Characteristic functions, systems, discriminant curves and vessels: the ideas of Moshe S. Livšic and some of their further developments

Victor Vinnikov

Affiliation: Ben Gurion University of the Negev, Department of Mathematics, Beer Sheva, Israel

email: vinnikov@math.bgu.ac.il

Moshe S. Livšic (1917–2007) was one of the founding fathers of the modern theory of nonselfadjoint and nonunitary operators, discovering both the fundamental notion of the characteristic function and its deep relation to scattering theory and system theory. In the early 1970s M. S. Livšic started considering the problem of linking system theory and the theory of nonselfadjoint operators with Riemannian geometry [LV74]. This led naturally to developing a spectral theory for tuples of commuting nonselfadjoint operators. The first major breakthrough here was achieved during the three years (1975–1978) that M. S. Livšic spent in Tbilisi and was published in a note in 1978 in the Proceedings of the Georgian Academy of Sciences [Li78]: M. S. Livšic proved that a pair of commuting nonselfadjoint operators with finite nonhermitian ranks satisfy an algebraic equation with constant coefficients. This developed later into the notion of a commutative operator vessel (an algebraic structure encoding the commutation relations between several nonselfadjoint operators) and its discriminant curve, and an extensive theory of commuting nonselfadjoint operators and related overdetermined multidimensional systems based on connections with algebraic geometry [LKMV95]. In algebraic geometry itself it led to a new detailed study of determinantal representations and related topics yielding a proof of the 1958 conjecture of Lax on homogeneous hyperbolic polynomials in three variables [HV07].

IWOTA 2015 in Tbilisi is an appropriate occasion to survey the genesis of these remarkable ideas and to describe some of their further developments, including the recent generalization of the notion of the vessel to the setting of nonselfadjoint representations of a real Lie algebra [SV].

References:

[HV07] J.W. Helton and V. Vinnikov: Linear matrix inequality representation of sets, *Comm. Pure Appl. Math.* 60, 654–674 (2007).

[Li78] M. S. Livšic: Commuting nonselfadjoint operators and solutions of systems of partial differential equations generated by them, *Soobshch. Akad. Nauk Gruzin. SSR* **91** (1978), no. 2, 281–284, **MR** 80m:47008. In Russian.

[LKMV95] M. S. Livšic, N. Kravitsky, A. S. Markus, and V. Vinnikov: *Theory of commuting nonselfadjoint operators*, Kluwer Academic Publishers Group, Dordrecht, 1995.

[LV74] M. S. Livšic, L. L. Vaksman: Open geometry, and operator colligations, *Ukrain. Geometr. Sb.* (1974), no. 15, 16–35, i, **MR** 51:11323. In Russian.

[SV] E. Shamovich and V. Vinnikov: Lie Algebra Operator Vessels, preprint `arXiv:1209.4224`.