On inverse continuity of the numerical range generating function

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The *numerical range*, a.k.a. the *field of values*, or the *Hausdorff set* of a linear bounded operator A on a Hilbert space $\mathcal{H}$, is the range of the map $f_A(x) = \langle Ax, x\rangle$ acting on the unit sphere in $\mathcal{H}$. We consider the continuity properties of the (multivalued) inverse function f_A^{-1}, distinguishing between weak continuity, strong continuity, and existence of single-valued continuous selections. It is established in particular that strong continuity holds on the interior of $F(A)$, and that in finite dimensional setting it may fail only at finitely many points, which have to be round multiply generated boundary points.

The talk is based in part on publications [1–5]. Some applications of the results obtained there to quantum mechanics are in [6].

References:

[1] D. Corey, C. R. Johnson, R. Kirk, B. Lins, and I. M. Spitkovsky: Continuity Properties of Vectors Realizing Points in the Classical Field of Values, *Linear and Multilinear Algebra* 61 (2013) 1329–1338.

[2] T. Leake, B. Lins, and I. M. Spitkovsky: Pre-Images of Boundary Points of the Numerical Range *Operators and Matrices* 8 (2014), 699–724.

[3] T. Leake, B. Lins, and I. M. Spitkovsky: Inverse Continuity on the Boundary of the Numerical Range *Linear and Multilinear Algebra* 62 (2014), 1335–1345.

[4] B. Lins and P. Parihar: Continuous Selections of the Inverse Numerical Range Map *Linear and Multilinear Algebra* (2015), to appear.

[5] T. Leake, B. Lins, and I. M. Spitkovsky: Corrections and Additions to "Inverse Continuity on the Boundary of the Numerical Range *Linear and Multilinear Algebra* (2015), to appear.

[6] L. Rodman, I. M. Spitkovsky, A. Szkoła, and S. Weis: Continuity of the Maximum-Entropy Inference: Convex Geometry and Numerical Ranges Approach *arXiv:1502.02018v1* (6Feb 2015).