Generic Sobolev Space (GSS) $\mathbb{GW}^m_p(G, d_G x)$, consisting of functions with the finite norm

$$\big\|\varphi \mid \mathbb{GW}^m_p(G, d_G x)\big\| := \Big[\sum_{|\alpha|\leqslant m} \big\|\mathfrak{D}^\alpha\varphi \mid \mathbb{L}_p(G, d_G x)\big\|^p\Big]^{1/p},$$

where $\mathfrak{D}^\alpha = \mathfrak{D}_1^{\alpha_1}\cdots\mathfrak{D}_n^{\alpha_n}$ and $\mathfrak{D}_1, \dots, \mathfrak{D}_n$ are generic differential operators, generated by vector fields of the Lie algebra, associated with G (they are convolution operators on G).

Our purpose is to prove solvability of Dirichlet, Neumen and mixed BVPs for the Generic Laplacian $\Delta_G = \mathfrak{D}_1^2 + \cdots + \mathfrak{D}_n^2$ on domains $\Omega \subset G$ in the generic Bessel potential spaces $\mathbb{GH}^s_p(G, d_G x)$. For this we Prove, Gaußformula, I and II Green formulae and write explicitely indicate the fundamental sokution for the generic Laplacian.

Examples of the Lie groups (there exist many other examples):

- $\mathbb{R} = (-\infty, \infty)$ with the Generic differential operator ∂ and the Haar measure dx.
- $\mathbb{R}^+ = (0, \infty)$ with the Generic differential operator $\mathfrak{D} = x\partial$ and the Haar measure $\frac{dx}{x}$.
- $I := (-1, 1)$-interval in $\mathbb{R}$ with Generic differential operator $\mathfrak{D} = (1-x^2)\partial$ and the Haar measure $\frac{dx}{1-x^2}$.
- Any number of direct products (also mixed ones) of the above Lie groups.

The report is based on the joint work which is prepared for publication with M. Ruzhansky, D. Cardona & A. Hendrix.

Acknowledgments

The work is supported by the grant, # FR-24-698 of Shota Rustaveli National Science Foundation.

Transition Polynomial as a Weight System

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To a singular knot K with n double points, one can associate a chord diagram with n chords. A chord diagram can also be understood as a 4-regular graph endowed with an oriented Euler circuit. L. Traldi introduced a polynomial invariant for such graphs, called a transition polynomial. We specialize this polynomial to a multiplicative weight system, that is, a function on chord diagrams satisfying 4-term relations and determining thus a finite type knot invariant.

Acknowledgments

This work was prepared within the framework of the Academic Fund Program at the National Research University Higher School of Economics (HSE) in 2020–2021 (grant 20-04-010) and supported within the framework of a subsidy granted to the HSE by the Government of the Russian Federation for the implementation of the Global Competitiveness Program.

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Combining Quantitative Computation and Deduction

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In this talk, we present quantitative rewriting logic [1,2], a quantitative extension of rewriting logic [4], a well-known logical and semantic framework that unifies computation and deduction. The extension is designed to support the modelling of systems exhibiting probabilistic, resource-sensitive, approximate, or otherwise graded behaviour. Its main features are as follows:

- *Abstract quantitative foundation.* Following [3], quantitative information is represented using Lawverean quantales, encompassing a range of instances, including the Boolean quantale, fuzzy quantales based on the minimum or product t-norm, the Lawvere quantale for various notions of distance, etc.
- *Quantitative rewriting modulo.* We introduce a notion of rewriting modulo quantitative relations, denoted by $\rightarrow_{R^\epsilon/E^\epsilon}$, in which both the rewrite relation R^ϵ and the background equational theory E^ϵ may be quantitative, with exact relations arising as a special case. Depending on whether R^ϵ and E^ϵ are empty, exact, or quantitative, $\rightarrow_{R^\epsilon/E^\epsilon}$ specializes to nine different relations, including several well-known ones as well as, to the best of our knowledge, novel combinations.
- *Deduction and operational correspondence.* We investigate deduction in (unsorted, unconditional) quantitative rewriting logic and establish its correspondence with both the reflexive-transitive closure of $\rightarrow_{R^\epsilon/E^\epsilon}$ and, under suitable conditions, its operational counterpart $\rightarrow_{R^\epsilon,E^\epsilon}$, which performs rewriting by matching modulo E^ϵ. This extends the classical correspondence between the proof-theoretic and operational semantics of rewriting logic to the quantitative setting.

Acknowledgments

Partially supported by the Austrian Science Fund (FWF) project # P 35530; Rustaveli National Science Foundation of Georgia project # FR-25-5957; grant CIPROM/2022/6 funded by Generalitat Valenciana; grant PID2024-162030OB-100 funded by MCIN/AEI/10.13039/501100011033 and ERDF A way of making Europe; The NATO Science for Peace and Security Programme project SymSafe (grant number G6133).

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Fuzzifying Derivatives of Regular Expressions

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There is growing interest in computer science and artificial intelligence in moving from *exact*, *qualitative* settings to *quantitative*, and ultimately *fuzzy*, ones. In this spirit, we extend the theory of finite automata, based on exact symbol equality, to fuzzy symbol matching. Following similarity-based reasoning [7], we enrich the alphabet with a *similarity relation* $\mathscr{R}$ and perform matching up to a fixed *cut* value $\mu \in (0, 1]$: two symbols match when their similarity is at least μ. Here $\mathscr{R}$ takes values in $[0, 1]$ endowed with a left-continuous T-norm (a residuated lattice [4]), and is reflexive, symmetric, and transitive. We generalize the theory of Brzozowski derivatives [1], revisited in [5], which constructs automata directly from regular expressions: after reading a word, the current state is the corresponding derivative of the expression. The similarity relation on symbols is extended to regular expressions, words, and languages, yielding in each case a similarity relation. Since intersection and complement do not fuzzify well, our results concern the *restricted regular expressions*: empty language, empty word, letters, union, concatenation, and Kleene star. We introduce *fuzzy language and expression derivatives*, in which each symbol of the input word is matched up to the cut value μ, and prove that they preserve regularity. We further establish a *semantic-syntactic correspondence* (the language of the fuzzy derivative of an expression coincides with the fuzzy derivative of its language) and a *finiteness* result: iterating fuzzy derivatives over all words yields only finitely many distinct languages. One of the main results connects our fuzzy theory to the classical, crisp one. For a fixed cut value μ and Gödel's minimum T-norm, the similarity relation induces an equivalence on the alphabet, whose quotient we call the *quotient alphabet* Σ^{μ}. We prove that ordinary Brzozowski derivative automata over Σ^{μ} capture exactly the fuzzy derivatives over the original alphabet. Most of these results are formalized in the Rocq prover [6], see [3], building on the Coquand–Siles [2] formalization of regular-expression equivalence via Brzozowski derivatives. Furthermore, we explore how to relate our similarity-based approach to the fuzzy-set construction of Stamenković and Ćirić [8].

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Pattern Calculus with Similarity Relations

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We introduce a pattern calculus extended with fuzzy similarity relations, allowing terms to reduce to other terms with associated similarity degrees. The calculus is parameterized by a similarity-based matching algorithm, which returns a maximal matcher together with its matching degree. Since pattern calculus is not confluent in general, we investigate sufficient conditions on the matching algorithm that guarantee confluence of the resulting reduction system. We identify two important fragments in which these conditions are satisfied. First, we prove confluence for arbitrary terms when the underlying T-norm is the minimum T-norm. Second, we establish confluence for the fragment of linear terms under arbitrary T-norms without zero divisors. These results provide a general framework for incorporating similarity-based matching into pattern calculus while preserving one of its fundamental rewriting properties.

Design and Implementation of a Proximity-Based Probabilistic Logic Programming Language

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Probabilistic Logic Programming (PLP) has emerged as a powerful paradigm for reasoning in environments characterised by uncertainty. At the heart of modern PLP lies the *distribution semantics*, which treats a probabilistic logic program as a probability distribution over *possible worlds*. ProbLog is the flagship language in this domain, providing a rigorous framework where facts are labelled with probabilities and inference is reduced to weighted model counting. In this context, ProbLog – and PLP in general – traditionally relies on strict unification, where two terms match if and only if there exists a variable substitution that makes them syntactically identical.

However, in real-world applications such as natural language processing, bioinformatics, and data integration, information is not only uncertain (probabilistic) but also vague or imprecise (fuzzy). Strict unification is often too strong for these domains; for instance, a system should be able to unify "Shakespeare" with "Shakespear" or "Cell" with "Tissue" to some degree of confidence. While ProbLog handles the probability of a statement being true, it lacks a native mechanism for handling the degree of similarity between terms.

This work considers the integration of fuzzy unification into the PLP framework. One possibility for this consists of introducing so-called *similarity relations*. However, the transitivity of these relations may be ill-suited for modelling certain problems. For example, if *grey* is similar to *white* and *grey* is also similar to *black*, then, by transitivity, *white* and *black* would also be considered similar, which may not make sense. Therefore, this work focuses instead on *proximity relations*, which are non-transitive tolerance relations (as in Bousi∼Prolog) that allow more flexible chains of similarity without forcing a global equivalence.

In particular, the expected contributions of this work are the following:

- Define an extended syntax for ProbLog that allows the declaration of fuzzy relations between constants (e.g., `0.8::apple∼pear`).
- Propose an extension of the distribution semantics to accommodate fuzzy unification. This involves redefining the satisfaction of a query in a possible world not as a binary value, but as a degree derived from the combination of probabilistic weights and fuzzy similarity degrees.
- Define a transformation (a form of compilation) that maps programs written in the extended language into standard ProbLog programs. This transformation eliminates fuzzy unification at the program level by introducing auxiliary probabilistic facts and rules.
- Develop a software tool (likely a pre-processor) that implements the aforementioned transformation, allowing users to execute fuzzy-probabilistic queries on existing PLP engines such as ProbLog.

Keywords: probabilistic logic programming; fuzzy unification; proximity relations; semantics; knowledge representation and reasoning.

Acknowledgments

This work is supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Project # FR-25-5957.

Plancherel identites for unbounded subsets of $\mathbb{R}^d$

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The classical Plancherel identity shows that the Fourier transform on $\mathbb{R}^d$ is an isometry between the L^2-spaces. We show that for certain unbounded subsets Ω of $\mathbb{R}^d$, one can associate a "dual subset" $\tilde{\Omega}$ with a renormalized Lebesgue measure μ on $\tilde{\Omega}$, so that the restriction of the Fourier transform to $L^2(\Omega)$ is an isometric isomorphism into $L^2(\tilde{\Omega}, \mu)$, and thus we have Plancherel identities for the pair $(\Omega, \tilde{\Omega})$, in the sense that

$$\int_{\Omega} |f(t)|^2 \, dt = \int_{\tilde{\Omega}} |\widehat{f}(\xi)|^2 \, d\mu(\xi) \quad (f \in L^2(\Omega)).$$

Artificial Intelligence Approaches for Smart City Planning: A Case Study of Tbilisi

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Artificial intelligence is becoming an increasingly important tool for data-driven urban planning, enabling the analysis of complex urban systems and supporting evidence-based decision-making. This contribution presents complementary AI-related approaches developed within the research project *Smart City Planning and Management Using Artificial Intelligence*, using Tbilisi as a case study.

The research combines machine learning, predictive analytics, geospatial modelling, environmental forecasting, and scenario-based analysis to address several smart-city planning challenges. In the mobility and spatial planning context, the study includes a machine-learning case study focused on traffic optimization, mobility patterns, and spatial site-selection. In parallel, the topics of municipal waste management, air-quality assessment, and selected mobility scenarios are addressed mainly through analytical, scenario-based, and framework-based approaches.

Although each study focuses on a distinct application domain, all share the common objective of transforming heterogeneous urban data into actionable knowledge for planning and management. Collectively, the presented approaches demonstrate how AI and data-driven methods can support urban planning through predictive modelling, spatial analysis, environmental assessment, and transportation scenario analysis. The contribution illustrates the breadth of AI methodologies applicable to smart-city planning and highlights their potential to support sustainable, evidence-based urban development, improved quality of life, and more effective decision-making under real-world institutional and resource constraints.

Keywords: Artificial Intelligence, Smart City Planning, Machine Learning, Predictive Analytics, Geospatial Modelling, Intelligent Transport Systems, Air Quality, Waste Management, Tbilisi

Acknowledgment

This work is supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [# FR-24-9597, *"Smart City Planning and Management Using Artificial Intelligence"*].

Study of Space Weather Driven Earth's Atmosphere Wind Dynamics using Machine Learning Tools

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Space weather, driven by solar activity such as solar flares, coronal mass ejections (CMEs), and variations in the solar wind, plays an important role in the Sun–Earth system by influencing the Earth's magnetosphere and upper atmosphere. This study investigates the relationship between space weather and atmospheric wind dynamics through numerical modeling and observational analyses. A non-hydrostatic global neutral wind model developed at the Polar Geophysical Institute was employed to simulate atmospheric circulation under varying geomagnetic conditions, while observational datasets included interplanetary magnetic field (IMF) and plasma beta data from NASA OMNIWeb, geomagnetic measurements from the Dusheti Observatory (Georgia), and meteorological observations from the Georgian National Environmental Agency. Statistical methods, including cross-correlation, wavelet coherence analysis, detrended fluctuation analysis (DFA), and machine learning techniques, were applied to examine the coupling between solar, geomagnetic, and atmospheric parameters. The results revealed significant relationships between solar wind variability, geomagnetic disturbances, and atmospheric dynamics. During the major geomagnetic storms of May 2024, the correlation between IMF fluctuations and the geomagnetic H-component reached approximately 70%, while wavelet coherence analysis identified coherent behavior before and during storm events, suggesting predictive potential. Numerical simulations and meteorological observations further indicate that geomagnetic activity can influence atmospheric circulation, wind variability, and pressure systems through magnetosphere–ionosphere–atmosphere coupling. These findings demonstrate that incorporating geomagnetic and solar activity indicators into weather forecasting frameworks may improve prediction accuracy and provide new insights into regional atmospheric variability.

Acknowledgement

The study was conducted by financial support of Shota Rustaveli Science Foundation Grants # FR 25-1725 and # FR 25-16-929.

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Machine Learning Approaches for Solving a System of Nonlinear Two-Dimensional Partial Differential Equations

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The initial-boundary value problems for a system of nonlinear two-dimensional partial differential equations are considered. The models of such type are studied in many works (see, for example, [1–3] and references therein). Numerical experiments have been conducted using machine learning methods, particularly Deep Neural Networks (DNNs) [4]. The aim of the present study is to apply a DNN to the system considered and to continue the investigation of these models in this direction.

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On the Convergence of Negative-Order Cesaro Means of Fourier and Fourier–Vilenkin Series

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In this talk we discuss the convergence properties of negative-order Cesàro means applied to Fourier and Fourier–Vilenkin series. We examine the convergence of these series both almost everywhere (a.e.) and in the L^1 norm, particularly after modifying the values of integrable functions on sets of arbitrarily small measure. Our results extend and generalize several previously established findings in this area (see [1–7]).

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Generalizing Mean–Variance Portfolio Optimization with a Quadratic Risk Measure

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In this work, we generalize the classical mean-variance framework of Markowitz by formulating a portfolio optimization problem based on a general quadratic risk measure. Using the method of Lagrange multipliers, we derive the fundamental propositions of portfolio theory within the resulting mean-quadratic-risk framework, thereby extending our previous results obtained for the classical mean-variance model and providing a mathematically complete treatment of the generalized setting [1]. We establish the corresponding optimality conditions, derive the efficient frontier and the Capital Market Curve (CMC), and characterize the associated efficient portfolios. The results provide a theoretical extension of modern portfolio theory in which variance is replaced by a more general quadratic measure of portfolio risk.

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An Algebraic Extension Lemma for Locally Free Sheaves

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The Serre–Swan correspondence provides a connection between algebra and geometry. It identifies vector bundles with finitely generated projective modules. In algebraic geometry, locally free sheaves of finite rank on an affine scheme correspond to finitely generated projective modules over its coordinate ring [1]. In topology, vector bundles over a compact Hausdorff space correspond to finitely generated projective modules over $C(X)$ [2]. In this talk, we present an algebraic analogue of Swan's extension lemma, which is used in the proof of the Serre–Swan correspondence.

Theorem *Let A be a Noetherian normal domain with fraction field K, and let $X = \mathrm{Spec}(A)$. Let $\mathscr{E}$ be a locally free $\mathscr{O}_X$-module of finite rank, and set $P = \Gamma(X, \mathscr{E})$. Fix a point $p \in X$, let $U \subseteq X$ be an open neighborhood of p, and let $s \in \Gamma(U, \mathscr{E})$. Then there exist an effective Weil divisor*

$$D = \sum_{\mathrm{ht}(q)=1} n_q[q] \ \ (n_q \in \mathbb{Z}_{\geq 0}),$$

such that $\mathrm{Supp}(D) \cap U = \varnothing$*, and a global section*

$$\widetilde{s} \in \Gamma\big(X, (\mathscr{E} \otimes \mathscr{O}_X(D))^{**}\big)$$

whose restriction to U identifies with the original section s.

Lemma *Let X be normal. Let U be a neighborhood of x, and let s be a section of a vector bundle E over U. Then there is a section s' of E over X such that s' and s agree on some neighborhood of x [2].*

Definition Let $X = \mathrm{Spec}(A)$ be an affine scheme. A sheaf of $\mathscr{O}_X$-modules $\mathscr{F}$ is called *locally free of rank r* if every point $p \in X$ has an open neighborhood $U \subseteq X$ such that

$$\mathscr{F}\big|_U \cong \mathscr{O}_U^{\oplus r}.$$

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The Isomorphic Mathematical Theory of the Georgian Language and Thinking as Main Aim of the Todays Georgian Schools of the Computer Sciences and Mathematical Logic

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Along with the elaboration of the Abkhazian computer "brain", the elaboration of the Georgian computer "brain" is the main aim of the Scientific Research Institute for Cultural Protection and Technological Development of the Georgian State Languages at the Georgian Technical University, which, according to our approaches, necessarily requires the development of an isomorphic mathematical theory of the Georgian language and thinking [1–4].

Based on the above, in our report, we will argued, that one of the most important task of the todays Georgian schools of mathematical logic and computer sciences is to develope the aforementioned isomorphic mathematical theory of the Georgian language and thinking (`https://drive.google.com/file/d/1eS3_3krRT9Xwsp9PSOBTjR9XhTW4Hqqt/view?usp=sharing`). – This is dictated by the understanding the particularly high-level national significance of this task, because all this, along with the aims of protecting and developing the Georgian language, directly serves the highest-level national goals of protecting and developing Georgia's digital sovereignty.

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Numerical Modeling of the Influence of Background Fields on Humid and Ecological Processes in the Mesoscale Atmospheric Boundary Layer

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Using our already developed two-dimensional (X–Z vertical plane) model of the mesoscale atmospheric boundary layer (MABL), we study the influence of background fields on humid (clouds and fog) processes and "aerosol" clouds (smog). We consider the background fields as atmospheric stratification, geostrophic wind, and relative humidity [1–4].

We obtained physically justified results, which guarantee that our numerical model adequately describes the MABL. In addition, our model makes it possible to provide both meteorological and ecological forecasts of the MABL through an external background, macroscale forecast.

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On the Expected Value of a Random Variable

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This article discusses the expected value (mathematical expectation) of a random variable as one of the fundamental concepts of probability theory. It presents the essence of a random variable, the classification of discrete and continuous random variables, as well as the mathematical definition of the expected value and the basic formulas for its calculation.

Particular attention is paid to the properties of mathematical expectation, including its linearity, the expected value of a constant, and the rules for calculating the expected value of the sum of random variables. The article also examines the interpretation of the expected value as the long-run average outcome, which makes it possible to evaluate the behavior of random phenomena under repeated trials.

The article demonstrates that the expected value is widely applied in economics, finance, insurance, medicine, engineering, and data analysis. It also analyzes the difference between the theoretical expected value and the sample mean and briefly discusses the role of the Law of Large Numbers in explaining the relationship between these two concepts.

Through practical examples and problem-solving tasks, the article illustrates how the expected value is used to solve real-world problems and support decision-making. It emphasizes that a thorough understanding of the expected value of a random variable is essential for the study of both mathematics and applied sciences, as it serves as one of the fundamental tools for the analysis of probabilistic models and forecasting.

On One Example of a Sub-Gaussian Random Variable

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Let $(\Omega, \mathscr{A}, \mathbb{P})$ be a probability space. We say that a random variable $\xi : \Omega \to \mathbb{R}$ is sub-Gaussian (see e.g. [1, 2]) if for some a, $0 \le a < \infty$, and for all real numbers $t \in \mathbb{R}$ the following inequality is satisfied

$$\mathbb{E}\, e^{t\xi} \le e^{\frac{t^2 a^2}{2}},$$

where $\mathbb{E}$ is a symbol of the mathematical expectation. For an arbitrary sub-Gaussian random variable ξ, $\tau(\xi)$ will denote the number defined by the equality

$$\tau(\xi) = \inf\Big\{a \in [0,\infty) : \ \mathbb{E}\, e^{t\xi} \le e^{\frac{t^2 a^2}{2}} \ \text{ for every } \ t \in \mathbb{R}\Big\}.$$

The number $\tau(\xi)$ is called the sub-Gaussian standard of ξ (see [2]; in [1], the term "Gaussian deviation" is used). It is known that if ξ is a sub-Gaussian random variable then $\mathbb{E}\,\xi = 0$ and $\mathbb{E}\,\xi^2 \le \tau^2(\xi)$.

A sub-Gaussian random variable ξ is called *strictly sub-Gaussian* if $\mathbb{E}\,\xi^2 = \tau^2(\xi)$. Gaussian random variable is strictly sub-Gaussian, but *not every sub-Gaussian random variable is strictly sub-Gaussian*. In connection with the last assertion in our talk we plan to discuss the following statement:

Proposition *For a fixed $\alpha, 0 < \alpha < 1$ write $\beta := 1 - \alpha$ and consider a random variable ξ_α such that $\mathbb{P}[\xi_\alpha = \beta] = \alpha$ and $\mathbb{P}[\xi_\alpha = -\alpha] = \beta$. Then we will have:*

(a) *ξ_α is a sub-Gaussian random variable and $\tau(\xi_\alpha) \le \max(\alpha, 1-\alpha)$.*

(b) *ξ_α is a strictly sub-Gaussian random variable if and only if $\alpha = 1/2$.*

Acknowledgments

The work are supported by the EU Horizon Europe project FORGE-AI and by the Shota Rustaveli National Science Foundation of Georgia, Grant # SC-24-753.

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Generalized Curaçao–Predator–Prey Model

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We consider a model for controlling an unwanted population by introducing sterile individuals (the Curaçao model) with the additional introduction of natural predators of this population:

$$\begin{cases} \dfrac{dN}{dt} = N(r - \alpha N - \beta S - \gamma P), \\ \dfrac{dS}{dt} = S(k - \eta N - \mu S - \rho P), \\ \dfrac{dP}{dt} = P(-d + cN + \sigma S - qP). \end{cases}$$

Here, N is the number of non-sterile individuals in a given population, S is the number of sterile individuals in the same population, and P is the number of predators. In this model, both non-sterile and sterile individuals are prey to predators.

The paper examines the dependence of the model's properties on its parameters. In particular, for certain values of the coefficients, the behavior of this model is chaotic.

On the Importance and Applications of Asymptotic Analysis in Mathematics

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In solving a mathematical problem, it is sometimes impossible to find an exact solution analytically, and sometimes, even if such a solution exists, our knowledge of the evaluation of such a solution or its behavior is incomplete, and its comparison with other functions becomes difficult. In such cases, an approximate description of these functions, or the determination of their asymptotic rates of growth or decrease, becomes of great importance. Asymptotic formulas sometimes provide more information than exact formulas. It is also important to study the issues of the hierarchy of infinitely large and infinitely small quantities and its further clarification, to determine the place of a particular function in the hierarchy of standard functions, and to use these issues in the study of the convergence of series. Sometimes the variable under consideration is not a natural number, but any real number, which sometimes leads to important generalizations.

We also discuss the problems of using Landau's symbols O and o in problems of asymptotic analysis, the importance of the context in which they are used, and the neglect of details that are not important when considering a given function or comparing functions.

The paper discusses a number of aspects of these and related issues.

Assessment That Survives AI: Designing Mathematics Assignments on Graphs, Functions, Limits, and Derivatives for AI-Permitted Learning

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The widespread availability of artificial intelligence (AI) tools capable of solving standard mathematical problems has fundamentally changed the conditions under which student knowledge can be assessed. Prohibiting AI is increasingly unrealistic and pedagogically counterproductive: students will inevitably use these tools in their professional lives, and assessment practices should reflect this reality rather than resist it. The genuine challenge, therefore, is to design assignments in which the use of AI is explicitly permitted – and even encouraged – while the assessment of the student's own understanding remains effective and reliable.

This paper presents a methodology for constructing such assignments for several core topics of mathematics: graphs, functions and their properties, limits, and derivatives. Building on our previous work, which introduced sample AI-permitted assessment tasks for a limited set of mathematical topics, this paper extends the methodology to new topics and introduces additional design approaches. The central idea is to shift the object of assessment away from the final answer – which AI can readily produce – toward those cognitive activities that require the student's own mathematical judgment: identifying the task, interpreting and justifying results, verifying and critically evaluating AI-generated solutions.

The proposed methodology rests on several complementary techniques. Assignments are personalized through parameterization, so that each student works on a conceptually identical but numerically and topologically (in case of graphs) distinct problem, which makes direct copying meaningless while preserving fairness of evaluation. Tasks are formulated so that a correct AI-generated output is only an intermediate step: the student must interpret, verify, or extend it in a way that reveals genuine understanding. Finally, evaluation criteria are designed to reward the quality of mathematical reasoning and the critical treatment of AI outputs rather than the mere correctness of computations.

The paper presents concrete sample assignments on functions, graphs, limits, and derivatives constructed according to this methodology, together with practical recommendations for their preparation and evaluation. The approach effectively eliminates the risk of plagiarism, reduces the incentive for uncritical reliance on AI, and transforms AI from a threat to academic integrity into a productive participant in the learning process.

In conclusion, the paper argues that thoughtfully designed AI-permitted assignments not only preserve the validity of assessment but also cultivate precisely those skills – critical evaluation, interpretation, and mathematical communication – that remain distinctly human contributions in an AI-assisted world.

Application of Dirichlet's Principle in Solving Mathematical Olympiad Problems

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The **Dirichlet** Principle, more commonly known as the **Pigeonhole Principle**, is one of the most fundamental and powerful tools in combinatorics. Despite its remarkably simple formulation, it has extensive applications in solving a wide variety of mathematical problems and plays a significant role in modern combinatorics, number theory, geometry, and graph theory.

This paper presents both the classical and generalized forms of the Dirichlet Principle, examining their theoretical foundations and principal areas of application. Particular emphasis is placed on the methodological aspects of applying the principle effectively, highlighting that the correct identification of the "objects" and the "boxes" within a problem is often the key step in constructing an elegant Olympiad solution.

The paper includes a range of classical examples illustrating the application of the Dirichlet Principle to problems involving graph theory, partial sums, residue classes, integer coordinate lattices, and the Fibonacci sequence. It also discusses the Erdős–Szekeres theorem, demonstrating how the Dirichlet Principle provides a natural framework for proving the existence of monotone subsequences. Each example illustrates how this principle enables the elegant and efficient solution of challenging Olympiad problems that might otherwise require considerably more sophisticated techniques.

In addition to its theoretical significance, the paper explores practical applications of the Dirichlet Principle in computer science and applied mathematics, including data structures, hashing algorithms, cryptography, computer networks, and optimization. These examples demonstrate that the principle is not merely a theoretical result but also a fundamental concept underpinning numerous modern computational methods and information technologies.

The novelty of this study lies in the systematic presentation of both the classical and generalized forms of the Dirichlet Principle within a unified methodological framework. This approach contributes to a deeper understanding of Olympiad problem-solving strategies and provides a valuable resource for both mathematics educators and students preparing for mathematical competitions.

The aim of this study is to present the Dirichlet Principle systematically as one of the fundamental techniques used in mathematical Olympiads, to examine its theoretical foundations, and to demonstrate its applicability across a broad spectrum of mathematical problems. The material presented may serve as a useful resource for mathematics Olympiad training as well as for teaching introductory combinatorics at the university level.

Regional Economic Birch–Swinnerton–Dyer Modelling: a Local-to-Global Framework for Structural Rank Formation, Corridor Stress and World-Economy Calibration

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This study develops a regional economic modelling architecture inspired by the local-to-global logic of the Birch–Swinnerton–Dyer (BSD) conjecture [1, 2]. It neither proves nor tests the classical conjecture; rather, it translates local information, global rank, Euler-type factors, regulator-like capacity, and friction terms into a reproducible economic measurement system. The framework combines economic-complexity and network approaches [3,4] for a World normalization anchor and ten macro-regional or corridor-sensitive nodes. It is one of seven architectures synthesized in the author's book *Millennium Economics* [5]. The empirical pipeline transforms productive-capacity, systemic-stress, flow, and network-position indicators into economic curve coefficients, finite-state trace analogues, Euler-factor analogues, a constructed economic L-function, and an L-order measure. A regional-year calibration layer with $N = 150$ observations supports sensitivity analysis, negative controls, and external-proxy diagnostics; replication materials are archived in Zenodo [6]. Finite-state reductions are controlled economic-state grids, not literal reductions over finite fields. Let $R_{r,t}$ denote the independently constructed structural rank of region r at time t, and let $\nu_{r,t}$ denote the order extracted from its economic L-function. The principal calibration relation is $\nu_{r,t} \approx R_{r,t}$, evaluated through the rank gap $G_{r,t} = |\nu_{r,t} - R_{r,t}|$, concordance measures, sensitivity checks, and placebo logic. Regulator-, Tamagawa-, torsion-, and BridgeStress-type analogues represent absorption capacity, local friction, locked capacity, corridor leverage, and transmission exposure. Results indicate that output mass alone does not determine structural importance: smaller corridor-sensitive regions may be systemically relevant because of network position, fragility, and shock-transmission exposure.

Theorem *Within the calibrated system, when local Euler-type factors are well-defined, global normalization is fixed, and rank components share a common scale, equality of local-to-global order and structural rank is equivalent to* $G_{r,t} = 0$.

Lemma *For fixed productive capacity, greater corridor centrality increases BridgeStress whenever its marginal transmission-exposure effect exceeds its marginal absorption-capacity effect.*

Definition The *regional economic BSD correspondence* is the model-based association between $\nu_{r,t}$ and $R_{r,t}$; it is a calibration device, not a claim about the arithmetic rank of an elliptic curve.

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R-Functions and 3-Valued Łukasiewicz Logic with Constants

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R-functions, introduced by V. L. Rvachev [1, 2], represent geometric objects by real-valued functions $f(x_1, \dots, x_n)$ with $f > 0$ in the interior, $f = 0$ on the boundary and $f < 0$ in the exterior. The sign of f is fixed by the signs of its arguments, so each R-function has a two-valued logical *companion*, and the R-functions sharing a companion form a *branch*; there are 2^{2^n} branches of n-ary R-functions [3]. We enrich this correspondence by passing from Boolean logic to Łukasiewicz logic with constants, in which the boundary carries its own truth value.

Three-valued logic with constants The logic $Ł_3^{1/2}$ has algebra $(\{0, \frac{1}{2}, 1\}, \oplus, \otimes, \neg, 0, \frac{1}{2}, 1)$, isomorphic via $x \mapsto 2x - 1$ to $(\{-1, 0, 1\}, \oplus, \otimes, \neg, -1, 0, 1)$ with $x \oplus y = \min(1, x + y + 1)$, $x \otimes y = \max(-1, x + y - 1)$ and $\neg x = -x$. The truth values $-1, 0, 1$ match the partition $(-\infty, 0)$, $\{0\}$, $(0, +\infty)$, so the boundary $f = 0$ becomes a distinct logical state rather than a zero-thickness transition. Since the constant $\frac{1}{2}$ is available, $Ł_3^{1/2}$ is functionally complete [4], whence every map $\varphi : \{-1, 0, 1\}^n \to \{-1, 0, 1\}$ is the companion of some branch, and the number of branches rises from 2^{2^n} to 3^{3^n}.

Differentiable companions The strong conjunction and disjunction admit smooth R-function companions $f_\otimes(x_1, x_2) = x_1 + x_2 - 2 + \sqrt{1 + z^2 - 2\alpha z}$ (with $z = x_1 + x_2 - 1$, $\alpha \geq 0$) and its dual $f_\oplus = -f_\otimes(-x_1, -x_2)$; the radicand never vanishes, so both are of class C^∞. They realise triple junctions – such as Plateau borders, where three soap films meet at 120° – as the logical state 0, and support ternary CAD operations combining three cylinders with X- or Y-junctions.

Higher-valued extensions The construction extends to n-valued logic whenever $n-1$ is even, so that $\frac{1}{2}$ lies in the chain and again plays the role of the self-negating boundary value. For $n = 5$ the truth values are $\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$ under the same min/max operations, and in general the number of branches grows to m^{m^n} for m truth values. This opens a route to modelling thick boundaries, transition zones and multi-state geometric objects within a single logical framework.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant # FR-25-19092].

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Eigenvalues of Non-Hermitian Banded Toeplitz Matrices. Approaching Simple Points of the Limiting Set

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For large non-Hermitian banded Toeplitz matrices, it is well known that their eigenvalues cluster along a limiting set, which is formed by a finite union of closed analytic arcs. We consider general non-Hermitian banded Toeplitz matrices and extend the simple-loop method to obtain individual asymptotic expansions for eigenvalues approaching simple and non-degenerate points of the limiting set as the matrix order increases to infinity. We also develop an algorithm to effectively compute these expansions.

A Decimal Theory of Real Numbers Based on Dirichlet's Pigeonhole Principle

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Several theories of real numbers are known (see [1]). They may be constructed explicitly or introduced axiomatically, as in [2]. In the latter approach, infinite decimals play a remarkable role: they serve as codes for the postulated real numbers (p. 63). However, a theory of real numbers can also be developed directly from infinite decimals; see [3, 4].

The theory presented here is of the same type but, in our opinion, is based on a more transparent approach. Let $\mathscr{D}$ be the set of all infinite decimals, and let $\delta_0, \delta_1, \delta_2, \delta_3, \dots \in \mathscr{D}$.

Definition We say that a sequence $\delta_1, \delta_2, \delta_3, \dots$ stabilizes at δ_0 –

$$\operatorname{stab}(\delta_n) = \delta_0,$$

if the following condition holds: from some index n_0 onward, the integer parts of δ_n remain constant and equal to the integer part of δ_0; from some index n_1 onward, the first digits after the decimal point remain constant and equal to the corresponding digit of δ_0; similarly for the second digits, then the third digits, and so on indefinitely.

Using Dirichlet's pigeonhole principle, one easily proves the following.

Theorem 1 *If a sequence $\delta_1, \delta_2, \delta_3, \dots$ of elements of $\mathscr{D}$ is nondecreasing and bounded above, then there exists a unique element $\delta_0 \in \mathscr{D}$ at which the sequence stabilizes.*

Let $\alpha \in \mathscr{D}$, and let $\underline{\alpha}_0, \underline{\alpha}_1, \underline{\alpha}_2, \dots$ be the sequence of its lower approximations. Clearly, this sequence is nondecreasing and bounded above by $\underline{\alpha}_0 + 1 \in \mathbb{N}$. By Theorem 1, the sequence (α_n) stabilizes at α.

Let $\beta \in \mathscr{D}$, and let $\underline{\beta}_0, \underline{\beta}_1, \underline{\beta}_2, \dots$ be the sequence of its lower approximations. Consider the sequences $(\underline{\alpha}_n + \underline{\beta}_n)$ and $(\underline{\alpha}_n \times \underline{\beta}_n)$. It is easy to see that both are nondecreasing and bounded above. We define $\alpha + \beta$ as the element of $\mathscr{D}$ at which the first sequence stabilizes, and $\alpha \times \beta$ as the element at which the second sequence stabilizes.

It can be shown that the algebra $(\mathscr{D}, +, \times, 0, 1, \leq)$ is an ordered semiring.

Moreover, stabilization of a sequence is equivalent to the existence of a limit in the sense of the $\varepsilon - N$ definition. Therefore, Theorem 1 expresses the sequential completeness of $\mathscr{D}$.

The field $\mathbb{R}$ is naturally obtained by adjoining negative infinite decimals to D and defining addition and multiplication by the usual rules.

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Asserting Stability for Switched Systems using Optimization and Lyapunov Functions

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Many useful and interesting systems in applications are hybrid in the sense that the dynamics are governed by differential equations, but at discrete times the dynamic mode is changed and the differential equation changes. Such systems are often referred to as switched systems. Asserting the stability of an equilibrium for such a system is considerably more difficult than for a single differential equation. We will discuss some different types of allowed switchings and how one can assert stability by using optimization to compute appropriate Lyapunov functions.

Dominant Zeros of Nekrasov–Okounkov Polynomials

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We give an exact finite-dimensional Perron–Frobenius realization of the dominant zero of the Nekrasov–Okounkov polynomials $\mathrm{NO}_n(z)$. For a normalized positive sequence $h = (h(n))_{n\geq 1}$ with $h(1) = 1$, define $\mathrm{P}_0^h(z) = 1$ and, for $n \geq 1$,

$$\mathrm{P}_n^h(z) = \frac{z}{h(n)} \sum_{k=1}^{n} \sigma(k) \mathrm{P}_{n-k}^h(z),$$

where $\sigma(k)$ denotes the sum of divisors of k. The Nekrasov–Okounkov polynomials are obtained from the specialization $h(n) = n$ by the shift $\mathrm{NO}_n(z) = \mathrm{P}_n^h(z+1)$. We derive a Hessenberg determinant representation for $\mathrm{P}_n^h(z)$. After separating the trivial zero at the origin, the remaining zeros of $\mathrm{P}_n^h(-z)$ are identified with the eigenvalues of an explicit $(n-1)\times(n-1)$ nonnegative matrix M_n^h. We prove that M_n^h is primitive and apply Perron–Frobenius theory to show that $\mathrm{P}_n^h(z)$ has a unique zero of maximal modulus; this zero is real, negative, and simple. As a consequence, the same property holds for the Nekrasov–Okounkov polynomials. We also prove strict monotonicity of the associated spectral radii.

This is a joint work with Markus Neuhauser.

Characterization of G-Lipschitz Spaces Via Commutators of the G-Fractional Maximal Function on G-Morrey Spaces

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The boundedness of the classical fractional maximal operator, the fractional integral, and its commutators plays an important role in harmonic analysis and their application. In recent decades, many authors have proved the boundedness of the commutators BMO functions of fractional maximal and fractional integral operators in some function spaces. In this abstract, we establish the boundedness of Gegenbauer fractional maximal operator in Lebesgue and Gegenbauer-Morrey spaces, when the commutator function belongs to Gegenbauer-Lipschitz space.

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q-Derivative and Artificial Neural Network Learning Algorithms

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The q-derivative, a generalization of the classical derivative rooted in quantum calculus, introduces an additional parameter q that governs the rate of change and offers greater flexibility compared to its ordinary counterpart. As q approaches 1, the q-derivative reduces to the classical derivative, while for other values it captures a broader family of variation patterns. This added degree of freedom makes it a natural candidate for enhancing learning dynamics in machine learning models. This work explores how the q-derivative concept can enhance the flexibility of artificial neural network training. Unlike prior studies that restrict the use of q-derivatives to activation functions alone, we propose replacing the classical derivative directly within backpropagation-based learning algorithms with its q-derivative counterpart. The resulting q-based algorithms were tested on three benchmark classification tasks (Digits, MNIST, and Iris) and benchmarked against their conventional versions. Experimental findings indicate that the q-derivative variants consistently outperform the classical algorithms across accuracy, precision, recall, and F1-score metrics, suggesting that this generalization offers a promising direction for more adaptable neural network optimization.

Acknowledgments

This study is derived from the first author's PhD dissertation conducted at Erzurum Technical University.

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On Quasi-Convex Functions and their Applications

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The talk is concerned with certain specific aspects of quasi-convex functions which underlie some of their applications in the optimization problems and in the theory of differential games of Apollonius type in the plane. We will present the core definitions and geometric properties of quasi-convex and quasi-concave functions (via upper and lower contour sets) and indicate their differences from standard convex/concave functions. Through specific illustrative examples (including the Arrow-Enthoven model), it will be shown that quasi-convexity is a genuine generalization of convexity, as not all quasi-convex functions can be represented as monotonic transformations of convex functions.

Concerning the applications to optimization problems, we give an analysis of the Kuhn–Tucker theorem under quasi-convex constraints and show that the first-order conditions are sufficient for determining a global optimum under some additional regularity condition. We will also explain how the quasi-convex functions can be used for geometric characterization of optimal trajectories in the planar differential games of Apollonius type.

A Hybrid Numerical Framework for Higher-Order Linear Volterra–Fredholm Integro-Differential Equations via Taylor Series and Block Pulse Functions

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This paper presents a novel hybrid numerical method that integrates Taylor series expansions with the orthogonal properties of Block Pulse Functions (BPFs) to solve higher-order linear Volterra–Fredholm IDEs. By leveraging the local approximation capability of Taylor polynomials alongside the piecewise-constant, orthogonal structure of BPFs, the proposed approach maps the integro-differential problem into a system of linear algebraic equations via a structured matrix representation. This transformation streamlines the computational process, allowing for the efficient determination of unknown coefficients through standard linear algebra techniques. The theoretical framework is rigorously established, encompassing the derivation of operational matrices, the formulation of the hybrid approximation scheme, and a comprehensive convergence analysis. We provide critical theorems regarding the existence, uniqueness, and convergence of the approximate solution, demonstrating that the method achieves high accuracy under appropriate smoothness conditions. Numerical experiments are conducted to validate the effectiveness of the technique; the results confirm that the proposed hybrid Taylor–BPF approach exhibits superior accuracy and computational efficiency when compared to conventional numerical methods.

Acknowledgments

The work was supported by the Karaj Islamic Azad University.

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A Novel Approach for Testing the Equality of Multiple Coefficients of Variation

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This paper introduces a novel framework for assessing the equality of coefficients of variation (CV) across multiple independent datasets. By building upon the foundational principles of Wald and Score statistics, the proposed methodology offers enhanced computational simplicity while maintaining statistical power comparable to existing methods, particularly in detecting subtle differences between groups. Central to this research is the development of a bootstrap-based technique, termed the Computational Approach Test (CAT), which is applied to Gradient, Wald, and Score tests to refine Type I error rate control. We conduct an extensive numerical evaluation, comparing the performance of our proposed tests against traditional likelihood ratio, Wald, and Score statistics. The coefficient of variation is a critical metric for assessing relative variability, especially when comparing datasets measured in different units. Given its versatility, the CV is widely employed across diverse disciplines. For instance, in economics and sociology, it serves as a primary index for quantifying social inequality across various demographics. Similarly, in remote sensing and agriculture, the CV is instrumental in distinguishing between agricultural terrains and stable land covers by analyzing fluctuations in reflectance values throughout crop cycles. Given these broad applications, the rigorous statistical testing of CV homogeneity across groups remains a fundamental challenge. This paper addresses this issue by providing a refined, computationally efficient alternative to existing approaches.

Acknowledgments

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A Physics-Informed Finite-Difference-Based Constrained Convolutional Neural Network Framework for 2D Stationary Convection Diffusion model with Boundary Enforcement

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The present study of dense layers aided convolutional neural networks as an application of deep learning techniques to solve elliptic partial differential equations. Convolutional neural networks are preferred over other deep learning architectures because they are free of translational invariance, and are placed among the prominent architectures in modern machine learning. The proposed architecture is evaluated by benchmarking two-dimensional elliptic partial differential equations appearing in convection dominated diffusion phenomena. The method is formulated within a physics-constrained learning paradigm, where the governing differential operator is enforced through a residual minimization strategy at interior collocation points. The incorporation of boundary data into convolutional neural networks is always challenging due to the architecture's nature as compared to the physics-informed neural network. Dirichlet boundary conditions are enforced directly via a soft penalty, and Neumann-type conditions are implemented using the forward difference method to ensure physical consistency. The loss for the interior collocation points is formulated by a finite-difference discretization technique. We have considered linear and nonlinear elliptic partial differential equations of physical importance to assess the performance of the proposed computational method.

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The Comparison of a Set of Normal Samples with Given Reliability

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The problem of solving the ordered one-way analysis of variance (ANOVA) (which consists of comparing a set of normal means) with restricted Type I and Type II error rates is considered in the paper. This case is more complicated than unordered one-way ANOVA because the detection of the monotonicity of means restrictions is necessary. To solve this issue, one of the possible formulations of the constrained Bayesian method (CBM) is applied here using the concept of directional hypotheses. The cases of known and unknown variances are examined. For unknown variances the maximum likelihood ratio and Stein's Methods are used for overcoming the problem connected with complexity of hypotheses. The correctness of the developed methods and high quality (in comparison with existing methods) of obtained results are demonstrated by computing results of the simulated concrete scenarios. Moreover, the offered method controls not only one Type of error as existing methods do but both Type I and Type II methods.

Acknowledgments

The work are supported by the EU Horizon Europe project FORGE-AI. The research of the second author PHDF-25-1783 has been supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG).

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Application of a Characteristic of the Sylvester Matrix in Digital Image Processing

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Digital image restoration and denoising remain foundational challenges in computer vision, signal processing, and medical imaging. During acquisition, transmission, and storage, digital images are frequently corrupted by various noise sources. The primary objective of image denoising is to suppress these unwanted high-frequency fluctuations while preserving the structural integrity, sharp boundaries, and textual readability of the underlying image. However, many popular methods require a large number of floating-point operations and matrix multiplications, which limits their applicability in real-time applications. To overcome these computational challenges, discrete orthogonal transforms based on binary matrix structures are employed, such as the Fast Walsh–Hadamard Transform. This work discusses an Adaptive Hadamard Method operating on the principle of transforming image blocks into the frequency domain and selectively suppressing noise. Based on the theoretical maximum of a functional defined on the Sylvester matrix, the algorithm processes overlapping windows. Edges are left unchanged to preserve image details, while in homogeneous regions where the transform coefficients exceed the theoretical bound, a special coefficient is applied to reduce high-frequency noise. Furthermore, a hybrid image denoising method is presented that adaptively fuses the strengths of the Hadamard transform, Haar wavelet transform, and total variation (TV) regularization based on the estimated noise level. Extensive evaluation on 20 retinal images from the DRIVE dataset demonstrates that the hybrid method achieves the highest overall peak signal-to-noise ratio (average PSNR of 31.00 dB) compared to individual methods. Statistical validation confirms significant improvements over individual Hadamard and TV approaches ($p < 0.001$), proving its robust performance and suitability for retinal medical image preprocessing when deep learning is not feasible.

Acknowledgments

The work are supported by the EU Horizon Europe project FORGE-AI and by the Shota Rustaveli National Science Foundation of Georgia, Grant # SC-24-753.

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Investigation of Short-Term Periodic Anomalies in Solar Activity and Galactic Cosmic Ray Time Series via the Kinematic Model of the Weighted Center of Solar Activity Regions

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This paper investigates short-term quasi-periodic variations in solar activity and Galactic Cosmic Ray (GCR) intensity, which reflect dynamical processes in the solar-interplanetary environment. The primary objective of this study is to identify anomalous frequencies within the observational data arrays of total sunspot areas and GCR intensity, and to provide a physical explanation for their kinematic nature.

We introduce the concept of the Weighted Center of Solar Activity Regions (WCSAR) as an original phenomenological tool to analyze the spatiotemporal distribution of solar active regions and the resulting changes in the WCSAR vector. For data processing, a complex mathematical cascade is utilized, incorporating digital filtering, the Hilbert transform, and localized wavelet analysis.

Our results demonstrate that the anomalous 20-45-day periods (excluding periods associated with solar differential rotation) are not permanent features, but rather manifest as distinct, short-term local bursts appearing concurrently and synchronously in both solar activity indices and cosmic ray flux variations. Based on the dynamics of the coordinates of the weighted centers of the solar activity regions on the solar surface, we show that these frequencies can be explained by the actual discrete changes in the direction of the WCSAR vector. The proposed kinematic model provides a reliable mathematical framework for investigating variations in global solar magnetic asymmetry and the resulting modulation effects on cosmic rays.

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On Beta G-Star Supplemented Modules

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In this work, every ring has a unity and every module is a unitary left module. Let M be an R-module. If every submodule of M is β_g^* equivalent to a g-supplement submodule that is essential in M, then M is called a β_g^*-supplemented module. In this work, some new properties of β_g^*-supplemented modules are investigated.

Key words: Small Submodule, Radical, β^* Relation, Supplemented Module.

2020 Mathematics Subject Classification: 16D10, 16D80.

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On the Existence of Bounded Solutions for Systems of Linear Difference Equations

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For the linear system of difference equations

$$y(k) - y(k-1) = G_1(k)y(k) + G_2(K-1)y(k-1) + \varphi_1(k) + \varphi_2(k-1), \quad k \in \mathbb{N}, \tag{1}$$

we consider existence of solutions that are bounded on $\widetilde{\mathbb{N}}$, i.e.,

$$\sup\left\{\|y(k)\| : \ k \in \widetilde{\mathbb{N}}\right\} < +\infty, \tag{2}$$

where $G_j(k) = (g_{jil}(k))_{i,l=1}^n \in \mathbb{R}^{n\times n}$ and $\varphi_j(k) = (\varphi_{ji}(k))_{i=1}^n \in \mathbb{R}^n$ $(j = 1, 2;\ k = 0, 1, \dots)$. Let $\det(I_n + (-1)^j G_j(k)) \neq 0$ $(j = 1, 2;\ k = 0, 1, \dots)$, where I_n is the identity $n\times n$-matrix and $\widetilde{\mathbb{N}} = \{0\} \cup \mathbb{N}$.

We give effective sufficient conditions for the existence of solution of the problem (1), (2).

For $k_i \in \widetilde{\mathbb{N}} \cup \{+\infty\}$ $(i = 1, \dots, n)$ we put $\mathbb{N}_0(k_1, \dots, k_n) = \{i; k_i \in \widetilde{\mathbb{N}}\}$.

Theorem *Let* $1 + (-1)^j g_{jii}(k) \neq 0$ $(j = 1, 2;\ i = 1, \dots, n)$, $k \in \widetilde{\mathbb{N}}$ *and let these exist* $k_i \in \widetilde{\mathbb{N}} \cup \{+\infty\}$ $(i = 1, \dots, n)$ *such that*

$$s_{ik} = \sup\Big\{ \sum_{l=0}^{k-1} |\gamma_i(s,l)| \left|(1 - g_{1ii}(l))^{-1} g_{1ij}(l)\right| + \sum_{l=1}^{k} |\gamma_i(s,l-1)| \left|(1 + g_{2ii}(l-1))^{-1} g_{2ij}(l-1)\right| : \ s \in \widetilde{\mathbb{N}}\Big\} < +\infty \quad (i \neq j;\ i, j = 1, \dots, n),$$

$$\sup\Big\{ \sum_{l=0}^{k-1} |\gamma_i(s,l)| \left|(1 - g_{1ii}(l))^{-1} \varphi_{1i}(l)\right| + \sum_{l=1}^{k} |\gamma_i(s,l-1)| \left|(1 + g_{2ii}(l-1))^{-1} \varphi_{2i}(l-1)\right| : \ s \in \widetilde{\mathbb{N}}\Big\} < +\infty \quad (i = 1, \dots, n)$$

and $\sup\{|\gamma_i(k, k_i)| : \ k \in \widetilde{\mathbb{N}}\} < +\infty$ *for* $i \in N_0(k_1, \dots, k_n)$, *where* $\gamma_i(s,k)$ $(i = 1, \dots, n)$ *defined by the Cauchy problem. Let, moreover,* $r(S) < 1$, *where* $S = (s_{ij})_{i,j=1}^n$, $s_{ii} = 0$ $(i = 1, \dots, n)$. *Then every* $c_i \in \mathbb{R}$ $(i \in N_0(k_1, \dots, k_n))$ *system (1) has at last one a bounded on* $\widetilde{\mathbb{N}}$ *solution satisfying the condition* $y(k_i) = c_i$ *for* $i \in N_0(k_1, \dots, k_n)$.

The analogous problems are studied in [1] for generalized ordinary differential systems, in [2] for impulsive differential systems, and in [3] for ordinary differential systems.

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The Group of Rational Points on a Cubic Curve

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The issues of determining the rank and torsion part of the abelian group of rational points on a cubic curve given in the Weierstrass form are discussed. By considering specific examples, it is shown how the basis of the aforementioned finitely generated abelian group can be determined.

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Models of Asymmetric Confrontation with Disinformation

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The information confrontation of two political groups in society with streams of disinformation is considered. In addition, one of the political groups has the ability to impose certain sanctions on the opposing political force for spreading disinformation. New mathematical and computer models of the presented information confrontation were built. The mathematical model is the Cauchy problem for systems of ordinary differential equations. A criterion for the information security of society in the context of this disinformation struggle has been proposed. Various scenarios of information confrontation are considered. Numerical solutions of the constructed model were found using MATLAB®. Visual realization of the solutions was obtained. The results were analyzed and appropriate conclusions were drawn.

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Quadratic-Phase Boas Transforms

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In this work, we introduce and investigate the quadratic-phase Boas transform (QPBT), a new extension of the classical Boas transform formulated within the quadratic-phase Fourier transform domain. Building upon the quadratic-phase Hilbert transform, we construct the QPBT, establish its structural properties, and derive its transform-domain representation together with a Parseval-type identity. The interplay between the QPBT, quadratic-phase convolution, and the associated complex signal is examined in detail, and closed-form expressions for kernel-specific cases are obtained. Numerical simulations support the theoretical findings and demonstrate that the QPBT yields a stable and accurate complex-valued representation for nonstationary signals with quadratic-phase characteristics. The results confirm that this quadratic-phase generalization significantly broadens the applicability of the Boas framework in advanced time–frequency and complex-signal analysis.

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On the Discrete Model of Tumor Growth in Connection with the Warburg Effect

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Cancer is one of the widespread diseases in the word. Modern technologies (MRI, PET, CT) enable to detect the cancer at an early stage. About of % 50 of cancers are of the virus origin, and others are developed in the human body within specific conditions such as toxic environment and stresses. Every day up to 10 000 cancerous cells are originated in the body, but immune system can fight with them until some prolonged stresses occurred and immune system is suppressed [1, 2]. The tumor becomes dangerous, when its diameter riches 2 mm and the immune system failed to fight with it [1]. The tumor volume squeeze the adjacent normal cells and its proteins change the interstitial fluid and plasma, consequently the osmotic pressure [1–3]. This factors inhibit nutrients and oxygen delivery from the bloodstream. In the absence of oxygen, the glucose undergoes fermentation to lactic acid and cell pH drops to 6.5 [1]. In the acid medium the cell loses its control mechanism, which leads to the normal cell death or their transformation to the cancerous cells. For normal functioning a body needs the constant energy supply. Glucose (C6H1206) is a primary source of energy [1, 4, 5]. Microelements and nutrients are consumed by body through the digestive system and are converted by means of oxygen, biphidrobacteries, stomach acid and definite enzymes and pherments. This process needs about 10 to 12 ours.About % 70 of glucose is absorbed into the blood and remains source of body energy for 6 ours. About % 30 of glucose is stored in muscle, adipose tissue, kidney and liver. In 1920 German biochemist Otto Warburg discovered, that tumor cells does not require oxygen for glucose consumption and through fermentation produce much less energy [5]. Cancer cells consume glucose at high rates, even at normal oxygen tension and large amount of glucose burned by these cells is exerted as lactate [5]. This process is known as Warburg Effect (anaerobic glycolysis). Recent studies showed, that "Warburg effect" is caused by the following factors: tumor suppressive gene (p. 53), activation of proto-oncogenes signaling pathways (p13k), and transcription factors (HIF-1) [4]. We propose the following formula $(g_T + g_N(t_0+t))V_0 = g_T\,V_T(t_0+t) + g_N(t_0+t)\,V_N(t_0+t)$, where V_0 is some volume of body consisted of the certain tissue type, $V_T(t_0)$ is the initial tumor volume inside V_0, $V_N(t_0)$ is the initial volume of normal cells, $g_T = const$ is the glucose consumption rate by tumor cells per day per volume unit, $g_N(t_0+t)$ is the glucose consumption rate by normal cells per day per volume unit $g_N(t_0+t) \to 0$.

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Window-Lag Bias in the Activity Interval Citation Index and a maturity Rule for Longitudinal Comparisons

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Cumulative citation indicators – the total citation count and the h-index – are monotone non-decreasing in time, record accumulated impact, but cannot signal when a researcher's recent output loses traction. The Activity Interval Citation Index (AICI) [1] addresses this by counting the citations received, up to an evaluation year t, by the works published within the preceding k-year window, so that a decline in relevance becomes visible. We study AICI as a longitudinal series on a stratified cohort of 417 researchers assembled from OpenAlex across four scientific domains and three career stages.

We first quantify the *window-lag bias* of windowed citation indices. Since citations accrue with a delay [2], the most recent window positions are systematically understated. On our cohort the pooled year-over-year ratio $\mathrm{AICI}_5(t)/\mathrm{AICI}_5(t-1)$ stabilizes within 1.4–1.6 at mature time points, but drops to 0.67 at the snapshot year. We therefore propose a *maturity rule* – values of AICI_k are comparable only at time points t with $T - t \geq k$, i.e. where the embedded publication window has fully matured.

Under the corrected comparison, 81 of 282 researchers with an established pre-2016 publication record (28.7%; 95% CI 23.8–34.3%) exhibit *relevance divergence*. The h-index [3] grows over 2016–2021 while the mature AICI_5 declines by more than 10% – and every flagged decline is also statistically significant (per-researcher binomial tests on the disjoint five-year windows, Benjamini–Hochberg corrected); the rate differs across domains ($\chi^2(3) = 8.78$, $p = 0.032$). A forecasting benchmark on annual citation series (5-year hold-out, 303 researchers, 3-year moving-average smoothing) shows that exponential smoothing and ARIMA models significantly outperform the naive baseline (median MASE 3.39 and 3.65 versus 6.15; Wilcoxon signed-rank tests with Benjamini–Hochberg correction, $p < 0.0001$), while adding parameters hurts on such short series (ARIMA(1,1,1) beats ARIMA(2,1,2), $p = 0.0002$). Absolute errors remain large (MASE $\gg 1$ at a five-year horizon), so the benchmark supports trend-level rather than point-level extrapolation of researcher-level citation dynamics.

Restricted to mature time points, AICI thus provides a dynamic complement to cumulative indicators for research-assessment decision support, consistent with the principle that quantitative indicators should inform, not replace, expert judgment.

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On an One Application of Hamel Basis

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Let φ denote the first ordinal number of cardinality continuum c. Let us consider $\mathbf{R^N}$ as vector space the field $\mathbf{Q}$ of all rational numbers let $\{x_\xi : \xi < \varphi\}$ be a χ-thick Hamel basis in $\mathbf{R^N}$, where χ denotes a σ-finite, invariant Borel measure in $\mathbf{R^N}$ (see, [1–3]). The existence of such basis can be proved by using the method of transfinite induction. Denote by G the group generated by φ-sequence $\{x_\xi : \xi < \varphi\}$. It is clear that:

(a) G is a group with a basis $\{x_\xi : \xi < \varphi\}$;

(b) the family $\{x_\xi : \xi < \varphi\}$ is everywhere dense in $\mathbf{R^N}$;

(c) for each closed subset F of $\mathbf{R^N}$ which has strictly positive measure with respect to χ, we have

$$card\big(F \cap \{x_\xi : \xi < \varphi\}\big) = c.$$

Let for each ordinal number $\xi \in [\omega, \varphi)$, denote by H_ξ the subgroup of G such that

$$h \in H_\xi \Longleftrightarrow \Big(h = \Sigma\alpha_\varsigma x_\varsigma : \alpha_\varsigma \text{ are even integer numbers, } \omega < \varsigma \leq \varphi\Big).$$

Theorem *In the space* $\mathbf{R^N}$ *there exists two families* $\{A^1_\xi : \omega \leq \xi < \varphi\}$ *and* $\{A^2_\xi : \omega \leq \xi < \varphi\}$ *such that:*

(1) $h \in H_\xi \Longrightarrow card(A^1_\xi \triangle h(A^1_\xi)) < c$, $card(A^2_\xi \triangle h(A^2_\xi)) < c$,
$h \in G \setminus H_\xi \Longrightarrow card(A^1_\xi \triangle h(A^1_\xi)) < c$, $card(A^2_\xi \triangle h(A^2_\xi)) < c$;

(2) $A^1_\xi \cap A^2_\xi = \varnothing$, $\omega \leq \xi < \varphi$;

(3) $A^1_\xi \cup A^2_\xi = A^1_\varsigma \cup A^2_\varsigma$, $\omega \leq \xi < \varphi$, $\omega \leq \varsigma < \varphi$;

(4) *if* $\{\xi_k : k \in \mathbf{N}\}$ *is an injective countable family of ordinal numbers from inter* $[\omega, \varphi)$, *then the intersection* $\{\cap A^{i_k}_{\xi_k}, k \in \mathbf{N}, i_k = 1, 2\}$ *is a* χ*-thick subset in* $\mathbf{R^N}$.

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Cotriple Cohomology and Two Term Extensions of Lie Superalgebras

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Let $L = L_{\overline{0}} \oplus L_{\overline{1}}$ be a Lie superalgebra over an associative ring with unit R of a characteristic $p > 0$, $M = M_{\overline{0}} \oplus M_{\overline{1}}$ a L-supermodule.

Let $H_i^n(L, M) = H_i^n(L, M)_{\overline{0}} \oplus H_i^n(L, M)_{\overline{1}}$, $i = 1, 2$, denotes the n-th cotriple cohomology groups of cotriples arising from the superderivations

$$Der(L, M) = Der(L, M)_{\overline{0}} \oplus Der(L, M)_{\overline{1}}, \;\; i = 1, 2,$$

and absolute and relative functors of free Lie superalgebras.

Definition A two term extension of L by M is an exact sequence of Lie superalgebras

$$0 \longrightarrow M \xrightarrow{\phi_2} X_1 \xrightarrow{\phi_1} X_0 \xrightarrow{\phi_0} L \longrightarrow 0\,,$$

where X_0 operates on X_1 in the following way [2] ($u, v \in X_1, x \in X_0$):

$$\varphi_2(\varphi_0(x)m) = x\varphi_2(m), \quad [u, v] = \varphi_1(u)v, \quad [x, \varphi_1(u)] = \varphi_1(xu).$$

Let $Ext_i^2(L, M)$ be the set of equivalence classes of two term extensions of L by M; when $i = 2$, the extensions are assumed to be R-split.

Theorem 1 *There exist the isomorphisms of abelian groups*

$$H_i^2(L, M)_{\overline{0}} \simeq Ext_i^2(L, M), \;\; i = 1, 2.$$

The parity shift of a L–module M is the L–module ΠM defined by $(\Pi M)_0 = M_1$, $(\Pi M)_1 = M_0$. The action of L is given by $x \cdot (\Pi m) = (-1)^{|x|}\Pi(x \cdot m)$ for homogeneous $x \in L$ and $m \in M$.

Lemma $Der_1(L, M) \simeq Der_0(L, \Pi M)$.

Theorem 2 $H_i^n(L, M)_{\overline{1}} \simeq H_i^n(L, \Pi M)_{\overline{0}}$.

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Franklin System in Variable Exponent Spaces

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In this paper, we study the properties of the Franklin system in variable exponent Lebesgue spaces $L^{p(\cdot)}([0,1])$. The classical Franklin system, consisting of piecewise linear continuous functions with dyadic knots, is a fundamental complete orthonormal basis in $C[0,1]$. In many contexts, the measurable exponent function $p(\cdot)$ is assumed to satisfy the local log-Hölder continuity condition; however, this assumption is not necessary for our results. The main result of our paper establishes that the Franklin system serves as an unconditional basis in $L^{p(\cdot)}([0,1])$ for $1 < p_- \le p_+ < \infty$ if and only if the exponent function $p(\cdot)$ belongs to the Muckenhoupt class $\mathscr{A}$. Furthermore, in the case where $1 \le p_- \le p_+ < \infty$, the Franklin system is a basis if and only if $p(\cdot)$ belongs to the Muckenhoupt class $\mathscr{A}$. We prove these equivalences using characterizations based on the Paley function and associated operators.

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Semi-Smooth Optimization Methods for Solution of Contact and Dynamic Problems

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We are motivated by solution of variational inequalities, and inverse problems subjected to inequality constraints [3], appearing in the context of non-smooth and non-convex optimization. The use of Lagrange multipliers and merit functions leads to semi-smooth Newton strategies and primal-dual active-set numerical techniques [2]. In the semi-smooth framework, we solve problems of mechanics stemming from railway applications [1], dynamic impact and contact problems describing multi-body contact [6], non-penetrating cracks [5], and fluid-driven fractures [4].

Acknowledgments

The author thanks the Georgian Mathematical Union for the long collaboration.

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A Simple Closed-Form Formula for Optimal Neural Network Regression on Metric Spaces

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Kernel ridge regression is both computationally convenient and analytically tractable largely because its estimator admits a simple closed-form expression; no analogous representation is generally available for neural networks. We address this problem by introducing a simple closed-form "two-layer" formula $\hat{f}$, with $\mathcal{O}(N)$ parameters, for reconstructing an unknown real-valued Lipschitz function f on a metric space (χ, ρ) from N i.i.d. noisy observations. Our main result is a high-probability uniform (L_∞) recovery guarantee, jointly controlling approximation, statistical, and optimization errors; not merely establishing convergence of the population risk.

Development of a Monitoring Dataset for Logic-Based Electromagnetic Field Risk Assessment

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Electromagnetic fields (EMF) have become an integral part of modern urban environments due to the increasing use of wireless communication systems, mobile base stations, public Wi-Fi networks, and electrical infrastructure. Continuous monitoring of these sources is important for understanding exposure patterns and providing reliable information for environmental planning. This research focuses on the Avlabari district of Tbilisi, where field measurements were carried out at locations representing different urban conditions, including areas near mobile base stations, metro infrastructure, healthcare facilities, university buildings, and open public spaces. To improve the reliability of the observations, measurements were repeated at different times and on different days. The collected information was organized into a structured database containing measurement values, geographic coordinates, measurement time, surrounding infrastructure, and land-use characteristics. The database was then used to examine the spatial distribution of electromagnetic field intensity across the study area. Measured values were compared with the exposure limits recommended by the International Commission on Non-Ionizing Radiation Protection (ICNIRP), as well as with published results from similar urban monitoring studies. The highest values were recorded near mobile communication infrastructure, while lower values were observed in open and less densely populated areas. In all monitored locations, the measured EMF levels remained well below the international exposure limits. Besides analyzing the measurement results, the study also examined the quality and completeness of the collected data. Repeated measurements helped identify temporal variations and improve confidence in the database, while also revealing locations where additional monitoring would be beneficial. The developed database provides a practical basis for the next stage of the project, which will apply fuzzy logic and spatial analysis methods to classify electromagnetic exposure levels and support urban environmental risk assessment. The proposed approach can be extended to other districts of Tbilisi and adapted for use in similar urban environments.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under the project # YS-25-4322.

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Multi-Criteria Problem of Optimal Service Center Location

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Creating service centers is associated with significant costs. Furthermore, each center has its own capacity. Center creation planning should be carried out so that costs are minimized, the area occupied by the centers covers the entire planned space, and the capacity of each center is utilized as efficiently as possible. Increasing capacity also entails costs. Since we can assume that the area of action of each center is generally a circle, the problem is reduced to the problem of covering a certain plane with circles, which in itself is an NP-hard problem. This article will examine the problem of covering a flat space with circles in a way that minimizes costs and optimally selects power. It will also discuss a specific multi-criterion discrete optimization problem and its solution algorithm.

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Craig Interpolation in the Logic of Temporal Linear Structures

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A **TES** (Temporal Event Structure, see [1]) is a structure

$$(F_1, \ldots, F_n, <_1, \ldots, <_n, f_1, \ldots, f_{n-1}),$$

where $(F_i, <_i)$ are finite strict linear orders, and $f_i : F_i \twoheadrightarrow F_{i+1}$ are surjective monotone maps, where monotone means $f_i(a) \leq_{i+1} f_i(b)$ for all $a \leq_i b$. Let $F := \bigcup_{i=1}^{n} F_i$, $< := \bigcup_{i=1}^{n} <_i$, and $f := \bigcup_{i=1}^{n-1} f_i$.

The language $\mathscr{L}$ is given by $\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \boxdot\phi \mid \boxplus\phi \mid \boxminus\phi \mid \boxminus\phi$, where p ranges over propositional symbols. The remaining logical connectives are defined as usual.

Let $\mathbf{T}_n$ denote the class of all **TES**s with fixed n. The language $\mathscr{L}$ is interpreted over a **TES** $(F, <, f)$ such that $\boxminus$ and $\boxminus$ are interpreted over $(F, <, >)$, while $\boxdot$ and $\boxplus$ are interpreted over (F, f, f^{-1}). Let $\mathbf{T}_n$ denote the class of all **TES**s with fixed n.

The logic of $\mathbf{T}_n$ is defined in a standard way.

$$\mathsf{Log}(\mathbf{T}_n) = \big\{\varphi \mid \ F \models \varphi \text{ for every } F \in \mathbf{T}_n\big\}.$$

A logic L has the *Craig interpolation property* if whenever $L \vdash \varphi \to \psi$, there exists a formula θ such that $L \vdash \varphi \to \theta$, $L \vdash \theta \to \psi$, and $\mathrm{Var}(\theta) \subseteq \mathrm{Var}(\varphi) \cap \mathrm{Var}(\psi)$.

The following theorem shows that the logic of **TES** frames does not have Craig Interpolation Property. The proof employs algebraic and model-theoretic techniques developed in [2].

Theorem 1 *The logic* $\mathsf{Log}(\mathbf{T}_n)$ *does not have the Craig Interpolation Property.*

Next, we study normal extensions of $\mathsf{Log}(\mathbf{T}_n)$. A normal modal logic L_2 is called a *normal extension* of a modal logic L_1 if $L_1 \subseteq L_2$ and L_2 is closed under uniform substitution, modus ponens, and the rule of necessitation. For a given logic L, the set of its normal extensions forms a lattice and is denoted by $\mathrm{NExt}(L)$.

Let us define a relation on the class K of Kripke frames as follows: $F_1 \leq F_2$ iff $F_2 = \mathrm{HS}(F_1)$, where H and S denote the class operations of taking homomorphic images and substructures, respectively. It is easy to show that, the class $\mathbf{T}_n$ forms a countable antichain with respect to this relation. The following result is a straightforward consequence of this fact.

Theorem 2 *There are uncountably many elements in* $\mathrm{NExt}(\mathsf{Log}(\mathbf{T}_n))$.

Acknowledgments

This talk was supported by TSU Andrea Razmadze Mathematical Institute.

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On the Convergence of a Series of Random Variables with a Chain Dependence

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On the probability space $(\Omega, \mathscr{F}, P)$, a two-component stationary (in the narrow sense) sequence $(\xi_i, Y_i)_{i\geq 1}$ is considered. The control sequence $(\xi_i)_{i\geq 1}$ $(\xi_i : \Omega \to \{b_1, b_2, \ldots, b_r\})$is a finite, homogeneous regular Markov chain. $\{Y_i\}_{i\geq 1}$ $(Y_i : \Omega \to R^1)$ is a sequence of random variables with a chain dependence. A sufficient condition for the convergence of the series $\sum\limits_{n=1}^{\infty} Y_n$ with probability 1 is established. A sufficient condition is established for the sequence $(Y_i)_{i\geq 1}$ to satisfy the strong law of large numbers.

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A Formal Framework for Synchronized Drone Swarms with Belief-Based Reinforcement

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Swarms of drones often operate under adversarial environments (e.g., wind), about which the drones may not have precise information. In this paper we propose a formal methodology in rewriting logic for modeling and analyzing drone swarms which take into account such adversarial environmental factors, yet can provide guarantees about the system behavior. Our approach follows a "split-world architecture" which separates each drone's physical state from its perceived state, and introduces belief as a confidence measure used to track the drone's certainty about its state estimate. We also propose methods for state estimation and drone control when each drone is operating based on belief, as well as methods for belief reinforcement. The framework provides functionality for measuring the success of the swarm, both epistemic and mission successes, as well as collision detection, taking into account the need for early pruning of branches to reduce the computational resources required for model checking.

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Minimal Universal Crossed Extensions of Superperfect Groups

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One of the basic facts in homological algebra is that every perfect group G, i. e. such that its first homology group $H_1(G)$ is trivial, possesses a central extension of G by $H_2(G)$.

Another basic fact is that the third cohomology group $H^3(G;M)$ classifies crossed extensions

$$0 \longrightarrow M \longrightarrow X \longrightarrow Y \longrightarrow G \longrightarrow 1\ ,$$

where $\partial : X \to Y$ is a crossed module, with the resulting action of G on M again given by the G-module structure on M. If this action is trivial and G is superperfect, that is, both $H_1(G)$ and $H_2(G)$ are trivial, then $H^3(G;M)$ becomes isomorphic to $Hom(H_3(G),M)$, so that there exists a universal crossed extension of G by $H_3(G)$.

However, unlike the universal central extension, the universal crossed extension determined by $H_3(G)$ is not unique anymore. There is an initial representative for it, with Y a free group and X freely (in a certain sense) built from Y and the universal cocycle on $H_3(G)$, but there are many others. In particular, it follows from the classical work of Holt that if G is finite, then there is a representative with also Y finite and X finite abelian. In fact, X can be taken the induced G-module $H_3(G)[G]$.

A natural question then arises whether there exists a smallest representative, i. e. one which is a quotient of any other representative. Indeed, we will see that for any Y-subgroup $Z \subset X$ such that $M \cap Z = 0$ one can further reduce to $X/Z \to Y/\partial(Z)$, so that if a smallest representative exists, there exists a largest Z with $M \cap Z = 0$.

Our main result is that such largest Z need not exist. Namely, we show that for G equal to the Mathieu group M_{11} there exist two distinct maximal submodules $Z \subset H_3(G)[G]$ with $H_3(G) \cap Z = 0$. As a corollary we deduce that in general there is no smallest in the above sense representative for the universal crossed extension.

Acknowledgments

The work was supported by Shota Rustaveli National Science Foundation of Georgia, grant # MR-25-315.

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Quot Schemes on the Projective Line

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We present a self-contained construction of the Quot scheme parametrizing quotients of a trivial vector bundle on $\mathbb{P}^1$. Unlike Grothendieck's general approach, this construction does not rely on the machinery developed in Fondements de la géométrie algébrique (FGA). Instead, it uses only basic linear algebra and elementary arguments, providing a conceptually transparent realization of a fundamental example of a Quot scheme.

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A Multi-Layer Architecture for the Georgian Language: From Natural Language Processing to Formal Knowledge Representation

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The rapid development of artificial intelligence and Natural Language Processing (NLP) increasingly requires architectures capable of not only performing linguistic analysis of text but also transforming linguistic information into formal knowledge representations. This challenge is particularly significant for low-resource languages, including Georgian, where the limited availability of linguistic resources restricts the development of intelligent dialogue systems.

This talk presents the concept of a multi-layer architecture for the Georgian language that integrates Natural Language Processing, semantic interpretation, and formal knowledge representation into a unified architectural model. The proposed approach is based on the stepwise transformation of textual information; whereby natural language units are successively converted into formal knowledge structures.

The lower layers of the architecture comprise morphological analysis, lemmatization, and semantic processing, whereas the upper layers include a knowledge representation metalanguage, knowledge base construction, and the structures required for dialogue reasoning. The proposed knowledge representation model defines the principal semantic entities-objects, actions, properties, semantic roles, and the relationships among them-through which natural language sentences are transformed into formal representations. Consequently, each layer provides the foundation for the next, more abstract level of representation.

The practical implementation of the proposed architecture is based on morphological analysis and lemmatization technologies developed for the Georgian language, enabling the transformation of textual data into internal representations suitable for formal knowledge representation.

The talk proposes a multi-layer architectural model that unifies the stages of Natural Language Processing, semantic interpretation, knowledge representation, and dialogue processing within a single conceptual framework. The proposed approach provides a foundation for the development of Georgian intelligent dialogue systems, knowledge bases, and other knowledge-driven artificial intelligence applications.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under the grant # FR-24-11637.

A QCD-Motivated Causal Model for the Heavy Quark Potential

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We construct a non-perturbative causal-analytic model for the QCD effective charge related to the static quark-antiquark potential. Our approach is based on the Källen–Lehmann representation for the effective charge $\alpha_V(Q^2)$. Hence, the model charge automatically satisfies the requirement of causal analyticity. The spectral function $\rho(s)$ of the representation, is represented as a sum of perturbative and non-perturbative terms. A new feature of our procedure is that we parameterize the non-perturbative contributions to the spectral function by the sum of one delta and one theta functions (we refer to this model as $1\delta 1\theta$-model). Respectively, the charge is represented by the sum of the "analytically improved" perturbative and non-perturbative components. The perturbative component is calculated within the Analytic Perturbation Theory [1]. The non-perturbative component describes the linearly rising quark-antiquark static potential. It also contains an additional term which has a logarithmic singularity in the momentum space at $Q^2 = 0$, we refer to this term as the theta term. To reduce the number of independent parameters, we impose physically motivated conditions on the asymptotic behavior of the charge at large momenta. The only parameter of the model is the QCD scale. Within this approach, we suggest a natural definition of the infrared boundary of perturbative QCD. We examine the stability of the predictions of the model with respect to higher loop corrections up to the three loop order. We observe that the introduction of the theta term appreciably improves predictions of the model. In the infrared regime the constructed approximation to the charge is monotonically decreasing function.

To set the values of the parameters, we use the phenomenological value of the string tension parameter $\sigma_{st} = (0.420, GeV)^2$. With the three-loop order perturbative component, calculated in the $\overline{\text{MS}}$ scheme for $n_f = 3$ quark flavors, the model-based analysis leads to the reasonable values for the QCD scale parameter and the infrared boundary of perturbative QCD: $\Lambda_{\overline{MS}} = 0.282\, GeV$, $\mu_{\text{B}} = 0.894\, GeV$.

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Design and Implementation of an Adaptive Neuro-Fuzzy Controller for Nonlinear Dynamic Systems

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This paper presents the design and analysis of an adaptive neuro-fuzzy control system for nonlinear dynamic processes [2]. The proposed controller integrates the learning capability of neural networks with the reasoning mechanism of fuzzy logic to improve control performance under system uncertainties [1]. An adaptive structure based on the Adaptive Neuro-Fuzzy Inference System (ANFIS) is developed to automatically tune the membership functions and fuzzy rules. The performance of the proposed controller is evaluated through simulation in a nonlinear system environment and compared with conventional fuzzy and PID controllers. The results demonstrate that the neuro-fuzzy controller provides improved stability, reduced overshoot, and faster response time [4]. The study shows that combining neural network learning with fuzzy inference can significantly enhance intelligent control strategies for complex engineering systems.

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Elliptic Systems in the Plane

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First attempts to the extension of the complex methods were connected with the generalization of Cauchy-Riemann system and the investigation of the properties of the space of solutions of such a system. Since the 1950-s, after the appearance of the monographs of I. Vekua and L. Bers, these problems have been the subject of investigation by many scientists. The main part of the present work is devoted to the study of the solutions of singular elliptic systems, specifically to the development of the fundamental theorems of the theory of analytic functions.

Hardy–Littlewood Inequalities for Fractional Integrals in Classical Morrey Spaces

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We present two-weight inequalities with power weights for fractional integrals (Hardy–Littlewood [3] type inequalities) in classical Morrey spaces defined on the semi-axis. Fractional integral operators considered by us involve one-sided (Riemann–Liouville and Weyl integral operators) and two-sided potential operators. We obtained both Adams [1] and Spanne (unpublished) type results.

In particular, we established the following inequality:

$$\|x^{-\beta}T_\gamma f(x)\|_{M^{q,\mu}(\mathbb{R}+)} \le C\|x^\alpha f(x)\|_{M^{p,\lambda}(\mathbb{R}_+)}$$

with some positive constant C and for all f, $\|x^\alpha f(x)\|_{M^{p,\lambda}(\mathbb{R}_+)} < \infty$, where T_γ is a fractional integral operator, and $M^{q,\mu}(\mathbb{R}+)$ and $M^{p,\lambda}(\mathbb{R}_+)$ are Morrey spaces.

Similar Adams-type results for Riesz potentials in R^n were derived in [2].

The presented results were announced in [4].

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First Results of Researches for Developing a Mathematical Theory of Abkhazian Language and Thinking: Declarative Sentences and 1-Place and 2-Place Conjugation in Natural Abkhazian Language System

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The report reviews the threats facing the Abkhazian language, which is much less studied than Georgian from a mathematical and engineering point of view.

That is cleare that, today, in order for the Abkhazian language not to completely lose its status as a carrier and producer of culture, it is necessary to develop an almost perfect Abkhazian computer "brain", which, in turn, necessarily requires the development of an isomorphic mathematical theory of the Abkhazian language and thinking, for which purpose researches has already begun at our institute, the first results of which we will present. In particular, at the conference, we will briefly reviewed:

1. Abkhazian Declarative Sentences, whose linguistic-logical nature coincides with the linguistic-logical nature of Georgian Declarative sentences in the sense that both are abbreviated forms of the Shalva Pkhakadze type;
2. Similar to Georgian, the Abkhazian language also naturally contains Konstantine Pkhkadze's 1-place and 2-place Conjugations, which, on the whole, was expected given the non-Indo-European nature of the Abkhazian language.

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Equations for the S-matrix of Liouville Theory and its Functional Integral Representation

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We derive equations for the S-matrix of Liouville theory on the basis of the conformal symmetry and construct the solution in a functional integral form.

Acknowledgments

This research has been supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) by the grant # FR-23-17899.

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Topological Foundations of Tariff Wars: Chipman's Model as a Test Economy for a Homotopy Argument

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We study a tariff war between two countries as a Nash equilibrium problem. Existence results of Brouwer and Kakutani type require compact convex strategy sets and quasi-concave payoffs; in the pure-exchange model of Chipman [1] augmented by an additively separable cross-country consumption externality, $U_1^E(x_{11}, x_{21}; x_{12}) = x_{11}x_{21} + a\,x_{12}$ with $a \in \mathbb{R}$ and symmetrically for country 2 with coefficient b, that convexity is lost. Being separable, the externality leaves market-clearing prices and trades unchanged and acts only through the indirect utility V_k^E maximised when choosing the tariff factor T_k. We follow the differential-topological approach of Smale and of del Mercato [2, 3]. With $\Delta_1 = \omega + (\omega+1)T_1 + T_1T_2$, $\Delta_2 = \omega + (\omega+1)T_2 + T_1T_2$ and $B_k = (\omega+1)(\omega+T_k)$, we first reduce the problem to a polynomial one on $\Xi := \mathbb{R}^2_{>0}$.

Lemma *For every $(T_1, T_2) \in \Xi$ and every $a \in \mathbb{R}$,*

$$\frac{\partial V_1^E}{\partial T_2} = \frac{(\omega+1)(\omega+T_1)}{\Delta_1^2\Delta_2^2}\left[B_1(\omega\Delta_1 - T_1T_2\Delta_2) - a\,\omega\,\Delta_1^2\right],$$

the prefactor being strictly positive on Ξ. The bracket is quadratic in T_2 and, whenever $|a| < 1$, has exactly one positive root, which is the unique global best response.

The second assertion justifies recasting the equilibrium problem as a search for zeros of a vector field once convexity is unavailable. In the variables $s = T_1T_2$, $r = \Delta_2/\Delta_1$ the system becomes $sr = \omega(1 - a\Delta_1/B_1)$, $s/r = \omega(1 - b\Delta_2/B_2)$; for $a = b = 0$ this gives $r = 1$, $s = \omega$, hence $T_1 = T_2 = \sqrt{\omega}$: a regular test economy of degree 1 modulo 2, which we then deform.

Theorem *Let $m = \max(|a|, |b|)$, $\kappa = 1/(\omega(\omega+1))$, $S = \omega(1+m)(\omega+1)/(\omega+1-m)$ and $\theta = m(1+\kappa S)$. If $m < \omega+1$ and $\theta < 1$, the zero set of the homotopy lies in an explicit compact subset of Ξ. Hence the degree modulo 2 of the equilibrium map is 1, and the set of Nash equilibria is non-empty and compact. For almost every (a, b) it is finite of odd cardinality and its points are regular, and the origin is itself a regular parameter, so the Chipman equilibrium deforms differentiably.*

The degree yields existence but not local uniqueness; the latter follows from Thom's parametric transversality theorem applied to the pair (a, b), which makes the parameter block of the Jacobian diagonal with strictly signed entries. Unlike the implicit function theorem, which gives only local persistence near $a = b = 0$, the degree argument covers the whole admissible range: for $\omega = 2$ and $|a|, |b| \le 1/2$ one has $\theta = 0.8$, and a strong positive externality drives the equilibrium below unity: an import subsidy arises endogenously, which is why the domain must be $\mathbb{R}^2_{>0}$ rather than $(1, \infty)^2$. The construction is a proof of concept for operationalising global analysis in non-convex economic systems.

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Special thanks to E. L. del Mercato for her guidance.

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Trace and Olsen's Sharp Inequalities: Diagonal Case

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Trace inequality characterization in terms of Adams (Frostman) condition and appropriate sharp Olsen's inequality are known only in the off-diagonal case (see [1, 2] respectively). We give a necessary and sufficient condition on an absolutely continuous measure μ governing the trace inequality

$$\|I_\alpha f\|_{L^p_q(\mu)} \lesssim \|f\|_{L^{p,1}_q},$$

where $L^p_q(\mu)$ is the Morrey space with absolutely continuous measure μ, and $L^{p,1}_q$ is the Lorentz–Morrey space (For this result in Lebesgue spaces we refer to [3] and [4]).

Consequently, the following sharp Olsen's inequality in the diagonal case holds:

$$\|gI_\alpha f\|_{L^p_q} \lesssim \|g\|_{L^{n/\alpha}_q} \|f\|_{L^q_{p,1}}.$$

The problem was studied in ball Banach function spaces introduced in [7].

The talk relies on the joint papers with Y. Sawano (see [5, 6]).

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On Multilinear Rellich Inequalities

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In [1] the authors introduced and studied $m-$ linear Rallich inequalities on the real line. Our aim here is to present refined inequalities of the same type for a wider range of exponents than in that paper. We are also focused on the bounds of the constants in the inequalities. In particular, we establish the inequality

$$\Big\| \prod_{j=1}^{m} u_j \Big\|_{L^q_{w(\delta(\cdot))}(I)} \le C \prod_{j=1}^{m} \|u_j''\|_{L^{p_j}(I)}, \quad I := (a,b), \quad -\infty \le a < b \le +\infty, \tag{1}$$

with the positive constant C independent of $u_k \in C_0^\infty(I)$, $k = 1, \dots, m$, where $1 < \min\{p_1, \dots, p_m\} \le q < \infty$ and $\delta(x)$ is the distance function on I given by the formula

$$\delta(x) = \min\{x - a, b - x\}$$

and $1 < p_k < \infty$, $k = 1, \dots, m$.

We also focus on upper bounds for the best possible constants in (1).

The results are obtained jointly with D. E. Edmunds and A. Meskhi [2] (see also [3]).

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Universal Coefficient Theorems for Noncommutative Spaces

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In this talk, we demonstrate how Universal Coefficient Theorems can be formalized within the framework of triangulated categories and homological algebra, building on the ideas of [1, 2, 6]. We then present specific applications of these methods to bivariant K-theory categories of C*-algebras, particularly those equipped with additional actions by a specific class of finite groups and topological spaces [3, 4, 5].

Acknowledgments

George Nadareishvili was supported by the Shota Rustaveli National Science Foundation of Georgia through grant # FR-25-10267.

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Topology of the Universe

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This paper introduces a novel theoretical framework termed the "Topology of the Universe", which models the structural interaction and separation of dark matter and dark energy. Originating from the premise that these two components once formed a unified, balanced space, the study posits that a structural defect led to their separation into dynamic elements. The behavior of these separated elements is governed by surface tension, which is also responsible for the linearity of the bounding planes within the proposed model. Through topological analysis, the framework demonstrates that the fundamental topology of the space remains invariant when subjected to the introduction of distinct substantial elements, whether dark matter or dark energy. Ultimately, the paper bridges mathematical, topological reasoning with physical cosmic processes to propose a unique, foundational structural paradigm for the universe

Theorem *Given the unified space, any spontaneous defect causing the separation of elements and results in the formation of boundary layers; the bounding planes separating these distinct elements must remain strictly linear, a topological property necessitated by the dynamic action of surface tension across the separated substances.*

Lemma *If a bounded subset (a "ball") of substance or substance is introduced into the existing space, it merges with its corresponding subset without altering the fundamental topological structure of the overall space.*

Definition Let the unified space be defined as a balanced topological manifold comprising two continuous, interacting elements: (dark matter) and (dark energy). A state of separation within is defined as a dynamic structural defect where elements and are divided by bounding planes governed by surface tension.

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Properties of Linear Mappings on Some Algebras Without a Unity

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While much of the existing literature focuses on unital algebras, many important algebraic structures lack a unit element. We study non-unital trivial generalized matrix algebras and provide a complete characterization of Lie maps. Specifically, we show that these linear mappings can be expressed in standard forms. We deduce previous results for module extension algebras as corollaries of our main findings.

Definition Let A, B be algebras, M an (A,B)-bimodule, and N a (B,A)-bimodule. The *trivial generalized matrix algebra* is

$$\mathscr{G} = \begin{pmatrix} A & M \\ N & B \end{pmatrix}, \quad \begin{pmatrix} a & m \\ n & b \end{pmatrix}\begin{pmatrix} a' & m' \\ n' & b' \end{pmatrix} = \begin{pmatrix} aa' & am'+mb' \\ na'+bn' & bb' \end{pmatrix}.$$

A linear map $\varphi : \mathscr{G} \to \mathscr{G}$ is a *Lie map* if $\varphi([x,y]) = [\varphi(x), \varphi(y)]$.

Lemma For a Lie map φ, the images of elementary matrices are

$$\varphi\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \alpha(a) & 0 \\ 0 & \beta(a) \end{pmatrix}, \quad \varphi\begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} \gamma(b) & 0 \\ 0 & \delta(b) \end{pmatrix},$$
$$\varphi\begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \mu(m) \\ 0 & 0 \end{pmatrix}, \quad \varphi\begin{pmatrix} 0 & 0 \\ n & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \nu(n) & 0 \end{pmatrix}.$$

Theorem (Main Result) Let $\mathscr{G} = \begin{pmatrix} A & M \\ N & B \end{pmatrix}$ be a trivial generalized matrix algebra over $\mathbb{F}$ with $\operatorname{char}(\mathbb{F}) \neq 2$, and let M be faithful. A bijective linear map $\varphi : \mathscr{G} \to \mathscr{G}$ is a Lie isomorphism iff there exist Lie isomorphisms $\alpha : A \to A$, $\delta : B \to B$, bijective bimodule homomorphisms $\mu : M \to M$, $\nu : N \to N$, and linear maps $\lambda : A \to Z_{\mathrm{Lie}}(B)$, $\rho : B \to Z_{\mathrm{Lie}}(A)$ such that

$$\varphi\begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} \alpha(a)+\lambda(b) & \mu(m) \\ \nu(n) & \delta(b)+\rho(a) \end{pmatrix}.$$

Conversely, any map of this form is a Lie isomorphism.

Corollary For a module extension algebra $\mathscr{E} = A \ltimes M$ with M faithful, every bijective Lie isomorphism $\varphi : \mathscr{E} \to \mathscr{E}$ has the form $\varphi(a,m) = (\alpha(a)+\lambda(m), \mu(m))$, recovering classical results.

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Application of an Alternative Potential Method to Mixed Initial-Boundary Value Problems of the Elasticity Theory

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We consider a special alternative potential method to investigate a three-dimensional mixed initial-boundary value problems (IBVP) of the elasticity theory of isotropic bodies. A bounded three-dimensional domain occupied by an elastic isotropic body, $\Omega \subset \mathbb{R}^3$, has a simply connected boundary surface $S = \partial\Omega$, which is divided into two disjoint parts, the Dirichlet part S_D and the Neumann part S_N, where the displacement vector (the Dirichlet type condition) and the stress vector (the Neumann type condition) are prescribed respectively. By the Laplace transform the mixed IBVP under consideration is reduced to the corresponding mixed boundary value problem (BVP) for an elliptic system of pseudo-oscillation equations containing a complex parameter.

Our alternative approach is based on the classical potential theory and has several essential advantages compared with the existing approaches. We look for a solution to the mixed elliptic BVP in the form of a linear combination of the single layer and double layer potentials with densities supported on the Dirichlet and Neumann parts of the boundary respectively. This representation reduces the mixed BVP under consideration to a system of boundary integral equations, which contain neither extensions of the Dirichlet or Neumann data nor the Steklov-Poincaré type operator involving the inverse of the single layer boundary integral operator, which is not available explicitly for arbitrary boundary surface. Moreover, the right-hand sides of the resulting boundary integral equations system are vector-functions coinciding with the given Dirichlet and Neumann data of the problem in question. We show that the corresponding matrix integral operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies unconditional unique solvability of the mixed BVP in the class of vector-functions belonging to the Sobolev space $[W_2^1(\Omega)]^3$. We also show that the obtained matrix boundary integral operator is invertible in the L_p-based Besov spaces and prove that under appropriate boundary data a solution to the mixed BVP possesses C^α-Hölder continuity property in the closed domain $\overline{\Omega}$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FR-23-267).

Hopf Bifurcation and Convergence Analysis of a Finite-Difference Scheme for a Nonlinear Diffusion Model

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A one-dimensional system of nonlinear parabolic partial differential equations based on Maxwell system [1] is studied. The models of such type are investigated in many works (see, for example, [2–6] and references therein). The asymptotic behavior of solutions as time tends to infinity is analyzed. Linear stability of the stationary solution is examined and an explicit condition for stability is derived. A critical bifurcation point is identified; beyond this point, Hopf bifurcation occurs and oscillatory behavior appears. A finite-difference scheme is constructed. Convergence on refined grids confirms the scheme's accuracy. Stability of the scheme is confirmed numerically.

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On Testing the Equality of Poisson Regression Functions

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The limiting distribution of the integral square deviation between two nonparametric estimators of Poisson regression functions is established. Based on this result, the test for the hypothesis testing of equality of two Poisson regression functions is constructed. The question of consistency is studied for the constructed test.

Integral Representation of Non-Smooth Functional of the Brownian Motion and its Running Maximum

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Let us fix the $a \geq 0$, $b \leq a$ and consider the functional of the Brownian motion and its running maximum: $F = I_{\{M_T \leq a, BT \leq b\}}$, where $M_T = \max\limits_{0 \leq s \leq T} B_t$. Despite the fact that both the Brownian motion B_T and its running maximum M_T are Malliavin differentiable – indeed, $D_u B_T = I_{[0,T]}$, and $D_u M_T = I_{\{u \leq \tau_T\}}$, where $tau_T := \arg\max\limits_{0 \leq s \leq t} B_s$ is the (a.s. unique) time at which the maximum is attained – the functional F is not Malliavin differentiable. Consequently, its its Itô–Clark stochastic integral representation cannot be obtained via the classical Clark–Ocone formula (which requires $F \in D_{1,2}$). Moreover, it turns out that the conditional expectation process $E[F \mid \mathscr{J}_t^B]$ is also not Malliavin differentiable. Therefore, in this case it is not even possible to use the Glonti–Purtukhia weakened (generalized) representation (see [1]), which requires only that $E[F \mid \mathscr{J}_t^B] \in D_{1,2}$ for a.e. $t \in [0, T)$ rather than $F \in D_{1,2}$ itself.

Theorem 1 *The following expansion holds:*

$$E\big[I_{\{M_T \leq a, B_T \leq b\}} \mid \mathscr{J}_t\big] = I_{\{M_t \leq a\}} \left[\Phi\Big(\frac{b - B_t}{\sqrt{T-t}}\Big) - \Phi\Big(\frac{B_t + b - 2a}{\sqrt{T-t}}\Big) \right],$$

where Φ *is the standard normal distribution function.*

Theorem 2 *The following stochastic integral representation takes place:*

$$F = \Phi\Big(\frac{b}{\sqrt{T}}\Big) - \Phi\Big(\frac{b-2a}{\sqrt{T}}\Big) - \int_0^T \frac{1}{\sqrt{T}-t}\, I_{\{M_t \leq a\}} \left[\varphi\Big(\frac{b - B_t}{\sqrt{T-t}}\Big) + \varphi\Big(\frac{B_t + b - 2a}{\sqrt{T-t}}\Big) \right] dB_t,$$

where φ *is the standard normal distribution density function.*

It should be noted that Theorem 2, when $a = b$, yields the result we previously obtained by another method for the running maximum of Brownian motion (see [2]).

Corollary 1 *The following stochastic integral representation holds true:*

$$I_{\{M_T \leq a\}} = 2\Phi\Big(\frac{a}{\sqrt{T}}\Big) - 1 - 2\int_0^T I_{\{M_T \leq a\}}\, \frac{1}{\sqrt{T}-t}\, \varphi\Big(\frac{b - B_t}{\sqrt{T-t}}\Big)\, dB_t.$$

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Iteration Method for the Neumann Type Problem of the Elasticity Theory

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We construct a convergent recurrence scheme for a solution of the three-dimensional Neumann type boundary value problem of the elasticity theory when on the boundary of a homogenous anisotropic elastic body the stress vector is prescribed. By the potential method, the boundary value problem is reduced to the second kind Fredholm integral equation with zero index generated by the single layer potential. The corresponding boundary integral operator has a six-dimensional null-space. Therefore, the nonhomogeneous integral equation possesses a solution if the right-hand side vector-function meets the necessary and sufficient solvability conditions, i.e., it is orthogonal to the space of rigid displacements, which are solutions to the corresponding homogeneous adjoint integral equation. First, we reduce the boundary integral equation to the equivalent first kind Fredholm integral equation with symmetric non-negative compact operator. Afterwards, we construct a uniquely solvable modified boundary integral equation with symmetric compact positive operator having the following property: if the above mentioned necessary conditions are satisfied then the solution of the modified boundary integral equation is a particular solution of the original integral equation. Further, we construct a recursive sequence of vector-functions which converges to the unique solution of the modified boundary integral equation in appropriate Bessel-potential spaces of functions defined on the boundary. Using these approximations as densities of the single layer potential, we construct a sequence converging to a particular solution (a particular displacement vector) of the Neumann type boundary value problem in the appropriate Sobolev-Slobodetskii spaces of functions defined in the region occupied by the elastic body. The general solution of the Neumann type boundary value problem can be constructed by adding an arbitrary rigid displacement vector to the obtained particular solution. Evidently, the corresponding strain and stress tensors are defined uniquely.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FR-23-267).

Symplectic Structures on Projectivised Tangent Bundles Over Complex Projective Spaces

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Given a complex tangent vector bundle $\tau : \mathbb{C}^n \to E \to B^{2n}$ over a symplectic manifold B^{2n}, its associated projectivisation bundle is $P(\tau) : \mathbb{C}P^{n-1} \to P(E) \to B^{2n}$. Our goal in this study is to determine the symplectic forms over the total space $P(E)$ of the projectivisation bundle $P(\tau)$ when B^{2n} is $\mathbb{C}P^n$, $\mathbb{C}P^1 \times \mathbb{C}P^{n-1}$ or $\overbrace{\mathbb{C}P^1 \times \cdots \times \mathbb{C}P^1}^{n\text{-times}}$.

$\oplus$-r-Supplemented Modules

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In this work, every ring has a unity and every module is a unitary left module. Let M be an R-module. If every submodule of M has an r-supplement that is a direct summand in M, then M is called a $\oplus$-r-supplemented module. It is clear that every $\oplus$-r-supplemented module is $\oplus$-supplemented. It is also clear that every $\oplus$-r-supplemented module is r-supplemented.

Acknowledgments

This research was in part supported by grants from Ondokuz Mayıs University (Project # BAP01-2026-6621).

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Polynomization of a Conjecture by Sun

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Let $p(n)$ denote the number of partitions of a natural number n. As $n \to \infty$, the nth root of $p(n)$ tends to 1, which is related to the Cauchy–Hadamard test for power series. Andrews also discovered an elementary proof [1]. Sun [7] conjectured that this happens in a certain way for $n \geq 6$:

$$\sqrt[n]{p(n)} > \sqrt[n+1]{p(n+1)}\,.$$

The conjecture was proved by Wang and Zhu [8]; shortly thereafter, Chen and Zheng [2] independently obtained a second proof. In this talk, we follow an approach by Rota [6]. We regard $p(n)$ as special values of the D'Arcais polynomials [3], known as the Nekrasov–Okounkov polynomials [5]. This identifies Sun's conjecture as a property of the largest real zero of certain polynomials. This leads to results toward k-colored partitions, overpartitions, and plane partitions. Moreover, we also consider Chebyshev and Laguerre polynomials. The main purpose of this talk is to offer a uniform approach.

The talk includes joint work with B. Heim (University of Cologne/RWTH Aachen University) [3–4].

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Bezout's Theorem as an Effective Tool for Preventing Typical Errors

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Algebraic transformations – specifically the factorization of polynomials – is an area where students frequently encounter "mental blocks". Traditional methods, which rely on the distributive law and grouping, require a high level of structural intuition and conjecture; a lack of these often leads to typical, recurring errors.

The most common error during factorization is "ineffective grouping". The learner attempts to combine terms based on random external similarities (e.g., identical coefficients) but fails to arrive at a common factor that can be factored out. This creates a psychological barrier: the student loses confidence in the method and begins to perform mechanical, mathematically unfounded transformations in an attempt to "force" the desired answer.

At this stage, a necessity arises to move beyond standard templates and apply non-traditional approaches that shift the problem to a more predictable and systematic dimension.

The application of Bezout's Theorem in algebraic analysis serves as a strategic advantage that allows the learner to avoid guesswork. The essence of the method lies in viewing a complex expression as a polynomial in a single variable.

For instance, if we have an expression of the type $P(x, y, z)$, we treat it as a polynomial $P(x)$, where y and z act as parameters. According to Bezout's Theorem, if we find a value $x = a$ such that $P(a) = 0$, then $(x - a)$ is one of the factors of the expression $P(x, y, z)$.

It is important to emphasize that the introduction of non-standard approaches should not be perceived as an unnecessary complication of the curriculum or an additional burden. On the contrary, it should be viewed as an enrichment of the intellectual arsenal, resulting in the learner gaining intellectual flexibility, the capacity for self-verification, and protection against errors.

Generally, from a scientific and methodological perspective, the emphasis in the teaching process must shift from formal calculations toward strategic thinking. The use of Bezout's Theorem or other non-traditional methods in algebraic transformations is an effective means of avoiding typical errors. It provides the learner with self-confidence and tools that allow them to view a mathematical problem not as a rigid exercise, but as a field for creative and logical analysis.

Acknowledgments

The research was carried out with the financial support of M. Nalbandyan State University of Shirak within the framework of the scientific project SUSh-PRIOR N01-SCI-2026.

The Backward Induction Method as an Effective Means for Developing Students' Creative Thinking

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In the modern educational paradigm, creative thinking is viewed as the ability to discover subjective novelty and to navigate non-standard situations. It implies an individual's readiness to move beyond established templates and forge their own cognitive path. From this perspective, logical problems solved from the end gain particular importance, serving as a unique platform for implementing the problem-solving and exploratory method.

Many problems that allow for a backward solution can theoretically be solved from the beginning through the introduction of variables, the construction of algebraic models, or the evaluation of all possible options. However, when moving from "beginning to end", the student typically applies deductive logic, which, although develops computational skills and algorithmic discipline, rarely demands mental flexibility or an inventive approach. In such cases, the student operates within the domain of already known rules, where the solution process is predictable and the creative effect is minimal, as thinking moves with a "natural" inertia and through direct causal links.

The development of creative thinking begins at the exact moment when the student is forced to abandon their conventional viewpoint and make a strategic shift. The uniqueness of problems solved from the end lies in the fact that each backward step is transformed into a new problem-solving situation; here, the student does not merely perform a mechanical action but analyzes the system of conditions, searching – through a retrospective lens-for the prior state from which the current point could have been reached. This process requires a deep understanding of the problem's structure, the discovery of "internal symmetry", and the intuitive grasping of patterns, which forms the basis of the creative leap.

Thus, the method of backward induction should not be viewed merely as a mathematical trick or an auxiliary tool. It is a systematic pedagogical strategy aimed at igniting the researcher's instinct and a "scientific spark" within the student. By emphasizing the backward course itself, we provide the student with the opportunity to see the "invisible" side of the problem, discover the reversibility of phenomena, and develop qualities such as analytical synthesis, intuition, and mental freedom. These are the fundamental qualities that serve as the key components of a creative mindset, preparing the student for independent scientific and life discoveries.

Acknowledgments

The research was supported by the Higher Education and Science Committee of MESCS RA (Research project # 25RG-5C032).

Two-Dimensional Dirac Equation with Matrix Potential: Solvable Models of Graphene in External Electromagnetic Field

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The excitations in graphene-like materials in external electromagnetic field are described by solutions of a massless two-dimensional Dirac equation including both Hermitian off-diagonal matrix and scalar potentials. In the literature, such two-component wave functions were calculated for different forms of external potentials, though as a rule depending on only one spatial variable. In the present work, we find analytically the solutions for a wide class of external potentials which physically correspond to applied mutually orthogonal magnetic and longitudinal electrostatic fields, both depending really on two spatial variables. The main tool for this progress is provided by supersymmetrical (SUSY) intertwining relations, specifically, by their most general – asymmetrical – form proposed recently in our previous papers. This SUSY-like method is applied in two steps, similar to the second order factorizable (reducible) SUSY transformations in ordinary quantum mechanics.

Bipartite Digraphs with Modular Concept Lattices

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This paper investigates the interaction between Formal Concept Analysis (FCA) and graph theory, with a focus on understanding the structure and representation of concept lattices derived from bipartite directed graphs. We establish connections between the complete formal contexts and their associated bipartite digraphs, providing a foundation for studying modular lattices. Particular attention is given to the structure of concept lattices arising from such contexts and their relationship to the combinatorial properties of the corresponding graphs.

Theorem 1 *Let $\mathbb{K} = (A, B, I)$ be a full formal context in which $A \cap B = \varnothing$. And let $\mathbf{G}$ be the corresponding context graph $(A \cup B; I)$. Then $\mathscr{L}(\mathbb{K}) \cong \mathscr{L}(\mathbb{K}_G)$.*

Theorem 2 *Let $\mathbb{K} = (A, B, I)$ be a full formal context in which $A \cap B = \varnothing$. Then the concept lattice $\mathscr{L}(\mathbb{K})$ is isomorphic to M_n for some $n \leq \omega$ if and only if the context graph $\mathbf{G}_K = (A \cup B; I)$ is a disjoint union of n complete bipartite digraphs.*

Acknowledgments

The authors are funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant # AP23485395).

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Two-Scale Mathematical Modeling of Failure Composite Laminates

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The development of mathematical and numerical models of the failure of composite laminates is key to solving a real-world problem in modern engineering.

The purpose of this report is to present new constitutive relations for the direct description of failure in anisogrid laminates in the form of closed-form expressions linking meso- and macro-level composite failure.

Definition A laminate whose elastic moduli in the direction perpendicular to the fibres and shear moduli in the fibre planes are negligible compared to the elastic modulus in the direction of the fibres will be called anisogrid.

The following Theorem 1 presents the main result of this report and establishes the laminate failure criterion.

Theorem 1 *Let the anisogrid multidirectional laminate be symmetrical and balanced, and be in a plane stress state under the action of arbitrarily directed uniaxial loading σ_α applied to the laminate at an angle α to the direction of the fibres in layer-0*

Then the failure stress $\sigma_{\varphi_l^*}(\alpha)$ of such laminate is determined as

$$\sigma_{\varphi_l^*}(\alpha) = \min_{\varphi_l} \left\{ X_{\varphi_l} F_{\varphi_l}(a, \overline{h}_{\varphi_l}) \right\}, \tag{1}$$

where X_{φ_l} is the longitudinal strength of the φ_l layer material, $\overline{h}_{\varphi_l} = h_{\varphi_l}/H$ is the relative thickness of the φ_l-layer, H is the laminate thickness, and F_{φ_l} is the coefficient proportional to which the stress σ_alpha is distributed within the layers of this laminate.

According to (1), under a given external load, the φ_l^* layer of the laminate fails first.

Equation (1) describes the failure envelope for the laminate.

For example, for an isogrid laminate, $\overline{h}_{\varphi_l} = h = idem$ with the plies stacking $(0_n, 45_n, 90_n, 135_n)$ these coefficients F_{φ_l} can be calculated using following formulas:

$$F_0 = \frac{2h}{1.5 - \sin^2\alpha}, \quad F_{90} = \frac{2h}{2\sin^2\alpha - 0.5}, \\ F_{45} = \frac{4h}{1 + 4\sin\alpha\cos\alpha}, \quad F_{135} = \frac{4h}{1 - 4\sin\alpha\cos\alpha}. \tag{2}$$

Failure envelope for IM7/8552 carbon–epoxy laminates is shown in Figure 1. The marker • indicates the measured value $\sigma_{\varphi_l^*}(0) =$ 842 MPa in the experiment [1].

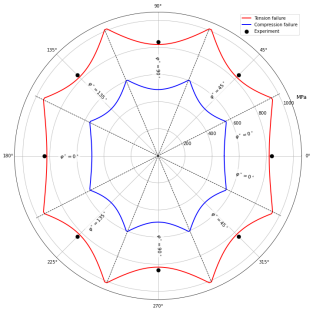

Figure 1: Failure envelope for IM7/8552 carbon–epoxy laminates

The calculated value of $\sigma_{\varphi_l^*}(0)$ according to (1) is 822 MPa, which is a 2% difference from the experiment. The calculated value according to laminated plate theory is 1074 MPa, which is a 22% difference from the experiment. The study concludes that the proposed two-scale mathematical modeling of composite laminate failure is effective.

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Counterexamples to Zygmund's Differentiation Conjecture for Córdoba-Type Bases

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Zygmund's conjecture on the differentiation of integrals, in its general form, was refuted by Soria. Later, Rey proposed a new method for constructing counterexamples. Using the crystallization technique for the accumulation of singularities of maximal operators, we exhibit a further class of counterexamples, thereby refuting Zygmund's conjecture even in the special case of Córdoba-type bases. This resolves a question raised by Soria.

Investigation and Numerical Solution of a Fourth-Order Nonlinear Integro-Differential System

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A system of fourth-order nonlinear integro-differential equations, which extends a well-established class of second-order models used to describe diffusion-type physical processes, is investigated. These higher-order systems generalize a range of classical equations that have been studied extensively in the mathematical literature (see, for instance, [1–3] and references therein), and have themselves been the subject of investigation in various contexts (see [3–5] and references therein). Uniqueness and stability conditions for solutions of the corresponding initial-boundary value problem in the fourth-order case are established, a finite-difference discretization algorithm is constructed, and its performance is demonstrated through numerical experiments.

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On the Numerical Solution of the J. Ball's Thermo-Viscous-Elastic Dynamic Beam Equation in the Case of External Damping

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An initial-boundary value problem is posed for the J. Ball integro-differential equation, which describes the dynamic state of a beam. A physical model that J. Ball uses in the article [1] is taken from the handbook of Engineering Mechanics written by E. Mettler (see [2]). The solution is approximated utilizing the Galerkin method, stable symmetrical difference scheme and the Jacobi iteration method(see [3]). Since 2019, our work has focused more on the physical content of the problem, namely the effective viscosity (depending on pressure, temperature...). This paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam. The presented paper discusses the case when an external impact acts of a beam. For approximate solving initial-boundary value problem several programs are composed in "Maple", several numerical experiments are carried out. In the presented work, we will consider three cases for the external impact coefficient: $\delta = 0$ Newton, $\delta = 10^3$ Newton, $\delta = 10^6$ Newton. For each variant, we will consider two essentially different cases with respect to temperature: 1. A non-extreme situation, when the temperature changes from $10°C$ to $20°C$, 2. An extreme situation, when the temperature changes from $10°C$ to $1000°C$. The results of numerical calculations qualitatively satisfactorily describe the process under consideration.

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On Nonlocal Boundary Value Problems for First Order Differential Equations with a Deviating Argument

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On a finite interval $[a, b]$, we consider the differential equation

$$u'(t) = \sum_{i=1}^{k} p_i(t) f_i(u(\tau(t))) \tag{1}$$

with the nonlocal boundary condition

$$\sum_{i=1}^{m} \int_a^b |u(s)|^{\lambda_i} d\sigma_i(s) = c_0, \tag{2}$$

where k and m are natural numbers, c_0 is a positive constant, $p_i \in L([a, b])$, $f_i \in C(\mathbb{R})$ $(i = 1, \ldots, k)$, $\lambda_i > 0$, $\sigma_i : [a, b] \to \mathbb{R}$ is a nondecreasing function $(i = 1, \ldots, m)$.

Sufficient conditions for the existence and uniqueness of a nonnegative solution to problem (1), (2) are established. Namely, the following theorem is proved.

Theorem *Let along with the inequality*

$$\tau(t) \leq t \ \text{ for } \ t \in [a, b],$$

the following conditions

$$p_i(t) \geq 0 \ \text{ for } \ t \in [a, b], \quad f_i(0) = 0, \quad f_i(x) \geq 0 \ \text{ for } \ x \in \mathbb{R}_+ \quad (i = 1, \ldots, k),$$
$$\sum_{i=1}^{m} (\sigma_i(b) - \sigma_i(b-)) > 0$$

hold, where $\sigma_i(b-)$ is a left limit of the function σ_i at the point b. Then problem (1), (2) has at least one nonnegative solution.

On Homology Monoids of Simplicial Abelian Monoids

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The present talk is devoted to studying the homology semimodules $H_n(S)$ of a presimplicial semimodule S, introduced in [1], in the case where S is a simplicial abelian monoid. In particular, we show that if a simplicial abelian monoid A satisfies the Kan condition and the monoid of path components of A is a group, then the homology monoid $H_n(A)$ is isomorphic to the homotopy group $\pi_n(A)$ for all n.

Acknowledgments

The author was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) through the grant # FR-25-10267.

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Some Methods for Constructing Approximate Solutions to Generalized Kirchhoff-Type Equations

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The report reviews most frequently used methods for constructing approximate solutions of generalized Kirchhoff-type equations.

The Gini Index in Markov Chains: Asymptotic Analysis of Socio-Economic Dynamics

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This paper explores the dynamic modeling of economic inequality using the Gini index within the framework of Markov chain theory. Although the Gini coefficient is traditionally treated as a static macroeconomic indicator, real-world socio-economic systems are characterized by continuous agent interactions and mobility between classes, fundamentally constituting a stochastic process.

In this study, the population is partitioned into discrete income/wealth groups, and their evolution over time is described by an ergodic Markov chain governed by a transition probability matrix $P = (p_{ij})_{n\times n}$. We demonstrate that the long-term (asymptotic) Gini index operates as a direct functional on the unique stationary distribution π of the Markovian operator.

Two fundamental scenarios are analyzed in detail:

1. *A Pure Stochastic Market (Discrete Analogue of the Yard-Sale Model):* Where the zero-wealth state is defined as an absorbing state of the Markov chain. We rigorously prove that without regulations, the absorption probability approaches 1, leading to the collapse of the system's phase space and the asymptotic growth of the Gini index towards 1.
2. *A Modified Stochastic System with Redistribution:* Where, using matrix perturbation theory, we illustrate how the stationary Gini index shifts under small variations of the transition probabilities p_{ij} (e.g., via stimulating social mobility).

The scientific novelty of this work lies in the integration of a rigorous probabilistic apparatus into economic inequality forecasting, allowing for the evaluation of optimal mathematical parameters of social mobility required to achieve a stable economic equilibrium.

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Declension of Nouns and Conjugation of Verbs According Konstantine Pkhkadze's Logical Grmmar of Georgian Langiage

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It is known that according to Konstantine Pkhakadze Logical Grammar of Georgian Language, there are 14 cases in Georgian instead of Akaki Shanidze's traditional 7 cases [1, 2].

Also, It is known that in the singular, Shanidze's traditional conjugation yields 3-member series, whereas according to Pkhakadze, 3-member series are produced only by Pkhakadze's 1-place Georgian conjugations, while Pkhakdze,s 2-place Georgian conjugation in the same singular number yields 7-member series.

Consequently, in the presentation, we will review the very important issues mentioned above. Specifically, here, we will provide Pkhakadze's definitions of logical declension and conjugation and, also, we will proved that it is necessary to teach Pkhakadze declension and conjugation in Georgian schools, as they are precisely what is natural for the non-Indo-European Georgian language. – It is worth emphasizing that this vision is shared by all members of the Institute's Scientific Council.

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One More Step Toward Semi Self Developing Georgian-Abkhazian Terminological Portal

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In presentation we will presented first experimental version of Georgian-Abkhazian semi self-developing terminology portal and, also, there is overviewed the goals and methods of developing of such portal.

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Products of Best Rational Approximation Errors and Meromorphic Continuation

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Let $E \subset \mathbb{C}$ be a regular compact set with connected complement, and let f be analytic on E. Denote by $\rho_{n,m}(f;E)$ the error of best uniform approximation of f on E by rational functions whose numerator and denominator have degrees at most n and m, respectively.

Let $g_\Omega(z,\infty)$ be the Green function of $\Omega := \widehat{\mathbb{C}} \setminus E$ with pole at ∞. We extend $g_\Omega(z,\infty)$ to E by setting $g_\Omega(z,\infty) = 0$ on E. For $R > 1$, set

$$D_R := \big\{ z \in \mathbb{C} : \ g_\Omega(z,\infty) < \log R \big\}.$$

For each $m \geq 0$, let R_m be the largest R such that f extends meromorphically to D_R with at most m poles, counted with multiplicity.

The following theorem describes the exponential rate of convergence for a fixed row of the Walsh table.

Theorem (Saff–Gonchar) *Let $E \subset \mathbb{C}$ be a regular compact set with connected complement, and let f be analytic on E. Fix $m \geq 0$. Then*

$$\limsup_{n\to\infty} \rho_{n,m}(f;E)^{1/n} = \frac{1}{R_m}\,.$$

The aim of this talk is to present a new product-type analogue of the Saff–Gonchar theorem, in which one approximation error is taken from each of the first $m+1$ rows of the Walsh table. The resulting product formula connects the rate of rational approximation with the radii of meromorphic continuation and gives a Hadamard-type extension of the classical Saff–Gonchar theorem. The proof uses Hankel operators and an AAK-type theorem for meromorphic approximation.

Theorem *Assume that a compact set E has connected complement and Jordan boundary, and let f be analytic on E. Then for every fixed $m \geq 0$,*

$$\limsup_{n\to\infty} \Big(\prod_{k=0}^{m} \rho_{n-m+k,k}(f;E) \Big)^{1/n} = \frac{1}{R_0 R_1 \cdots R_m}\,.$$

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A Generalization of Artin's Theorem on Characters of Finite Groups for Separable Hopf Algebras

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Let A be a finite dimensional separable Hopf algebra over a field k, $chark = 0$.

Lemma 1 *A bilinear form over A defined as the trace of the regular representation $(u, v) = tr(uv)$ is a symmetric invariant non-degenerate form.*

Lemma follows from the fact that such separable algebras are Frobenius algebras ([1, 2]).

Let $B \subset A$ be a separable subalgebra, $\chi(A)$ – the ring of characters of A, $res(B) : \chi(A) \to \chi(B)$, $ind(B) : \chi(B) \to \chi(A)$ – the homomorphisms of restriction and induction respectively. Let M and N are A-modules (B-modules) accordingly. Let φ_M and ψ_N their characters.

Lemma 2 $(\varphi, ind\psi) = (res\varphi, \psi)$.

Theorem *Let $\mathfrak{B}$ be some finite family of separable subalgebras of A such that*

$$\bigcup_{a \in S, B \in \mathfrak{B}} aBa^{-1} = A$$

for some set S of units in A. Let $\chi(A)_{\mathfrak{B}} = \sum_{B \in \mathfrak{B}} indB$. Then $\chi(A)_{\mathfrak{B}}$ has finite exponent in $\chi(A)$, i.e. there is a nonzero positive integer d such that $d\chi(A) \subset \chi(A)_{\mathfrak{B}}$.

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Pre-A^*-Algebraic Framework for Formal Reasoning under Uncertainty in Networked Systems

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We develop a formal algebraic framework for modeling uncertainty and influence in networked systems using Pre-A^*-algebras with a non-classical truth ordering $2 \leq 0 \leq 1$, representing indeterminate, false, and true states, respectively. We focus on Pre-A^*-posets that are locally finite, possess a least element 2, and satisfy the Jordan–Dedekind Chain Condition (JDCC). Under these assumptions, we prove the existence and uniqueness of a rank function ρ that assigns to each element the length of any maximal chain from the least element, thereby providing a path-independent and structurally stable measure of hierarchical influence depth.

Theorem *In a locally finite Pre-A^*-poset with least element satisfying the JDCC, there exists a unique rank function $\rho : A \to \mathbb{N}$ such that $\rho(2) = 0$ and $\rho(y) = \rho(x) + 1$ whenever y covers x.*

We establish that this rank function is computable via a breadth-first traversal of the Hasse diagram, ensuring algorithmic tractability for large-scale networks. Furthermore, we characterize valuation propagation using Pre-A^*-meet and join operations to model consensus and influence aggregation. The proposed framework offers a rigorous foundation for stable influence ranking and formal reasoning in incomplete information systems, with direct applications to social networks, trust propagation, communication systems, and intelligent decision-support environments.

Acknowledgments

The authors thank the United States International University, Nairobi, for supporting this research.

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A Combined Numerical Approach for Solving a Nonlinear Dynamic Timoshenko Beam System

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In this talk, we consider a nonlinear, dynamic, spatially one-dimensional Timoshenko beam coupled system. For its numerical treatment, a symmetric three-layer semi-discrete time-stepping scheme is developed. The proposed scheme yields second-order linear ordinary differential equations for each unknown at every time level, enabling parallel computation of the solution.

The convergence of the scheme is analyzed, and second-order accuracy in time is established. For spatial discretization, a Legendre–Galerkin spectral approximation is employed, in which differences of Legendre polynomials are used as basis functions. Within this combined framework, the original problem is reduced to subsystems of linear equations whose coefficient matrices are sparse, symmetric, and positive definite. Moreover, these subsystems can be efficiently decoupled.

The convergence of the Galerkin method is also investigated. Several numerical experiments are conducted on carefully selected test problems, and the results are consistent with the developed theoretical findings. This talk is based on arXiv:2602.02068 (see [1]).

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Quantale-Based Automated Reasoning for Symbolic AI

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Modern intelligent systems must reason in the presence of uncertainty, vagueness, incomplete information, and context-dependent knowledge. While classical logic provides a rigorous foundation for automated reasoning, its binary notion of truth is often too restrictive for real-world applications. Over the past decades, fuzzy logic and related quantitative reasoning frameworks have significantly extended the expressive power of symbolic reasoning by introducing graded notions of truth and similarity. Nevertheless, most existing approaches remain limited in the types of quantitative information they can represent and are primarily confined to first-order reasoning.

This talk presents preliminary results of a new research project that aims to establish a general framework for quantitative automated deduction based on quantales, a rich algebraic structure capable of representing not only fuzzy truth values but also distances, costs, resources, probabilities, accessibility relations, and other quantitative measures within a unified mathematical setting. By replacing traditional fuzzy relations with quantale-valued semantics, the project seeks to substantially broaden the scope of symbolic reasoning while preserving rigorous logical foundations.

The project addresses three complementary research directions. First, it develops novel algorithms for quantale-valued approximate unification and constraint solving in both first-order and restricted higher-order languages. These algorithms constitute the computational core required for automated reasoning under graded similarity and approximation. Second, the project designs new resolution- and tableau-based proof systems that incorporate quantitative information directly into the inference process, enabling deduction over vague and context-sensitive knowledge. Finally, the developed methods will be applied to Natural Logic, creating a quantitative reasoning framework capable of handling linguistic vagueness and approximate semantic inference in natural language.

A distinguishing feature of the project is its interdisciplinary character, combining mathematical logic, automated reasoning, algebra, artificial intelligence, and computational linguistics. The proposed methods are intended to remain largely independent of any specific application domain, making them suitable for knowledge representation, explainable AI, decision-support systems, logic programming, and natural language understanding. Moreover, the framework naturally complements emerging neuro-symbolic approaches, where neural models provide quantitative information while symbolic reasoning guarantees transparency, correctness, and interpretability.

The talk will provide an overview of the mathematical motivation behind quantale-based reasoning, discuss the main scientific objectives of the project, and outline the expected theoretical and practical contributions. It will also illustrate how extending automated reasoning beyond fuzzy logic opens new opportunities for developing expressive, explainable, and mathematically well-founded AI systems capable of reasoning with the complexity and uncertainty inherent in real-world information.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation of Georgia under the project # FR-25-5957.

Application of Inequalities to the Non-Oscillatory Properties of a Class of High-Order Difference Equations

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Let us consider the problem of non-oscillatory of the higher-order Sturm-Liouville difference equation

$$(-1)^n \Delta^n (\rho_k \Delta^n y_k) - v_k y_{k+n} = 0, \tag{1}$$

where $n \in \mathbb{N}$, $n > 1$, $\Delta y_k = y_{k+1} - y_k$, $\rho = \{\rho_k\}_{k=0}^{\infty}$ is a sequence of positive numbers, $v = \{v_k\}_{k=0}^{\infty}$ is a sequence of non-negative numbers, and $\mathbb{N}$ is a set of natural numbers.

The non-oscillatory properties of the equation (1) are determined by the behavior of its solutions in a neighborhood of infinity. However, ultimately, the behavior of these solutions depends on the behavior of the equation coefficients. Thus, the determination of the non-oscillatory properties of equation (1) in terms of the coefficients ρ, v is important. We work directly with the variation principle ([1–3]). We establish conditions equivalent to the fulfillment of the variational principle, on the basis of which the non-oscillatory conditions of equation (1) are determined.

Our main result reads as follows:

Theorem *If*

$$\max_{0\le i\le n-1} \left[\binom{n-1}{i}\right]^2 \limsup_{N\to\infty} \left(\sum_{k=s}^{\infty} v_{k-(n-1)} \left[k^{(n-1-i)}\right]^2 \sum_{j=N}^{s} \left[(j+i-1)^{(i)}\right]^2 \rho_j^{-1} \right) < \frac{1}{8n^2},$$

then the equation (1) is non-oscillatory.

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Double Almost Statistical Convergence of Order α with Respect to Seminorms

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Let $\lambda = (\lambda_m)$ and $\mu = (\mu_n)$ be two non-decreasing sequences of positive real numbers both of which tends to ∞ as m and n approach ∞, respectively. Also let $\lambda_{m+1} \leq \lambda_m + 1$, $\lambda_1 = 1$ and $\mu_{n+1} \leq \mu_n + 1$, $\mu_1 = 1$. We write the generalized double de la Valèe-Poussin mean by

$$t_{mn}(x) = \frac{1}{\lambda_m \mu_n} \sum_{i \in I_m,\, j \in I_n} x_{ij}.$$

In this paper we define and study λ-double almost statistical convergence of order α with respect to seminorm. Furthermore some inclusion relations have been considered. We further introduce a new sequence space by taking double almost statistical convergence and Orlicz function in seminorm.

Analysis of a Mixed Problem for the One-Dimensional Wave Equation with Strong Dissipation and Functional Transmission Conditions

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The present study is devoted to the investigation of a transmission problem with a dynamic functional transmission condition arising in the mathematical modeling of composite media and coupled physical processes. We consider a mixed problem in the domain $Q_T = [0,T] \times [0,2]$:

$$u_{tt} - (\mu_1(x)u_x)_{xt} - \eta_1(x)u_{xx} = f(t,x), \quad 0 \le t \le T, \quad 0 \le x \le 1, 1 \le x \le 2,$$
$$v_{tt} - (\mu_2(x)v_x)_{xt} - \eta_2(x)v_{xx} = g(t,x), \quad 0 \le t \le T, \quad 1 \le x \le 2$$

with boundary conditions

$$u\,(t,0) = 0, \quad v(t,2) = 0,$$

functional transmission conditions

$$u(t,1) = v(t,1) = \phi(t),$$
$$\phi_{tt}(t) + \gamma_1 u_{xt}(t,1) - \gamma_2 v_{xt}(t,1) + K[u(t,\,\cdot\,), v(t,\,\cdot\,)] = h(t) \tag{1}$$

and initial conditions

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad 0 \le x \le 1,$$
$$v(0,x) = v_0(x), \quad v_t(0,x) = v_1(x), \quad 1 \le x \le 2,$$
$$\phi(0) = \phi_0 = u_0(1) = v_0(1),$$
$$\phi_t(0) = \phi_1 = u_1(1) = v_1(1).$$

Let us introduce the following space

$$X_p = \big\{w :\ w = (u,v,\alpha),\ u \in L_p(0,1),\ v \in L_p(1,2),\ \alpha \in C\big\},$$

with a norm

$$\|w\|_{X_p} = \left[\int_0^1 \|u(x)\Big\|_p^p dx\right]^{\frac{1}{p}} + \left[\int_1^2 \|v(x)\|_p^p\, dx\right]^{\frac{1}{p}} + |\alpha|,$$

and

$$Y_p = \Big\{w :\ w = (u,v,\phi),\ u \in W_p^2(0,1) \cap W_p^1((0,1);0),$$
$$v \in W_p^2(1,2) \cap W_p^1((1,2);2),\ u(1) = v(1) = \phi\Big\}$$

with a norm

$$\|w\|_{Y_p} = \|u_{xx}\|_{p,1} + \|u_x\|_{p,1} + \|v_{xx}\|_{p,2} + \|v_x\|_{p,2}.$$

Note that in (1), $K[\,\cdot\,,\,\cdot\,]$ is a linear continuous functional acting from Y_P to R.

The mixed problem is reduced to the Cauchy problem for an operator equation in the space X_p. Based on the theory of evolution equations, the existence and uniqueness of a global solution are established.

Construction of a Mathematical Model of One Class of Nonlinear Dynamic Systems with Feedback

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The problem of constructing a mathematical model for nonlinear dynamic systems operating with positive feedback is considered. Such systems are widespread in manufacturing, particularly in the mining and metallurgical, chemical, cement, pulp and paper industries, ecology, and other fields.

During model development, the determination of the model structure is carried out in accordance with L. Zadeh's classical definition of system identification. It is assumed that the classes of models and the system's input signals are known, and it is necessary to develop a criterion for determining the model structure from the set of models. With a known model structure, data on the input-output signals of the system are used to estimate the parameters.

The problem of constructing a mathematical model of nonlinear systems with feedback is considered on the set of block-oriented models. This class comprises a linear model with nonlinear feedback, a nonlinear model with linear feedback, and the Hammerstein and Wiener models with unity feedback. The equations of the models, which can be reduced to Riccati ordinary nonlinear differential equations, taking into account the specific characteristics of the operation of such systems functioning in manufacturing.

The model structure is identified, and its parameters are estimated in the frequency domain using harmonic input signals applied to the system. By taking into account the necessary conditions for obtaining a stable motion in a steady-state at the output of a system operating with positive feedback, the method of a small parameter can be employed to solve the Riccati equations. An analysis of the analytical expressions for the output signals of the models, obtained from the solutions of these equations, has led to the development of a criterion for selecting the appropriate model from the considered set of models. The model parameters are estimated using the least squares method based on the Fourier approximation of the models' forced output oscillations. The accuracy of the obtained results has been investigated through both theoretical analysis and computer simulation.

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Geometry of Ideal Fluid

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In dimensions 2 and 3 the Euler equations $(u \cdot \nabla)u + \nabla p = 0$, $\nabla \cdot u = 0$ describe steady flows of incompressible ideal fluid. The equations are of some mathematical interest in any dimension. Here $u = \big(u_1(x), \dots, u_n(x)\big)$ is a vector field on $\mathbb{R}^n$ (fluid velocity) and p is a scalar function (pressure). We restrict ourselves by considering solutions such that the functions u_i $(i = 1, \dots, n)$ and p are defined on the whole of $\mathbb{R}^n$, are sufficiently smooth and sufficiently fast decay at infinity. In the most cases we assume that u_i, p belong to the Schwartz space $\mathscr{S}(\mathbb{R}^n)$.

The following question was open during a long time: Does there exist a non-trivial (i.e., not equal identically to zero) solution to the Euler equations on $\mathbb{R}^3$ such that u_i, $p \in \mathscr{S}(\mathbb{R}^3)$? The existence of such a solution was recently proved by A. Gavrilov [1]. Such solutions can be easily found in any even dimension. The solution found by Gavrilov satisfies the additional equation $u \cdot \nabla p = 0$. We use the name *Gavrilov flow* for a solution satisfying this equations. Gavrilov flows can be completely described in terms of classical differential geometry of hypersurfaces in $\mathbb{R}^n$. Namely, the restriction of u to every regular *isobaric hypersurface* $M_{p_0} = \{x \in \mathbb{R}^n \mid p(x) = p_0 = const\}$ is a tangent to M_{p_0} vector field that does not vanish and satisfies the equations

$$\nabla_u u = 0, \quad \operatorname{div} u = u(\log |\nabla p|), \quad \mathrm{II}(u, u) = -|\nabla p|, \tag{1}$$

where II is the second quadratic form of the hypersurface M_{p_0}. The first of these equations means that u is *a geodesic vector field* on M_{p_0}, i.e., integral lines of u (= particle trajectories) are geodesics of the hypersuface M_{p_0}. We can now forget the Euler equations and investigate the pure geometric system (1). But the investigation turns out to be a not easy business. So far more or less complete results are obtained in the 3D case for surfaces of revolution, i.e., for axisymmetric Gavrilov flows [3].

The following observation belongs to Nadirasvili–Vladuts [2]. If a solution (u, p) to the Euler equations is such that u_i, $p \in \mathscr{S}(\mathbb{R}^n)$, then $\int\limits_P (\xi \cdot u(x))(m \cdot u(x))\, dx = 0$ for every affine hyperplane $P \subset \mathbb{R}^n$ where m is the normal vector of P, ξ is an arbitrary vector parallel to P, and dx is the $(n-1)$-dimensional Lebesgue measure on P. Due to the observation, methods of integral geometry (= of tensor tomography) can be applied to investigating the Euler equations [4].

Acknowledgments

The work was performed according to the Government research assignment for IM SB RAS, project # FWNF-2026-0026.

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Quaternionic Banach Frames and their Localization

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Quaternionic Banach frames are introduced with respect to quaternionic Banach sequence spaces, together with their associated analysis and reconstruction operators. Several examples illustrate tightness, exactness, and the role of the associated sequence space. Perturbation results are established, and it is shown that every quaternionic Banach space admitting a quaternionic Banach frame also admits a normalized tight exact quaternionic Banach frame. Finally, a localization theory for quaternionic Banach frames is developed. For polynomially localized frames in right quaternionic Hilbert spaces, inverse-closedness is established for the corresponding localized quaternionic matrices, localization of the canonical dual frame is obtained, and boundedness of the associated analysis and synthesis operators on weighted quaternionic Banach spaces is proved. As a consequence, the main localization theorem for quaternionic Banach frames is obtained.

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A Frame-Theoretic Framework for Comparative Analysis of FTIR Spectra

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Fourier Transform Infrared (FTIR) spectroscopy is one of the most widely used analytical techniques for investigating molecular structures and chemical interactions. Despite its extensive applications, the mathematical analysis of FTIR spectra is predominantly based on signal-processing methods, chemometric techniques, and machine learning algorithms, with comparatively little attention devoted to rigorous functional-analytic representations. In this work, we propose a frame-theoretic framework for the comparative analysis of FTIR spectra by modeling experimental spectra as elements of a Hilbert space. The proposed framework employs frame representations to obtain stable and redundant decompositions of spectral signals, enabling exact reconstruction and systematic comparison through frame coefficient sequences. A finite-dimensional implementation is developed for discretely sampled FTIR data, making the methodology directly applicable to experimental measurements. The proposed approach is illustrated using experimentally measured FTIR spectra of 1-(pyridyl-2-azo)-2-naphthol (PAN) before and after its interaction with a mild steel surface in acidic medium. The framework provides a mathematically rigorous basis for spectral representation, reconstruction, and quantitative comparison while complementing conventional peak-based interpretation. The proposed methodology demonstrates the potential of frame theory as an effective mathematical tool for FTIR spectral analysis and offers a new direction for integrating functional analysis with experimental spectroscopy.

Keywords: FTIR spectroscopy, frame theory, Hilbert spaces, frame coefficients, spectral reconstruction, comparative spectral analysis.

Singular Integro-Differential Equations with Fixed Singularities and their Applications

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A singular integro-differential equation with fixed singularities, a variable coefficient, and prescribed a certain condition is studied. A theorem describing the asymptotic behavior of the solution, provided that it exists, is established. Several choices of the equation coefficient are considered, and, by exploiting the properties of orthogonal polynomials, the equation is reduced to an infinite system of linear algebraic equations. Using the properties of the Mellin integral transform and the Gamma function, the quasi-complete regularity of the resulting systems is proved, thereby justifying a reduction method for constructing approximate solutions of the integro-differential equation. In several cases of applied importance, both exact (closed-form) and approximate solutions of the corresponding problem are obtained.

About the Second Recurrent form of Representation for Parameter Estimate by the Method of Least Squares of Relative Errors Under Non-Classical Assumptions for Systems in the Presence of Uncertainties

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In most cases, when estimating the parameters of mathematical models of objects, the main attention is focused on minimizing some norm of the vector of absolute model errors. But in practice, there is also a need to use the vector of relative errors of the mathematical model instead of the vector of absolute errors. This is especially relevant for areas such as economics, finance, physics, chemistry, environmental and Earth sciences, etc.

In the latter situation, when solving the problem of estimating model parameters, you can use the method of least squares of relative errors. For a linear regression model, the problem of estimating unknown parameters by the least squares method of relative errors in the classical case, when this estimate is unique, has already been solved earlier. This study considers the problem of estimating unknown parameters of a linear regression model, but in the non-classical case, when the estimate may not be unique.

The aim of the study was, based on own results on explicit and recurrent forms of representation for parameter estimate by the method of least squares of relative errors for the linear regression model, to obtain for this estimate a new second recurrent form of representation for the above-mentioned model in the non-classical case.

Consider the above-mentioned estimation problem of unknown parameters α for a linear regression model

$$y(k) = x^T(k)\,\alpha + e(k), \quad k = \overline{1, N}, \quad N \in \mathbb{N},$$

where $y(k)$ – scalar observation of the dependent variable, $e(k)$ – model error, $x(k)$ – known regressor vector, $\alpha \in \mathbb{R}^p$.

Let's assume that $y(k) \neq 0$, $k \in \mathbb{N}$. Then the set of all estimates by the method of least squares of relative errors for this regression model under non-classical assumption when this estimate may be not unique is defined as $\operatorname{Arg}\min\limits_{\alpha} Q(\alpha, N)$, where $Q(\alpha, N) = \sum\limits_{k=1}^{N} (\frac{e(k)}{y(k)})^2$.

An explicit form of representation for the desired set of these optimal parameter estimates can only be written using the Moore–Penrose matrix pseudo-inversion operator. This fact significantly complicates the computation of the desired estimate.

The way out of this situation is to use a recurrent form of representation for the desired optimal estimate, which no longer requires the use of Moore–Penrose pseudo-inversion of matrices. The parameter estimate with the least norm $\widehat{\alpha}(N)$ can be used as a unique estimate on the entire set of optimal estimates. For this estimate, the author has already proposed the first recurrent form of representation.

For parameter estimate $\widehat{\alpha}(N)$ by the method of least squares of relative errors, a second recurrent form of representation is proposed, which no longer requires the use of either the Moore–Penrose pseudo-inversion of matrices or even the usual inversion of matrices. This algorithm is also supplemented with a recurrent form of representation for the corresponding weighted residual sum of squares $q(N) = Q(\widehat{\alpha}(N), N)$.

2-Grupoid Actions and Liftings of Crossed Modules Over Groupoids

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This paper investigates the higher-dimensional categorical relationship between the algebraic actions of strict 2-groupoids and the lifting structures of crossed modules over groupoids. Within this framework, we formalize how a strict 2-groupoid action corresponds systematically to a lifting structure of the associated crossed module. Furthermore, we develop a quotient theory via groupoids and analyze covering frameworks in this higher setting. By utilizing these covering and quotient structures, we explicitly construct new and illustrative examples of strict 2-groupoid actions. To ensure structural coherence, all constructions are developed within an identity-on-objects (object-preserving) setting, where the underlying functors and morphisms act strictly as the identity on the object class. Finally, we establish a natural equivalence between the category of strict 2-groupoid actions and the category of liftings of crossed modules over groupoids, providing a unified algebraic pipeline for higher-dimensional lifting theory.

Acknowledgments

The work was supported by Recep Tayyip Erdogan University under Project # FBA-2025-2210 and by Recep Tayyip Erdogan University Development Foundation.

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Analyticity of the Djerbashyan Canonical Product on the Boundary of the Unit Circle

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Djerbashyan canonical product (Djerbashyan M. M., 1945) has a form

$$B_p(z,(a_n)) = z^{\lambda}\prod_{n=1}^{\infty}\Big(1-\frac{1-|a_n|^2}{1-\overline{a_n}z}\Big)\exp\Big(\sum_{k=1}^{p}\frac{1}{k}\Big(\frac{1-|a_n|^2}{1-\overline{a_n}z}\Big)^k\Big),$$

where $\lambda+1$ and p are natural numbers,

$$0<|a_n|\le|a_{n+1}|<1,\ \ \lim_{n\to\infty}|a_n|=1,\ \ |z|<1,$$
$$\sum_{n=1}^{+\infty}(1-|a_n|)^p<+\infty.$$

Let $A=\{a_1,a_2,\dots,a_n,\dots\}$ and denote by A' the set of its accumulation points.

Theorem *If $A=\{a_1,a_2,\dots,a_n,\dots\}$ and $e^{i\theta}\notin A'$, then there exists a neighborhood of the point $e^{i\theta}$ in which the Djerbashyan canonical product converges absolutely and uniformly, and hence it is an analytic function in this neighborhood.*

Result 1 *The point $z=e^{i\theta}$ is a singular point of $B_p(z,(a_n))$ if and only if $e^{i\theta}\in A'$.*

Result 2 *If $B_p\big(z,(a_n)\big)$ is analytic at the point $z=e^{i\theta}$, then there exists a neighborhood of $e^{i\theta}$ in which $B_p(z,(a_n))$ is absolutely and uniformly convergent.*

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Using τSR Logic Methods in Programming Languages

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It is known that computer technology is used in almost all areas of human activity. It would be good to use the same method for creating a general theory of programming that was used for creating notation theory [1].

The report reviews the τSR logic developed based on Shalva Pkhakadze's notation theory [1], in which Shalva Pkhakadze's Sx and Rx insertion or replacement meta-operators are considered the main operators of the theory. In addition, the report focuses on the use of Shalva Pkhakadze's sufficiently general mathematical language and methods of notation theory [2] in programming languages.

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R-Functions and 3-Valued Łukasiewicz Logic with Constants

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This paper studies bipartite graphs as a natural representation of formal contexts and the concept lattices that arise from them within Formal Concept Analysis (FCA). In this paper we recall the basic apparatus of FCA, discuss the structural properties of concept lattices with a particularly simple, flat partial order, and consider practical applications of such lattices, including data analysis, clustering, and the identification of relations between objects and attributes. Using a concrete bipartite graph, we build the corresponding formal context table, extract the formal concepts, and construct the resulting star-shaped, modular concept lattice. The results are illustrated with examples from accesscontrol systems, *e*-commerce, and security monitoring. The paper is intended for students, graduate students, and researchers working on graph theory, formal concept analysis, and data-mining methods.

Keywords: Bipartite graphs, formal concept analysis, formal context, formal concepts, concept lattice, modular lattice.

Some New Sharp Boundedness Result Concerning Linear functionals on Banach Spaces

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The convergence and summability of Fourier series with respect to classical orthonormal systems (ONS) – such as the trigonometric, Haar, and Walsh systems – are well-established in mathematical analysis, particularly for differentiable functions. Due to the favorable localization and structural properties of these classical systems, regular functions yield well-behaved Fourier series. However, for general ONS, the behavior of Fourier expansions is markedly different and highly irregular. As demonstrated by a classical result of Banach [2], Fourier series with respect to certain ONS may fail to be summable even for constant functions.

In this paper, we bridge this gap by investigating the Cesàro summability of order $\alpha > 0$ of Fourier series with respect to general ONS for the class of functions C_L, defined as functions whose derivatives belong to the Lipschitz class Lip_1. Treating Lip_1 as a Banach space, we analyze a sequence of linear functionals $B_n(f,x)$ and introduce an auxiliary sequence of functions $H_n(x)$ constructed from the kernel functions $Q_n(u,x)$.

The primary contribution of this work is the establishment of precise, pointwise boundedness criteria for the Cesàro sums $\sigma_n^{\alpha}(x,f)$ on a subset of full measure $G \subset [0,1]$. We prove that the condition $H_n(x) = O(1)$ is sufficient to guarantee the boundedness of Cesàro sums at any point $x \in G$ for all functions in C_L. Crucially, we show that this result is sharp: if $\limsup_{n\to\infty} |H_n(x_0)| = +\infty$ at some point $x_0 \in G$, there exists a function whose Cesàro sums diverge at that point. Finally, we present an existence theorem proving the existence of an ONS and a function $g \in C_L$ for which the corresponding Cesàro sums diverge almost everywhere on $[0,1]$. These results are shown to be the best possible in a special sense, offering new insights into the summability of smooth functions under general orthonormal expansions.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia under grant # YS-25-169.

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Theoretical and Practical Aspects of the Unified Mathematics Curriculum

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There was a period in Georgian reality when algebra and geometry were taught as separate subjects in school; textbooks were also separate, and, accordingly, students were assessed with two marks. However, the unity of algebra and geometry has a long history, and after Descartes and Fermat, when they introduced the rectangular coordinate system, algebra and geometry were actually united, and thus a new, powerful analytical method of research was created. Thus, it is impossible to bypass this unity in the educational process and it is necessary to refine, modernize, and search for new methods in this direction in the educational process. In addition to the elements of algebra and geometry, the mathematics curriculum in Georgia also combines elements of probability and statistics. It is also noteworthy that this direction should also be integrated with others and taught with a unified vision.

This unity should be familiar to the student from the elementary grades: division into proportional parts, teaching fractions with geometric representations. Historical context: geometric method of solving quadratic equations.

It is important to consider optimization problems: determining the maximum area with a minimum perimeter; choosing the dimensions of spatial figures so that the volume is maximum and the surface area is minimum (i.e., the material consumption is minimal). Proving the properties of geometric figures using coordinates; connecting the equation of a line with similarity problems, connecting graphs with motion problems. Geometric probability and many other issues directly relate to this thesis.

In general, the modern approach to understanding the world is based on integrated learning. How much more important is it not only to find connections between objects, but also to find intra-subject connections – to consider solving the same problem from different perspectives. This develops the ability to use knowledge in a combined (complex) way, which is important for the practical application of knowledge and skills acquired at school.

Approximation Algorithm for One Nonlinear Integro-Differential Equation

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The variational and difference methods are used respectively for spatial and time variables to solve a nonlinear integro-differential Timoshenko dynamic beam equation, with the initial-boundary conditions. The resulting algebraic system of cubic equations is solved by the iterative method. The iteration process error is estimated. Galerkin method, Krank–Nicolson difference scheme, Jacobi iteration method, Cardano formula, was used.

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Infomorphisms in Lattice-Theoretic Contexts

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Infomorphisms give the standard contravariant way to compare classifications and formal contexts: an object map and an attribute map are coupled by one incidence-preservation law [1]. We extend this idea to lattice-theoretic contexts of Yao–Han–Wang [2], where object and attribute domains are complete lattices and every context carries a Galois connection. The constructive problem is to decide when a meaningful map on one side canonically determines the missing contravariant component, and then to read the resulting transport on concept lattices.

The main construction is expressed by two complementary characterization theorems. The first starts with an object-side map and constructs the attribute map by residuation; the second starts with an attribute-side map and constructs the object map dually. These results turn closure-continuity, residual bounds, and closure-monotonicity into explicit formulas for infomorphisms. In particular, closure-continuous join-homomorphisms extend to canonical infomorphisms, and every such infomorphism induces an adjunction between the corresponding concept lattices.

Theorem 1 *For lattice-theoretic contexts $(\mathscr{X}, \mathscr{U}, I)$ and $(\mathscr{Y}, \mathscr{V}, J)$ and a map $f : \mathscr{X} \to \mathscr{Y}$, an infomorphism (f, g) exists if and only if f satisfies extent-closure continuity, the residual bound, and target-closure monotonicity. The canonical map is $g = F_I f^* G_J$.*

Theorem 2 *Dually, for a map $g : \mathscr{V} \to \mathscr{U}$, an infomorphism (f, g) exists exactly when g satisfies the corresponding attribute-closure conditions. The canonical map is $f = G_J g^* F_I$.*

The motivating examples come from sparse surrogate activations of large language models [3–5]. A transcoder or sparse autoencoder assigns sparse latent vectors to token positions; coordinatewise joins aggregate them into sequence summaries. In the resulting context, objects are token or sequence sets and attributes are activation constraints: $F(A)$ extracts common sparse features, while $G(u)$ retrieves summaries dominating u. Minimal-nonzero truncation is a fixed-meet contraction $m(u) = u \wedge \mu$, hence the identity on objects extends to an infomorphism from the original sparse-surrogate context to its truncation. Choosing atoms $\mathscr{A}$ gives an atomic contraction: $\downarrow m(u) \cap \mathscr{A}$ records the selected binary features retained by u and yields an infomorphism to $(\mathscr{X}, \mathscr{P}(\mathscr{A}), R_{\mathscr{A}})$. For product attribute lattices, one atom per coordinate identifies this powerset with $\mathscr{P}(D)$, giving a controlled passage from a lattice-theoretic formal context to an ordinary binary formal context. Token-based concept lattices use Theorem 2 in the dual direction, because token intents generate activation intents.

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Dynamic Metahypergraphs: A New Mathematical Model for Reconfigurable Systems Based on Multifunctional Elements

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The talk presents a method we developed for describing the state space of complex reconfigurable systems composed of multifunctional elements, as well as the dynamic processes of partial failure and functional recovery of such elements, using a new type of metahypergraph.

The state space of a multifunctional element is described by a bipartite hypergraph, in which the vertex representing the element is connected by edges to the vertices representing the functions assigned to it. The loss of an element's ability to perform a given function is represented by the removal of the edge connecting the element and that function. As long as at least one edge connecting the element to its functions remains, the element is considered to be in an operable state. The dynamic transition of an element from one state to another can be described using a Markov model (Markov chain), employing either the probabilities of the element performing individual functions or the transition probabilities between states, enabling advance prediction of degradation processes at different points in time.

The graph representing the functional capabilities of a reconfigurable system composed of multifunctional elements is likewise a bipartite hypergraph, where element vertices connect to the function vertices they are capable of performing. Reconfigurable systems are characterized by the ability to perform an assigned function through multiple pathways. These pathways and the connections among them can be represented by a metahypergraph graph whose vertices are themselves bipartite hypergraphs, with edges connecting these hypergraphs representing the dynamic processes of switching from one operating pathway to another.

By using the incidence matrices of the bipartite hypergraphs and the metahypergraph, we can identify the operable states from the full set of possible system states and assess the system's reliability and fault tolerance. Furthermore, by assigning weights (probabilistic estimates for each pathway) to the edges of such a graph, we can achieve optimal control of the system's reconfiguration and recovery processes. The result is a flexible, adaptive, highly reliable reconfigurable system with strong fault tolerance against degradation processes.

Acknowledgments

The work was supported by the Set of States Shota Rustaveli National Science Foundation of Georgia, Grant # SC-24-753.

The Conducting Liquid Flow Between Porous Walls with Heat Transfer

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Has been studied the flow of viscous electroconductive fluid between porous walls with heat transfer, when in perpendicular to walls is applied external homogeneous magnetic field. The fluid flow is caused by pulsation fall and pulsation motion of porous walls.

The physical characteristics of fluid flow are found.

Extreme Value Domains of Iterated Equilibrium Distributions and their Connection to Stop-Loss Moments

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Let X be a non-negative random variable with distribution function F and tail $\overline{F}(x) = 1-F(x)$. If $\mu_1 = \mathbb{E}[X] < \infty$, the equilibrium distribution associated with F is defined by

$$\overline{F}_e(x) = \frac{1}{\mu_1} \int_x^{x_F} \overline{F}(u)\, du,$$

where x_F denotes the right endpoint of F. Iterating this transformation yields the sequence $F^{(1)} := F$ and $F^{(s+1)} := (F^{(s)})_e$, $s \geq 1$. These distributions arise naturally in renewal theory, reliability, and actuarial mathematics, and they provide a convenient framework for describing higher-order residual lifetime behaviour. In this work we study iterated equilibrium distributions from the viewpoint of Extreme Value Theory. Our main objective is to understand how the three classical maximum domains of attraction – Fréchet, Weibull and Gumbel – behave under repeated equilibrium transforms. We show that the extreme-value type is preserved in all three cases, although the associated tail parameter changes in a systematic way. In the Fréchet case, the tail index decreases linearly with the iteration order; in the Weibull case, the right endpoint is preserved while the endpoint index increases linearly; and in the Gumbel case, the same auxiliary function continues to govern the asymptotic scale. Thus, the equilibrium transform does not destroy the underlying extreme-value structure, but modifies it in a precise and explicit manner.

A second goal of the paper is to connect these tail results with actuarial risk functionals. For this purpose we use the identity

$$\overline{F}^{(s)}(x) = \frac{1}{\mathbb{E}[X^{s-1}]}\, \mathbb{E}\big[(X-x)_+^{s-1}\big],$$

which links the tail of the s-th iterated equilibrium distribution to the $(s-1)$-th order stop-loss moment. This makes it possible to transfer the extreme-value asymptotics of iterated equilibrium distributions directly to asymptotic formulas for high-layer stop-loss premiums and related layer-loss moments. As a consequence, we obtain polynomial asymptotics in the Fréchet case, polynomial decay near the finite endpoint in the Weibull case, and auxiliary-function-driven asymptotics in the Gumbel case. These results provide a unified connection between equilibrium transformations, maximum domains of attraction and higher-order stop-loss functionals. In particular, they show that the asymptotic behaviour of actuarially meaningful quantities can be read directly from the extreme-value structure of the underlying claim size distribution.

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On the Extension and Realization of the (P)-(Q) Apparatus for Some Differential Equations

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In the works of T. Vashakmadze "The theory of anisotropic elastic plates" (Kluwer Academic Publishers, 1999), a field adjacent to the theory of differential equations and numerical analysis is presented (for brevity, the (P)-(Q) apparatus), a brief description of which will be given in the introduction of the report. The main part is devoted to the extension and application of this apparatus to some boundary value problems in the case of ordinary and partial differential equations.

1. Boundary value problems are considered for a typical second-order nonlinear ordinary differential equation describing the optimal control theories of R. Bellman and L. Pontryagin, to which the (P)-(Q) apparatus is extended. In particular, the criterion for the existence and uniqueness of the solution is refined, and numerical results compared with the exact solution are presented for test problems.

2. The corresponding boundary value problem for the basic, ordinary nonlinear second-order differential equation of structural mechanics is studied. Unlike classical approaches, no approximation of the nonlinear part or simplification of the model by any physical or geometric assumptions is made. An algorithm and a program, along with a numerical table, are provided. For illustration purposes, a comparative analysis of the obtained methodology is conducted against some modern works.

3. Three-dimensional nonlinear boundary value problems of elasticity theory are considered, which are reduced to the corresponding boundary value problems of the so-called refined theories and hierarchical (Vekua-type) two-dimensional systems of partial differential equations. In the report, a continuous analog of the Peaceman–Rachford alternating direction implicit (ADI) method with respect to the (P)-(Q) apparatus is developed for the latter.

4. The issue of the approximate solution of a second-order stochastic elliptic partial differential equation with Dirichlet boundary conditions in a rectangle (presented by I. Babuška, who used the "p-h finite element method") is considered. While reviewing this work, we noticed that the method is effective when the number of finite elements is relatively small and the coefficients satisfy sufficiently rigid requirements. If we use the continuous analog of the ADI, then for the problem, the process converges under relatively less rigid requirements on the coefficients.

Acknowledgments

This work was performed together with my young colleagues G. Buzhghulashvili and N. Jorbenadze, without whom the functioning of the corresponding algorithms and obtaining numerical results would be unimaginable.

Analogy and its Use in the Educational Process

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Analogy is one of the methods of mathematical research. As a rule, it is considered together with generalization and specialization. In the methodological literature, its essence is formulated as follows: from the similarity of two objects in some respect, we conclude their similarity in another respect. The first example of a mathematical discovery by analogy belongs to Euler. Using this method, he expressed a conjecture about the sum of a series composed of the reciprocals of the squares of natural numbers. He subsequently rigorously proved the result obtained. Analogy, as well as other methods of mathematical research, is used in the process of teaching mathematics, the work provides examples of the use of discussing discoveries using analogy, among them, at the elementary level.

A Comprehensive Study of Fourier Series for the Generalized I-Function

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This paper presents a comprehensive study on the Fourier series expansions of the generalized I-function of several complex variables. By evaluating two integrals involving the I-function, we derive the corresponding Fourier sine and cosine series representations. These results generalize and extend existing work on the H-function and I–function, incorporating earlier findings by N. Bhati and R. K. Gupta as particular cases. The analytical approach leverages Mellin–Barnes contour integrals and orthogonality properties of trigonometric functions to establish convergence and derive explicit series expressions. The outcomes of this study are significant in the broader application of special functions in mathematical analysis and theoretical physics.

Keywords: I-function, Fourier series, H-function, Mellin–Barnes integral, generalized special functions, multiple complex variables, sine series, cosine series.

On the Concept of Countable Error Measurement Theory

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In a Banach space, the error of approximation is measured by a single quantity, namely the norm of the difference between an element and its approximation. In a Fréchet space generated by a countable family of seminorms $\{p_n\}_{n=1}^{\infty}$, approximation naturally produces a countable family of errors $p_n(x-y)$. Consequently, one seeks a unified error measure that faithfully represents this countable family. Since every Fréchet space is metrizable, one may measure approximation errors by means of a metric generating the topology of the space. Classical metrics due to Mazur, Birkhoff, Kakutani, Albinus and others were introduced primarily to describe the topology of a Fréchet space by a metric.

The purpose of Countable Error Measurement Theory is different. It seeks to replace a countable family of seminorm errors by a single metrically generated error measure while preserving approximation-theoretic and geometric properties of the underlying space. In particular, the metric should satisfy the following requirements:

(i) best approximations with respect to the metric coincide with best approximations generated by an appropriate seminorm;

(ii) the same equivalence holds for ε-approximations;

(iii) the geometric structure is preserved in the sense that metric balls are obtained from the unit ball of a generating seminorm by homothetic scaling, similarly to the Banach-space case;

(iv) when the construction is applied to a Banach space, the resulting quasinorm coincides with the original norm. Among the known metrics on Fréchet spaces, only the metric constructed in [1] (see also [2, Section 2.5]) possesses all of the above properties.

The process of replacing a countable sequence of seminorm errors by a single metrically generated error measure while preserving approximation and geometric structures is called Countable Error Measurement Theory in Fréchet Spaces.

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Applying a Grammatical, Set-Theoretic and Circuit Framework to LLM Logical Validation: Detecting Logical Inconsistencies in Generated Text

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Large Language Models (LLMs) can generate fluent and grammatically coherent text, yet they may still produce conclusions that are logically inconsistent. This problem is especially visible in statements involving conjunction, inclusive and exclusive disjunction, implication, and negation. This paper applies a previously developed grammatical–set-theoretic–circuit framework to the problem of LLM logical validation. The framework establishes correspondences between natural-language connectives, formal set-theoretic representations, and 3-input digital logic circuits.

Building on earlier work on the grammatical interpretation of logical operations in the Georgian language, the paper focuses on statements involving three propositions and their readings through constructions such as "and," "or," "either ... or," and "if ... then." The central argument is that such linguistic constructions can be mapped onto deterministic logical forms and corresponding digital circuits. This mapping makes it possible to compare an LLM-generated conclusion with the output of a formally defined truth-functional circuit.

The paper proposes this framework as a verification layer for detecting connective-based logical inconsistencies in generated text. It does not claim to verify factual truth or eliminate all forms of hallucination. Rather, it addresses a specific class of logical hallucinations that arise when an LLM mishandles the structure of logical connectives, conditional statements, or negations. By connecting linguistic interpretation, set-theoretic notation, and circuit-based validation, the paper contributes to discrete mathematics, computational linguistics, digital electronics, and neuro-symbolic approaches to AI reasoning. Keywords: Large Language Models; logical validation; linguistic connectives; Georgian language; set-theoretic interpretation; digital logic circuits; neuro-symbolic AI; logical hallucination.

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Strongly Best Approximation and Moore–Penrose Generalized Solution

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Let the continuous injective linear operator $A : E \to F$ of a Fréchet–Hilbert space E into Fréchet–Hilbert space F with closed range $A(E) = G$ be considered. Let the topology of the space F is generated with the non-decreasing sequence of hilbertian norms $\|x\|_n$ and with metric [1, p. 167].

The subspace $A(E) = G$ is strongly proximal, if for each $f \in F$ there exist strongly best approximation element g_0 of f in $A(E) = G$ ([1], p. 167).

Since the operator A is injective, then it is monomorphism of a Fréchet–Hilbert space E into Fréchet–Hilbert space F and by equality $A_n x = Ax$ the continuous injective operator

$$A_n : (E, \|\cdot\|_n) \to (F, \|\cdot\|_n)$$

from pre-Hilbert space $(E, \|\cdot\|_n)$ into pre-Hilbert space $(F, \|\cdot\|_n)$ is defined for any $n \in \mathbb{N}$.

In this case is valid

Theorem *G is strongly proximal in F iff for equation $A_n u = f$, $f \in F$, $d(f, G) = r \in I_n = [2^{-n+1}, 2^{-n+2}[$ $(n \in \mathbb{N})$, there exist generalized solution in the sense of Moore–Penrose.*

The obtained results are used for the Radon operator $\mathfrak{R} : S(\mathbb{R}^p) \to S(S^{p-1} \times \mathbb{R})$, where $S(\mathbb{R}^p)$ is the Schwartz's Fréchet space of rapidly decreasing functions on $\mathbb{R}^p$, S^{p-1} is unit ball in $\mathbb{R}^p$.

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On the Relationship Between the Offset and the Coupled Fractional Fourier Transforms: Continuous and Discrete Models

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The fractional Fourier transform, which was introduced by V. Namias in 1980 [1], is a generalization of the Fourier transform. The transform did not receive much attention until the early 1990s when it was discovered to have numerous applications in optics and time-frequency representations.

The fractional Fourier transform, denoted by F_θ, depends on a parameter $0 \leq \theta \leq 2\pi$, so that when $\theta = 0$, F_0 is the identity transformation and when $\theta = \pi/2$, $F_{\pi/2}$ is the standard Fourier transform. In addition, $F_\alpha F_\beta = F_{\alpha+\beta}$.

This transform has been generalized in a number of different ways. For example, by introducing a space-shifted and a frequency-modulated terms into the fractional Fourier transform, S. Pei and J. Ding obtained a transform called *the offset-fractional Fourier transform* [2]; see also [3]. The fractional Fourier transform was also extended to higher dimensions using tensor products of one-dimensional transforms. In [4, 5], a two-dimensional fractional Fourier transform that is not a tensor product of two one-dimensional fractional Fourier transforms, called the *coupled fractional Fourier transform* was introduced.

In this talk, we will discuss different generalizations of the fractional Fourier transform and the relationships between them from continuous and discrete points of view.

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On Finitely Generated Quasivarieties

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The main result is the determination of an equational basis for the variety of lattices generated by a finite non-modular and non-semidistributive lattice. It is established that the resulting quasivariety is a variety defined by three identities. Structural constraints are determined for minimal non-trivial covers of join-irreducible elements in lattices satisfying the weak modularity identity, and the possible closures of individual join-irreducible elements are described.

Keywords: Variety, finite lattice, quasivariety, non-modular lattice, non-semidistributive lattice.

2020 Mathematics Subject Classification: 06B10, 06B20, 06B05.

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Mini-Symposium:
Algebraic Geometry and Model Theory of Groups and Rings

Abstracts of Talks

Equivalences of Algebras in Universal Algebraic Geometry

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Investigations in universal algebraic geometry give rise to various types of equivalences of algebras related to the geometry of algebras under consideration. The main idea of this approach is *to compare the abilities of algebras with respect to solving systems of equations* (see [3–5]).

This point of view leads to the various types of equivalences of algebras: *geometric equivalence, geometrically automorphic equivalence, geometric similarity* and *geometric compatibility* (see [5]). In this talk we will discuss some results concerning the first two types of equivalences [1, 2].

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On the R-Structure of Tensor R-Completions of Torsion-Free Nilpotent Groups

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In this report we study tensor completions $G \otimes_{\mathscr{N}_{2,R}} R$ of finitely generated torsion-free nilpotent groups G of class 2 in the quasivariety $\mathscr{N}_{2,R}$ of R-exponential 2-nilpotent groups over a binomial integral domain R. We show that the classical Hall completion $G \otimes_{\mathscr{H}} R$ embeds as an abstract group (the embedding is not an R-homomorphism) into $G \otimes_{\mathscr{N}_{2,R}} R$ such that $G \otimes_{\mathscr{N}_{2,R}} R \cong (G \otimes_{\mathscr{H}} R) \otimes D$, where D is an R-module and the direct product is a product of abstract groups (not R-groups!). In particular, the canonical R-epimorphism $\mu : G \otimes_{\mathscr{N}_{2,R}} R \to G \otimes_{\mathscr{H}} R$ is a retract on $G \otimes_{\mathscr{H}} R$ with abelian kernel D. Moreover, in addition to the algebraic structure, we describe precisely how raising to an R-power works in the group $G \otimes_{\mathscr{N}_{2,R}} R$. To do this, we introduce a new type of commutators, the so-called c-commutators, which are interesting in their own right. These results answer an old question of Remeslennikov about the algebraic structure of free 2-nilpotent R-group in the quasivariety $\mathscr{N}_{2,R}$. Indeed, it was shown in [1] that if G is a free 2-nilpotent group with base X (in the variety of abstract 2-nilpotent groups), then $G \otimes_{\mathscr{N}_{2,R}} R$ is a free 2-nilpotent R-group in $\mathscr{N}_{2,R}$ with base X. Note that in this case $G \otimes_{\mathscr{H}} R$ is a free 2-nilpotent Hall R-group with base X. As an illustration, for a free 2-nilpotent group G of rank 2, we describe the group $G \otimes_{\mathscr{N}_{2,R}} R$, the action of R or $G \otimes_{\mathscr{H}} R$, and the module D in the case where R is either the polynomial ring $\mathbb{Q}[t]$ on the field of rational functions $\mathbb{Q}(t)$ with coefficients in the field of rational numbers $\mathbb{Q}$ [2].

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Lattice Isomorphisms of Nilpotent Lie Rings

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The study of lattice isomorphisms in group theory has been the subject of a vast literature. Significant results have been obtained in this area. It is known that any lattice isomorphism of a locally nilpotent torsion-free group is induced by a group isomorphism. This problem was solved by L. E. Sadovsky [1]. For W-power groups lattice isomorfisms was study in [2, 3].

In the presented paper, we have studied the lattice isomorphisms of Lie rings and Lie algebras.

Proposition *If a lattice isomorphism φ of a pure non-Abelian algebra L is induced by an isomorphism, then there is exactly one.*

Theorem 1 *If L and L^{φ} are pure non-Abelian locally nilpotent Lie rings, then every lattice isomorphism φ between them is induced by exactly one isomorphism.*

Using Baer's theorem [4], we obtain the following theorem.

Theorem 2 *Every lattice isomorphism φ of a pure finitely generated nilpotent Lie ring L onto L^{φ} is induced by two one-to-one correspondences φ_1 and $\varphi_2 = -\varphi_1$, which are isomorphisms on every pair of Abelian subrings $A \subset L$ and $A^{\varphi} \subset L^{\varphi}$.*

Acknowledgments

The first author was supported by the Shota Rustaveli National Foundation of Georgia, project # FR-25-7138.

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Centroids of Partially Commutative Two-Step Nilpotent Groups

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Let us fix an arbitrary associative ring with identity A and a group G. Let A have an action on G, that is, a mapping $G \times A \to G$. The result of the action of an element $a \in A$ on $g \in G$ will be denoted by g^a. Let e denote the unit of the group G.

Definition ([5]) A group G is called an *A-exponential group* (or an *A-group*) if the action of A on G satisfies the following axioms:

(1) $\forall\, a \in A\ \forall\, g \in G\ g^1 = g,\ g^0 = e,\ e^a = e$;

(2) $\forall\, a, b \in A\ \forall\, g \in G\ g^{a+b} = g^a g^b,\ g^{ab} = (g^a)^b$;

(3) $\forall\, a \in A\ \forall\, g, h \in G\ (h^{-1}gh)^a = h^{-1}g^a h$;

(4) $\forall\, a \in A\ \forall\, g, h \in G\ [g, h] = e \to (gh)^a = g^a h^a$.

The concept of an *exponential group* goes back to the idea of extending exponents in groups, which was originally arrived at by Ph. Hall, A. I. Mal'tsev, and R. Lyndon. Ph. Hall [2] began to consider exponents in a *binomial domain* for nilpotent groups. A. I. Mal'tsev used the completion of the ring $\mathbb{Z}$ to $\mathbb{Q}$ to obtain the solvability of the equations $x^n = g$ for all $n \in \mathbb{Z}$. R. Lyndon [4] defined the *R-exponential groups* as groups satisfying axioms (1)–(3) from the Definition given above. A. G. Myasnikov and V. N. Remeslennikov [5] refined the concept of an exponential group by introducing an additional axiom (axiom (4) from the Definition). Subsequently, such groups were called *MR*-groups.

In [3] S. Lioutikov and A. Myasnikov introduced the concept of *centroid of a group G* as the set of all normal quasi-endomorphisms acting on G. In the same paper it was shown that the centroid of G is an associative ring with identity, and that it is a maximal ring of scalars whose action on G satisfies all axioms of exponential groups [5]. In [3], the centroids of finitely generated free nilpotent R-exponential groups and groups of unitriangular matrices $UT_n(R)$, where R is a binomial domain, were described. Using the description obtained, S. Lioutikov and A. Myasnikov proved the rigidity of such groups and, thus, positively solved the problem of F. Grunewald and D. Segal from [1].

The main goal of our work is to study the rigidity of finitely generated partially commutative (or graph) two-step nilpotent R-exponential Hall groups, where R is the binomial domain. To do this, we calculate the centroid of such groups and use the idea described in [3] for free nilpotent groups.

Acknowledgments

The research was funded in accordance with the state task of the IM SB RAS, project # FWNF-2026-0033.

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Sha-Rigidity of Chevalley Groups Over Rings

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A group is called Sha-rigid if every locally inner endomorphism of the group is in fact an inner automorphism. Thus, the question is whether conjugations that exist locally can always be represented by a single global conjugation. In this talk I will discuss Sha-rigidity for Chevalley groups over rings. I will present results establishing this property for several classes of Chevalley groups and explain the main ideas behind the proofs.

Theory of Interpretations and Geometry of Constructions with a Compass and Straight Edge

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Interpretations from algebraic structures of one nature to algebraic structures of another emerged in the mid-20th century in the works of Tarski and Maltsev as a tool for proving the undecidability of theories. Nowadays, this tool is being refined and imbued with new expressive power (see [1–3]). As is well known, the undecidability of classical compass-and-straightedge construction problems reduces to algebraic problems over fields of rational and real numbers. In this talk, we will discuss whether a general model-theoretic mechanism for interpretations lies behind these transitions and explore the implications that can be drawn from them.

We will consider two algebraic structures. The first is the set of real numbers $\mathbb{R}$ in the standard ring signature $\{+, -, \cdot, 0, 1, <\}$ with order. The second is the set of all points of the real plane $\mathbb{R}^2$ in the signature $\{CC^{(5)}, LL^{(5)}, CL^{(5)}\}$, consisting of three predicate symbols of the following meaning:

1. $\mathbb{R}^2 \models CC(x_1, x_2, x_3, x_4, x_5)$ if and only if x_1 is an intersection point of the circle centered at x_2 and passing through x_3 and the circle centered at x_4 and passing through x_5;
2. $\mathbb{R}^2 \models LL(x_1, x_2, x_3, x_4, x_5)$ if and only if x_1 is an intersection point of the straight line passing through x_2, x_3 and the straight line passing through x_4, x_5;
3. $\mathbb{R}^2 \models CL(x_1, x_2, x_3, x_4, x_5)$ if and only if x_1 is an intersection point of the circle centered at x_2 and passing through x_3 and and straight line passing through x_4, x_5.

In these notations, the following theorem is valid, which is of a model-theoretical nature.

Theorem *Algebraic structures $\mathbb{R}$ and $\mathbb{R}^2$ are strongly regularly injectively bi-interpretable in each other.*

The theorem gives the opportunity to describe models of elementary theory of compass-and-ruler construction geometry $\mathbb{R}^2$ through non-standard models of real numbers $\mathbb{R}$, which are well described. Let us list other consequences that follow from the theorem above.

1. The elementary theory of $\mathbb{R}^2$ is decidable.
2. The algebraic structure $\mathbb{R}^2$ has elimination of imaginaries.
3. The algebraic structure $\mathbb{R}^2$ is not saturated, is not definable by types, is not prime, is not rich.
4. The algebraic structure $\mathbb{R}^2$ is stable.

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Riemann–Roch Bases in Kummer Extensions II

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This talk gives me the occasion to discuss about Mumford representation of divisors

$$\Delta = n_1 P_1 + \cdots + n_t P_t - (n_1 + \cdots + n_t)\Omega, \quad P_k = (x_k, y_k),$$

of degree zero on a hyperelliptic curves. Thereafter, I will present some of the technical details underlying the given plenary talk with the same title. After stressing that, by Riemann–Roch, for $m \geq 2g-1$, it holds $\dim(\mathscr{L}(\Delta + m\Omega)) = m - g + 1$, we note that in the table

m	0	$\cdots$	$\cdots$	$2g-1$	$2g$	$2g+1$	$\cdots$
dim	0 or 1	$\cdots$	$\cdots$	g	$g+1$	$g+2$	$\cdots$

we have to fill $2g-1$ cells with g values from 0 to $g-1$, and we will see what strategies are available.

Gelfand–Graev Problem for Infinite-Dimensional Representations of Lie Algebras

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We will discuss the state of the Gelfand–Graev continuation problem for Gelfand–Tsetlin modules for classical Lie algebras.

On the First-Order Genus of Wreath Products and their Central Extensions

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We prove that groups of the form $\mathbb{Z}^m \wr \mathbb{Z}^n$, where $m, n \in \mathbb{N}$, are regularly bi-interpretable with $\mathbb{Z}$ and therefore are first-order rigid: every finitely generated group elementarily equivalent to $\mathbb{Z}^m \wr \mathbb{Z}^n$ is isomorphic to $\mathbb{Z}^m \wr \mathbb{Z}^n$. On the other hand, we show that $\mathbb{Z}^2 \wr \mathbb{Z}$ admits $2^{\aleph_0}$ elementarily equivalent, pairwise non-isomorphic central extensions with finite kernel.

These are joint results with A. Miasnikov and D. Osin.

Nonstandard Free Groups and Exponential Completions

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For every model $\widetilde{\mathbb{Z}} \equiv \mathbb{Z}$ and every finite rank $n \geq 2$, we give an explicit construction of a group $F_n(\widetilde{\mathbb{Z}})$ from arithmetically coded $\widetilde{\mathbb{N}}$-long reduced words. The construction is obtained by applying the absolute interpretation of F_n in the list superstructure $S(\mathbb{Z}, \mathbb{N})$ to the nonstandard list superstructure $\widetilde{S}_E(\mathbb{Z}, \mathbb{N})$; hence $F_n(\widetilde{\mathbb{Z}}) \equiv F_n$. It is independent, up to a definable isomorphism, of the chosen arithmetic enumeration of finite lists. The standard free group embeds elementarily in $F_n(\widetilde{\mathbb{Z}})$. We exploit this construction to show that under mild assumptions, ultrapowers of a group can be viewed as nonstandard models of that group. This leads us to describe the structure of the ultrapowers in terms of structure of nonstandard models of natural numbers, offering insight into a longstanding question of Malcev.

We also record the naturale $\mathbb{Z}$-exponentiation and show that the centralizer of every nontrivial element is isomorphic to the additive group $\mathbb{Z}^+$. Finally, we compare $F_n(\widetilde{\mathbb{Z}})$ with the universal $\widetilde{\mathbb{Z}}$-exponential completion $F_n^{\widetilde{\mathbb{Z}}}$ and prove that the canonical homomorphism $F_n^{\widetilde{\mathbb{Z}}} \to F_n(\widetilde{\mathbb{Z}})$ is injective. The proof combines the canonical centralizer-extension tower with an internal infinite-word separation lemma whose cancellation bounds are ordinary integers.

Word Structures and their Automatic Presentations

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We study automatic presentations of the structures: (N, S), (N, E_S), $(N, \leq)$, and their expansions by a unary predicate U. Here S is the successor function on N, E_S is the undirected version of S, and $\leq$ is the natural order on N. We call these structures word structures.

Our goal is three-fold. First, we study the isomorphism problem for automatic word structures by focusing on the following three problems. The first problem asks to design an algorithm that, given an automatic structure A, decides if A is isomorphic to (N, S). The second problem asks to design an algorithm that, given two automatic presentations of (N, S, U_1) and (N, S, U_2), where U_1 and U_2 are unary predicates, decides if these structures are isomorphic. The third problem investigates if there is an algorithm that, given two automatic presentations of $(N, \leq, U_1)$ and $(N, \leq, U_2)$, decides if $U_1 \cap U_2 Neq \emptyset$. We show that all these three problems are undecidable.

Next, we study intrinsic regularity of the successor relation S in the structure (N, E_S). We construct an automatic presentation of (N, E_S) in which S is not regular. This implies that S is not intrinsically regular in (N, E_S).

Finally, we build some exotic examples of word structures. For $U \subseteq N$, let d_U be the function that computes the distances between the consecutive elements of U. We build automatic presentations of $(N, \leq, U)$ where d_U can realise logarithmic, radical, intermediate, and exponential functions.

This is a joint work with Xiao Yang.

Inclusive (Universal Positive) Theory of Abelian Groups

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Model theory of Abelian groups is extensively studied in the literature also in recent years. An *identical inclusion* is a formula that can be expressed as a (possibly infinitary) *disjunctive identity*

$$u = v_1 \vee u = v_2 \vee u = v_3 \vee \cdots ,$$

or, equivalently, as a universally closed identical equality of subsets of words (terms). For groups and rings, the classes defined by identical inclusions and by infinitary disjunctive identities are coincide, for semigroups they do not coincide. A class of algebras defined by a set of identical inclusions is called an *inclusive variety*. An inclusive variety that can not be defined by first order formulas is called a *nonelementary* inclusive variety. An inclusive variety defined by a system of identical inclusions – each depending on a finite set of variables – is called a *quasielementary* inclusive variety.

We describe elementary, nonelementary and quasielementary inclusive varieties of Abelian groups. There exist continuum many inclusive varieties of each of these kinds. We also determine Abelian groups defined by identical inclusions up to isomorphism, classify Abelian groups up to inclusive equivalence and characterize some special sublattices of the lattice of inclusive varieties of Abelian groups.

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Centroids and Exponentiation in Groups

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In this talk, I will introduce various group centroids and discuss their relations to exponentiation in groups and algebras.

Varieties of Exponential MR-Groups

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The notion of an exponential R-group, where R is an arbitrary associative ring with unity, was introduced by R. Lyndon [1]. Myasnikov and Remeslennikov refined the notion of an R-group by introducing an additional axiom [2]. In particular, the new concept of an exponential MR-group (R-ring) is a direct generalization of the concept of an R-module to the case of noncommutative groups. We come up with the notions of a variety of MR-groups and of tensor completions of groups in varieties. Abelian varieties of MR-groups are described, and various definitions of nilpotency in this category are compared. It turns out that the completions of a 2-step nilpotent MR-group is 2-step nilpotent [3, 4].

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On Moufang Loops with Certain Automorphisms

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A Moufang loop is the generalization of a group, such that instead of the law of associativity the identity

$$(xy)(zx) = x(yz)x$$

holds. Let M be a Moufang loop. Left and right translations of M generate a group $Mult(M)$. The elements of $Mult(M)$ which preserve the neutral element of M form a subgroup of inner mappings $Int(M)$. We will discuss the properties of Moufang loops with certain inner automorphisms. In particular, we will show that the Restricted Burnside Problem has a positive solution for the class of left automorphic Moufang loops (i.e., such that $L_{xy}^{-1} \circ L_x \circ L_y$ are automorphisms for all x, y). In general for Moufang loops this is an open problem.

Representations of Grassmann Poisson Superalgebras

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We prove that every irreducible Poisson supermodule over the Grassmann Poisson superalgebra G_n is either isomorphic to the regular Poisson supermodule $Reg\, G_n$ or to its opposite supermodule. Moreover, every unital Poisson supermodule over G_n is completely reducible. If P is a unital Poisson superalgebra which contains G_n with the same unit then $P = Q \otimes G_n$ for some Poisson superalgebra Q. Furthermore, we classify the supermodules over G_n in the category of dot-bracket superalgebras with Jordan brackets and prove that every irreducible Jordan supermodule over the Kantor double $Kan\, G_n$ is isomorphic to the supermodule $Kan\, V$ where V is an irreducible dot-bracket supermodule with Jordan bracket over G_n.

It is a joint work with U. Umirbaev.

Potentially Positive and Partially Positive Elements of a Free Group

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We say that a word w in a free group F is positive if all generators of F occur in w with positive exponents only. G. Baumslag proved that all one-relator groups with a positive relator are residually solvable. D. Wise proved that all one-relator groups with a positive relator are residually finite (under an additional small cancellation condition). We say that w is partially positive if there is at least one generator that appears in w with positive exponents only. Finally, we say that w is potentially positive if there is an automorphism of F that takes w to a positive word. In this talk, I will survey recent results on counting potentially positive words and on using potentially and partially positive words to generalize the results of G. Baumslag and D. Wise mentioned above. A particular result that can be singled out here is: all one-relator groups with a partially positive relator are residually solvable. This was recently proved by my PhD students L. Koch-Hyde, S. O'Connor, and E. Olive. Other people who have been involved in this line of research include E. Dinowitz, A. G. Miasnikov, and myself.

Equational Noethericity for Graphs, Groups and Algebras and Kotov's Lemma

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M. Kotov in his 2010 paper formulated a lemma – a convenient criterion for when an algebraic system is not equationally Noetherian. The talk will present results on the equational Noethericity for graphs and some groups and algebras obtained with help of this lemma.

Algebraic Complexity of Computations Over Baumslag–Solitar Metabelian Groups

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Problems of computation over various algebraic structures are related to the estimation of the complexity of computing terms in the language of the algebraic system under consideration. One of the main objects of research in algebraic complexity are algebraic schemes and nonbranching programs, which allow one to study the computational properties of polynomials, rational functions, and operations in various algebraic structures. In classical algebraic complexity theory, similar questions are studied for rings and fields; see the works of A. A. Razborov [3] and W. Strassen [4]. It should be noted that the computational aspects of Baumslag–Solitar groups have previously been studied in the context of algorithmic problems in group theory, normal forms, and the complexity of the dictionary problem; see the works of [1, 2]. However, questions of the algebraic complexity of computations in these groups have been much less studied. The paper presents results on the algebraic complexity of computing normal forms and basic group operations over the family of Baumslag-Solitar groups of type $BS(1,k)$ by interpreting these groups in the ring $\mathbb{Z}$.

Let us denote the family of groups parameterized by $k \in \mathbb{Z} \setminus \{-1, 0, 1\}$ by $BS(1,k)'$. Then the main result can be formulated as follows.

Theorem *Let $F_2 \cong \langle a, b\rangle$ be a free group. Then*

1. *For any $g = a^{a_1}b^{b_1}a^{a_2}\cdots b^{b_{s-1}}a^{a_s}b^{b_s} \in F_2$ the algebraic complexity of reducing g to normal form in $BS(1,k)'$ is $f(n) = \Omega(\log_2 n)$, where $n = \max\{|a_i|\}$, $i = \overline{2,s}$;*
2. *For any set of elements $g_1, \ldots, g_s \in G \in BS(1,k)'$ the algebraic complexity of calculating the product $g = g_1 g_2 \cdots g_s = a^{a_1}b^{b_1}a^{a_2}\cdots b^{b_{s-1}}a^{a_s}b^{b_s}$ is $f(n) = \Omega(\log_2 n)$, where $n = \max\{|a_i|\}$, $i = \overline{2,s}$.*

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Infinite-Dimensional Superalgebras

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We will discuss superconformal algebras, their generalizations and representations.

Mini-Symposium:
Computer Science, Data Science and Artificial Intelligence: Theory, Practice, and Education

Abstracts of Talks

Impact of Statistics Updates and Index Optimization on Cardinality Estimation Accuracy and Query Performance in Microsoft SQL Server 2022

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This study investigates the impact of statistics updates and index optimization on Cardinality Estimation (CE) accuracy and query performance in Microsoft SQL Server 2022. Since the SQL Server Query Optimizer relies on accurate statistics to generate efficient execution plans, outdated statistics and inadequate indexing may lead to poor cardinality estimates and suboptimal query performance. The study evaluates the effects of statistics updates and index optimization on estimation accuracy, execution plans, and resource utilization, while also assessing the usefulness of Query Store for execution plan analysis.

The experiments were conducted using Microsoft SQL Server 2022 and the StackOverflow2010 benchmark database. Four scenarios were analyzed: baseline, stale statistics, updated statistics, and index optimization. Query performance was evaluated using estimated and actual row counts, CE Error Ratio, logical reads, CPU time, elapsed time, execution plans, and Query Store metrics.

The results show that stale statistics significantly increased cardinality estimation errors, with the CE Error Ratio rising from 0.39 to 15.95 in one query and from 0.61 to 5.03 in another. Updating statistics substantially improved estimation accuracy, particularly for Q1, while index optimization produced the greatest performance gains by reducing logical reads from 7,405 to 4 and from 801,171 to 335 in the most expensive queries, leading to lower CPU time and execution latency. Query Store enabled effective comparison of execution plans and performance metrics and revealed no evidence of execution plan regression. These findings demonstrate that maintaining up-to-date statistics improves estimation accuracy, whereas workload-specific index optimization is essential for achieving optimal query performance in Microsoft SQL Server 2022.

Integrating Adaptive Machine Learning with Stochastic Risk Models for Software Input Security

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Software vulnerabilities related to input validation and sanitization remain a critical concern for modern web applications. Traditional defensive paradigms often treat security investment and mitigation effort as static properties, failing to adapt to fluctuating threat behaviors. This paper presents a conceptual approach that bridges classical stochastic risk modeling with predictive machine learning to enhance runtime software defense.

We model the interaction between automated cyber threats and software defenses by combining arrival process characteristics with engineering deployment costs. To introduce dynamic responsiveness, we discuss how machine learning can be utilized to estimate attack intensities and predict exploitation probabilities based on historic payload patterns. This allows the system to shift from rigid security rules to data-driven investment and engineering decisions.

The practical relevance of this integrated approach is demonstrated through a layered defensive blueprint within Ruby-based web architectures. We show how theoretical risk boundaries translate into practical, adaptive middleware filters and sanitization mechanisms, providing a sustainable balance between computational performance and application security.

Keywords: Software security, Stochastic models, Machine learning applications, Input validation, Ruby web security, Risk management.

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Modelling and Optimisation of the Photovoltaic Chain: an Approach using Fuzzy Logic MPPT Control with Dynamic Adaptation of the Output Gain

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This article presents an advanced approach to optimising photovoltaic energy production based on fuzzy logic. Because of climatic variations, particularly irradiance and temperature, and the nonlinear characteristics of solar panels, conventional maximum power point tracking (MPPT) methods have limitations in terms of speed and accuracy. To address these limitations, we propose a controller based on two-level fuzzy logic: the first level locates the maximum power point, while the second dynamically adapts the controller's output gain. This dual approach improves responsiveness to meteorological transients and enables faster convergence to the maximum power point. Simulation results obtained using MATLAB/Simulink under various irradiance and temperature scenarios demonstrate improved performance compared with a single-level fuzzy logic controller, including higher energy efficiency and reduced oscillations around the optimal operating point.

Keywords: Photovoltaics, MPPT, fuzzy logic, adaptive gain, modelling.

Acknowledgments

This study was conducted at the Laboratoire TAMAYOUZ de Technologie, Industrielle et de l'Information, Faculty of Technology, University of Bejaia, Algeria. The authors thank the laboratory director, Prof. T. Rekioua, and the head of the renewable energy research team, Prof. Dj. Rekioua, for providing the equipment necessary to complete this work.

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Startup Evaluation System Using AI

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Assessing startup success at an early stage remains one of the most challenging tasks in venture capital.

Modern assessment systems increasingly rely on large language models (LLM). Among the approaches proposed in this direction, SSFF (Startup Success Forecasting Framework) stands out – a system that combines

web information and LLM analysis for startup evaluation. SSFF uses RAG for web information retrieval – an approach where the model relies on information obtained from external sources. However, in the original

system this search is performed in only two directions using predefined keywords, which limits the completeness of the assessment. This paper proposes extending the SSFF RAG approach using Reasoning RAG.

This approach means that the system itself formulates search queries for each assessment component separately, performs the search, then evaluates what information is missing, and repeats the search if necessary.

This multi-step process ultimately provides a more complete picture than conventional single-pass retrieval. To demonstrate this approach, a prototype was developed that uses a local LLM without an external API. The prototype was tested on real startup data and, through the example of three specific companies, confirms that the use of Reasoning RAG improves assessment quality compared to the original SSFF.

From Data to Algorithms: Practical Approaches to Machine Learning Model Selection

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This presentation discusses practical approaches to machine learning model selection based on the type of data, the learning task, and the evaluation criteria. Using examples from well-known datasets, it presents typical applications of Linear Regression, Logistic Regression, K-Nearest Neighbors, Decision Trees, Random Forests, and Support Vector Machines. Particular attention is paid to the distinction between regression and classification tasks, data preprocessing, feature scaling, class imbalance, and hyperparameter selection. The presentation also demonstrates that overall accuracy alone is not always sufficient for model evaluation and that metrics such as precision, recall, F1-score, and regression-specific measures should be selected according to the practical objective of the task. The main argument is that there is no universally best machine learning algorithm. An appropriate model should instead be selected by considering the characteristics of the dataset, the cost of different prediction errors, interpretability requirements, and available computational resources.

Acknowledgments

The talk is supported by HORIZON Europe Project *"Fostering Innovation Culture to Boost Careers of New Generation of Georgian Artificial Intelligence Researchers"* (FORGE-AI), HORIZON-WIDERA-2024-TALENTS-03-01, Grant # 101216703.

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Solving Screening and Ranking Problems of Innovative Ideas using Multi-Criteria Methods and Artificial Intelligence: a Two-Cycle Integrated Model from Problem Formulation to Outcome

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The paper presents a two-cycle integrated model for solving screening and ranking problems of innovative ideas using multi-criteria decision-making methods and artificial intelligence. In contemporary research and managerial practice, AI-based tools increase the number of generated ideas; therefore, the main problem is not only idea generation, but the transformation of ideas into comparable alternatives, their evaluation by justified criteria, and the preparation of a transparent decision. The proposed model combines two interrelated cycles. The first cycle describes an AI-supported workflow from problem formulation to the preparation of structured alternatives: task clarification, definition of preferences and constraints, context provision, role distribution, problem decomposition, agent-based planning, intermediate review, refinement, and consolidation of results. AI is not treated as an autonomous decision-maker, but as a methodological assistant that supports organization, explanation, comparison, and iterative improvement.

The second cycle follows the logic of multi-criteria expert evaluation. It includes the formulation of alternatives, construction of criteria and indicators, preparation of an evaluation matrix, aggregation of expert judgments, consensus analysis, and application of a ranking method. Multi-criteria methods preserve the formal and justified logic of decision-making, while AI strengthens the organization and explainability of the process.

The connection between the two cycles is achieved through functional delegation. AI can be assigned such roles as moderator, analyst, criteria extraction assistant, consensus checker, or interpreter of results. These roles make it possible to model a simulated expert council, compare different perspectives, and reveal inconsistencies before the final decision is made. The model remains transparent, repeatable, and human-centered: AI supports the preparation of the decision, while the final interpretation, judgment, and responsibility remain with the human decision-maker.

The novelty of the paper lies in presenting AI workflow and multi-criteria expert evaluation as a single integrated decision-support model. The approach can be applied in the evaluation of innovative ideas, project selection, education, managerial decision-making, and other tasks where alternatives must be compared according to several criteria.

Keywords: artificial intelligence, multi-criteria decision support, MCDM, innovative ideas, screening, ranking, AI workflow, expert evaluation, decision matrix, consensus analysis, TOPSIS, simulated expert council, functional delegation.

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Robust Gaussian Mixture Modeling under Cellwise Contamination

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Gaussian Mixture Models (GMMs) are widely used probabilistic models for clustering and density estimation, but classical estimation methods are highly sensitive to cellwise contamination. In this paper, we propose a robust estimation algorithm for GMMs that explicitly accounts for cellwise outliers during parameter estimation and clustering. The proposed approach is based on the L_2 distance estimation framework, which reduces the influence of contaminated cells while preserving the underlying cluster structure. An iterative optimization procedure is employed to jointly update cluster memberships and robust parameter estimates. The performance of the proposed method is evaluated through simulation studies under different contamination and dimensional settings and compared with classical GMM and recent robust mixture approaches. The results demonstrate that the proposed framework provides more stable parameter estimation and improved clustering performance, particularly under moderate and high contamination settings.

Keywords: Robust clustering, Gaussian mixture model, cellwise outliers, contaminated data, robust estimation.

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Fuzzy VIKOR Approach for Multiple Criteria Group Decision-Making

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The solution to the multi-criteria group decision-making (MCGDM) problem involves selecting the best of the possible alternatives or ranking all alternatives evaluated by a group of decision makers (DMs) based on multiple criteria. A complex MCGDM problem typically uses both quantitative and qualitative criteria, which may conflict with each other and contain incomplete and uncertain information.

The fuzzy VIKOR approach is a well-known MCGDM technique. It focuses on finding one or more compromise solutions and ranking and selecting from a range of alternatives in the presence of contradictory and non-commensurable criteria.

This study proposes an application of the VIKOR approach to solve the MCGDM problem in a hesitant fuzzy environment. To evaluate the criteria and their weights in this approach, decision-makers' assessments are used. The weights and values of the established criteria are first evaluated with linguistic expressions (linguistic terms), since they are the most natural and convenient representations of DMs' preferences. Triangular fuzzy numbers are then used to represent these linguistic assessments. The combined vector of weights specified by all DMs is a hesitant triangular fuzzy set. After joining each hesitant fuzzy element of this set, the aggregated weights vector is obtained, whose elements are triangular fuzzy numbers. A fuzzy decision matrix is constructed in a similar manner, each element of which is a hesitant fuzzy set of the triangular fuzzy numbers that are combined evaluations of the criteria. It is then transformed into an aggregated triangular fuzzy decision matrix.

Following the VIKOR algorithm to rank the alternatives, first, the best f_i^* and the worst f_i° values for criteria are determined. The presence of both the benefit type criteria and the cost type criteria is considered. Then, calculated the values of S_i and R_i, where S_i is the aggregated value of the ith alternatives with a maximum group utility, and R_i is the aggregated value of the ith alternatives with a minimum individual regret of the 'opponent.' Next, the values of Q_i are calculated by $Q_i = v((S_i - S_imin)/(S_imax - S_imin)) + (1 - v)\,(R_i - R_imin)/(R_imax - R_imin)$, where v is introduced as the weight of the strategy of 'the maximum group utility' (or majority of criteria). Its value lies between 0 and 1. The compromise can be selected with 'voting by majority' ($v > 0.5$), with 'consensus' ($v = 0.5$), or with 'veto' ($v < 0.5$).

After this, defuzzification of S_i, R_i and Q_i into crisp values is made by the center of area (COA) method. Then, performed ranking of the alternatives by sorting each S, R and Q values in ascending order. Finally, identified as a compromise solution is the alternative A_1, which is the best ranked by the Q (minimum) if the following two conditions are met:

C1. The alternative A_1 has an acceptable advantage: $Q(A_2) - Q(A_1) \geq 1/(N-1)$, where A_2 is the alternative with the second position in the ranking sequence by Q, and N is the number of alternatives.

C2. Acceptable stability in decision-making:

Alternative A_1 must also be the best ranked by S and/or R.

If one of the conditions is not met, then a set of compromise solutions is proposed:

1. Alternatives A_1 and A_2 if only the condition C2 is not satisfied, or
2. Alternatives $A_1, A_2, \ldots, A_M$ if the condition C1 is not met; A_M is determined by the relation $Q(A_M) - Q(A_1) < 1/(N-1)$ for maximum M (the positions of these alternatives are "in closeness").

An example of selecting the investment projects using the hesitant fuzzy VIKOR method is presented to illustrate the application of the proposed approach.

Bridging Data Science and Social Research in Higher Education

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The rapid digitalization of modern society has generated an unprecedented volume of complex social data, establishing a critical demand for specialists capable of both rigorous mathematical modeling and nuanced domain interpretation. Traditional academic curricula often suffer from a structural disconnect: computer science programs frequently lack socio-political context, while social science tracks rarely provide advanced computational and statistical frameworks.

This paper presents the educational design and methodological architecture of the Master's program "Social Data Analysis", newly launched at Taras Shevchenko National University of Kyiv. Built at the intersection of Systems Analysis, Data Science, and Empirical Sociology, the curriculum introduces a matrix model that ensures a deep synergy between mathematical rigor and applied domain research.

The core intellectual contribution of this curriculum lies in its structured synthesis of four computational and substantive dimensions. First, it establishes a solid mathematical and statistical foundation (covering advanced multivariate statistics, factor analysis, structural equation modeling, time series forecasting, etc.) led directly by mathematicians. Second, this foundation is paired with traditional methods of data collection and preprocessing. Third, it integrates cutting-edge computational tools for automated data acquisition, including OSINT, web scraping, and big data mining frameworks. Finally, these technical skillsets are grounded in targeted sociological courses that focus critically on research design, ethical considerations, and the substantive formulation of complex empirical problems. We further discuss how this integrated approach, supported by a two-tier practical training framework, enables students to bridge the gap between abstract algorithmic tools and real-world societal challenges.

Keywords: *Social Data Analysis, Data Science Curricula, Computational Social Science, Applied Statistics, Curriculum Design.*

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An Intelligent Computational Framework for Developing an AI-Driven Online Educational Platform for the Georgian Language

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The Georgian language presents one of the most challenging environments for Natural Language Processing (NLP) due to its agglutinative–fusional morphology, extensive system of affixation, lexical ambiguity, and exceptionally rich paradigm formation. These characteristics demand computational models capable of integrating morphological processing, lexical knowledge representation, semantic interpretation, and intelligent educational support within a unified architecture. Existing language-learning systems primarily focus on educational content and user interaction, whereas computational linguistic technologies are typically developed independently of educational environments. This separation limits both linguistic functionality and the adaptability of intelligent learning systems.

To address these challenges, this research proposes an intelligent computational framework for developing an AI-driven online educational platform for the Georgian language. Rather than focusing solely on the implementation of an educational application, the proposed research establishes the computational foundation upon which intelligent language-learning systems can be developed. The framework integrates computational linguistics, artificial intelligence, lexical databases, semantic technologies, and educational services into a unified architecture capable of supporting both research and practical applications. A distinctive scientific contribution of the framework is the integration of a tree-based lexical architecture with the author's original Wave Search strategy for hierarchical lexical retrieval. The proposed approach is conceptually inspired by the semantic network model introduced by William A. Woods while extending its principles through computational methods specifically developed for the hierarchical organization of Georgian lexical data and morphological processing. Unlike conventional graph traversal techniques, the framework combines controlled wave propagation, hierarchical lexical structures, and semantic transitions between lexical subnetworks. This enables efficient lemma retrieval, morphological disambiguation, semantic processing, and real-time linguistic analysis within a single computational environment. The proposed framework is designed as a reusable technological foundation rather than a single-purpose application. It enables the development of AI-driven educational platforms, intelligent dialogue systems, digital lexicographic resources, semantic knowledge bases, and real-time language technologies. The bilingual educational environment built upon the framework supports interactive learning through immediate morphological and semantic feedback, making Georgian language acquisition more efficient for both native speakers and international learners. Beyond its educational applications, the proposed framework contributes to the digital transformation of the Georgian language and establishes a sustainable technological infrastructure for future AI-based language technologies. Owing to its modular architecture, the framework can be adapted to other morphologically rich and low-resource languages, extending its applicability beyond Georgian and contributing to multilingual artificial intelligence research.

Acknowledgements

This work was supported by the Shota Rustaveli National Science Foundation of Georgia (Grant #FR-25-8755).

Parallel Processing of Augmented Transition Networks using Functional Programming in Haskell

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This paper explores efficient implementation strategies for natural language processing (NLP) architectures, specifically focusing on William Woods' Augmented Transition Networks (ATNs) [1], using the purely functional language Haskell. Grammatical parsing via ATNs inherently involves non-deterministic path exploration and backtracking, which introduces significant computational overhead when processing large volumes of text.

We present a novel approach to parallelizing ATN state traversal by leveraging GHC's multi-core runtime environment and deterministic parallelism models, specifically the Eval monad and evaluation strategies as detailed by Simon Marlow [2]. Our findings demonstrate that Haskell's purity eliminates data races by design, allowing high-level, declarative parallelization of semantic networks with minimal code modification and robust scalability.

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Artificial Intelligence and Quality Assurance in Higher Education

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The rapid adoption of artificial intelligence (AI) in higher education is reshaping expectations for computer science programs and their quality assurance. Emerging accreditation requirements increasingly emphasize the responsible, ethical, and pedagogically appropriate integration of AI into curricula, learning outcomes, assessment, and academic integrity policies. This study examines how institutional AI strategy is translated into academic practice at Ivane Javakhishvili Tbilisi State University (TSU), whose Computer Science program is ABET-accredited, and identifies the support mechanisms needed for sustainable implementation.

The analysis draws on two complementary empirical sources: TSU's 2025 institutional AI survey, covering students, academic and administrative staff, and employers, and a focus-group study involving academic staff from the Faculty of Exact and Natural Sciences. The results confirm widespread AI adoption: 94% of students report using ChatGPT, while 64.3% of administrative staff use AI tools in their work. Employers identify data analysis (60%), content generation (55%), and process automation (40%) as important graduate competencies. Respondents also express concerns about academic integrity, unreliable AI-generated information, and the possible weakening of critical thinking.

Although TSU has adopted an Artificial Intelligence University Strategy for 2026–2029, the findings reveal an implementation and communication gap. All focus-group participants consider basic AI competencies and institutional guidelines necessary for academic staff; however, only 17% report access to relevant training, 17% recognize a clear institutional policy, and 33% believe that the university provides effective support for integrating AI into teaching and learning. The study concludes that alignment with emerging accreditation standards requires clear policies, systematic faculty development, accessible guidance, effective communication, and quality assurance mechanisms that convert institutional strategy into measurable educational practice.

Keywords: artificial intelligence, quality assurance, higher education, accreditation, computer science education, ABET.

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Mini-Symposium: Formal Groups, Cobordism and Related Structures

Abstracts of Talks

Real K-Theory of Bounded Rank

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In this joint project with Ralf Meyer we construct a geometric K-theory for finite-dimensional $\mathbb{Z}/2$-CW-complexes, denoted $KR_{(k)}(X)$, where the rank parameter k is explicitly determined by the maximal dimensions of the fixed-point set and the free cells of X. By adopting Atiyah's definition and applying our recent stability theorems, we prove that this finite-rank theory is isomorphic to the stable theory for spaces that are not necessarily compact. This bridges the gap between abstract stable homotopy theory and the concrete classification of topological quantum systems.

Acknowledgments

This work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) grant # FR-23-779.

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Two-Valued Groups and the Bielliptic Genus

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The subject of this talk lies at the intersection of two classical and modern lines of mathematics: the theory of Abelian integrals and algebraic addition theorems, and the theory of formal groups, complex cobordism, and Hirzebruch genera. A central object connecting these directions is a polynomial defining a two-valued algebraic group, introduced in 1990 [1] and subsequently known as the *Buchstaber polynomial*.

The formal-group part of the story goes back to the modulus-square construction introduced by Buchstaber and Novikov in 1971. In a series of papers of 1975–1976, Buchstaber developed the algebraic theory and obtained a classification of one-dimensional two-valued formal groups. In particular, a two-valued formal group has trivial involution, $\mathrm{inv}(x) = x$, if and only if it is obtained by the modulus-square construction from a one-dimensional single-valued formal group.

An important feature of the universal construction is that the exponential of the universal one-dimensional *odd* formal group whose modulus square gives the universal two-valued formal group is described by a formal Abelian integral of hyperelliptic type

$$t = \int_0^{f(t)} \frac{dX}{\sqrt{1 + c_1X^2 + c_2X^4 + c_3X^6 + \cdots}}, \quad f(-t) = -f(t).$$

In [1], Buchstaber studied the modulus-square construction for formal groups associated with the Krichever genus. In the sign convention used in this talk, the resulting three-parameter polynomial is

$$B_{\mathbf{a}}(x, y, z) = (x + y + z - a_2xyz)^2 - 4(1 + a_3xyz)(xy + xz + yz + a_1xyz).$$

The universality statement: this polynomial is universal among the two-valued groups obtained by applying the modulus-square construction to the corresponding class of Krichever formal groups.

Buchstaber and Kornev [4] introduced the class of symmetric 2-algebraic two-valued laws and obtained its universal law. After the neutral-element, inverse, and associativity axioms, as well as the symmetry conditions, have been imposed, the universal polynomial has the form

$$P(x, y, z) = (x + y + z)^2 - 4(xy + xz + yz) + xyz\big[k_2 + k_4(x + y + z) + k_6(xy + xz + yz) + k_8xyz\big]$$

with the single relation $4k_8 = k_4^2 - k_2k_6$.

The Buchstaber polynomial is obtained by the specialization $k_2 = -4a_1$, $k_4 = -2a_2$, $k_6 = -4a_3$, $k_8 = a_2^2 - 4a_1a_3$. The unrestricted universality problem for arbitrary algebraic two-valued formal groups remains open.

Vladimir Rubtsov observed that the associativity relation for $P(x, y, z)$ has the form of a classical relation familiar from the theory of modular and quasimodular forms. This observation opened the way to the applications developed in the joint work of Buchstaber, Kornev, and Rubtsov [5]. There the associativity condition of the universal symmetric 2-algebraic two-valued law is related to the Ramanujan system, the Chazy III equation, Gauss–Manin connections, Dubrovin–Frobenius structures, and the quantum Yang–Baxter equation.

The second main part of the talk concerns the genus-two reduction of the universal construction of 1975. Kornev [6] showed that when the infinite-genus hyperelliptic construction underlying the

universal odd formal group is restricted to genus two, the modulus-square law becomes exactly the Buchstaber polynomial. The relevant curve is $C_{\mathbf{a}} : Y^2 = 1 - a_1X^2 + a_2X^4 - a_3X^6$. If $f(t)$ is the inverse germ of the Abelian integral, then

$$f(0) = 0, \quad f'(0) = 1, \text{ and } (f'(t))^2 = 1 - a_1f(t)^2 + a_2f(t)^4 - a_3f(t)^6.$$

The one-dimensional formal group with exponential $f(t)$ defines a Hirzebruch genus with characteristic series $Q(t) = \frac{t}{f(t)}$. Kornev [6] denoted this genus by $Bc : \Omega_U \to \mathbb{Z}[\frac{1}{2}][a_1, a_2, a_3]$. In view of its natural origin from the bielliptic genus-two curve, we refer to it here as the *bielliptic genus*. Thus the same Buchstaber polynomial has two essentially different one-valued origins, where both arrows are given by the modulus-square construction. Their common specialization at $a_3 = 0$ is the Ochanine case.

The first group of applications concerns Spin manifolds. In the classical normalization, the Ochanine elliptic genus of a Spin manifold of real dimension $4k$ takes values in the integral ring $\mathbb{Z}[8\delta, \varepsilon]$. For the normalization, $a_1 = 2\delta$, $a_2 = \varepsilon$, $a_3 = 0$, so this ring becomes $\mathbb{Z}[4a_1, a_2]$. Witten's work placed elliptic genera in the framework of quantum field theory and loop-space geometry.

We compute the bielliptic genus on natural families of complex Spin manifolds, including the even-dimensional complex quadrics Q^{2m} and the complex Grassmannians $\mathrm{Gr}(2, 2m+2)$. For these families the values satisfy the integral refinement $Bc(M) \in \mathbb{Z}[a_1, a_2, a_3]$.

The second group of applications concerns SU-manifolds. For a compact toric manifold with its standard complex structure one has $c_1 \neq 0$; equivalently, such a toric manifold cannot itself be an SU-manifold. This obstruction is part of the toric-genus framework of Buchstaber, Panov, and Ray [2]. The quasitoric category is more flexible.

Lü and Panov [3] constructed explicit families of quasitoric SU-manifolds by changing the stable complex structure so that $c_1 = 0$. For the present talk we restrict attention to their real $4k$-dimensional family $N_e(n_1, n_2)$, $n_1 = 2k_1$, $n_2 = 2k_2 + 1$, whose moment polytope is $\Delta^1 \times \Delta^{n_1} \times \Delta^{n_2}$. The Krichever genus vanishes on specially omnioriented quasitoric SU-manifolds; hence it vanishes on the manifolds $N_e(n_1, n_2)$.

For the N_e-series we obtain a stronger arithmetic statement: $Bc\big(N_e(n_1, n_2)\big) \in \mathbb{Z}[4a_1, a_2, 4a_3]$.

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Universal Formal Group for Elliptic Genus of Level N

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The elliptic function of level N determines the elliptic genus of level N as a Hirzebruch genus. It is known that the elliptic function of level N is a specialization of the Krichever function that determines the Krichever genus. The Krichever function is the exponential of the universal Buchstaber formal group.

The paper [1] gives a specialization of the Buchstaber formal group, such that it determines formal groups that correspond to the elliptic genus of level N. In the talk we will present old and new results on this topic.

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Bordism Classes of Symmetric Products of Surfaces

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For a topological space X its *n-th symmetric product* $SP^n(X)$ is the quotient $X^{\times n}/\Sigma_n$ of the Cartesian n-th product by the natural action of the permutation group. It is a classical object of study in geometry and topology. It is known that if X is a surface, then $SP^n(X)$ has a canonical smooth structure. Moreover, if X is oriented, its symmetric product admits a complex structure (depending on the chosen complex structure on X). In the talk I will discuss the bordism classes of $SP^n(X)$ (complex for orientable X, and unoriented for non-orientable X) and give explicit formulas expressing these bordism classes as polynomials in (complex or real) projective spaces.

This is the joint work with D. V. Gugnin.

π-Regular Spaces, Continuous Mappings, and their Universality Problem

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Regular spaces are one of the most studied and significant concepts in topology and related fields of mathematics giving several results and characterizations. In a formal definition, in regular spaces, closed sets and points not in them can be separated by disjoint open neighborhoods. A modification of such spaces meets the realm of the so-called π-regular spaces where for every non-empty open set U there exists a non-empty open set V whose closure belongs to U.

In this talk, we develop a new theory around π-regular spaces referring to the universality problem, whether there exist universal elements in the class of π-regular spaces. Especially, if $\mathbb{L}$ is a class of topological spaces, then a topological space T is called *universal* in this class if the following conditions are satisfied:

(1) $\mathrm{T} \in \mathbb{L}$,

(2) for every $X \in \mathbb{L}$, there exists an embedding of X into T.

The question whether there are universal spaces in a given class of spaces is called the *universality problem* for that class [1].

We mainly prove that classes of π-regular spaces have universal elements and study the ascertainment of the existence of universal mappings in classes of continuous mappings with domains and ranges either the π-regular spaces or the Euclidean space $\mathbb{R}^n$, $n = 1, 2, \ldots$.

Acknowledgments

Results of this talk are given in the paper titled *"π-Regular spaces, continuous mappings, and their universality problem"* written by D. Georgiou, Y. Hattori, A. Megaritis and F. Sereti.

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On Completness and Cocompletness of Categories of Mixed Bimodules

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In this talk, we look for the existence of limits and colimits of some type in the category of mixed bimodules arising from a mixed distributive law of a monad over a comonad. We will also briefly discuss some applications of our results.

Acknowledgments

The work was supported by the Shota Rustaveli National Science Foundation Grant # FR-23-271.

Geometric Representatives in SU-Bordism Classes

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The SU-bordism theory was the subject of intensive study in algebraic topology of the 1960s. Many leading topologists of the time contributed to the description of the SU-bordism ring, combining the classical geometric methods of Conner–Floyd, Wall and Stong with the Adams–Novikov spectral sequence and formal group law techniques that emerged after the fundamental 1967 work of Novikov.

Thanks to toric topology, a new geometric approach to calculations with SU-bordism has emerged, which is based on representing generators of the SU-bordism ring and other important SU-bordism classes by quasitoric manifolds and Calabi–Yau hypersurfaces in toric varieties.

Acknowledgments

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Chromatic Euler Characteristics and Duality for Infinite Groups

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We study a family of generalizations of the notion of Euler characteristic of discrete groups (or of orbifolds, depending on one's perspective) indexed on the natural numbers. For $n = 0$ this is the classical orbifold Euler characteristic as studied by Wall and Serre, whereas for $n \geq 1$ and finite groups this is the chromatic cardinality as studied by Ben-Moshe–Carmeli–Schlank–Yanovski. For general n we show that our generalized Euler characteristic admits a natural interpretation in terms of the Morava E-theories. Our work involves showing that the generalized cohomology of infinite groups G with finite universal space for proper actions $\underline{E}G$ has a good theory of duality, as expressed by a new duality functor on the category of proper G-equivariant spectra. In particular, for such groups we prove the vanishing of (generalized) Farrell–Tate cohomology with $K(n)$-local ($T(n)$-local) coefficients. We compute our generalized orbifold Euler characteristics in a large number of examples. This includes many mapping class groups, where the classical calculation is a result of Harer–Zagier, and many arithmetic groups, whose classical orbifold Euler characteristics were computed by Harder.

This is joint work with Gijs Heuts.

Acknowledgments

The author was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) through the grant # FR-25-10267 and by the EPSRC grant EP/X038424/1 *"Classifying spaces, proper actions and stable homotopy theory"*.

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