Toeplitz determinants and lattice theory

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A lattice is a discrete subgroup in a finite-dimensional Euclidean space. Lattice theory has numerous applications, for instance, in discrete optimization or coding theory. One of these applications consists in associating lattices with elliptic or related curves over finite fields, which are of prominent use in coding theory, and then to connect arithmetic properties of the curve with geometric properties of the lattice. A basic quantity of every lattice is the volume of its fundamental domains. This volume is the determinant of some matrix, and in several interesting cases this matrix is just a perturbed Toeplitz matrix. The generating function of this Toeplitz matrix is in general not well-behaved, so that the classical Szegő limit theorem for Toeplitz determinants cannot be used. It rather turns out that the generating function is often a so-called Fisher-Hartwig symbol. As such symbols are also emerging in statistical physics, their determinants have been thoroughly studied for decades. The results of these studies now prove to be of use in lattice theory. The talk is an introduction to some aspects of lattice theory and Toeplitz determinants and also exhibits some recent results obtained in joint work with Lenny Fukshansky, Stephan Ramon Garcia, and Hiren Maharaj in [1],[2].

References:

[1] Albrecht Böttcher, Lenny Fukshansky, Stephan Ramon Garcia, and Hiren Maharaj: Toeplitz determinants with perturbations in the corners. *J. Funct. Analysis*, 268 (2015), 171–193.

[2] Albrecht Böttcher, Lenny Fukshansky, Stephan Ramon Garcia, and Hiren Maharaj: On lattices generated by finite Abelian groups. *SIAM J. Discrete Math.*, 29 (2015), 382–404.