Operator relations in boundary value problems

Frank-Olme SPECK

Técnico, Universidade de Lisboa, Departamento de Matemática, Portugal

email: fspeck@tecnico.pt

We study relations between operators which appear in the context of linear boundary value problems. Typically the operator associated to a BVP is "equivalently reduced" to a another "simpler" operator that, e.g., appears in boundary integral equations or in a corresponding semi-homogeneous problem etc. Some questions are: I. Which kind of relations appear? What are their properties? II. How can this knowledge be used to make the reasoning more transparent and efficient? III. What are the consequences concerning explicit solution, regularity, asymptotic results, normalization of ill-posed problems etc.?

Particular attention is devoted to problems which are not well-posed and even not Fredholm. Examples are given from diffraction theory, leading to Wiener-Hopf plus Hankel and Fourier integral operator matrices. In short: Strong relations (like the equivalence after extension relation) transfer nice properties such as an explicit computation of generalized inverses or allow conclusions such as the discovery of compatibility conditions for finding convenient space settings.

The talk is based upon joint work with L.P. Castro, A. Moura Santos and F.S. Teixeira.

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