The string density problem and nonlinear wave equations

Aleksey KOSTENKO

Faculty of Mathematics, University of Vienna, Vienna, Austria

email: Oleksiy.Kostenko@univie.ac.at

Classical objects in spectral theory are the differential equation

$$-y'' = z\,\omega\,y, \quad x \in [0, L), \tag{1}$$

(here $L \in (0, \infty]$, ω is a positive Borel measure on $[0, L)$ and z is a spectral parameter) and the Weyl–Titchmarsh m-function, which encodes all the spectral information about (1). In a series of papers in the 1950s, M. G. Krein investigated the direct and inverse spectral problems for this equation. Viewing these problems as a natural generalization of investigations of T. Stieltjes on continued fractions to the moment problem, M. G. Krein identified the totality of all possible m-functions with the class of the so-called Stieltjes functions in a bijective way. Recently, the string density problem has come up in connection with some completely integrable nonlinear wave equations (e.g., the Camassa–Holm equation) for which the string spectral problem serves as an underlying isospectral problem. In contrast to the KdV equation, the Camassa–Holm equation possesses peaked solitons, called peakons, and models breaking waves. The latter happens when ω is a signed measure, i.e., the string is *indefinite*.

In this talk, we review the direct and inverse spectral theory for indefinite strings and relate it to the conservative Camassa–Holm flow. As one of our main results we are going to present the indefinite analog of M. G. Krein's celebrated solution to the string density problem. A special attention will be given to multi-peakon solutions.

The talk is based on joint work with Jonathan Eckhardt.