Limit spectrum graph for some non-self-adjoint differential operators with small physical parameter

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The celebrated Orr-Sommerfeld equation

$$\big\{(D^2-\alpha^2)^2 - i\alpha R[q(x)(D^2-\alpha^2) - q''(x)]\big\}\, y = -i\alpha R\lambda(D^2-\alpha^2)y,$$

comes from the linearization of the Navie-Stokes equation in the infinite 3-dim spacial layer $|x| \leqslant 1, \xi, \eta \in \mathbb{R}^2$. Here R is the Reynolds number (which characterizes the viscosity of the liquid), α is the wave number, $q(x)$ is the velocity profile, λ is the spectral parameter, and $D = d/dx$. It is the old problem in hydrodynamics to understand the spectrum behavior of the Orr-Sommerfeld equation as $R \to \infty$ (when the liquid becomes close to the ideal one). It turns out that the simpler model problem

$$-i\varepsilon y'' + q(x)y = \lambda y, \quad y(\pm 1) = 0, \quad \varepsilon = 1/R \to 0,$$

is closely related to the above one.

There is a great amount of works devoted to both problems. However, there are not too many rigorous results on this topic. The spectrum behavior of the above problems depends dramatically on the analytical properties of the profile q. Our aim is to describe the spectral portraits of the model problem as $\varepsilon \to 0$ for the case when q is a polynomial. It turns out that in this case the spectrum concentrates along some critical curves in the complex plane which form "the limit spectral graph". The problem is to understand the geometry of this graph, to find analytic formulae for the curves describing its parts and to write the asymptotic eigenvalue distribution (uniformly, as $\varepsilon \to 0$) along the limit spectral curves. All these problems will be discussed in the talk and the obtained results will be formulated.

The talk is based on the joint works of the author with S.N.Tumanov.