Commuting dilations and linear matrix inequalities

Igor Klep

The University of Auckland, Department of Mathematics, Auckland, New Zealand

email: igor.klep@auckland.ac.nz

Given a tuple $A = (A_1, ..., A_g)$ of real symmetric matrices of the same size, the affine linear matrix polynomial $L(x) := I - \sum A_j x_j$ is a monic linear pencil. The solution set S_L of the corresponding linear matrix inequality, consisting of those x in R^g for which $L(x)$ is positive semidefinite (PsD), is a spectrahedron. It is a convex semialgebraic subset of R^g. The set D_L of tuples $X = (X_1, ..., X_g)$ of symmetric matrices (of the same size) for which $L(X) := I - \sum A_j \otimes X_j$ is PsD, is called a free spectrahedron. We explain that any tuple X of symmetric matrices in a bounded free spectrahedron D_L dilates, up to a scale factor, to a tuple T of commuting self-adjoint operators with joint spectrum in the corresponding spectrahedron S_L. The scale factor measures the extent that a positive map can fail to be completely positive. In the case when S_L is the hypercube $[-1, 1]^g$, we derive an analytical formula for this scale factor, which as a by-product gives new probabilistic results for the binomial and beta distributions.

This is based on joint work with Bill Helton, Scott McCullough and Markus Schweighofer.