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სახელმწიფო უნივერსიტეტი
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Contents

Roin Nadiradze

Roin Nadiradze was born November 31, 1949 in Samtredia, Georgian Republic of the Soviet Union. His father Givi Nadiradze was a farmer, his mother Zinaida Tevzadze was a housewife. Family lived at Chaikovsky Street (now the street is named after Roin Nadiradze).

Roin entered the school in 1956, and he graduated Samtredia No 3 Public School in 1966 with a Golden Medal. His first mathematics teacher was Eskha Guldedava. The same year Roin entered the Faculty of Mechanics and Mathematics of Tbilisi State University (now named Ivane Javakhishvili). He participated in the works of the seminars of Professors G. Janashia, G. Berishvili and B. Kartsivadze.

In 1968, Professor Janashia asked the President of Tbilisi University Ilia Vekua to address to the President of the Lomonosov Moscow State University I. G. Petrovsky to accept a small group of talented Georgian students, including Roin, in the Lomonosov University. The specialization was supposed to be in algebraic and differential topology under the leadership of S. P. Novikov. Graduated from Lomonosov Moscow University

in 1972 Roin Nadiradze started his work in A. Razmadze Mathematics Institute of the Georgian Academy of Sciences where he finally became a leading specialist. In 1973 Academician G. Chogoshvili initiated Roin's research fellowship in Moscow Mathematical Steklov Institute. In 1980 he defended his thesis under guidance of S. P. Novikov and V. M. Buchstaber. The thesis of Doctor of Sciences was defended in 1994 at Tbilisi University.

Among his hobbies there were bridge, astrology and music. He participated in international bridge tournaments.

Roin died July 31, 1996 in Samtredia.

Roin Nadiradze mathematical works

In Mathematics Roin worked in the domain of Topology, basically in Cobordism Theory and the Theory of Formal Groups connected with cobordisms.

In his first article Roin studied the *Stong manifolds* and cobordism of self-conjugate manifolds. Stong manifolds $M(n,m)$ are examples of symplectic manifolds, i.e. manifolds whose stable tangent bundle is a quaternionic vector bundle; they are defined as submanifolds of $\mathbb{C}P^{2n+1} \times \mathbb{C}P^{2m+1}$ dual to a symplectic line bundle, where one first notes that complex projective spaces of odd complex dimension can be viewed as symplectic manifolds. In this note, models for $M(n,m)$ are described by equations, and a fixed point free involution T on $M(n,m)$ is described. Then it is asserted that the stable tangent bundle of $M(n,m)/T$ has a complex self-conjugate structure; this is a useful fact for the study of self-conjugate cobordism. This work he continue in several subsequent articles.

One of his most interesting works is about elements of finite order in the symplectic cobordism ring Ω^*_{Sp}. Roin constructed geometrically and examined a new sequence of torsion elements in this ring. Among these elements is one of dimension 111. It was proved latter by the Adams–Novikov spectral sequence that there is an element of order 4 in Ω^*_{Sp} in this dimension. It is not proved yet whether it coincides with Roin's element. He also made a conjecture which implies that all torsion elements in Ω^*_{Sp} have order 2 or 4. This is not proved also.

In 1987 Nadiradze in the joint work with his student Malkhaz Bakuradze introduced negative characteristic classes in SC-theory. The Nadiradze classes satisfy the Whithey formula, the top class is the Euler class, but in contrast with usual characteristic classes the Nadiradze classes are of dimensions from $-\infty$ till n. These negative classes give undecomposable elements in SC-cobordism. The article was published in 1991. Later some of these classes were realized geometrically.

In 1992 Nadiradze introduced a class of formal group laws, which now is called the Nadiradze formal group laws. This class is important because as it was clarified later the Nadiradze formal group laws coincides with the Buchstaber and Krichever formal group laws and therefore give another derivation of related genera.

In the paper published in 1994 in a geometrical way series of generators were constructed in the self-adjoint cobordism theory. Although these series do not form a complete system, they give interesting information about the self-adjoint cobordism ring.

Prof. Malkhaz Bakuradze (Tbilisi State University, Tbilisi, Georgia)
Prof. V. V. Vershinin (Montpellier 2 University, Montpellier, France)

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Abstracts of Plenary and Invited Speakers

პლენარული და მოწვეული მომხსენებლების თეზისები

On Some Questions of Algebraic Topology and General Topology

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The main aim of report is the constructions of some new (co)homological, shape and dimensional like invariants and their applications in the investigations of some open questions of algebraic topology, geometric topology and cohomological dimension theory.

The report envisages to study the following contrite problems.

Question I (V. Baladze)

Is it possible or not to make a new, different, simple description and axiomatic characterization of homology theory without using the relative homology groups?

We give the axiomatic characterization of the Čech spectral (co)homological groups of spaces [7] and the Čech spectral (co)homology groups of continuous maps only in the terms of absolute groups, induced homomorphism and suspension isomorphism.

Question II (T. Watanabe)

Let X and Y be topological spaces of some class and $X$and Ybe inverse systems consisting of "good" spaces, such that $X = \varprojlim\mathbf{X}$ and $Y = \varprojlim\mathbf{Y}$. Then:

- **for each continuous map $f : X \to Y$whether exists or not a map $\mathbf{f} : \mathbf{X} \to \mathbf{Y}$ of inverse systems, such that $f = \varprojlim\mathbf{f}$.**

We investigate an approximation problem of continuous maps of compact Hausdorff spaces by ANR($\mathbf{Mor_M}$)-maps and of continuous maps of general topological spaces by ANR($\mathbf{Mor_M}$)-resolutions.

We also construct and investigate a fiber shape theory of continuous maps of compact Hausdorff spaces.

Question III (T. Miyata)

Find the conditions under which the results of the classical cohomological dimension theory are true for the completely regular spaces.

We construct and give an axiomatic characterization of functional (co)homology groups $(\check{H}^n_F(X,G))$ $\hat{H}^n_F(X,G)$ of completely regular spaces based on functionally open finite coverings.

We define functional cohomological dimension $d_F(X;G)$ of completely regular spaces and establish defining its main properties.

Besides, we find out whether the equations $(\hat{H}^n_F(X;G) = \hat{H}^n_F(\beta X;G))$ $\check{H}^F_n(X;G) = \check{H}_n(\beta X;G)$, $d_F(X;G) = d(\beta X;G)$ hold, where $(\hat{H}^n_F(\beta X;G))$ $\check{H}_n(\beta X;G)$ is Čech cohomology groups and $d(\beta X;G)$ is the classical cohomological dimension.

Question IV (P. S. Alexander, Yu. M. Smirnov, J. Asrts)

Find a necessary and sufficient condition, under which the space of the given class has a compactification or completion, which, or whose reminder, has a given topological property.

We give characterizations of (co)homology groups, coefficients of cyclisity, cohomological dimensions of some compactifications and their remainders.

Remark. V. Baladze and A. Beridze also constructed the axiomatic characterization of (co)homology groups of Chogoshvili for topological spaces and continuouse maps.

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On Some Trends and Principles Related to Arbitrary Meromorphic or Analytic Functions in a Given Domain

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The first results (principles) related to arbitrary meromorphic, particularly analytic, functions in a given domain were established by Cauchy (19-th century), while the next results arisen a century later in Ahlfors theory of covering surfaces (1935).

In this survey we present some other (diverse type) results of the same generality which were obtained since 1970s.

The majority of these results occur in three trends in theory of meromorphic functions: Gamma-lines, proximity property, and universal version of value distribution theory.

Each of these trends complements the classical Nevanlinna value distribution theory or Ahlfors theory and also reveals some new type of phenomena.

Content: list of sections.

(The results in each section relate to arbitrary meromorphic or analytic functions in a given domain.)

1. Two principles related to derivatives.
2. Results related to level sets and Gamma-lines.
3. Three simple consequences related to a-points.
4. Ahlfors fundamental theorems in terms of windings and a new interpretation of deficient values.
5. Universal version of value distribution.

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On the Burau Representation for $n = 4$

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The Burau representation is a well–known matrix representation (or rather: a sequence of representations) of the braid group B_n. For quite a time it was conjectured that these representations might be faithful. The problem is now solved for all n except $n = 4$ (faithful for $n = 1, 2, 3$, not faithful for $n \geq 5$. The problem of faithfulness of the (reduced) Burau representation for $n = 4$ is known to be equivalent to the problem of whether certain two matrices A and B generate a free group of rank two (which is in itself a much studied classical problem).

More specifically:

$$A = \begin{bmatrix} 0 & 0 & -t^{-1} \\ 0 & -t & -t^{-1}+t \\ -1 & 0 & -t^{-1}+1 \end{bmatrix}, \quad B = \begin{bmatrix} -t^{-1} & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -t \end{bmatrix}.$$

In [1] we gave a simple proof that $\langle A^3, B^3 \rangle$ is a free group of rank two, the result known earlier from [1]. Also, we obtained a nice generalization of this result. In this talk I will show some further insight into the problem leading to an even more simple solution of the quoted theorem. The surprising starting point for this will be an observation that for a certain very simple matrix $T = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$, which may be easily checked to satisfy $T^4 = I$ we have $B^{-1} = T^2BT^2$, $A = T^{-1}BT$, $A^{-1} = TBT^{-1}$.

Given this it is very easy to prove the A^3, B^3 statement just by analyzing how the minimum degree of the variable t that appears in the considered product of matrices rapidly decreases with the number of matrices multiplied which means that the product cannot ever be equal to the unit matrix. We are currently working on the A^2, B^2 case.

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Superoscillations (Faster than Fourier): Vorticulture, Fractals, Escape...

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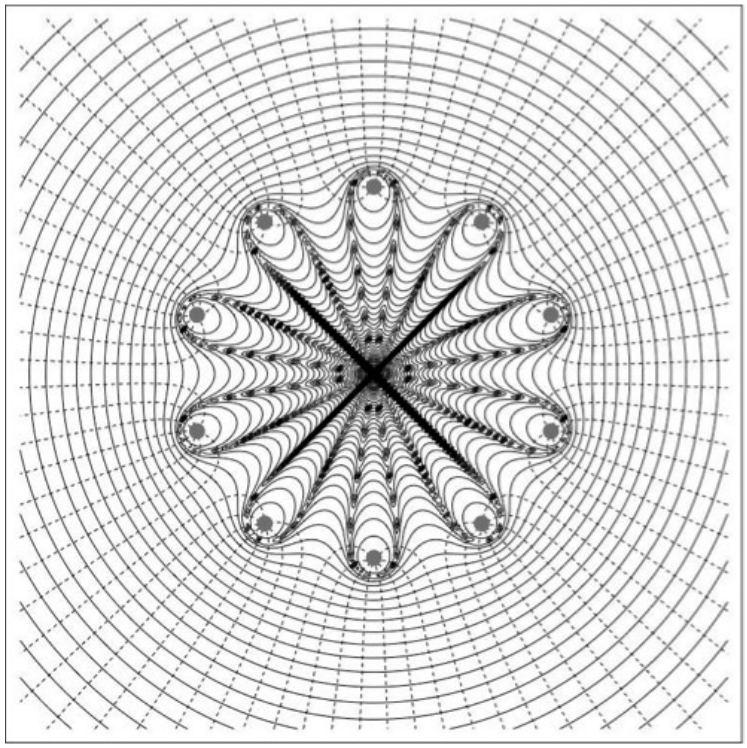
Band-limited functions can oscillate arbitrarily faster than their fastest Fourier component over arbitrarily long intervals: they can 'superoscillate'. In physics, this mathematical phenomenon is associated with almost-destructive interference, and occurs near phase singularities in optics and on the world's ocean tides; and it is associated with quantum weak measurements. Where superoscillations occur, functions are exponentially weak. They are an unexpectedly compact way of representing fractals. Superoscillations in red light can escape as gamma radiation.



Sharply Multiply Transitive Groups

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Let G be a group acting on a set X. If for every pair of ordered n-tuples of distinct elements $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ in X, there exists a unique $g \in G$ such that $gx_i = y_i$ for all $i = 1, \ldots, n$, then we say G acts *sharply n-transitively* on X. If a group has a sharply n-transitive action on a set, then we say that the group is *sharply n-transitive*.

The first important result in the study of sharply n-transitive groups was obtained in 1872 by Jordan; he proved that alternating groups, symmetric groups and the two Mathieu groups M_{11}, M_{12} are the only finite sharply n-transitive groups if $n \geq 4$. Later in 1936, Zassenhaus classified *finite* sharply 2-transitive and sharply 3-transitive groups, and hence settled the question in the finite case. In 1950's Hall and Tits, independently, proved that there is no *infinite* sharply n-transitive group for $n \geq 4$. Classification of infinite sharply 2-transitive and sharply 3-transitive groups are still open.

In my talk, after summarizing the known results in the area, I shall show how these concepts can be redefined in groups of finite Morley rank (which can be thought as groups with a dimension function) and hence many more examples of groups fall into this new category. Finally, I shall mention two recent classification results (one is joint with Alexandre Borovik, and the other is joint with Tuna Altınel and Frank Wagner).

Are Elementary Particle Being Truly Elementary?

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It is now almost clear that there is no meaningful symmetry scheme for classification of all observed quarks and leptons which are presently considered as elementary particles in the framework of the conventional Standard Model. Any attempt to describe all three quark-families leads to higher symmetries with enormously extended representations which contain lots of exotic states that never been detected in the experiment. This may motivate us to seek solution in some subparticle or preon models for quark and leptons

just like as in the nineteen-sixties the spectroscopy of hadrons required to seek solution in the quark model. By that time there was very popular some concept introduced by Murray Gell–Mann and called the Eightfold Way according to which all low-lying baryons and mesons are grouped into octets. We find now that this concept looks much more adequate when it is applied to the level of elementarity, namely, to elementary preons and composite quarks and leptons. It follows that under some natural circumstances just eight preons and their generic local $SU(8)$ symmetry may determine the fundamental entities of the Physical World and its total internal symmetry at very small distances.

Maxwell's Classical Theory in the Framework of Geometric Reductions Method

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In the framework of the generalized Hamiltonian formalism, canonical variables of the electromagnetic field are constructed, two of which are equated with constraints:

$$\prod\nolimits^{0}(t,\vec{x}) = \pi^{0}(t,\vec{x}), \quad \prod\nolimits^{1}(t,\vec{x}) = \operatorname{div}\pi(t,\vec{x}).$$

In this representation, the following gauge conditions are selected:

$$Q^{0}(t,\vec{x}) = 0, \quad Q^{1}(t,\vec{x}) = 0.$$

The relationship of this gauge with other known gauges has been studied.

Weissler's Conjecture on the Hamming Cube

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Let $1 \le p \le q$ and z be a complex number. The necessary and sufficient condition for L^p to L^q boundedness of the Hermite semigroup $e^{z\Delta}$ on the boolean cube of an arbitrary dimension equipped with uniform counting measure will be shown to be $|p-2-e^{2z}(q-2)| \le p - |e^{2z}|q$. This solves an old open problem in complex hypercontractivity theory on the Hamming cube. Certain cases of the triples (p,q,z) were characterized by Bonami (1970); Beckner (1975); and Weissler (1979). Several applications will be presented.

Work in progress with Fedja Nazarov.

A Class of Hausdorff–Berezin Operators

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We introduce and study the class $\mathfrak{K}$ of the defined below in (1) operators on the unit disc in the Lebesgue spaces with the Haar measure. Given a measurable function K (kernel) on the unit disc for a function f denote

$$\begin{aligned}\mathbb{K}f(z) &= \int\limits_{\mathbb{D}} K(w) f(\alpha_z(w))\,\mathrm{dH}(w) \\ &= \int\limits_{\mathbb{D}} K(\alpha_z(w)) f(w)\,\mathrm{dH}(w),\end{aligned} \tag{1}$$

where dH stands for Moebius invariant Haar measure

$$\mathrm{dH}(z) = \frac{dA(z)}{(1-|z|^2)^2}\,, \quad z \in \mathbb{D}.$$

Here

$$dA(z) = \frac{1}{\pi}\, dx\, dy, \quad z = x + iy \in \mathbb{D}.$$

An immediate example of such operators is given by the well known Berezin transform:

$$\mathbb{B}f(z) = \int\limits_{\mathbb{D}} f(\alpha_z(w))\, dA(w).$$

We find it convenient to call such operators as Hausdorff-Berezin operators.

We discuss certain algebraic properties of such operators, and also give sufficient and necessary boundedness conditions for such operators. We also reformulate the obtained results for the case of spaces and operators with usual Lebesgue measure.

The talk is based on joint work with co-authors Stefan Samko (Portugal), and Kehe Zhu (USA).

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Transmutation Operator Method for Practical Solution of Forward and Inverse Spectral Problems

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The transmutation (transformation) operators are one of the main theoretical tools of the spectral theory [6-8]. In the talk a new approach is presented for solving the classical forward and inverse Sturm–Liouville problems on finite and infinite intervals. It is based on the Gel'fand–Levitan–Marchenko integral equations and recent results on the functional series representations for the transmutation (transformation) operator kernels [1–5]. New representations of solutions to the Sturm–Liouville equation are obtained admitting the following feature important for practical applications. Partial sums of the series admit estimates independent of the real part of the square root of the spectral parameter which makes them especially convenient for approximate solution of spectral problems. In particular, a new representation for so-called Jost solutions is obtained reducing all the calculations related to spectral and scattering data to finite intervals instead of the half line or the whole line. This is the case of the spectral density function as well as of the reflection coefficient in the scattering problem. In a sense this reduction trivializes the classical spectral and scattering problems on infinite intervals previously considered as numerically challenging problems.

Numerical methods based on the proposed approach for solving forward problems allow one to compute large sets of eigendata with a nondeteriorating accuracy. Solution of the inverse problems reduces directly to a system of linear algebraic equations. In the talk some numerical illustrations will be presented.

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Hardy Type Inequalities in the Category of Hausdorff Operators

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Hardy type inequalities in the category of Hausdorff operators Classical Hardy's inequalities are concerned with the Hardy operator and its adjoint, the Bellman operator. Hausdorff operators in their various forms are natural generalizations of these two operators. We adjust the scheme used by Bradley for Hardy's inequalities with general weights to the Hausdorff setting. It is not surprising that the obtained necessary conditions differ from the sufficient conditions as well as that both depend not only on weights but also on the kernel that generate the Hausdorff operator. For the Hardy and Bellman operators, the obtained necessary and sufficient conditions coincide and reduce to the classical ones. For some types of Hausdorff operators, necessary and sufficient conditions coincide for power weights as well.

Instability and Stabilisation of the Chemotaxis Dynamics

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Name 'chemotaxis' designates the interaction of substances or species (often biological or chemical) which forces one of them to move on the macroscopic scale in response to another one. For instance, one can consider 'the predators' which pursue 'the prey' which, in turn, evade them. A common model for such interactions known as Patlak–Keller–Siegel' law (PKS) states that the pursuers direct their moving along the concentration gradients of the stimulus or signal they are searching.

A PDE' system describing the spatial-temporal dynamics of several species with chemotaxis (briefly, PKS system) usually joins the contributions from the kinetics, diffusion and advection. At that, describing the advective velocities with the use of PKS law puts the advection into the form of nonlinear cross-diffusion.

The present talk is about the excitation of wave motions in PKS systems. We'll be discussing some milestones in this area and some new results concerning the excitation/suppressing of waves by a signal emitted externally. For example, one can consider the predator-prey dynamics assuming the predators to be perceiving both the signal emitted by the prey and some external signal. The latter can represent, for instance, an environmental characteristic such as temperature, salinity, terrain relief, etc. We'll be considering the short-wavelength signals and discussing the drifts they give rise to upon homogenizing the specific non-linearity contributed by the PKS cross-diffusion.

Also, we'll be comparing the excitation of the self-oscillatory wave motions in the homogenized systems to the homogeneous system with no signal. The homogeneous system possesses the homogeneous equilibria the instabilities of which accompanied by the local bifurcations give rise to the wave motions. We'll see that similar transitions are possible in the homogenized system too but depending on the mentioned drift to a great extent. For instance, let the external signal represent a travelling wave having the amplitude a and wave speed c. Then there exists a critical value of the external wave speed, c_*, such that increasing the values of a enhance the excitation of waves in the homogenized system for $c > c_*$ and suppresses them for $c < c_*$.

The talk is based on joint work with Dr. K. Ilin, The university of York, UK.

Algebras and Varieties with Formulas of Second Order Logic

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In this talk:

(a) we present Schauffler's type theorems for new formulas of second order logic;

(b) we characterize the hyperidentities of lattice varieties;

(c) the structures of algebras and varieties with hyperidentities of lattice varieties are given;

(d) we present the varieties and hypervarieties of algebras with functional representations of free finitely generated algebras via generalized Boolean functions;

(e) a number of open problems are formulated.

The Bohrs Banach Algebraic Operator

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A brief history shall be given, basic properties on various domains shall be presented, in particular, the unit disk and half planes.

On the Calderón Operator for Maxwell's Equations

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The Calderón (also known as the Poincaré-Steklov) operator and its variants have been and are studied extensively in the mathematical theory of Electromagnetics, see, e.g., [1, 3, 6], and references therein.

The first part of the talk deals with the exterior Calderón operator (that maps the tangential scattered electric surface field to the corresponding magnetic surface field) for not necessarily spherical domains. A new approach of finding the norm of the exterior Calderón operator for a wide class of surfaces is presented. The basic tool in the treatment is the set of eigenfunctions and eigenvalues to the Laplace–Beltrami operator for the surface. The norm is obtained in view of an eigenvalue problem of a quadratic form containing the exterior Calderón operator. The connection of the exterior Calderón operator to the T-matrix for a perfectly conducting surface is also analyzed.

In the second part, we discuss a Steklov-type problem for Maxwell's equations which is related to an interior Calderón operator and an appropriate Dirichlet-to-Neumann map. The corresponding Neumann-to-Dirichlet map turns out to be compact and this provides a Fourier basis of Steklov eigenfunctions for the associated energy spaces. With an approach similar to that developed by Auchmuty [2] for the Laplace operator, we provide natural spectral representations for the appropriate trace spaces, for the Calderón operator itself and for the solutions of the corresponding boundary value problems subject to electric or magnetic boundary conditions on a cavity.

The talk is based on the works [4] and [5].

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Locomotion and Forcing of Rigid Bodies in a Fluid. An Overview of the Subject

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This lecture is devoted to the central problem of fluid dynamics: motion and forcing of rigid bodies in a fluid. It consists of two parts. In the first part we consider locomotion in a fluid. We give a review of basic achievements including pioneering steps by Benjamin (1966) and Saffman (1967), who considered self-propulsion of bodies and bubbles in an inviscid fluid. Then we analyse a key paper by Childress and all (2011), devoted to the generalizations of that results to a viscous fluid. As the original part, we present the results by V. A. Vladimirov (2019) and D. Kapanadze & V. A. Vladimirov (2019). We study 'recoil' locomotion of a robot (or an underwater vehicle) in an inviscid incompressible fluid. The robot's body is rigid, and its locomotion ability is due to an internal actuator. This can perform controlled translational and rotational oscillations. Our attention is focused on two classes of exact plane solutions, describing rectilinear locomotion. The solutions of the first class describe *tumbling locomotion*, while the second class corresponds to *zigzag locomotion*, without tumbling. We show that tumbling locomotion is more efficient. Both classes of solutions show that the use of an actuator allows to choose any desired direction and any speed of locomotion. As a special case, we consider the self-propulsion caused by small-amplitude and high-frequency oscillations. Due to the exactness of our solutions, the results are potentially useful to test and to verify physical and engineered models, as well as numerical results and asymptotic approximations. We also discuss generalizations of our results, which appear after taking into account such factors as the increasing of

dimension, controllability, viscosity, and flow separation. In the second part of the lecture we consider the factor of chirality (absence of mirror symmetry) in a system solid+fluid based on H. K. Moffatt & V. A. Vladimirov (2019) and the recent monograph '*Self-Exited Fluid Dynamos*' by H. K. Moffatt (2019). Suppose that viscous fluid is contained in the space between a fixed sphere S_2 and an interior sphere S_1 which moves with time-periodic velocity $\boldsymbol{U}(t)$ and angular velocity $\boldsymbol{\Omega}(t)$, with $\langle \boldsymbol{U}(t) \rangle = \langle \boldsymbol{\Omega}(t) \rangle = 0$. It is shown that, provided this motion is chiral in character, it can drive a flow that exerts a non-zero torque on S_2. Thus angular momentum can be transferred through this mechanism. Both considered directions show that classical areas of fluid dynamics are perspective in current research and faraway of its completion.

Abstracts of Participants' Talks

მონაწილეთა მოხსენებების თეზისები

Solution of the Elliptic Equations with m-Point Bitsadze–Samarskiĭ Boundary Conditions Using MEDG Method

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The present paper deals with optimal control problems whose behavior is described by elliptic equations with m-point Bitsadze–Samarskiĭ [3] boundary conditions. Necessary optimality conditions are established by using the approach worked out in [2] for controlled systems of general type. To investigate the conjugate problem, we use the algorithm reducing nonlocal boundary value problems to a sequence of Dirichlet problems. Such a method makes it possible to solve the problem numerically. In paper [4], for the numerical solution of the Dirichlet boundary value problem, the relaxation method is used. Modified Explicit Decoupled Group (MEDG) method uses a skewed difference formula which leads to lower computational complexities since the iterative procedure need only involve nodes on half of the total grid points in the solution domain and thus a reduced system of linear equations is attained. A MEDG method is presented for numerical solving an optimal control problem for elliptic equations by means of the Mathcad.

The results of solution of nonlocal boundary value and adjoint problems are presented in graphical form. Numerical experimentations of this new formulation shows significant improvement of computational complexity over the original relaxation method, which was introduced in work for elliptic Equations [1].

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Fluid Flow Motion Fluid Motion in Conjugate Reservoir–Well–Pipeline System

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There are frequent cases of connecting or withdrawing gas, liquid from existing pipelines. In this case, all changes occurring in the pipeline are transferred to the reservoir and thus the wells are transferred to another mode of operation.

The definition of transient modes of operation of wells associated with the consideration of the movement of fluid in the associated system of the reservoir-well-pipeline. Until now, unsteady fluid flow in the interfaced reservoir-well system remains poorly understood. Therefore, modeling and studying the movement of fluid in the interfaced reservoir-well-pipeline system presented to both the large scientific and practical interest to which this work is devoted.

Consider the movement of fluid in the associated reservoir-well-pipeline system. At some point in time to the main pipeline at a distance of l_2 from the wellhead was connecting the pipeline with a capacity of Q.

At the same time, we will determine what changes will occur in the well operation mode.

In the first approximation, we take the liquid homogeneous. Then the equation of the flat-radial filtration of the fluid will be:

$$\frac{\partial^2 \Delta P}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta P}{\partial t} = \frac{1}{\chi}\frac{\partial \Delta P}{\partial t}, \quad r_c \le r \le R_k, \quad t > 0, \tag{1}$$

The movement of fluid in the riser has the form

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - 2a\frac{\partial u}{\partial t}, \tag{2}$$

The movement of fluid in the pipeline is

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x^2} - 2a_1 \frac{\partial P}{\partial t} - \frac{2a_1 c^2 G\delta(x - l_2)}{f_1}, \tag{3}$$

Initial and boundary conditions

$$\begin{aligned} \frac{\partial P}{\partial t}\Big|_{t=0} &= -\frac{c^2 G\delta(x - l_2)}{f_1}, \\ P\big|_{t=0} &= P_{yc}(0) - 2a_1 Q_0 x, \\ P\big|_{x=0} &= P_{yc}(t), \\ P\big|_{x=l_1} &= P_2 = const, \end{aligned}$$

The definitions of a change in the mode of operation of a well can be determined by jointly solving the related equations (1)–(3) and this is done in representative work.

Research of Stressed State at Bending of Having Uncompensated Cuts Cylindrical Shells

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The analysis of mode of deformation of structure will be obtained on various stages of design process. In the engineering practice are applied various approximation methods related with features of objects under study.

For the lot of applied tasks is required to valuate stress in the cross-sections that have various arrangement and size cuts. As example let's determine the and compare variations in stresses in fixed cross-sections of shells: when the border of long cut passes on it and in order for determining of mode of deformation in according cross-sections with taking into account the different type cuts.

Erdösh–Mordell Inequality and Use of it to Solve the Tasks of School Mathematics Olympiads

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In school mathematics Olympiads we often meet the approval geometric inequalities, which are essentially a great effort to provide a thorough knowledge of the well-known and less common geometric facts for the broad masses. Our goal is to present one of the most important, compact and mathematical terms of a wonderful geometric factor, the inequality that was first officially published in 1935 by two well-known mathematician Paul ö and Louis Joel Mordell. It is true that the first version of the inequality was not so simple, but the better, relatively elementary approval was published in 1957. It is logical to know our inequality as the inequality of the Erdös–Mordell and this is how it is formed: For any triangle and point inside triangle, the sum of the distances from point to the sides is less than or equal to half of the sum of the distances from point to the vertices.

The work deals with the inequality of the ö–Mordell and its approval. The geometric tasks that come to the various mathematical Olympiads are also discussed, which are solved with this inequality. It should be noted that two of the tasks discussed in the work are the objectives of the International Mathematical Olympiad (IMO).

Existence of Consistent Criteria for Hypotheses Testing for Strongly Separable Statistical Structures

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Let (E, S) be a measurable space with a given family of probability measures $\{\mu_h, h \in H\}$.

Definition 1. An object $\{E, S, \mu_h, h \in H\}$ is called a statistical structure.

Definition 2. A statistical structure $\{E, S, \mu_h, h \in H\}$ is called strongly separable if there exist a disjoint family of S-measurable sets $\{X_h, h \in H\}$ such that the realations are fulfilled: $\mu_h(X_h) = 1$, $\forall\, h \in H$.

Let H be a set of hypotheses and let $B(H)$ be a σ-algebra of subsets of H which contains all finite subsets of H.

Definition 3. We will say that the statistical structure $\{E, S, \mu_h, h \in H\}$ admits a consistent criterion for hypothesis testing if there exists at least one measurable mapping $\delta : (E, S) \to (H, B(H))$ such that $\mu_h(\{x : \delta(x) = h\}) = 1$, $\forall\, h \in H$.

Here we give the construction of strongly separable statistical structure that does not have any consistent criterion for hypotheses testing. Let ω_1 be the first uncountable ordinal number. It is well known that the space $[0, \omega_1]$ is compact and the space $[0, \omega_1)$ is locally compact. Denote by $B([0, \omega_1))$ the Borel σ-algebra of the topological space $[0, \omega_1)$ and define the probability measure on it as follows:

$$\mu(Z) = \begin{cases} 1, & \text{if } Z \text{ contains some unbounded closed set;} \\ 0, & \text{if } [0, \omega_1) \setminus Z \text{ contains some unbounded closed set.} \end{cases}$$

We suppose that $E = [0, \omega_1) \times [0, \omega_1)$, $S = B([0, \omega_1) \times [0, \omega_1))$ and take as a set of hypotheses the set $H = (\{0\} \times [0, \omega_1)) \cup ([0, \omega_1) \times \{0\})$.

If $h = (0, \xi) \in H$ $\xi < \omega_1$ we can define the probability measure μ_h as $\mu_h(Z) = \mu(Pr_1(Z \cap [0, \omega_1) \times \{\xi\}))$, and if $h = (\xi, 0)$ we define the probability measure μ_h as $\mu_h(Z) = \mu(Pr_1(Z \cap (\{\xi\} \times [0, \omega_1)))$. The statistical structure $\{E, S, \mu_h, h \in H\}$ is strongly separable, and there are no a consistent criteria on it.

Remark 1. In the constructed statistical structure $\{E, S, \mu_h, h \in H\}$ all measures are two-valued. If we consider the probability measures $(\mu_h \times b)_{h \in H}$ then this statistical structure will be strongly separable and will not have any consistent criterion for hypotheses testing, at the same time all measures are nonatonic and separable.

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On the Uniformization of Planar Curves and the Two-dimensional Jacobian Problem

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Some algebraic properties of the pairs of polynomials on C^2 are established. It is proved that the homogeneous forms of the highest degree of two polynomials with the constant non-zero Jacobian have a non-trivial common divisor. This enable one to obtain a simple proof of the well-known result for the case where the degrees of given polynomials have non-trivial common divisors. A particular case of the Keller Jacobian hypothesis for gradient pairs will also be presented.

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Varieties of Exponential MR-Groups

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In this report we introduce the notion of a variety of exponential MR-groups and tensor completions of groups in varieties. We study relationships between free groups of a given variety under different rings of scalars and describe varieties of abelian MR-groups. Moreover, in the category of MR-groups, we consider several analogs of n-class nilpotent groups. We got that the completion of a 2-class nilpotent group is a 2-class nilpotent.

About One Heuristic Algorithm of Solution of a Problem of Optimization

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The main complexity of solution of optimization problems is related to non-linearity of functions and scales of problems [2]. Searching of optimal values with classical techniques sometimes finishes without results. In the given paper heuristic algorithm of solution of a problem of optimization is offered. The algorithm is based on main principles of classic genetic algorithm [1]. Genetic operators are considered. Selection is made with technique of inbreeding. Modified Operators of crossover and mutation is offered. Algorithm is approved for test problems. The quantitative results prove efficiency of our modified genetic operators and their combinations.

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Identification of Biomass Growth Model

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In model of growth of plants it is possible to allocate two blocks which should be present at all dynamic models: the block of accumulation or growths biomass which describes increase in a biomass due to assimilation of carbonic acid of air and due to

receipt of elements of a mineral feed from ground; the block describing in the form of system of the dynamic equations redistribution of these substances on bodies. These blocks make a basis of model of formation of a crop [2].

In order to determine the parameters of the intensity of photosynthesis in the biomass growth model, we select the criterion, which is useful to calculate the search parameters. Such a criterion can be used for square functionality.

$$S = \sum_{t\lambda} \left\{ {}_{t\lambda}(k_1, k_2, k_3\alpha\gamma) - m_{et\lambda} \right\}^2,$$

where t_λ is moments of measuring biomass; $m_{t\lambda}$ is theoretical implications are from the daily dose of dry biomass on the unit area of the field; $m_{et\lambda}$ is experimental values of dry biomass daily growth on the area of a single field.

During the operation of the model with the operational adjustment of irrigation management, it is advisable to achieve the best approximation not only of the total biomass, but also the biomass of individual organs [1],

$$S = \sum_{i=1}^{4} \sum_{t_\lambda=t_1}^{t_N} \left\{ M_i(t_\lambda) - M_{ei}(t_\lambda) \right\}^2 \longrightarrow \min,$$

where i is plant organisms index: 1 – leaves, 2 – stalk, 3 – ears of corn, 4 – roots.

This image is used to find unknown parameters of biomass growth.

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Modern Forecasting Models in Economy

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A new approach for business forecasting is discussed in the article. This means usage of parallel data paradigm of programming. Parallel data are different kind of former data, which give us chance to predict an event in dynamic mode. Also, functioning of forecasting process online is being discussed. This method helps us to use super computers not only for original purpose – calculation big amounts, but for processing parallel data online.

Business-forecasting tasks may include: demand, intermittent demand, time and space hierarchies, shares, macroeconomic indicators, commodity groups, new products and more. Consider two tasks: predicting demand and interruption request. That's the solution of these two tasks will be discussed in the example when "parallel data".

The matrix, corresponding to the model of predictable processes is dynamically variable, which means a change of its sizes. The matrix constantly expanses upwards, new data is added to it and the number of columns varies, the data is added or taken away according to the event function, which will be considered below. It is also possible that a matrix is expanded downwards – the data is added, corresponding to old, already occurred events or archive data.

Each expandable matrix for each business forecasting task may describe the one territorial region, therefore, for the certain task of prediction, more than one expandable matrix can exist, which will be built for certain period of time.

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On the Estimating the Bernoulli Regression Function Using Berstein Polynomials

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The estimate for the Bernoulli regression function is constructed using the Bernstein polynomial. The question of its consistency and asymptotic normality is studied. Testing hypothesis is constructed on the form of the Bernoulli regression function. Also, the test is constructed for the hypothesis on the equality Bernoulli functions. The question of consistency of the constructed tests is studied.

On the Limit Distribution of the Integral Square Deviation of a Nonparametric Estimator of the Bernoulli Regression Function

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The limiting distribution of the integral square deviation of a kernel-type nonparametric estimator of the Bernoulli regression function is established. The criterion of testing the hypothesis about the Bernoulli regression function is constructed. The question as to its consistency is studied. The asymptotic power of the constructed test is also studied for certain types of close alternatives.

On Uniform Convergence of Rearranged Fourier Series

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Dedicated to 80-th birthday anniversary of Zaur Chanturia (1939–1989)

In 1905 H. Lebesgue has shown that the Fourier series of a continuous periodic function f may not converge uniformly even if it converges point-wise to f. For some known conditions for uniform convergence see [1]. In 1964 B. S. Stechkin [5] has found a continuous periodic function f, whose Fourier series diverges at some points, but the rearranged Fourier series converges uniformly to f. Later D. V. Pecherskij [3] and Sz. Gy. Révész [4] independently have found an additional condition on the Fourier series of a continuous periodic function f, which guarantees that some its rearrangement converges uniformly to f.

We plan to discuss the question whether Pecherskij–Révész's condition is always satisfied for a differentiable function.

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Elastic-Plastic State of Thin-Walled Structures

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In the paper is developed such method of analysis in the conditions of elastic-plastic state of having orthogonal cuts thin-walled structures that provides the determination of elastic-plastic state on arbitrary stage of loading. Are created design diagrams including from plates having orthogonal cuts of thin-walled structures. Is developed the method of analysis of having cuts and orthogonal holes orthogonal in plane plates and shells with taking into account different boundary conditions on various type contours as well as arrangement of various size and location holes.

Sinkhorn Algorithm and Problem of Regularization for FreeForm Optics Applications

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FreeForm Optics is the branch of Optics concerned with the design of non-conventional asymmetric refractive and reflective optical elements or systems of such elements. On the academic side, two classes (collimated or point source illuminance) of idealized tailoring irradiance problems can be exactly modeled and solved using Optimal Transport theory. Optimal Transport defines unique map or a coupling between prescribed distributions

representing given illuminance and irradiance. This map then can be used to construct the optical element [3].

There are several different approaches for finding numerical solution of Optimal Transport problems, varying in efficiency, accuracy and complexity ([1] and references therein). This work concentrates on Sinkhorn algorithm. Main advantage of Sinkhorn algorithm is it's simple structure of implementation, involving only simple basic linear algebra operations, and it's fastness both from mathematical foundation and from wide selection of fast linear algebra libraries [2].

Although this advantage comes with price of accuracy: Sinkhorn algorithm is based on Entropic regularization, that is, altering cost functional which is minimized, by adding entropic term. On one side this modification makes sure new target is strictly convex, and has the unique solution which has structure that provides way of finding it using Sinkhorn algorithm, but on the other hand, it alters the map which should be used to construct Optical Element.

This work discusses and proposes ways of lessening this regularization effect while trying to maintain beneficial properties of Sinkhorn algorithm.

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On Cardinality Numbers of Certain Classes of Measures

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We study the measurability properties of sets and functions with respect to certain classes of measures. In the presented talk we consider various families of the measures

and their characterizations in the sense of the set theory. More precisely, we investigate the cardinality of some classes of measures.

For example, it is well known, that there exists continuumly many measures ν on $\mathbf{R}$, which extend λ, where λ is standard Lebesgue measure, are quasi-invariant under the group of all translations of $\mathbf{R}$ and satisfy the relation $H \in dom(\nu)$.

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On the Axiomatic Systems of Singular Cohomology Theory

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On the category of pairs of topological spaces having a homotopy type of CW complexes the singular (co)homology theory was axiomatically studied by J. Milnor [3]. In particular, Milnor gave *additivity axiom* for a (co)homology theory and proved that any additive (co)homologe theory on the given category is isomophic to the singular (co)homology. On the other hand, the singular homology is a homology with compact support [1]. In the paper [2], L. Mdzinarishvili proposed *partially compact support property* for a cohomology theory and gave another axiomatic characterization of the singular cohomology theory [2]. In this paper, we will give additional different axiomatic characterizations of the singular cohomology theory. Moreover, we will study connections of the mentioned axiomatic systems.

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An Application of Yu. V. Prokhorov's SLLN

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Dedicated to 90-th birthday anniversary of Yuri Vasilyevich Prokhorov (1929–2013)

A sequence $(\xi_n)_{n\in\mathbb{N}}$ of real-valued random variables is said to satisfy

- *the weak law of large numbers*, for short, $(\xi_n)_{n\in\mathbb{N}} \in WLLN$ if the sequence $\big(\frac{1}{n}\sum_{k=1}^{n}\xi_k\big)_{n\in\mathbb{N}}$ converges *in probability* to zero;
- *the strong law of large numbers*, for short, $(\xi_n)_{n\in\mathbb{N}} \in SLLN$ if the sequence $\big(\frac{1}{n}\sum_{k=1}^{n}\xi_k\big)_{n\in\mathbb{N}}$ converges *almost surely* to zero.

In 1949 Yu. V. Prokhorov has announced and then in 1950 published with a complete proof the following result [2, 3]:

Theorem 1. *For a sequence $(\xi_n)_{n\in\mathbb{N}}$ of independent of real-valued centered Gaussian random variables the following statements are equivalent:*

(i) $(\xi_n)_{n\in\mathbb{N}} \in SLLN$.

(ii) *For every $\varepsilon > 0$ we have $\sum_{r=1}^{\infty} e^{-\frac{\varepsilon}{H_r}} < \infty$, where*

$$H_r = \frac{1}{2^{2r}} \sum_{k=2^r+1}^{2^{r+1}} \mathbb{E}\xi_k^2, \quad r = 1, 2, \dots .$$

A generalization of Theorem 1 and a survey of related results can be seen in [1].

Next we recall a classic result of P. L. Chebyshev:

Theorem 2. *Let $(\xi_n)_{n\in\mathbb{N}}$ be a sequence of second order pair-wise uncorrelated real-valued centered random variables such that*

$$\lim_n \frac{1}{n^2}\sum_{k=1}^{n}\mathbb{E}\xi_k^2 = 0. \tag{1}$$

Then $(\xi_n)_{n\in\mathbb{N}} \in WLLN$.

We plan to derive from Theorem 1 that the condition (1) of Theorem 2 may not imply the validity of the stronger conclusion $(\xi_n)_{n\in\mathbb{N}} \in SLLN$ even for a sequence $(\xi_n)_{n\in\mathbb{N}}$ of *independent* of real-valued centered Gaussian random variables.

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Approximate Solution to the Plane Dynamic Problem of Thermodiffusion

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The paper deals with the investigation of a plane dynamic problem of the conjugate theory of thermodiffusion with mixed boundary conditions for multiply-connected domains. The approximate solution to the plane dynamic problem of the conjugate theory of thermodiffusion is constructed by the method of generalized Fourier series.

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Breadth-First Search Algorithm

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The search algorithm is a step-by-step procedure to find specific data and retrieve elements from any data structure. The breadth-first search is one of the examples of the search algorithm, which is the simplest traversing algorithms for searching a graph, it can handle a graph with billions of vertices and edges. One of the main problems with the breadth-first search is that it consumes a lot of time. However, it is used widely for various data structure and has many interesting applications.

The aim of this thesis is to give an overview of the graph search algorithm. In particular, we study the breadth-first search algorithm, implement it and describe some of the applications.

Depth First Search Algorithm

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Data structure plays an important role in raising the performance of a program or software. The software should store and retrieve the data needed as fast as possible. The hardest part for the programmers is to choose the appropriate data structure for a program, according to the type of operation is required. There are several types of data structures help us to make different operations on data. The depth-first search is an

algorithm for traversing or searching tree or graph data structures. It is widely used in practice and has many interesting applications. The purpose of this thesis is to provide an overview of the graph search algorithm, especially, we are studying the depth-first search algorithm, implementing it and describing some of the applications.

Credit Card Fraud Detection Using Machine Learning: a Realistic Modeling and a Novel Learning Strategy

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Detecting frauds in credit card transactions is perhaps one of the best testbeds for computational intelligence algorithms. In fact, this problem involves a number of relevant challenges, namely: concept drift (customers habits evolve and fraudsters change their strategies over time), class imbalance (genuine transactions far outnumber frauds) and verification latency (only a small set of transactions are timely checked by investigators). However, the vast majority of learning algorithms that have been proposed for fraud detection, relies on assumptions that hardly hold in a real-world Fraud Detection System (FDS). This lack of realism concerns two main aspects: i) the way and timing with which supervised information is provided and ii) the measures used to assess fraud-detection performance. There are three major contributions. First will test different methods on skewed data. The idea is to compare if preprocessing techniques work better when there is an overwhelming majority class that can disrupt the efficiency of our predictive model. We also illustrate the most appropriate performance measures to be used for fraud-detection purposes. Second, we design and assess a novel learning strategy which effectively address class imbalance, concept drift and verification latency. Third, in our experiments we demonstrate the impact of class unbalance and concept drift in a real-world data stream containing more than 75 millions transactions, authorized over a time window of three years. Besides these transactions, we will use various predictive models to see how accurate they are in detecting whether a transaction is a normal payment or a fraud. As described in the dataset, the features are scaled and the names of the features are not shown due to privacy reasons. Nevertheless, we can still analyze some important aspects of the dataset.

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Credit Card Fraud Detection Using Naïve Bayes Model

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Machine Learning is the technology, in which algorithms which are capable of learning from previous cases and past experiences are designed. It is implemented using various algorithms which reiterate over the same data repeatedly to analyze the pattern of data. The techniques of data mining are no far behind and are widely used to extract data from large databases to discover some patterns making decisions. This paper presents the Naïve Bayes improved K-Nearest Neighbor method (NBKNN) for Fraud Detection of Credit Card. Naïve Bayes is simple but powerful algorithm for predictive modeling and classification. Experimental results illustrate that both classifiers work differently for the same dataset. The purpose is to enhance the accuracy and enhance the flexibility of the algorithm.

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Boundary Value Problems for an Infinite Layer with Voids

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In the present paper the basic three-dimensional BVPs of equilibrium linear theory for materials with voids are investigated for an infinite layer. The representation of regular solution of the system of equations in the considered theory is constructed by means of the elementary (harmonic, bi-harmonic and meta-harmonic) functions. Using the Fourier transform the BVPs are solved effectively,(in quadratures) for the infinite layer.

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The Boundary Value Problem for One Equation Describing Processes Taking Place in Magneto Hydraulic Pusher

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In the work is studied the boundary value problem for one equation, which is received by mathematical modeling of processes taking place in the magneto hydraulic pusher of new construction taking into account the hydraulic resistance [1]. There is shown the uniqueness of solution, which is written in an explicit form.

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On Nilpotent Power MR-Groups

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The notion of a power MR-group, where R is an arbitrary associative ring with unity, was introduced by R. Lyndon. A. G. Myasnikov and V. N. Remeslennikov gave a more precise definition of a R-group by introducing a complementary axiom. In particular, the new notion of a power MR-group is the direct generalization of the notion of a R-module to the case of noncommutative groups. In the present paper, central series and series of commutants in MR-groups are introduced. Three variants of the definition of nilpotent power MR-groups of step n are discussed. It is proved that for $n = 1, 2$ all these

definitions are equivalent. The question of the coincidence of these notions for $n > 2$ remains open. Moreover, it is proved that the tensor completion of a 2-step nilpotent MR-group is 2-step nilpotent.

About Behavior of Transformation Asymmetric of Medium Square Integral Deviation Laplace Assessment of Distribution Density

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Is discussed unknown behavior of transformation asymmetric of medium square integral deviation Laplace assessment of distribution density. Proposed method gives an opportunity to unsubscribe stranger quotients by the limit until the law as an apparent form.

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On Queuing System with Bifurcation of Arrivals

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The Investigation subject of this paper is a multi-component redundant system with unreliable repairable units. The system consists of identical m active and n redundant units. We consider here the case, where m is a large number and we will suppose $m = \infty$, n is non-negative integer number.

The total failure rate of all active units is α. The failure rate for redundant units is β. A failed active unit is replaced by a serviceable redundant one if there is an available unit in the system. Otherwise the replacement will be performed after the availability of the redundant unit. The repaired failed units become identical with the new ones and pass to the group of redundant units. The system has infinite number of replacement and repair facilities. The replacement time and the repair time are random variables with exponential distribution functions with parameters λ and μ, respectively. As we see, in a natural way we have queuing system with two types of maintenance operations – replacement and repair. The request for the replacement arises due to failure of the active units. The same event generates a request for repair. In this way, the necessity of two parallel service operations arose and this is the bifurcation of failure flow which is the most important feature of our investigation. Besides, the failure of redundant units also generates request for repair.

As mathematical model we obtain infinite system of differential equations. In steady state it reduces to infinite system of linear algebraic equations. At present the system is still being investigated.

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On the Integral-Type Functional of Multidimensional Probability Distribution Density

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Asymptotical behavior of Laplace transformation of mean square integral deviation of non-parametric Chentsov-type projection estimate of multidimensional probability distribution density relating Lebesque measure in square integrable function space is considered.

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Mixed Boundary-Transmission Problem of Pseudo-Oscillation for Metallic-Electro-Magneto-Elastic Composite with Interface Crack

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We investigate the Pseudo-Oscillation mixed boundary-transmission problem of the generalized thermo-electro-magneto-elasticity theory for homogeneous anisotropic composite solid consisting of electro-magneto-elastic medium and metallic inclusion and containing an interface crack. The distribution of the thermo-mechanical and electromagnetic fields in the electro-magneto-elastic medium is described by generalized thermo-electro-magneto-elasticity theory associated with Green-–Lindsay's model. The thermo-mechanical field in the metallic inclusion is modelled with Lord–Shulman theory of generalized thermoelasticity.

By the potential method the considered problem is reduced to equivalent strongly elliptic system of pseudodifferential equations on manifolds with boundary. We study the solvability of this system in appropriate function spaces and prove uniqueness and existence theorems for the original problem.

We also analyse the regularity properties of the corresponding thermo-mechanical and electric fields near the crack edges and near the curves where the Dirichlet and Neumann boundary conditions collide. In particular, we characterize the stress singularity exponents and show that they can be explicitly calculated with the help of the principal homogeneous symbol matrices of the corresponding pseudodifferential operators.

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The Applied Character of Financial Mathematics

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Significant changes occur in mathematics throughout Georgia, in particular: mathematics development, mainly related to the relevant tasks of natural sciences and the fields of science with the requirements of the military-industrial complex.

Today, there is a field of mathematical knowledge that explores the mathematical problems of finances. A researcher of this field of mathematics is called actuarius, and the math direction itself is called actuarial.

Actuarius assesses the expected risks and randomness and uses the knowledge to solve financial problems. It is necessary for actuarius to be not only good mathematician but economist and legal education representative, the knowledge obtained in all three directions creates an actuarial science.

As a modern view, the actuarius can be called an insurance mathematics specialist, which implies not only life insurance but also includes financial risks insurance, including the game on the paid shield market.

Students who are interested in this direction will be obliged to care for revenue and they will face a lot of unresolved tasks, including those where it is necessary to study the potential financial risks and be able to analyze it. This problem is solved by the knowledge of financial mathematics, which uses the methods of probability theory and mathematical statistics.

On the Riesz Basisness of the Root Functions of a Sturm–Liouville Operator with a Conjugate Conditions

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In this study we interested in the discontinuous Sturm–Liouville operator on the interval $[-1, 1]$ with periodic boundary conditions which are not strongly regular and with

conjugate boundary conditions at the point zero. We obtain the asymptotic formulas of eigenvalues and eigenfunctions of the boundary value problem, when the potentials are complex valued function. By the aid of these asymptotic formulas, Riesz basisness in $L_2(-1,1)$ of the root functions are proved.

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The Method of Probabilistic Solution for Determination of Electric and Thermal Stationary Fields in Conic and Prizmatic Domains

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In our report, for determination of the electric and thermal stationary fields the Dirichlet ordinary and generalized harmonic problems are considered. The term "generalized" indicates that a boundary function has a finite number of first kind discontinuity curves. For numerical solution of boundary problems we apply the method of probabilistic solution (MPS), which in its turn is based on a modeling of the Wiener process. The algorithm suggested by us does not require an approximation of a boundary function, which is main of its important properties. For examining and to illustrate the effectiveness and simplicity of the proposed method four numerical examples are considered on finding the electric and thermal fields. Namely, in order to test the algorithm the following non-trivial domains: finite right circular cone and truncated cone, a rectangular parallelepiped are taken. Numerical results are presented.

On Probabilistic Methods of Scheduling One Task From Discrete Optimization Problems

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One task from discrete optimization problems, which contains some stochastic parameters is considered. Our goal is to found efficient heuristic and optimal (in some special cases) polynomial time algorithms.

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Superdiffusive Transport in Near Earth Plasmas with Shear Flows

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Analytical and numerical simulations' data show that the transport of energetic particles in the presence of magnetic turbulence can be superdiffusive. The so-called anomalous transport has gained growing attention during the last two decades in many fields including laboratory plasma physics, and recently in astrophysics and space physics. Here the examples, both from laboratory and from astrophysical plasmas are shown, where superdiffusive transport has been identified, with a focus on what could be the main influence of superdiffusion on fundamental processes like diffusive shock acceleration and heliospheric energetic particle propagation. The use of fractional derivatives in the diffusion equation is also discussed, and directions of future investigations are indicated.

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On a Conjecture of Eusebio Corbacho

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Dedicated to 70-th birthday anniversary of Eusebio Corbacho Rosas

In 2011 professor *Eusebio Corbacho Rosas (University of Vigo, Spain)* had conjectured the validity of the following statement:

Proposition. *Let $x > 0$ be a real number such that*

$$\sqrt{x + \frac{1}{x} + 2}$$

is a natural number.

Then for every natural number k

$$\sqrt{x^k + \frac{1}{x^k} + 2}$$

is a natural number as well.

We plan to discuss two proofs of this proposition. We plan to show also that among *countably many* real numbers $x > 0$ satisfying the assumptions of this proposition *the only rational number is* 1.

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ქართული უნივერსალური ჭკვიანი კორპუსი როგორც ერთიანი ქართული საინტერნეტო ჭკვიანი ქსელის პირველი ლაბორატორიული პროტოტიპი

მერაბ ჩიქვინიძე, კონსტანტინე ფხაკაძე, გიორგი ჩიჩუა, დავით კურცხალია, შალვა მალიძე, კონსტანტინე დემურჩევი

ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

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მოსაუბრე თვითგანვითარებადი ქართული უნივერსალური ინტელექტუალური კორპუსი, რომელიც ჩვენ ავაგეთ შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ 2017 წელს წარმატებულად გამოცხადებული AR/122/4-105/14 პროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ" ფარგლებში, გამომდინარე მასში ჩადგმული სახით არსებული ქართული და აფხაზური ინოვაციური ენობრივი ტექნოლოგიური სისტემებიდან, ჩვენს მიერ განიხილება აგრეთვე როგორც ერთიანი ქართული საინტერნეტო ქსელის ლაბორატორიული პროტოტიპი [1, 2].

ამგვარად, გარდა ზემოთ უკვე აღნიშნულის წარმოჩენისა, მოხსენების მიზანია აგრეთვე ზემოაღნიშნული სახის ერთიანი ქართული სახელმწიფო საინტერნეტო ქსელის ფორმირების აუცილებლობის დასაბუთება. — საქმე ისაა, რომ, დღეს, უკვე კარს მომდგარ ციფრულ ეპოქაში ანუ, სხვა სიტყვებით, უკვე კარს მომდგარ სხვადასხვა ბუნებრივი ენების სრულყოფილად მცოდნე მანქანების ეპოქაში, ხომ სრულიად ცხადია რომ ქართული ენის სრულყოფილად მცოდნე ამძრავი მანქანა არ შეიძლება არ იყოს ქართული სახელმწიფოს ანუ ქართველი ერის კუთვნილებაში!

ასევე, მაშინ როდესაც უკვე დღეს ქართული და აფხაზური კულტურული პროდუქციის 80% პროცენტზე მეტი ელექტრონული სახით იქმნება, რაც ახლო მომავალში 100%-ით ასე იქნება, რა უფლება გვაქვს არ ვიზრუნოთ ამ ყველაფრის ანუ საქართველოს სახელმწიფო ენებით ქმნადი კულტურლი პროდუქციის ქვეყნის შიგნით „დასერვირება" — შენახვაზე იმგვარად მოწესრიგებული სახით, რომ მომავალ თაობებს არ გაუჭირდეთ საჭირო დროს მათთვის საჭირო ცოდნისა და ინფორმაციის დროული მოძიება.

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ლიტერატურა

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სასწავლო-სამეცნიერო ცენტრის სამეცნიერო-საგანმანათლებლო ჟურნალის „ქართული ენა და ლოგიკა“ დამატების სახით, 2017, 4-320.

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Nonlinear Mathematical Model of the Competition Between Two Universities Considering Process of Mobility of Students

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Earlier we considered mathematical models of interference of fundamental and applied researches [1, 2] and so two or three-stage trainings of the diplomaed scientific experts [3, 4].

In this work a nonlinear mathematical model of competition for micro-constraints between two universities (micro model, for one region, where only two choice) is discussed. The micro-model of the competition considers the change in the contingent of students in these universities during the year.

In particular, the model provides a new stream of first-year students (applicants join the PR administration); cooperation (collaboration) between graduates and university applicants; students at this university advertise their university to mobilize students from a second university. For special cases of parameters of model in quadratures exact analytical solutions are found.

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Problems of Minimization in Mathematical Models of Resolution of Conflicts

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In the recent years, mathematical modeling has penetrated into social sphere, such as linguistic globalization processes [1].

Earlier Prof. T. Chilachava offered [2] and we discussed nonlinear mathematical models of economic cooperation for resolution existing political conflict between two opposing political sides [3-–6].

In the given work are discussed two nonlinear mathematical models: in the first one the process of economic cooperation is free from political pressure, in the second – the governments of both sides encourage the process of economic cooperation.

In case of some ratios between constant coefficients of mathematical models exact analytical solutions of Cauchy's tasks for nonlinear two-dimensional dynamic systems are found. Through management parameters existing in mathematical models it is possible to determine the conditions for which the conflicts are peacefully solvable i.e. for fixed coefficient of aggressiveness, for quantities of the population of sides (zero demographic factor), minimum values of coefficients of cooperation and coercion for cooperation (minimum economic expenses for peaceful settlement of the conflict) are found.

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Numerical Modeling of Fracture of Reinforced Concrete Shells

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The feature of reinforced concrete shells lays in that their analysis without taking into account the crack origination don't gives the possibility to obtain rather complete imagination on work of loaded structure and on it's rational reinforcement. Rather perspective direction is presented by numerical modeling of origination and propagation of fracture zones in the process of increasing of loading. As result of such analysis is possible to determine the cracks resistance of structure, observe on formation of it's development scheme and determine the load bearing ability.

The Systems of Ordinary Differential Equations on Graphs

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The different processes in networks of electrical power systems, gas transmission and distribution pipelines, other pipelines carrying material such as water, etc. can be described using mathematical models with nonstationary systems of nonlinear partial differential equations given on graphs. But for practical realisation, the linear models are used, which do not depend on the time.

In the present work, the boundary value problem is considered for the system of linear second order ordinary differential equations, given on graphs. The existence and uniqueness of the solution of the formulated problem are proved. The numerical method for solving this problems is proposed. In the case of constant coefficients, the analytical solution of the problem is constructed.

Logical Proving Applications in Meteorological Tasks

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Automated reasoning is one of the most important research area in logic and computer science. It is also considered as a sub-field of artificial intelligence. It studies different aspects of reasoning. The most important tools of automated reasoning are different calculi for classical logic.

We study TSR-logic based methods, that can be used in automated theorem proving and develop TSR solver. The main targe to solver is to be used in real life applications. One of such applications is enviromental contamination and weather forecast. The expected results will have as practical as well theoretical character.

Study of Local Scale Convection Forecasts by Different Physical Options of WRF Model and READY System above Complex Terrain

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At present climate change problem is associated with increased anthropogenic environment pollution, more frequent heavy precipitations, hails, floods and droughts with a growing desertification processes in the territory of Georgia. Namely for the last four decades the number of the natural hazards has increased about three times in comparison with 60 years period of the last century on the territory of Georgia. So the question of studding formation of hazardous precipitations on the background of modern climate change is an urgent issue for Georgia.

In this article a comparison study of the results of numerical calculations of three cumulus parameterization and five micro physics schemes of the Weather Research Forecast (WRF) v.3.6 model and the Real-time Environmental Applications and Display System (READY), against the radar's observational data, on the background of four exceptional local scale precipitation events occurred in the capital city of Georgia Tbilisi during summertime of 2015 and 2016 years is presented. Also for evaluation of summer time short term, local scale, heavy showers prediction in Tbilisi area READY System is used. Aerological diagrams of READY system for discussed cases precisely showed instability of atmosphere on local territory despite of the fact that in all four cases we had different level of instability. Predicted accumulated total precipitations (24 h) are evaluated by careful examination of meteorological radar and radio zoned data against the WRF simulated fields. Some results of the numerical calculations executed for warm season convective events are presented and analyzed.

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Modeling of Dew Point Temperature Localization in Gas Pipeline

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The main causes of gas line constipation (emergency shutdown) are the formation of hydrates, freezing of water jams, pollution, and so on. In order to take timely measures against the formation of hydrates, it is necessary to know better the distribution of humidity, pressure and gas temperature in the pipeline. It is well known that a favorable condition for the formation of hydrates in the main pipeline is the place where the dew point occurs (depending on pressure and humidity). Indeed, the dew point is the temperature below which liquid droplets begin to condense, and dew can form. In this paper, we study the problem of predicting the possible points of hydrate occurrence in main pipelines, taking into account unsteady gas flow and heat exchange with the medium. To solve the problem, a system of partial differential equations has been investigated that controls unsteady gas flow in the gas pipeline. The problem solution for adiabatic gas flow is presented. Numerical calculations showed the effectiveness of the proposed method. Some results of numerical calculations are presented.

Selection of Motivational Tasks and Formulating the Purpose on the Mathematics Lesson

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Proper planning of the mathematics lesson makes the learning process effective and it should respond to the needs of each student, and the teacher must be able to conduct the

learning outcome oriented learning process. When planning a math lesson, the teacher will first select the theme of the lesson and shape the goal. Observing a lot of lessons learned that the teacher uses two ways of formulation: One, traditionally, will introduce the subject of the lesson at the beginning of the lesson and conforms to the goal of it, and the second – by bringing the motivational task, by answering the relevant questions and the possible answers given by the pupils, try to bring the students with the formulation of the goals. Unfortunately, sometimes a poorly designed task is not able to help students demonstrate what the goal of the lesson is.

The work is offered to discuss motivating tasks that will give the teacher an opportunity to choose a motivational tasks for the lesson goal.

სადოქტორო თემა — „ქართული ტექსტების ავტომატური ინტელექტუალური კლასიფიკაციის მეთოდები და ინსტრუმენტები“ — მიზნების, ამოცანებისა და მეთოდების ზოგადი მიმოხილვა

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2017 წლიდან საქართველოს ტექნიკური უნივერსიტეტის „ინფორმატიკის“ სადოქტორო პროგრამის ფარგლებში ამოქმედდა სადოქტორო თემა „ქართული ტექსტების ინტელექტუალური კლასიფიკაციის მეთოდები და ინსტრუმენტები“ [1] (დოქტორანტი — კ. დემურჩევი, ხელმძღვანელი — სტუ ქართული ენის ტექნოლოგიზების ცენტრის დირექტორი პროფ. კ. ფხაკაძე). სადოქტორო კვლევა, რომელიც ძირეულად ეყრდნობა სტუ ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [2] ორწლიანი ქვეპროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ ფარგლებში ფხაკაძის ქართული ენის ლოგიკურ გრამატიკაზე დაყრდნობით შემუშავებულ მოსაუბრე ქართულ ინტელექტუალურ ვებ-კორპუსს [3], მიზნობრივად მიმართულია ქართული ტექსტების ავტომატური ინტელექტუალური კლასიფიკაციის მეთოდებისა და ინსტრუმენტების შემუშავებაზე, რაც გასაგებს ხდის ამ კვლევის მჭიდრო კავშირს ქართული ენის დაცვის მიზნით 2012 წლიდან გრძელვადიანი პროექტით „ქართული ენის ტექნოლოგიური ანბანი“ მიმდინარე კვლევებთან [2].

ამგვარად, აღნიშნულთა გათვალისწინებით, მოხსენებისას, მიმოხილული იქნება სადოქტორო კვლევის ფარგლებში უკვე შემუშავებული ქართული ტექსტების ავტომატური ინტელექტუალური კლასიფიკაციის პირველი საცდელი ვერსია.

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About one Algorithm of Currency Arbitrage

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The Algorithmizing matter is one of the biggest issues when solving decision making problems. Algorithm can be realized by different means and may describe processes drastically different from each other. The goal of this paper is to create an effective algorithm that can be used in solving arbitrage problems. The algorithm presented in this work is realized via using linear programming models. Abovementioned algorithm provides an opportunity to define priority (prevalent) currency towards other currencies in the region. The actual process of currency conversion is simulated. Mathematical model of currency arbitrage is presented, and after its programming realization we have received the maximum profit gain strategy of currency arbitrage, which in fact is the algorithm created to determine the maximum effectiveness. We, created an algorithm,which will define the best result of transformation for which we have to create mathematical model and analyse it.

Programming an Algorithm Definition the Critical Force of the Rod in Excel

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Engineers of a wide profile of navigation (and not only) often have to solve algebraic and transcendental equations, which can be an independent task, or be an integral part of more complex tasks. n both cases, the practical value of the numerical method is largely determined by the speed and efficiency of the solution. The choice of a suitable algorithm for solving equations depends on the nature of the problem under consideration.

According to this method, it is possible to solve some mathematical, physical, mechanical and other types of problems, the solution of which is reduced to nonlinear or transcendental equations.

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On the Fair Price of European Option

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On the filtered probability space $(\Omega, \mathcal{F}, P)$ consider binomial model of complete market with $(m+1)$ assets $(B_n^{(1)}, \ldots, B_n^{(m)}, S_n)$, $n = 0, 1, \ldots, N$, with m bonds and one stock

$$B_n^{(i)} = (1 + r^{(i)})B_{n-1}^{(i)}, \quad S_n = (1 + \rho_n)S_{n-1}, \tag{1}$$

where ρ_n is the sequence of independent identically distributed random variables, that take two values a and b, $a < b$ [1, 2].

Let $f_N^{(1)} = (S_N - K^{(1)})^+$ and $f_N^{(2)} = (K^{(2)} - S_N)^+$ be the payoffs of European standard *call* and *put* options with terminal date N and strikes $K^{(1)}$, $K^{(2)}$. For the fair prices $C(N)$ and $P(N)$ the following Theorem is valid

Theorem. *Let the financial (B, S) market is described by* (1)*, then the following inequalities are true*

$$C(N_1) \le C(N_2), \quad P(N_1) \le P(N_2), \quad N_1 \le N_2.$$

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Shell Equations in Terms of Günter's Derivatives, Derived by Γ-Convergence

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We consider the problem of bending elastic isotropic thin media $\Omega^h : \mathcal{C} \times [-h, h]$ around a surface $\mathcal{C}$ with the Lipshitz boundary $\Gamma := \partial\mathcal{C}$, described by the boundary value problem (BVP) for the Láme equation with the unknown displacement vector-function $\boldsymbol{U} = (U_1, U_2, U_3)^\top$:

$$\begin{cases} \mathcal{L}_{\Omega^h}\boldsymbol{U}(x) = -\mu\Delta\boldsymbol{U}(x) - (\lambda+\mu)\nabla\mathrm{div}\,\boldsymbol{U}(x) = \boldsymbol{F}(x), & x \in \Omega^h := \mathcal{C} \times (-h, h), \\ \boldsymbol{U}^+(t) = 0, & t \in \Gamma_L^h := \partial\mathcal{C} \times (-h, h), \\ (\mathfrak{T}(x, \nabla)\boldsymbol{U})^+(x, \pm h) = \boldsymbol{H}(x, \pm h), & x \in \mathcal{C}. \end{cases} \quad (1)$$

Here $\mathfrak{T}(x, \nabla)$ is the traction operator, Δ is the Laplacian, ∇ is the gradient and div is the divergence.

The object of the investigation is what happens with the boundary value problems (1) when the thickness of the layer converges to zero $h \to 0$. Is proved the following.

Theorem 1. *Let the weak* $\mathbb{L}_2$ *limits*

$$\lim_{h\to 0} \boldsymbol{F}(x, h\tau) = \boldsymbol{F}(x), \quad \lim_{h\to 0} \frac{1}{2h}\big[\boldsymbol{H}(x, +h) - \boldsymbol{H}(x, -h)\big] = \boldsymbol{H}^{(1)}(x)$$

exist. Then the boundary value problem (1) converges in the sense of Γ*-limit to the following BVP on the mid surface* $\mathcal{C}$*:*

$$\begin{cases} \mu\Big[\Delta_{\mathcal{C}}\overline{U}_\alpha + \mathcal{D}_\beta\mathcal{D}_\alpha\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\beta\mathcal{D}_\alpha\overline{U}_\beta - \mathcal{D}_\gamma(\nu_\alpha\nu_\beta\mathcal{D}_\gamma\overline{U}_\beta)\Big] \\ \qquad\qquad + \dfrac{4\lambda\mu}{\lambda+2\mu}\Big[\mathcal{D}_\alpha\mathcal{D}_\beta\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\alpha\mathcal{D}_\beta\overline{U}_\beta\Big] = \dfrac{1}{2}F_\alpha + \boldsymbol{H}^{(1)}_\alpha \ \ \text{on } \mathcal{C}, \\ \overline{U}_\alpha(t) = 0 \ \ \text{on } \Gamma = \partial\mathcal{C}, \ \ \alpha = 1, 2, 3. \end{cases}$$

Here the vector-function

$$\overline{\boldsymbol{U}}(x) := (\overline{U}_1(x), \overline{U}_2(x), \overline{U}_3(x))^\top, \quad \overline{U}_\alpha(x) := U_\alpha(x, 0), \ \ \alpha = 1, 2, 3$$

is the spacial displacement of the surface points $x \in \mathcal{C}$*,* $\mathcal{H}_{\mathcal{C}}$ *is the mean curvature of the surface* $\mathcal{C}$*,* $\boldsymbol{\nu} = (\nu_1, \nu_2, \nu_3)$‘$\top$ *is the unit normal vector field on the surface,* $\mathcal{D}_j := \partial_j - \nu_j\mathcal{D}_4$*,* $j = 1, 2, 3$ *are the Günter's tangential and* $\mathcal{D}_4 = \partial_\nu = \sum\limits_{k=1}^{3} \nu_k\partial_k$ *the normal derivatives.*

The proofs are based on theorems on Γ-convergence and the following representations of the classical Laplace operator, the divergence and the gradient with the Günter's derivatives:

$$\Delta = \partial_1^2 + \partial_2^2\partial_3^2 = \sum_{j=1}^{4}\mathcal{D}_j^2 + 2\mathcal{H}_{\mathcal{C}}\mathcal{D}_4, \ \ \nabla\varphi := \big\{\partial_1\varphi, \partial_2\varphi, \partial_3\varphi\big\} = \big\{\mathcal{D}_1\varphi, \mathcal{D}_2\varphi, \mathcal{D}_3\varphi, \mathcal{D}_4\varphi\big\}^\top,$$

$$\operatorname{div}\mathbf{U} := \sum_{k=1}^{3}\partial_k U_k = \sum_{j=1}^{4}\mathcal{D}_j U_j^0 + \mathcal{H}_{\mathcal{C}}U_4^0,$$

$$\mathbf{U} = (U_1, U_2, U_3)^\top, \quad U_j^0 := U_j - U_4^0, \quad U_4^0 := \sum_{k=1}^{3}\nu_k U_k, \ \ j = 1, 2, 3.$$

The work is carried out in collaboration with T. Buchukuri (Tbilisi).

Buckling of Column with Non-Ideal Boundary Conditions

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Buckling of slender elastic columns under static compressive load is a fundamental and classical problem in structural stability. It has been examined over more than 250 years, starting with Euler and up to present day. For combinations of the ideal boundary conditions – simple, fixed, sliding or free – exact analytical solutions for critical loads are available. In real structures, however, end fixations of columns are not ideal but rather restrained by elastic end supports. For elastic supports, numerous numerical methods are available, but even though it is possible to evaluate the buckling load more accurately than in the past, a complete mapping of all critical loads is still missing.

In this work, the realistic boundary conditions are represented by a couple of translational and rotational springs at each end of the column, implying a total of four end supports parameters. Mathematically, the buckling loads are obtained as roots of a transcendental equation that includes the four parameters of the end restraint springs. The present research aims at a complete (once and for all) mapping of the buckling loads for entire possible combinations of elastic boundary conditions. Specifically, compact design charts, in a user-friendly format, are presented along with several elegant new formulae for buckling loads with definite limits of validity. Analysis is within the framework of Euler buckling theory of homogeneous columns with constant bending stiffness.

We start with a modification of an earlier reformulation of the governing transcendental eigenvalues equation, where the four spring constants are regrouped into just three parameters: two equivalent spring parameters, which are essentially the harmonic means of the actual parameters, and one nondimensional measure of rotational supports asymmetry. The equation is then factorized into two distinct branches that are examined separately. New closed form expressions are derived for several cases and numerical findings are supported by accurate asymptotic expansions. Comparison with available solutions confirms the efficiency of suggested approach to the buckling problem. A detailed sensitivity analysis reveals the nonlinear effect of elastic end supports and ends asymmetry on the buckling load.

Some important cases are examined in detail, leading to simple formulae and analytical approximations. Specifically, the cases of rigid body instability, symmetrical rotational boundary conditions, one rotation free end and weak columns (applicable to MEMS) will be discussed. In addition, the conditions for multiple root buckling are presented.

Results from elastic buckling are extended into the plastic range, where pre-buckling stresses are higher than the yield stress.

Finally, energy considerations are given together with an analysis using special energy related parameters, and it is shown that the ratio between the energy absorbed in the column to the energy absorbed in the end restraints is determined by the two equivalent springs parameters along with the measure of asymmetry.

The Smoothness of Functions of Two Variables

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The function $f(x, y)$, defined in the neighbourhood of the point $(x_0, y_0) \in R^2$, is called smooth at (x_0, y_0) when the equality [1]

$$\lim_{(h,k)\to(0,0)} \frac{f(x_0+h, y_0+k) + f(x_0-h, y_0-k) - 2f(x_0, y_0)}{|h| + |k|} = 0$$

is fulfilled.

The function, differentiable at the point (x_0, y_0) is smooth at (x_0, y_0).

To the function $f(x, y)$, continuous on the square $[-\pi, \pi]^2$ there correspond two one-dimensional Fourier series $S[f]_1$ (coefficients depend on $y \in [-\pi, \pi]$) and $S[f]_2$ (coefficients depend on $x \in [-\pi, \pi]$) with respect to the variables x and y, respectively.

The equalities $S[f]_1(x_0, y_0) = f(x_0, y_0) = S[f]_2(x_0, y_0)$ hold, when (x_0, y_0) is the point of smoothness for the function f [2].

In the report, the sufficient and also the necessary and sufficient conditions to the function $f(x, y)$ in order for this function to be smooth at the given point (x_0, y_0), will be indicated.

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Some Issues of Conducting Fluid Unsteady Flows in a Circular Tube

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In this article is considered the unsteady flow of a viscous incompressible electrically conducting fluid in annular pipe under external radial magnetic field. An exact solution of the problem in general form and its extreme case are obtained.

The developed flows of conducting fluid in an annular pipe are recently rather detailed studied [1–5], and the possibility of obtain of new exact analytical exact solutions seems quite limited. However, such opportunities do exist and it is possible, as will be shown below, to find even simple new solutions that however, have a rather interesting qualitative features.

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Construction of Theory of Sandwich Orthotropy Plates

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Is constructed the sandwich linear geometric plate linear geometric model and is determined relation between stress and deformation. Then based on the Hamilton–Ostrogradski variation principle is carried out the derivation of equations of motion and natural boundary conditions The constructed and expanded expressions for normal, transversal forces and bending moments, high order forces and moments, as well as for inertia forces and moments.

For the solution of equations of neutral equilibrium and proper boundary conditions is applied the elastic stability variation principle of V. Bolotin that gives the possibility to taking into account the high order parametric terms.

Analysis of Multi-Wave Plate Coverings

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Is stated the method of analysis of multi-wave coverings from depressed, cylindrical and corrugated shells based on generation and solution of equations of theory of shells with discontinuous parameters. The multi-wave coverings from depressed and cylindrical shells are widely distributed worldwide during last two decades at construction of buildings with large spans. Were developed united precast elements of shells, contour girders, arcs, beams, columns, is developed the technology of production and erection. are stipulated the variants of shell edges joint, providing rigid connection or cylindrical hinge at transfer of forces and moments.

The development of such type structures, often based on the engineering intuition, was accompanied with intense development of approximation and exact methods of their analysis.

Despite on this issues of optimal dimensioning of these system's elements still are rather contradict and in the aspects of stress distribution in adjacent of angular points and in the zones of contact are not completely clear.

It would be mentioned that developed and currently applied in the design practice methods of analysis of multi-wave coverings are based on the analysis of containing the system separate shells, with further consideration of their joint deformation that represents the natural distribution of methods of disclosure the static indeterminacy of structural mechanics of rod systems on the theory of thin shells, compiling of program and features of programming at analysis of systems with singularities.

From all listed methods most rapid convergence reveals the successive loadings method, but it requires lot of auxiliary time on formation of Jacobian matrix for each approximation. As simplest is presented the method of simple but it has the worse convergence.

Numerical Simulation of Smog Against Aerosol-"Humidity" Thermohydrodynamics Model

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The full cycle of fog- and cloud formation in the mesoscale boundary layer of the atmosphere (MBLA) is simulated by numerical methods.

Aerosol diffusion in MBLA from a point source is also simulated.

It is possible to simulate smog formation by 'superposition' models of humidity processes and aerosol diffusion in MBLA.

All three tasks are stated in both $(x, -z)$ and (r, z) vertical planes.

Simulation of 1D Spin Glasses From the First Principles of Classical Mechanics and Foundations of Statistical Mechanics

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We study the classical $1D$ Heisenberg spin glasses assuming that spins are spatial. Minimizing the Hamiltonian at the nodes of the $1D$ lattice, a system of recurrent equations is derived. It is proved that at each lattice node there is a probability of bifurcation of the recurrent equations' solution. This leads to the fact that, performing consecutive node-by-node calculations on the *n-th* step instead of a single stable spin-chain we get a set of spin-chains which form Fibonacci subtree (graph). We have assessed the complexity of computation of one graph and have shown that it is $\propto 2^n K_s$, where n and K_s denotes the subtree's height (the length of spin-chain) and the Kolmogorov's complexity of a string (the branch of subtree), respectively. It is shown that the statistical ensemble may be represented as a set of random graphs, where the computational complexity of each graph is $\mathbb{NP}$ hard. It is proved that all the strings in the ensemble have the same weight regardless of which tree they belong to. The latter circumstance allows in the limit of statistical equilibrium with predetermined accuracy to reduce the $\mathbb{NP}$ hard problem to the $\mathbb{P}$ problem. Comparison of the statistical distributions of various parameters have performed using $\mathbb{NP}$ and $\mathbb{P}$ algorithms, which showing a perfect match of the corresponding curves. This allows to claim that it is possible to calculate all parameters and the corresponding distributions of the statistical ensemble from the first principles of classical mechanics without using any additional considerations. Finally, using the formal similarity between the ergodic dynamical system and the ensemble of spin chains, we propose a new representation for the partition function in the form of a one-dimensional integral over the energy distribution of spin-chains. The latter leads to the revision of some fundamental axioms and concepts of statistical mechanics.

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A New Generalized Function Defined by the Euler Integral of the First Kind

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The study of the Euler integral of the first kind in the region of its divergence is associated with many new items [1, 2].

In this abstract, we propose the consequences related to the integral

$$B(i\gamma - m, -i\gamma - n) = \int_0^1 t^{i\gamma-m-1}(1-t)^{i\gamma-n-1}\,dt, \tag{1}$$

where m and n are integers ($m \geq 0$ and $n \geq 0$).

The integral (1) defines a generalized function which can be expressed in terms of the Dirac delta function as follows:

$$\begin{aligned} B(i\gamma - m, -i\gamma - n) &= 2\pi \exp\Big(-im\frac{d}{d\gamma}\Big)\Big(1+\exp\Big(-i\frac{d}{d\gamma}\Big)\Big)^{m+n}\delta(\gamma) \\ &= 2\pi\Big(1+\exp\Big(-i\frac{d}{d\gamma}\Big)\Big)^{m+n}\exp\Big(-im\frac{d}{d\gamma}\Big)\delta(\gamma). \end{aligned} \tag{2}$$

The functional (2) has the following analytic representation

$$B(iz - m, -iz - n) = -\frac{1}{i}\sum_{k=0}^{m+n}\frac{A_{m+n}^k}{z - i(k-m)}\,, \tag{3}$$

where z is a complex variable and A_m^k denotes the binomial coefficient

$$A_m^k = \binom{m}{k} = \frac{m!}{k!(n-k)!}\,, \quad z = x + iy.$$

Let note that (3) is a holomorphic function, with the exception of a finite number of points of the complex plane.

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The Euler Integral of the Second Kind. New Calculations

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It is known that the well known Euler integral of the second kind is an analytic function in the domain of its definition ($\operatorname{Re} z > 0$). The specified Integral in this area can be expressed by the function $\Gamma(z)$ (see e., g., [1, p. 551]):

$$\Gamma(z) = \int_0^\infty t^{z-1} \exp(-t)\, dt, \quad \operatorname{Re} z > 0 \tag{1}$$

which satisfies the functional equation (see e., g., [1, p. 548] and [2, p. 16])

$$\Gamma(z+1) = z\Gamma(z)$$

and represents an analytic function with the exception of points of the complex plane $z = -k$ $(k = 0, 1, 2, \dots)$.

Recent investigations showed, that the integral (1) is integrable in the sense of generalized functions and defines the generalized function at the points $\operatorname{Re} z = 0$

$$G(z) = \int_0^\infty t^{z-1} \exp(-t)\, dt, \quad \operatorname{Re} z = 0. \tag{2}$$

The functional $G(z)$ which is defined by the representation (2), can be expressed via the Dirac delta function as follows

$$G(i\tau) = 2\pi \exp\Big(- \exp i\Big(\frac{d}{d\tau}\Big)\Big)\delta(\tau).$$

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Subgaussian Random Elements and 2-Summing Operators

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Motivation of this presentation is the paper [3]. Recall that a (real-valued) random variable ξ is Subgaussian if there exists a number $a \geq 0$ such that $\mathbb{E}\, e^{t\xi} \leq e^{\frac{1}{2}a^2t^2}$ for every t, $-\infty < t < +\infty$, where $\mathbb{E}$ is the symbol of expectation [2].

For a random variable ξ let us define the following (not necessarily finite) quantity

$$\tau(\xi) = \inf\Big\{a \geq 0 :\ \mathbb{E}\, e^{t\xi} \leq e^{\frac{1}{2}t^2a^2} \text{ for every } t,\ -\infty < t < +\infty\Big\}.$$

Note that a random variable ξ is Subgaussian if and only if $\tau(\xi) < +\infty$.

Let $SG(\Omega)$ be the set of all real-valued Subgaussian random variables defined on a fixed probability space $(\Omega, \mathcal{A}, \mathbb{P})$. $SG(\Omega)$ is a vector space with respect to the natural point-wise operations, the functional $\tau(\,\cdot\,)$ is a norm on $SG(\Omega)$ (provided the random variables which coincide a.s. are identified) and, moreover, $(SG(\Omega), \tau(\,\cdot\,))$ is a Banach space [1].

Let X be a Banach space and X^* be its dual. We say that a random element η in a Banach space X is *weakly Subgaussian*, if for every $x^* \in X^*$ random variable $x^*(\eta)$ is Subgaussian [5]. For every weakly Subgaussian random element η let us define the induced operator $T_\eta : X^* \to SG(\Omega)$ by the equality $T_\eta x^* = x^*(\eta)$ for all $x^* \in X^*$. Random element ζ in a Banach space X is called *T-Subgaussian*, if there exists a Gaussian random element γ in X such that $\mathbb{E}\, e^{x^*(\zeta)} \leq \mathbb{E}\, e^{x^*(\gamma)}$ for every $x^* \in X^*$ [4]. Every T-Subgaussian random element clearly is weakly Subgaussian. Inverse statement in general is not true.

The main results of the presentation are the following two theorems.

Theorem 1. *For a random element η in the Hilbert space H with the inner product $\langle\,\cdot\,,\,\cdot\,\rangle$ the following statements are equivalent:*

(i) *η is T-Subgaussian.*

(ii) *For every orthonormal basis (e_n) of H the following condition holds*

$$\sum_{n=1}^{\infty} \tau^2\big(\langle e_n, \eta\rangle\big) < \infty.$$

Theorem 2. *For a weakly Subgaussian random element η in a Banach space X consider the assertions:*

(i) *η is T-Subgaussian.*

(ii) *Induced operator $T_\eta : X^* \to SG(\Omega)$ is a 2-summing operator.*

Then (i) $\longrightarrow$ (ii). *The implication* (ii) $\Longrightarrow$ (i) *is true provided X is a type* 2 *space.*

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Generating Sets of the Complete Semigroup of Binary Relations Defined by Semilattices of the Finite Chains

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We study generating sets of the complete semigroups of binary relations defined by X-semilattices unions of the finite chains. We found uniquely irreducible generating sets for the given semigroups.

Let $\Sigma_m(X,m)$ be a class of all X -semilattices of unions whose every element is isomorphic to an X-semilattice of unions $D=\{Z_1,Z_2,Z_3,\ldots,Z_{m-2},Z_{m-1},Z_m=\breve{D}\}$, which satisfies the condition

$$Z_1\subset Z_2\subset Z_3\subset\cdots\subset Z_{m-2}\subset Z_{m-1}\subset Z_m=\breve{D}$$

Let

$$C(D)=\left\{P_0,P_1,P_2,P_3,\ldots,P_{m-2},P_{m-1}\right\}$$

be a family of sets, where $P_0P_1,P_2,P_3,\ldots,P_{m-2},P_{m-1}$ are pairwise disjoint subsets of the set X.

The formal equalities of the semilattice D have a form:

$$\begin{aligned}
\breve{D}=Z_m&=P_0\cup P_1\cup P_2\cup P_3\cup\cdots\cup P_{m-1},\\
Z_m&=P_0\cup P_1\cup P_2\cup P_3\cup\cdots\cup P_{m-2},\\
Z_{m-1}&=P_0\cup P_1\cup P_2\cup P_3\cup\cdots\cup P_{m-3},\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
Z_3&=P_0\cup P_1\cup P_2,\\
Z_2&=P_0\cup P_1,\\
Z_1&=P_0
\end{aligned}$$

Here the elements $P_1,P_2,P_3,\ldots,P_{m-2},P_{m-1}$ are bases sources, the element P_0 is sources of completeness of the semilattice D. Therefore $|X|\geq m-1$, since $|P_i|\geq 1$, $i=1,2,\ldots,m-1$.

We are learning irreducible generating sets of the semigroup $B_X(D)$ defined by semilattices of the class $\Sigma_m(X,m)$.

In the sequel, by symbol $\Sigma_{m,0}(X,m)$ we denote all semilattices

$$D=\left\{Z_1,Z_2,Z_3,\ldots,Z_{m-2},Z_{m-1},Z_m\right\}$$

of the class $\Sigma_m(X,m)$ for which Z_1 is not empty. From the last inequality and from the formal equalities it follows that $Z_1 = P_0 \neq \varnothing$, i.e. in this case $|X| \geq m$.

We denote the following sets by symbol $\mathfrak{A}_k$ and $B(\mathfrak{A}_k)$

$$\mathfrak{A}_k = \Big\{V(X^*,\alpha) \mid \ |V(X^*,\alpha)| = k \text{ for any } \alpha \in B_X(D)\Big\},$$
$$B(\mathfrak{A}_k) = \big\{\alpha \in B_X(D) \mid \ V(X^*,\alpha) \in \mathfrak{A}_k\big\},$$

where $k = m, m-1, m-2, \ldots, 3, 2, 1$.

Theorem. *Let $D \in \Sigma_{m,0}(X,m)$ and D is a finite set. If $|X \setminus \breve{D}| \geq 1$ and*

$$\mathfrak{A}_m = \Big\{V(X^*,\alpha) \mid \ |V(X^*,\alpha)| = m \textit{ for any } \alpha \in B_X(D)\Big\},$$
$$B(\mathfrak{A}_m) = \big\{\alpha \in B_X(D) \mid \ V(X^*,\alpha) \in \mathfrak{A}_m\big\},$$

then the set $B(\mathfrak{A}_m)$ is irreducible generating set for the semigroup $B_X(D)$.

Corollary. *Let $D \in \Sigma_{m,0}(X,m)$. If $|X \setminus \breve{D}| \geq 1$ and*

$$\mathfrak{A}_m = \Big\{V(X^*,\alpha) \mid \ |V(X^*,\alpha)| = m \textit{ for any } \alpha \in B_X(D)\Big\},$$
$$B(\mathfrak{A}_m) = \big\{\alpha \in B_X(D) \mid \ V(X^*,\alpha) \in \mathfrak{A}_m\big\}.$$

If X and D are finite sets $|X| = n$ and $|D| = m$, then the number $|B(\mathfrak{A}_m)|$ elements of the set $B(\mathfrak{A}_m)$ is equal to

$$|B(\mathfrak{A}_m)| = n^m + \sum_{i=1}^{n-1}(-1)^i \cdot C_n^i \cdot (n-i)^m.$$

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On the Almost Everywhere Convergence of Multiple Fourier Series for Square Summable Functions

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It is proved that if for each one-dimensional orthonormal system $\Phi_1, \dots, \Phi_d$ partial sums (lacunary partial sums) of Fourier series of every square summable function converge almost everywhere then the product system $\Phi_1 \times \cdots \times \Phi_d$ also possesses similar property for a quite general type partial sums.

On the Genera of the Positive Definite Diagonal Quaternary Quadratic Forms

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Quadratic forms with integer coefficients, given number of variables and determinant can be arranged in classes and genera. Each genus is a union of classes of forms. The number of the classes in the genera (and therefore in each genus) is finite. One-class genera, i.e. genera which contains only one class of the forms, are of special interest. Among many results obtained earlier we note the one by G. Watson [2]. He solved the problem of determining one-class genera for special type positive definite primitive (PDP) quaternary quadratic forms. He listed all of the such genera. In our some earlier papers, for example in [1], we studied one-class genera problem for PDP quadratic forms with various variables.

In this talk I will introduce estimates for one-class genera of all types of PDP diagonal quaternary quadratic forms according to their determinant.

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On the Principles of Creation of Secondary School Math Course

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At present time a considerable number of our teachers (older than middle age) suppose that integrating the courses of algebra and geometry (sometimes even trigonometry) in secondary schools is one of the main reasons for the problems in the school mathematical education system in Georgia and so the unified textbooks of such courses are inadmissible. In their opinion, geometry now is "lost" in this integrated math course. They idealize the mathematics of the "traditional" school, the level of separate subjects teaching and even the period when the geometry was taught by Pogorelov's famous textbook, which for a long time did not encourage teaching of geometry in schools. Moreover, most of the teachers at that time replaced geometry by algebra lessons.

Proponents of representation of school mathematics as separate courses of geometry and algebra may be less aware that the course, which they referred as algebra, is actually composed of not only algebraic issues – it is a mixture of algebra, geometry (analytical geometry), function theory, probability theory and statistics. Thus, to declare them as algebra it creates an unjustified idea of algebra for teachers and students.

The supporters of the introduction of the separate geometric course think that the course provides continuous and consistent teaching of geometry. We should note that, while teaching geometry twice and algebra thrice in a week, student has to immediately switch to completely different issues from lesson to lesson – from geometric to "non-geometric" issues and back to geometric ones.

In modern integrated courses of mathematics, the difficulty is omitted. Issues are presented as blocks and students will consider them without any interruptions. Also, there is a high connection between different issues in such kind of teaching.

In general, now mathematics is presented with many subjects. But they are studying at university level.

The talk also discusses some other issues of mathematical education.

Ranking Theory Methods for MCDM Problems

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The Pareto optimality is a widely used concept for the multicriteria decision-making (MCDM) problems, [1]. However, this concept has a significant drawback – the set of Pareto optimal alternatives usually is large. Correspondingly, the problem of choosing a specific Pareto optimal alternative for the decision implementation is arising. This study proposes a new approach to select an "appropriate" alternative from the set of Pareto optimal alternatives. The proposed approach is based on ranking-theory methods used for ranking participants in sports tournaments, [2, 3]. In the framework of the proposed approach, we build a special score matrix for a given multicriteria problem, which allows the use of the mentioned ranking methods and to choose the corresponding best-ranked alternative from the Pareto set as a solution of the problem. The proposed approach is particularly useful when no decision-making authority is available, or when the relative importance of various criteria has not been evaluated previously. The proposed approach is tested on an example of a materials-selection problem for a sailboat mast.

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Gluon Pole Mass

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We argue that QCD is quantum field theory with the dynamically/spontaneously broken gauge symmetry in its ground state. This makes it possible to derive the dynamical equation of motion for the full gluon propagator containing the mass scale parameter, called the mass gap. The explicit presence of the mass gap separates the massive gluon propagator from its massless counterpart. The gauge fixing parameter is uniquely defined and it is the the generalized t' Hooft gauge. We identify the dynamical source of the mass gap with the tadpole/seagull term, which is explicitly present in the full gluon self-energy. Its contribution does not survive in the perturbative regime when gluon momentum goes to infinity, while becomes dominant at small and finite gluon momenta. The non-perturbative renormalization program has been done, and the mass gap approach to QCD has been formulated. It predicts the existence of gluons with exactly defined gluon pole masses. These are different from the excitations with effective gluon masses.

Singularly Perturbed Vector Fields

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A coordinate free concept of singularly perturbed systems (singularly perturbed vector fields) will be discussed. Singularly perturbed vector fields can be represented locally as singularly perturbed systems (for corresponding coordinate system choice. The talk focuses on possible ways of fast and slow manifolds evaluations. A special algorithm for the evaluation is proposed. The algorithm is called as a global quasi-linearization procedure. A practical application of the proposed algorithm will be discussed for kinetic and combustion models.

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ღერძსიმეტრიული ამოცანა ბიპოლარული ცილინდრისთვის

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წარმოდგენილ მოხსენებაში დასმულია და ანალიზურად ამოხსნილია ღერძსიმეტრიული ამოცანა ბიპოლარული ცილინდრისათვის.

ბიპოლარულ კოორდინატთა სისტემში განიხილება ტრანსტროპული (ტრანსვერსალურ-იზოტროპული) ერთგვაროვანი სასრული ცილინდრი, რომლის ფუძეებზე მოცემულია სიმეტრიის ან ანტისიმეტრიის პირობები, ხოლო ცილინდრულ ზედაპირზე მოცემული ან ძაბვები ან გადაადგილებები. გადაადგილების ვექტორისა და ძაბვის ტემზორის კომპონენტები გამოისახება ექსპონენციალურად კრებადი მწკრივებით, რომელთა კოეფიციენტები ცხადი სახითაა მოცემული.

Application of Mathematical Regulation Methods to Assess the Optical State of Urban Air

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This is an assessment of the optical state of the city air with mathematical regulation methods. Here, the optical position of the city air is investigated in different optical intervals and the various aspects of observation. We will use mathematical methods that are widely utilized in practice in public decision-making technologies. The essence of these methods is to determine the order of preference among the elements of a set of properties [2, 3].

For the purpose of evaluating the optical state of the city air, a series of measurements are first done with the use of mobile media (pyrometer, actinometer, photocenter, etc.) to obtain reprinted information about the air pollution spatial change. The results are generated primarily by statistical data. It is known that the use of entropy indicators is a common approach to evaluating informativeness in many research areas. The process of obtaining any information can be explained as a result of uncertainty changes in transmitting signals. If any manifestation passes from a situation (eg the state of the situation), transitional information is understood to be the difference between these situations and the uncertainties [1]:

$$I(A, B) = H(A) - H(B),$$

here $H(A)$ and $H(B)$ in accordance with the entropy prices of manifestations A and B situations. As an example, the informality of the optical state of different destinations of Sumgayit city air has been assessed, and the importance of informativeness among the directions has been determined. The calculations use the Condorcet, Schulze, Borda, Copeland and Simpson rules of mathematical regulation.

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The Computer Realization of Approximate Solution of the Initial-boundary Problem of Two-dimensional Parabolic Equation by the Algorithm of Splitting a Multi-layer Difference Scheme

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We consider a following initial-boundary problem for a parabolic equation:

$$\frac{\partial u(x,y,t)}{\partial t} = \frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} + f(x,y,t), \quad x \in [0,1], \quad y \in [0,1], \quad t \in]0,1], \tag{1}$$

$$u(x,y,0) = \varphi(x,y), \quad u(0,y,t) = u(1,y,t) = u(x,0,t) = u(x,1,t) = 0, \tag{2}$$

where f, φ is a continuously differentiable function, u is the function to be found.

Using the perturbation algorithm [1, 2], purely implicit three and four-layer difference scheme for an approximate solution of the initial-boundary problem (1), (2) is reduced to two-layer schemes. An approximate solution of the original problem is constructed by means of the solutions of these schemes. The first two-layer scheme gives an approximate solution with an accuracy of first order, where the solution of each subsequent scheme is the refinement of the preceding solution by one order. Note that this method gives the approximate solution of the problems (1), (2) by time variable. As for spatial variable, we assume that they are defined with a higher accuracy than with time variable.

Based on the considered method, we construct an algorithm and graphic scheme for solving the problem (1), (2). At the and of the work graphics of the results of the numerical experiments is constructed.

In our research we also showed that the algorithm can be parallelized. The graphic scheme for the approximation solution of the problem (1), (2) by the parallel algorithm is constructed.

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The Axisymmetric Problem of the Theory of Thermoelasticity in the Elliptic Coordinate System

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In the report of the elliptic coordinate system, the thermoelastic equilibrium of the objects bounded by the coordinate surfaces of the system is discussed. The report formulates a mathematical model of the thermoelasticity problems. Specifically, the area of determination of the vector components can be as confocal elliptical cylinder as well as its quarter, or three-quarters of elliptical cylinders etc. Four different boundary conditions may be given on the Surface of these areas. The solution of these tasks by the general solution and the method of variable separation will be replaced by the solution of the corresponding linear algebraic equation system. After the system is solved, the components of the movement vector and voltage tensor can be given by exponentially convergent series, whose coefficients are given in clear form.

Some Properties in Weak Absolute Geometry

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M. Hajja and H. Martini proved Theorem in the Hilberts absolute geometry(whose axioms are the plane axioms of incidence, order, and congruence of groups I, II, and III of Hilbert's *Grundlagen der Geometrie*) about characterization interior points of triangle, but in proof they used Zorn's lemma and the Bolzano–Weierstrass theorem and raised question "whether such a heavy machinery is indeed inevitable?". Moreover, since they can only prove the existence of the point, they also ask "whether there is a procedure (an

algorithm) to construct the point". In this paper we define axiom system for weak absolute geometry, and prove some properties in weak absolute geometry, and finally using that properties we prove two theorems which will answer to all questions raised in that paper.

Theorem 1. *For any point P inside or on the boundary of triangle ABC, there is no point Q, different from P, such that Q and P satisfy.*

$$AQ \leq AP, \;\; BQ \leq BP, \;\; CQ \leq CP. \tag{1}$$

Theorem 2. *For every point P outside of triangle ABC there exists a point Q inside of triangle ABC, such that Q and P satisfy $(1)_<$.*

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L_1-Biharmonic $\delta(2)$-Ideal Hypersurfaces in Euclidean Spaces

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Let $K(\pi)$ be the sectional curvature of a Riemannian n-manifold M associated with a plane section $\pi \subset T_pM$, $p \in M$. For any orthonormal basis $\{e_1, \ldots, e_n\}$ of the tangent space T_pM, the scalar curvature τ at p is defined by $\tau(\pi) = \sum_{i<j} K(e_i \wedge e_j)$. Let L be a subspace of T_pM of dimension $r \geq 2$ and $\{e_1, \ldots, e_r\}$ an orthonormal basis of L. The scalar curvature $\tau(L)$ of L is defined by

$$\tau(L) = \sum_{\alpha<\beta} K(e_\alpha \wedge e_\beta), \;\; 1 \leq \alpha, \beta \leq r.$$

For an integer $r \in [2, n-1]$, the δ-invariant $\delta(r)$ of M is defined by

$$\delta(r)(p) = \tau(p) - \inf\{\tau(L)\},$$

where L runs over all r-dimensional linear subspaces of T_pM. For any n-dimensional submanifold M in $\mathbb{E}^m$ and any integer $r \in [2, n-1]$, Chen proved the following general sharp inequality

$$\delta(r) \leq \frac{n^2(n-r)}{2(n-r+1)} H^2,$$

where $H^2 = \langle \vec{H}, \vec{H} \rangle$ is the squared mean curvature. A submanifold M^n in $\mathbb{E}^m$ is called $\delta(r)$-ideal if it satisfies the above inequality. Let $x : M^n \to \mathbb{E}^{n+1}$ is an isometric immersion of a Riemannian manifold M^n into the Euclidean space $\mathbb{E}^{n+1}$. In this paper, we consider the chen conjecture for L_1-operator. The L_1-conjecture is formulated as follows: if $L_1^2 x = 0$ then $H_2 = 0$. Where H_2 is the 2-th mean curvature of M and L_1 is the linearized operator of the 2-th mean curvature of the Euclidean hypersurface M. We prove the L_1-conjecture for the $\delta(2)$-ideal hypersurface M.

Synthesis and Comparison of Hybrid Cryptographic Algorithms

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In cryptography, encryption is called the method of encoding information in order to provided security against unauthorized access and to convert any type of data into the encoded version. Nowadays, encryption plays a vital role to provide data security, especially for the data transmitted via networks. Cryptography aims to provide high-security level there are different types of cryptography methods. But, each of cryptography algorithms has its own weak and strong points. In modern cryptography mathematical theory and computer science practice are used as a basis.

The objective of this research was to make comparative analyses on AES, Twofish, RSA ciphers and their hybrid models like Twofish + RSA and AES + RSA.JAVA (version SE programming language was used to analyze those decryption algorithms. Developed software code allows us to make different experimental researches. The program outcomes give information about the amount of time and system memory usage during the performance of each algorithm (Nanoseconds).

The paper presents results on comparative analyses of above defined cryptography algorithms and hybrid models. Those algorithms are evaluated by terms of encryption speed, memory usage, encrypted file size and ensured security level. After conducted experimental testing on those algorithms, we can conclude that among new hybrid models AES + RSA is significantly secure (because AES was selected as a finalist for the Advanced Encryption Standard contest, NIST) because it takes all advantages and strength of symmetric and asymmetric systems, but Twofish + RSA is faster.

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Multilinear Fefferman–Stein Type Inequalities for Strong Fractional Maximal Operators with Variable Parameters

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We derived Fefferman–Stein type inequalities for multi(sub)linear strong fractional maximal operators with variable parameters. Some related two-weight problems are also

investigated. Necessary and sufficient condition for the boundedness of multi(sub)linear fractional maximal operators defined with respect to a measure in Lebesgue spaces is also obtained.

The investigation was carried out jointly with Q. Xue.

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Different Approaches to the Parallelization of the Program for Solving of Electromagnetic (EM) Problems

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The conventional method for solving electromagnetic problems is the method of moments (MoM), which leads to the solution of a system of linear equations with complex coefficients [1]. Numerical solution of large such industrial problems is very time-consuming. One of the possibilities of speeding up the process is the parallelization of the algorithm and the usage of modern multiprocessor computing cluster systems.

The MoM algorithm is quite complex and it may be divided into the following parts: data reading and geometry processing, preprocessing, filling the matrix of the system of equations, solving the system, post-processing (calculating of fields, currents, etc.). Moreover, these actions, except of data reading and processing of geometry, can be repeated several times for different wave frequencies.

The structure of the algorithm allows different approaches to parallelization to be applied. For example, process of filling and solving of system of equations for individual frequency can be parallelized or whole frequency loop can be parallelized. In the both cases, memory of the cluster node may be shared (or not) between different parallel branches. Each approach has its advantages and disadvantages. These approaches are discussed in this report.

In addition, this algorithm is implemented on ARM clusters and the comparison of the obtained times with the results of "ordinary" clusters is also given in this report.

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On an Iteration Method of Solution of a Non-Homogeneous System for a Dynamic Beam

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Let us consider the system of equations

$$\begin{gathered}\omega_{tt} = \left(cd - a + b\int\limits_0^1 \omega_x^2\,dx\right)\omega_{xx} - cd\psi_x + \alpha(x,t),\\ \psi_{tt} = c\psi_{xx} - c^2\,d(\psi - \omega_x) + \beta(x,t),\\ 0 < x < 1,\ \ 0 < x < T,\end{gathered} \tag{1}$$

with the initial and boundary conditions

$$\begin{gathered}\omega_t(x,0) = \omega^{(1)}(x), \quad \omega_x(x,0) = \omega^{(2)}(x),\\ \psi_t(x,0) = \psi^{(1)}(x), \quad \psi(x,0) = \psi^{(2)}(x),\\ \omega_t(0,t) = \omega_t(1,t) = 0, \quad \psi_t(0,t) = \psi_t(1,t) = 0,\\ 0 \le x \le 1,\ \ 0 \le t \le T.\end{gathered} \tag{2}$$

In [1] it is considered the homogeneous system of equations with $\alpha(x,t) = 0$ and $\beta(x,t) = 0$, which is well known Timoshenko model of geometrically nonlinear vibration of a beam. An error estimate is derived for the iteration process used to solve the nonlinear system of algebraic equations which are obtained by discretizing the problem.

In this paper, we use the same iteration process to solve the problem (1), (2). Then in order to estimate error of the method we consider numerical examples. For this purpose we use GNU Octave.

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Dynamical Stability of Sandwich Plate

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Is studied dynamical stability of sandwich plate at action of periodic compression loading that is applied in its middle plane. Are obtained general formulae for frequencies of own oscillations, critical loading and disturbance coefficients that provides the impact of in-plane shear and high order symmetrical terms. With taking into account of these factors are obtained dynamical non-steadt zones for transversal-isotropic plates.

One of the Methods of Analysis of Text Data

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The work presents a new method that allows us to classify and analyze various texts. The presented method is based on the results obtained from the frequency analysis of the symbols used in the study text, generating images and processing them through neuronal networks. Modern classifiable tasks are well developed in the direction of the recognition of the words. Under the classification, it is understood that the grouping of the same objects (text fragments in our correspondence) will be sorted by any common qualities or properties and such groups are called clusters. Here, it is quite successful to use convolutional neuronal networks [1, 10]. Artificial Neuron Networks are widely used for classification such as recognition of the subject, recognition of manuscript text [3, 7], face recognition, etc. Further studies have shown that using neuronal networks can also classify textual data [5, 6, 8, 11]. Based on the above, it is very promising to link the matrix generated by the method described in the conventional neuron networks in terms of their further classification analysis. To achieve this goal, the following scheme of data processing, built using the classroom and visual control elements built in Orange (see the pic).

In the paper, based on the proposed method of research, with numerical experiments, quite high probability, we can assume that the textual data analysis can be based on the text style level, based on the processing of frequency analysis of the characters in the text. The obtained results allow us to categorically categorize the fragments of textual data by authors and periods, even if we do not know the other works of the author.

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Numerical Research of Thin-Walled Structures

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For the solution of multi-layer depressed shells and plates bending and oscillations general system of equations by finite differences method is developed the row formation algorithm that provides the succession of matrix operations and gives the possibility to effectively apply up-to-date computers with its mathematical maintenance. For stability tasks is developed the private method of numerical research that is based on finite difference presentation of system. The method gives the possibility to research the stability of multilayer plates at action of local and concentrated loadings.

On Some Properties of Certain Discrete Point Sets in Euclidean Space

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A system X of point in the Euclidean space $\mathbf{R}^n$, $n \geq 1$, is called discrete if every ball in $\mathbf{R}^n$ contains only finitely many points from X. In general, a set X is discrete in topological space E if every point $x \in X$ has a neighborhood U such that $X \cap U = \{x\}$ [2].

Let D be a point-set (finite-infinite) in the n-dimensional Euclidean space $\mathbf{R}^n$. We say that this D is a Diophantine set if the distance between any two points from D is a natural number [1, 3, 4].

Theorem 1. *Let D be a Diophantine subset of an n-dimensional sphere of integer radius r, where $r \geq 2$. Then in the Euclidean space $\mathbf{R}^{n+2}$ there exists a Diophantine set D_1 containing D and such that $card(D_1 \setminus D) = 2$.*

Theorem 2. *For any natural number $n \geq 2$, there are Diophantine set in the Euclidean space $\mathbf{R}^n$ which have arbitrary many point and do not lie in a hyperplane of $\mathbf{R}^n$.*

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Physical Lorentz Invariance Violation

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Now, it is already not a big surprise that due to the spontaneous Lorentz invariance violation (SLIV) there may emerge massless vector and tensor Goldstone modes identified as photon and graviton. Emergent theory is non-linear and contains CPT and Lorentz violating interaction, but unexpectedly, Lorentz violation is superficial. While this is the case in gauge invariant theory, Lorentz violation becomes physically significant if even the

tiny gauge non-invariance is introduced. This leads to the modified dispersion relation for the fields involved. While modifications are negligible in low energy physics they may play crucial role at sufficiently high energy energies and lead towards plethora of new effects, which are characteristic to the theory and could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the GZK cutoff for UHE cosmicray nucleons, stability of high-energy pions and W bosons, modification of nucleon beta decays, and some others.

Few of such models will be discussed with accompanied Lorentz violating effects.

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Asymptotic Nets in the Improper Three-dimensional Affine Hyperplane

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The two-dimensional surface is considered in the four-dimensional extended affine space. A moving frame s attached to the surface according to certain rule. We choose a conjugated net arranged by geodesic lines on the surface. When the point moves on the surface then each of the points describes two-dimensional surfaces in the hyperplane, on which naturally arise nets of lines. The cases when the nets are asymptotic are considered.

Control and Stabilization of Fractional Order Chaotic Systems with Nonlinear Disturbances

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In this paper, the control and stabilization of uncertain fractional-order chaotic systems with disturbances are investigated. For this purpose, an appropriate fractional-integer integral type sliding surface is defined and an adaptive fractional-order sliding mode controller is designed to assure that asymptotical stability of the uncertain fractional-order chaotic systems in the presence of nonlinear disturbances occurs and the unknown parameters are updated. Numerical simulation results are given to demonstrate the validity of the proposed control method.

An n-dimensional fractional-order system with unknown parameters can be described as [1]

$$D^q x = f(x)\alpha + g(x) + d(x,t), \tag{1}$$

where $q = [q_1, q_2, \ldots, q_n]^T$ for $0 < q_i < 1$ $(i = 1, 2, \ldots, n)$, $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is the state vector of the system, $f : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is a function matrix, $\alpha \in \mathbb{R}^m$ is the parameter vector, $g : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function and $d(x,t)$ is the disturbance that can include parameter uncertainties, unmodeled dynamics, and external noise disturbances. To control chaos in the fractional-order chaotic system (1), the control input $u(t)$ is added to the system

$$D^q x = f(x)\alpha + g(x) + d(x,t) + u(t).$$

Let $x^\star$ is any equilibrium point of system (1). The aim of this paper is to design an appropriate fractional-order sliding mode controller such that the equilibrium point $x^\star$ is asymptotically stable,

$$\lim_{t\to\infty} \|e\| = \lim_{t\to\infty} \|x - x^\star\| = 0.$$

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Isospin Asymmetric, Chiral Imbalanced Dence Quark Matter in the Framework of Nambu–Jona–Lasinio Model

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Isospin asymmetry is the well-known property of dense quark matter, which exists in the compact stars and is produced in heavy ion collisions. On the other hand, the chiral imbalance between left- and right- handed quarks is another highly anticipated phenomenon that could occur in the dense quark matter.

To investigate quark matter under these conditions, we take into account baryon – μ_B, isospin – μ_I and chiral isospin – μ_{I5} chemical potentials and study QCD phase diagram using NJL_4 model generalised to two massive quarks that could condense into the pion condensation. Also we introduce non-zero temperature into consideration in order to make our investigation applicable to hot dense quark matter and compare our NJL_4-model analysis with the known lattice results.

Finite Endomorphisms and the Full Transitivity of a Cotorsion Hull

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To study the ring of endomorphisms of a separable primary group, W. Liebert considered finite endomorphisms which map the group in to a finite subgroup. In the talk it is shown how these endomorphisms can be used to study the full transitivity of the cotorsion hull of a separable primary group.

New Mathematical and Computer Models of Non-Permanent Information Warfare

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The report deals with the system of Non-Permanent Information Warfare, in which the opposition of the antagonistic parties is due to a certain event. For it, new both mathematical and computer models are constructed. The models take into account different scenarios for the development of the system after the occurrence of some event. On the basis of a computer experiment with various values of system parameters, a forecast is made of the further course of the Information Warfare. The question of the systems controllability by the peacekeeping side and the definition of recommendations for the completion of the Information Warfare by third-party efforts are studied [1–5].

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On q-Capability Properties of Groups and Lie Algebras

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For a positive integer q, there are two different notions of q-capability of groups. We analyse interrelationship between them, q-exterior center and q-precise center of a group. Then we study two different Lie algebra versions of q-capability. Both of them are described via the non-abelian q-exterior product of Lie algebras, by introducing two different notions of q-exterior center of a Lie algebra. It is shown that one of them coincides with the q-precise center of a Lie algebra.

Some Local and Nonlocal Boundary Problems for One Class of First Order Hyperbolic Systems

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The work deals with some local and nonlocal boundary problems for one class of first order hyperbolic systems that are studied by the method of characteristics.

These systems arise in continuous environmental mechanics, mathematical modeling of many processes in physics and technology.

Examples of these systems are: Maxwell's differential equations of electromagnetic field in vacuum, Dirac differential equations, differential equations of crystalline optics, isentropic single-dimensional flow equations, equations of barotropic gas motion, acoustic three-dimensional and flat waves spread in immovable environments.

The correctness of the periodic problem is shown and its solution is constructed in explicit form. The questions of correctness of some local boundary problem are also considered.

On the Quantum Properties of Nanostructures

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In the present paper the mathematical model of quantum properties of different nanostructures is studied. Nanostructures properties are significantly different from their similar materials at macro scale. The influence of the surface atoms becomes important, the thermal, optical and electrical properties change dramatically at the nanoscale dimension, the dimensions are comparable with the wavelength of electrons which causes quantum confinement of electrons and quantization of their energy [1].

The properties and atomic configuration of nanomaterials also depend on different materials they are deposited on and the temperature [1]. For example, in 2D carbon nanostructures carbon atoms are arranged periodically in hexagonal rings, i. [1]. Besides, close packing hexagonal in- plane structure form gold-colloid particles chemically self-assembled onto the surface of gold via thiol group [1]. The cubic structure is shown by gold nanoparticles deposited on the hollow carbon surface, gold colloid LS (Langmuir–Schaeffer) films deposited onto glass slides etc., [1].

We have studied the energy levels of electrons in the cubic and prismatic (with the hexagonal cross-section) nanostructures from the non-relativistic viewpoint. We consider the Schrödinger Equation for the wavefunction of the electron with the homogeneous boundary condition at those areas [3, 4]. The eigenfunctions and the corresponding eigenvalues are obtained and consequently the possible numerical values of the energy levels of electrons are estimated. We have obtained the possible energy levels of the electrons in the prism of height 1nm and with the hexagonal cross-section of the diameter $0.28\,nm$. These levels are about $0.32\,eV$, $28.3\,eV$ and $139.7\,eV$, the possible energy levels of the electrons in the box with size $0.23\,nm \times 0.23\,nm \times 1\,nm$ are about $5.1\,eV$, $7.7\,eV$, $10.2\,eV$, $20.4\,eV$, $28.9\,eV$. Here we have studied inorganic nanostructures. The quantum properties of organic nanostructures are much more complicated. Some aspects of quantum biophysics was studied in [2].

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New Method of Analysis on Stability of Shells and Plates with Discontinuous Parameters

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In this paper is developed the method of analysis of shells and plates with discontinuous parameters on deformability and stability. Based on it are obtained the results of specific calculations. The realiability of obtained results are provided due that for linearization of non-linear equations is applied the known and widely approbated method of successive loading. The effectiveness of method is in that on each stage of loading is applied the standard set of approximation functions that represents the solutions of linear tasks and would be constructed by arbitrary known method of analysis. The application of discontinuous approximation functions gives the possibility to reflect the characteristic singularities and behavior of components of mode of deformation in adjacent of singularities.

Numerical Methods of Analysis of Shells and Plates with Singularities

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Constructed methods of analysis of shells and plates with taking into account the geometrical and physical non-linearity gives the possibility to evaluate variation of all

components of mode of deformation, values of critical loadings and shapes of buckling in process of loading and are more effective in comparison with solutions, obtained by other, numerical-analytical methods. Are developed and researched numbers of simplifying variants of solutions, in particular the variant of reduction to single-layer plate with certain bending stiffness that leads to significant simplifications in of analysis without important losses in accuracy of calculation especially at determination of displacements.

On a Liouville Theorem for a Ornstein–Ulenbeck Operator

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We present a one-side Liouville theorem for a Ornstein–Ulenbeck Partial Differential Operator, as a consequence of a Liouville-type theorem at $-\infty$ for the corresponding Kolmogorov operator.

The talk is based on joint work with E. Lanconelli and E. Priola.

Effective Use of Games in Math Education

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Every person, regardless of profession, knows how to add natural numbers. Do we observe a similar situation, when it comes down to addition of integer numbers? Technically, addition of negative numbers is no more complicated than summing up positive ones. Then why do we experience difficulties with the former, while the latter seems to be pretty easy? The reason may be, that we learn how to deal with positive numbers and develop corresponding skills over the course of many years, while we are supposed to

learn how to deal with negative numbers in some three weeks? In the middle school many topics are taught without any prior preparation, rather by introducing large chunks of information and requiring students to perform complicated operations. Experience tells us, that these topics are perceived as "difficult" from the very beginning and remain to be such for many students. Is it possible to lay the foundation for learning such topics at the elementary level? Instead of teaching concepts and abstract structures, one could introduce students to more sophisticated mathematical concepts and topics through fun activities and games. Such topics could include algebraic operations on negative numbers, vector addition, translation. The authors are convinced, that certain topics can be introduced to the students already in the elementary school through games. It's necessary to prepare the series of games with common starting rules and increasing difficulty level. The methods of teaching math through games described in this paper are based completely on practical experience of the authors in various schools. The topics taught while making elementary school students play include addition of integers, vector addition and multiplication by a scalar. Students were taught the technical side of the topics and were not introduced to terms and abstract constructs. For example, in the "journey" of a point along a horizontal line, if the point moves two units to the right, the corresponding number is "$+2$", while the motion to the left by three units is denoted by "-3". If we combine several steps and assign one value to the result, the students have managed to add up several integers (positive and negative). The inverse problem of replacing one particular displacement of a point along a horizontal line with two or more consecutive steps involves higher level of analytical skills. Similar games can be played on a plane. Based on the experience of the authors the evidence is that students of this age group acquire all the necessary skills to perform such operations without any major difficulties. Moreover, when these students reached the 7th grade, they managed to learn and perform algebraic operations on integers quite easily, including multiplication of negative numbers, though they had not had any prior experience with this particular operation. Most probably, students were able to distinguish "$+$" and "$-$" signs from the symbols representing the algebraic operation. According to the authors, the confusion between those is the most frequent reason of students making mistakes while performing algebraic operations on integers. It's important to note, that the results of this method of teaching are still being observed and the teaching techniques are being further sophisticated.

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About Modelling MaTRU-based Protocols in Maude-NPA

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There has been increasing demand to post-quantum cryptographic schemes with their security based on the computational hardness of the lattice problems. Post-quantum cryptographic schemes refer to the algorithms that are resistant to quantum attacks. These are the alternatives of public key cryptographic schemes such as RSA, DLP-based schemes and elliptic curve based protocols. These schemes have been proposed for public key encryption, signature schemes and hash functions. Post-quantum cryptographic schemes are classified into the following approaches: code-based, hash-based, multivariate and lattice-based cryptography. Lattice-based cryptographic schemes are one of the most widely studied post-quantum cryptographic protocols. NTRU is the first proposed lattice-based cryptosystem. MaTRU can be considered as the matrix version of NTRU and differs mostly on the agreement phase due to the matrix multiplication property.

Formal analysis of cryptographic protocols is used to model a cryptographic protocol and its security properties to verify that a protocol is secure. In the last decades, such a formal analysis revealed security flaws in some protocols. For example, Needham–Schroeder Public Key protocol (NSPK) has been formally proved to be insecure by Lowe and Meadows.

The Maude-NRL Protocol Analyzer (Maude-NPA for short) is a simple yet powerful tool based on Rewriting Logic. It uses backwards search engines, which means it searches backwards from a final insecure state to determine whether or not it is reachable from an initial state. The Maude-NPA tool has been successfully used for the formal specification and analysis of cryptographic protocols satisfying different algebraic properties of cryptosystems.

In this talk we present one approach to model MaTRU-based cryptographic protocols in Maude-NPA. We discuss limitations and possible extension of the system to formally analyze MaTRU-based cryptosystems.

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ერთი ეკონომიკური ამოცანის მათემატიკური მოდელის შესახებ

ქეთევან კუთხაშვილი, ლიანა ყარალაშვილი
მეცნიერებისა და ტექნოლოგიების სკოლა, საქართველოს უნივერსიტეტი
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ამა თუ იმ კომპანიის მიერ გრძელვაიან პერიოდში პროექტების დაგეგმვა არსებულ რესურსებზე (განსაკუთრებით ფინანსურ) არის დამოკიდებული. რესურსების ოპტიმალური განაწილება და მართვა რთული საქმეა და ფირმისა თუ რაიმე დიდი ორგანიზაციის სტაბილურობისთვისა და ეკონომიკური განვითარებისთვის სწორი გადაწყვეტილების მიღებას ენიჭება მთავარი მნიშვნელობა. ძირითადად, გადაწყვეტილების მიღება მენეჯერის გამოცდილებაზეა დამოკიდებული და რაიმე დასაბუთებულ მსჯელობას არ ეყრდნობა. ამიტომ, ძალზედ მნიშვნელოვანია თუ გადაწყვეტილების მისაღებად გამოყენებული იქნება ზუსტი გამოთვლები და მეთოდოლოგია, რომელიც საშუალებას მოგვცემს უფრო კომპეტენტური და დასაბუთებული გადაწყვეტილება მივიღოთ.

ნაშრომში განხილული იქნება ერთ-ერთი ასეთი ამოცანა და ამ ამოცანის მათემატიკური მოდელი. კერძოდ, ვთქვათ, ფირმას დასაგეგმი აქვს წლის განმავლობაში განსახორციელებელი პროექტების შესრულების გრაფიკი. პროექტებში შეიძლება ვიგულისხმოთ ახალი პროექტების შექმნა, არსებულ პროექტებზე სარემონტო სამუშაოების განხორციელება, საქველმოქმედო ღონისძიებების დაგეგმვა, სარეკლამო ღონისძიებების დაგეგმვა და სხვა. თითოეული პროექტის განხორციელება გარკვეულ ფინანსურ ხარჯებს ფირმის შემოსავალი ყოველი თვის დასაწყისისთვის წინასწარ ცნობილია. პროექტების რაოდენობა იმდენად დიდია, რომ ყველა მათი განხორციელებისათვის საჭირო ჯამური ხარჯები წლის განმავლობაში ფირმის ჯამურ შემოსავალს აჭარბებს. საჭიროა:

1. ამოვარჩიოთ მაქსიმალური რაოდენობა იმ პროექტებისა, რომელთა განხორციელებაც ფირმას წლის განმავლობაში შეუძლია;
2. შევადგინოთ ამორჩეული პროექტების განხორციელების უწყვეტი გეგმა-გრაფიკი მათი ფინანსური ხარჯების გათვალისწინებით. დასმული ეკონომიკური ამოცანის გადასაჭრელად ავაგებთ მათემატიკურ მოდელს, რომელიც წარმოადგენს დისკრეტული ოპტიმიზაციის ამოცანას. შემოთავაზებული იქნება ამოხსნის ალგორითმი.

Construction of a Kernel Density Estimate of Rosenblatt–Parzen Type by Conditionally Independent Observations and the Accuracy of Approximation to Determine L_1 Metrics

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On the probabilistic space (Ω, F, P) considered a two-component, stationary in the narrow sense, a sequence of random variables $\{\xi_i, Y_i\}_{i\geq 1}$, where $\{Y_i\}_{i\geq 1}$ sense of $(Y_i : \Omega \to R^1$, $i = 1, 2, \dots)$, conditionally independent sequence $\{\xi_i, \}_{i\geq 1}$, $(\xi_i : \Omega \to \{b_1, b_2, \dots, b_r\}, b_i \in R^1$, $i = 1, 2, \dots)$ control sequence whose elements are independent, randomly distributed random variables

$$\xi_j = \sum_{i=1}^{r} b_i I_{(\xi_j = b_i)}, \quad P(\xi_j = b_i) = p_i, \ \ i = \overline{1, r},$$

$$\sum_{i=1}^{r} p_i = 1, \ \ j = 1, 2, \dots .$$

Conditional distributions $P_{Y_1|\xi_1=i}$ have unknown densities $f_i(x)$, $i = \overline{1, r}$, respectively. Using observations $\{Y_i\}_{i\geq 1}$, a kernel estimator density

$$\overline{f}(x) = \sum_{i=1}^{r} p_i f_i(x)$$

of Rosenblatt–Parzen type is constructed. Under certain conditions, the accuracy of the approximation of the density $\overline{f}(x)$ constructed estimate metric L_1.

Renorm-Group Analysis of the Chiral Effective Field Theory

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Renorm-group equations are derived for currents in the chiral effective field theory. The solutions to these equations are used to construct observables. Power counting analysis of the corresponding perturbation theory is carried out.

Modeling and Forecasting Exchange Rates

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This paper investigates models for the Georgian LARI exchange rate against the USD. The objective of this paper is to compare different methods of modeling and out-of-sample forecasting. We will discuss following types of time series methods: Moving average, exponential smoothing, exponential smoothing adjusted for trend, time-series decomposition models, and ARIMA models. The forecasting ability of these models is subsequently assessed using the symmetric loss functions which are the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). In many cases, predicting the direction of exchange rate change would be valuable and profitable. According this point, we will also estimate correctness of these models by percent of forecasted change direction correct number made by each model.

Enterprise Cost Minimization Using Cobb–Douglas Function

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The aim of this paper is to study behavior of firms, that are oriented to maximize profit in terms of competitive and not competitive market. To achieve this goal we find the minimum of the Cobb-Douglas function $f(x_1, x_2) = x_1^a x_2^b$ using Lagrangian method.

Suppose, we have two factors of production, with price ω_1 and ω_2, and we want to find cheapest way of producing production on y level. If x_1 and x_2 are amounts of factors, and $f(x_1, x_2)$ is firms production function, then minimization can be written in the following way [1]:

$$\min_{x_1,x_2}(\omega_1 x_1 + \omega_2 x_2), \quad \text{when} \quad f(x_1, x_2) = y.$$

For this method let's use Lagrangian:

$$\begin{gathered} L = \omega_1 x_1 + \omega_2 x_2 - \lambda(f(x_1, x_2) - y), \\ \omega_1 - \lambda \frac{\partial f(x_1, x_2)}{\partial x_1}, \\ \omega_2 - \lambda \frac{\partial f(x_1, x_2)}{\partial x_2}, \\ f(x_1, x_2) - y = 0. \end{gathered}$$

The costs function can be found by writing those costs that firm has while making costs minimizing choice:

$$c(\omega_1, \omega_2, y) = \omega_1 x_1(\omega_1, \omega_2, y) + \omega_2 x_2(\omega_1, \omega_2, y).$$

After algebraic calculations we obtain:

$$c(\omega_1, \omega_2, y) = \left[\left(\frac{a}{b}\right)^{\frac{b}{a+b}} + \left(\frac{a}{b}\right)^{\frac{-a}{a+b}}\right] \omega_1^{\frac{a}{a+b}} \omega_2^{\frac{b}{a+b}} y^{\frac{a}{a+b}}.$$

So, the minimal costs of production will be ω_1 y or ω_2 y, depending on which is less. In other words

$$c(\omega_1, \omega_2, y) = \min\{\omega_1 y, \omega_2 y\} = \min\{\omega_1, \omega_2\} y.$$

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აფხაზური ენის სრული ტექნოლოგიური უზრუნველყოფის მიზნებზე მიმართული PHDF-18-1228 პროექტის პირველი შედეგები

შალვა მალიძე, კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია

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2016 წლიდან სტუ „ინფორმატიკის“ სადოქტორო პროგრამის ფარგლებში ამოქმედდა სადოქტორო კვლევა „ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“ [1-3], რომელიც ქართული ენის ტექნოლოგიზების ცენტრის 2012 და 2015 წლებიდან მოქმედი გრძელვადიანი პროექტების „ქართული ენის ტექნოლოგიური ანბანი“ და „აფხაზური ენის დაცვისა და განვითარების გეგმა-პროგრამა“ ქვეპროექტებია. 2018 წელს რუსთაველის ფონდმა სადოქტორო კვლევით მაშინ უკვე მიღწეულ შედეგების გათვალისწინებით დაფინანსება გამოყო PHDF-18-1228 პროექტზე „ქართული აფხაზური ენებით ევროკავშირში ანუ სადოქტორო თემა – ქართული უნივერსალური ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“ (დოქტორანტი – შ. მალიძე, ხელმძღვანელი – კ. ფხაკაძე).

ამგვარად, აღნიშნულთა გათვალისწინებით, მოხსენებისას მიმოვიხილავთ PHDF-18-1228 პროექტის იმ პირველ შედეგებს, რომლებიც მიმართულია აფხაზური ენის სრული ტექნოლოგიური უზრუნველყოფის ანუ, რაც იგივეა, აფხაზური ენის ციფრული კვდომის საფრთხისგან დაცვის მიზნებზე [2].

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ PHDF-18-1228 პროექტზე გაღებული ფინანსური მხარდაჭერით.

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ქართული ენის სრული ტექნოლოგიური უზრუნველყოფის მიზნებზე მიმართული PHDF-18-1228 პროექტის პირველი შედეგები

შალვა მალიძე, კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია

ქართული ენის ტექნოლოგიების სასწავლო-სამეცნიერო ცენტრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

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2016 წლიდან სტუ „ინფორმატიკის" სადოქტორო პროგრამის ფარგლებში ამოქმედდა სადოქტორო კვლევა „ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება" [1-3], რომელიც ქართული ენის ტექნოლოგიების ცენტრის 2012 და 2015 წლებიდან მოქმედი გრძელვადიანი პროექტების „ქართული ენის ტექნოლოგიური ანბანი" და „აფხაზური ენის დაცვისა და განვითარების გეგმა-პროგრამა" ქვეპროექტებია. 2018 წელს რუსთაველის ფონდმა სადოქტორო კვლევით მაშინ უკვე მიღწეულ შედეგების გათვალისწინებით დაფინანსება გამოყო PHDF-18-1228 პროექტზე „ქართული აფხაზური ენებით ევროკავშირში ანუ სადოქტორო თემა – ქართული უნივერსალური ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა

და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“ (დოქტორანტი – შ. მალიძე, ხელმძღვანელი – კ. ფხაკაძე).

ამგვარად, ზემოაღნიშნულთა გათვალისწინებით, მოხსენებისას მიმოვიხილავთ PHDF-18-1228 პროექტის იმ პირველ შედეგებს, რომლებიც მიმართულია ქართული ენის სრული ტექნოლოგიური უზრუნველყოფის ანუ, რაც იგივეა, ქართული ენის ციფრული კვდომის საფრთხისგან დაცვის მიზნებზე.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ PHDF-18-1228 პროექტზე გაღებული ფინასური მხარდაჭერით.

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ქართული უნივერსალური ჭკვიანი კორპუსის გაძლიერება PHDF-18-1228 პროექტით შემუშავებული მომხმარებელთა კომპიუტერების მიმაერთებელი ახალი ინსტრუმენტებით

შალვა მალიძე, კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია

ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

ელ. ფოსტის მისამართი: *gllc.ge@gmail.com*

2016 წლიდან სტუ „ინფორმატიკის“ სადოქტორო პროგრამის ფარგლებში ამოქმედდა სადოქტორო კვლევა „ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“ [1, 2], რომელიც ქართული ენის ტექნოლოგიზების ცენტრის 2012 და 2015 წლებიდან მოქმედი გრძელვადიანი პროექტების „ქართული ენის ტექნოლოგიური ანბანი“ და „აფხაზური ენის დაცვისა და განვითარების გეგმა-პროგრამა“ ქვეპროექტებია. 2018 წელს რუსთაველის ფონდმა სადოქტორო კვლევით მაშინ უკვე მიღწეულ შედეგების გათვალისწინებით დაფინანსება გამოყო PHDF-18-1228 პროექტზე „ქართული აფხაზური ენებით ევროკავშირში ანუ სადოქტორო თემა – ქართული უნივერსალური ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“ (დოქტორანტი – შ. მალიძე, ხელმძღვანელი – კ. ფხაკაძე).

ამგვარად, ზემოაღნიშნულთა გათვალისწინებით, მოხსენებისას მიმოვიხილავთ PHDF-18-1228 პროექტით ქართული უნივერსალური ჭკვიანი კორპუსის გაძლიერების მიზნით შემუშავებულ მომხმარებელთა კომპიუტერების მიმაერთებელ ახალ ინსტრუმენტებს და ამ ინსტრუმენტების შემუშავების მიზნებს.

გამოქვეყნება მომზადდა შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის მიერ PHDF-18-1228 პროექტზე გაღებული ფინასური მხარდაჭერით.

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A Different Approach to the Definition of a Stochastic (Malliavin) Derivative of Poisson Functionals

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It is known that the stochastic (Malliavin) derivative one can define as an “inverse” of the Ito stochastic integral (with deterministic integrand) in the sense that $D^W W(h) = h$. In the white noise case, as well as normal martingale case, we can go a bit deeper in this way by showing that the Malliavin derivative of an iterated integral of order n is an iterated integral of order $n-1$. A martingale M is called normal if its predictable square characteristic $\langle M, M\rangle_t$ is deterministic.

But in the case of a normal martingale with jumps, in contrast to the Wiener case, it is impossible to define the stochastic differentiation operator in a generally adopted manner to obtain the structure of the Sobolev space $D^M_{2,1}$. Ma, Protter, Martin gave an example showing that two possible ways of determination of a stochastic derivative coincide if and only if the quadratic martingale characteristic $[M, M]$ is the deterministic function. Consequently, the second definition does not apply in the case of a Compensated Poisson process $M_t = N_t - t$ (because $[M, M]_t = N_t$), where N_t is the Poisson process, and we proposed a new definition, which does not contradict the above-mentioned example.

Definition 1. The class of smooth Poisson functionals S^M is the class of a random variables which has the form

$$F = f(M_{t_1}, \dots, M_{t_n}), \;\; f \in C_p^\infty(R^n), \;\; t_i \in [0,T], \;\; n \geq 1,$$

where $C_p^\infty(R^n)$ is the set of all infinitely continuously differentiable functions $f : R^n \to R$ such that f and all of its partial derivatives have polynomial growth.

Definition 2. The stochastic (Malliavin) derivative of a smooth random variable $F \in S^M$ is the stochastic process $D_t^M F$ given by

$$D_t^M F = \sum_{k=1}^{n} \sum_{i_1 < \dots < i_k} \Delta_+^{i_1}\big(\cdots(\Delta_+^{i_k} f(M_{t_1}, \dots, M_{t_n}))\big) I_{[0,t_{i_1}]}(t) \cdots I_{[0,t_{i_k}]}(t),$$

where

$$\Delta_+^i f(x_1, \dots, x_i, \dots, x_n) = f(x_1, \dots, x_i + 1, \dots, x_n) - f(x_1, \dots, x_i, \dots, x_n).$$

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On the Application of Certain Informative-Communicative Technologies in the Learning Process

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The speed of spreading digital technologies in today's world is extraordinarily rapid. Developing of the appropriate infrastructure and getting involving in the informative space of the world is considered a foreground task in our country.

As high technologies radically change the living conditions, it is essential, that during the learning process, students must develop the skills, the modern world requires from them. IT plays an important part in the development, also stimulating motivation.

However, today the teacher's personal initiative plays a considerable part in creating diverse and interactive educational course. With the aim, numerous useful programs and resources are actively and effectively used in the educational space of the leading countries. The interesting, powerful, available and consecutively renewed resource is the interactive and dynamic program GeoGebra, created only about 10 years ago.

During the report I'll be reviewing:

- Short history of developing the program and its achievements;
- Effectiveness of its use and advantages towards other programs;
- Presenting many of its potentials and discussing the usage, the original methodological findings and the expected success;
- Presenting, for illustration, the tasks solved by me using the program. For example, the optimization task will be solved using algebraic and geometric methods. Then, using the experiment – adjusting the parameters – I'll check the correctness of the solution. The software "tricks" of GeoGebra will enable me to visualize, simplify and effectively solve one of the stereometric tasks.

On a Nonlocal Problem and its Discrete Analogy

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We investigate a nonlocal problem and its discrete analogue for non-strictly hyperbolic equation

$$u_{xx} + (1 + u_x + u_y)u_{xy} + (u_x + u_y)u_{yy} = 0. \tag{1}$$

The characteristic roots of (1) are $\lambda_1 = 1$ and $\lambda_2 = u_x + u_y$. (1) is a hyperbolic equation, but in the case $u_x + u_y = 1$ it degenerates parabolically. Therefore the class of hyperbolic solutions of the considered equation should be defined by the condition $u_x+u_y-1 \neq 0$. If we know the value of the sum $u_x(x_0, y_0)+u_y(x_0, y_0) \equiv \alpha(x_0, y_0)$ for some set of points (x_0, y_0) from the plane $\mathbb{R}^2$, then the characteristics of the family of the root λ_2 are representable

as $y-y_0=\alpha(x_0,y_0)(x-x_0)$. If $\alpha(x_0,y_0)=1$, then the equation parabolically degenerates all over the straight line $y-y_0=x-x_0$. If the condition

$$\alpha(x,0)\neq 1,\quad x\in[0,a],\qquad \alpha(x,0)\equiv u_x(x,0)+u_y(x,0) \tag{2}$$

is fulfilled, the characteristics of the family of the root λ_2 have the form

$$y=\alpha(x_0,0)(x-x_0),\quad x_0\in[0,a],$$

and they intersect with the straight line $y=x$ at the point $(\mu(x_0),\mu(x_0))$, where $\mu(x)=\frac{x\alpha(x,0)}{\alpha(x,0)-1}$.

The Darboux type nonlocal problem. *Find a regular solution $u(x,y)$ of equation (1) and, along with it, the domain of its propagation if it satisfies condition (2) and the nonlocal condition*

$$u(x,0)+\beta(x)\,u(\mu(x),\mu(x))=\varphi(x),\quad x\in[0,a], \tag{3}$$

where $\alpha,\beta,\varphi\in C^2[0,a]$ are given functions.

The similar problems were studied by different authors (see, for example, [1]). We have proofed theorems in which the solution of the problem (1), (3) are investigated. These theorems and their proofs one can find in [2].

We also considered the discrete analog of the problem (1), (3) and have proved the convergence theorems for the difference scheme.

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About Teaching Problems of Inverse Function

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The concepts corresponding to teaching of inverse function are presented. The necessity to introduce the notion of inverse function is connected to solution of exponential and simplest trigonometric equations. Therefore, we suggest that inverse function in general and first of all inverse mapping among finite sets with their properties should be studied earlier than above mentioned subjects. In order to find the inverse function we have in fact to state the domain of definition, range of function, intervals of monotonicity. Hence, different interesting examples of this type are investigated. We word recommend to teach inverse trigonometric functions, their graphs and properties desirable in school course of mathematics.

Investigation of Multi-Field Basic Transmission Problems for Composed Elastic Structures

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We investigate multi-field basic transmission problems for complex elastic anisotropic structures when in different adjacent components of the composed body different refined models of elasticity theory are considered. In particular, we analyse the case when we have the generalized thermo-electro-magneto elasticity model (GTEME model) in one region of the composed body and the generalized thermo-elasticity model (GTE model) in another adjacent region. Both models are associated with Green–Lindsay's model [1, 2]. This type of mechanical problem mathematically is described by systems of partial differential equations with appropriate transmission and boundary conditions. In the GTEME model part we have six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function), while in the GTE model part we have four dimensional unknown physical field (three components of the displacement vector and temperature distribution function). The diversity in dimensions of the interacting physical fields complicates mathematical

formulation and analysis of the corresponding boundary-transmission problems. We apply the potential method and the theory of pseudodifferential equations and prove uniqueness and existence theorems of solutions to different type basic boundary-transmission problems in appropriate Sobolev spaces and analyse their smoothness properties.

This is a joint work with David Natroshvili.

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Solving the Problem of Boundary Layer Flow of Viscous Fluid

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The investigation of the laws of motion of non-Newtonian fluids has acquired much importance in industry as well as in the development of technologies promoting a wider application of new materials and stimulating the study of flows of various biological fluids. MHD flows on flat surfaces have many important industrial and technological applications, for example, in micro-MHD-pumps, devices for the micro-mixing of physiological specimens, the delivery of drugs and so on. we consider the problem of the stationary MHD flow of a viscous non-Newtonian fluid on the flat plate in the presence of a transverse magnetic field when the electrical conductivity coefficient depends on the fluid flow velocity. The problem is solved by using the integral method. Using the function that depends on an automodel variable, the solution is found for the problem of free convection of a non-Newtonian fluid. The expressions are obtained for the boundary layer thickness, surface friction force and friction coefficient. It is shown that fluid flow and the friction force can be controlled by an appropriate choice of parameters defining the rheological

properties and electrical conductivity of a non-Newtonian fluid. for pseudoplastic fluids the boundary layer thickness more than for dilatant fluids.

Faltings Finiteness Dimention of Abelian Subcategory $\mathcal{I}_M$

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This paper is devoted to the subcategory $\mathcal{I}_M$ of R-Mod whose objects consist of all direct summands of every direct product of copies of M, where M is an injective module over a Noetherian ring R. We prove that the finiteness dimension of this subcategory with respect to an ideal I of R, is equal to the finiteness dimension of the class

$$\mathcal{V} = \Big\{ N \in \mathrm{Obj}(R - Mod) \mid \text{ there exists an injective map } f : M \to N \Big\}$$

with respect to I, in category of R-Mod. In other words,

$$F_I(\mathcal{I}_M) = F_I(\mathcal{V}).$$

In addition, we check the conditions under which this subcategory is abelian.

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An Operator Defined by Convolution Involving the Komatu Operator

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In this article, we studied the Komatu operator and define a new operator and introduced a new class of function. Some inclusion results, covering theorem, coefficients inequalities and several other interetsing properties and sufficient conditions for subordination were also studied.

Investigation of Multi-Field Mixed Problems for Composed Elastic Structures by the Integral Equation Method

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We investigate multi-field mixed problems for complex elastic anisotropic structures when in different adjacent components of the composed body different refined models of elasticity theory are considered. In particular, we analyse the case when we have the generalized thermo-electro-magneto elasticity model (GTEME model) in one region of the composed body and the generalized thermo-elasticity model (GTE model) in another adjacent region. Both models are associated with Green–Lindsay's model [1, 2]. This type of mechanical problem mathematically is described by systems of partial differential equations with appropriate transmission and boundary conditions. In the GTEME model part we have six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function), while in the GTE model part we have four dimensional unknown physical field (three components of the displacement vector and temperature distribution function). The diversity in dimensions of the interacting physical fields complicates mathematical formulation and analysis of the corresponding boundary-transmission problems. We apply the potential method and the theory of pseudodifferential equations and prove uniqueness and existence theorems of solutions to different type basic and mixed

boundary-transmission problems in appropriate Sobolev spaces. We analyse the smoothness and singularity properties of solutions to mixed and interfacial crack type problems.

This is a joint work with Tengiz Buchukuri, Otar Chkadua, and Maia Mrevlishvili.

Acknowledgement. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant # FS-18-126).

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Amply g-Radical Supplemented Modules

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In this work, amply g-radical supplemented modules are defined and some properties of these modules are investigated. Let M be an R-module and $U \leq M$. If for every $V \leq M$ such that $M = U + V$, U has a g-radical supplement V' in M with $V' \leq V$, then we say U has ample g-radical supplements in M. If every submodule of M has ample g-radical supplements in M, then M is called an amply g-radical supplemented module.

Results

Proposition 1. *Let M be an R-module, $U_1, U_2 \leq M$ and $M = U_1 + U_2$. If U_1 and U_2 have ample g-radical supplements in M, then $U_1 \cap U_2$ has also ample g-radical supplements in M.*

Lemma 1. *Every factor module of an amply g-radical supplemented module is amply g-radical supplemented.*

Corollary. *The homomorphic image of an amply g-radical supplemented module is amply g-radical supplemented.*

Proposition 2. *Let M be an R-module. If every submodule of M is g-radical supplemented, then M is amply g-radical supplemented.*

Lemma 2. *Let M be a π-projective and g-radical supplemented module. Then M is amply g-radical supplemented.*

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Essential Injective Modules

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Let M and N be R-modules. N is said to be essential M-injective if for every exact sequence $0 \longrightarrow K \xrightarrow{g} M$ with $f(K) \trianglelefteq M$ and every R-module homomorphism $f : K \to N$, there exists an R-module homomorphism $h : M \to N$ with $h \circ g = f$. In this work, some properties of these modules are investigated.

Results

Lemma 1. *Let M and N be R-modules. Then N is essential M-injective if and only if for every $K \trianglelefteq M$ and every R-module homomorphism $f : K \to N$, there exists an R-module homomorphism $h : M \to N$ with $h \circ i = f$ (here $i : K \to M$ is inclusion).*

Proposition. *Let M be an R-module and $\{U_\lambda\}_\Lambda$ be a family of R-modules. Then the product $\prod_\Lambda U_\lambda$ is essential M-injective if and only if U_λ is essential M-injective for every $\lambda \in \Lambda$.*

Lemma 2. *Let N be an essential M-injective R-module. Then N is essential T-injective for every $T \trianglelefteq M$.*

Lemma 3. *Let N be an essential M-injective R-module. Then N is essential M/T-injective for every $T \leq M$.*

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A Theoretical Cryptanalysis of Quantum-Resistant WalnutDSA Digital Signature Algorithm

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WalnutDSA [1] is a digital signature algorithm that has been accepted by the National Institute of Standards and Technology for evaluation as a standard for quantum-resistant public-key cryptography. Its security is based on the hardness of *reversing* E-Multiplication. E-multiplication [2] is a one way function first introduced by I. Anshel, M. Anshel, D. Goldfeld and S. Lemieux in 2005 and uses a combination of braids, matrices, and finite fields. In this study, we analysis WalnutDSA, review the attack of Matvei Kotov, Anton Menshov and Alexander Ushakov [4] on the protocol and give some countermeasures to defeat this attack and finally introduce our theoretical cryptanalysis of WalnutDSA. In [4], authors uses message-signature pairs to recover a substitute for the signer's private key by removing cloaking elements. As a countermeasure against

this attack, we give a new reconstruction of cloaking elements in WalnutDSA protocol and thus show that how to defeat this attack. Finally in our attack, we first analyze the colored burau matrices [5] of Artin generators [3] then reduce the problem of *reversing* E-Multiplication to the factorization problem over invertible matrices. This means that if an efficient algorithm to compute short factorizations over General Linear Group exists, then by evaluating the factors of Public Key of the Signer we can easily reverse the E-Multiplication.

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On the Numerical Solution of a Problem of a Nonlinear Timoshenko Plate

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In the present report we consider the static deformation problem of the plate, described by the nonlinear Timoshenko equations system.

The existence of the generalized solution for the described problem and the convergence of the projection method is proven. The numeric solution for the concrete cases is implemented in the parallel computing system and its convergence against exact solutions is shown.

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On the General Mathematical Model of Autoimmune Diseases

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In this report we propose the general mathematical model of autoimmune diseases obtained by generalization of mathematical model of rheumatoid arthritis [1]. By offered model can be described progress and treatment of various autoimmune diseases. According the model, there are target cells that are recognized as "foreign" cells by immune system and it begins to attack them. In this process B-, $Treg$- and Th- lymphocytes are involved. The model is a system of non-linear ordinary differential equations. Equations determine change of size of target cell population, B-, Th-, $Treg$- cell populations and amount of drug in the blood. The model assumes that the disease occurs when the number of B-lymphocytes exceeds the limit value. The model involves the drug, that promotes target cells proliferation and/or reduction of B-lymphocytes to the limit value by influence on Th- and $Treg$- cells. Based on the model, it is possible to set a control problem with respect to the dosage and effectiveness of the drug.

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On One Contact Problem of Plane Elasticity Theory

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The article addresses the contact problem of plane elasticity theory for doubly-connected domain which outer boundary presents the square boundary, while the inner boundary is sought full-strength hole. Let to every link of the broken line be applied absolutely smooth rigid punches with rectilinear bases which displace to the normal under the action of concentrated normally compressive forces. There is no friction between the given elastic body and punches. Under these assumptions tangential stresses are zero along the entire boundary of the domain and normal displacements of every link of broken line of the outer boundary are the piecewise – constant functions.

Using the methods of complex analysis, the body stress state and the equation of unknown boundary of full-strength hole are determined. The solvability of this problem provides controlling stress optimal distribution selecting the appropriate hole boundary.

On a Numerical Computation for One Non-Linear Beam Equation of Timoshenko Type

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The present work is direct continuation of the following articles [2–4], in which considered the construction of algorithms and corresponding numerical computation for the approximate solution of non–linear integro–differential equation of the Kirchhoff and the Timoshenko types. In particular, we consider an initial–boundary value problem for the J. Ball integro–differential equation, which describes the dynamic state of a beam [1]. We

look for the approximate solution of the stated problem by applying the Galerkin method, the symmetric stable scheme and the Jacobi iterative method. The algorithm has been approved for tests. The results of computation are represented by tables and graphics.

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The Newton Iterative Method for a Discrete System of an Integro-Differential Beam Equation

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We consider the initial boundary value problem

$$u_{tt}(x,t)+\delta u_t(x,t)+\gamma u_{xxxxt}(x,t)+\alpha u_{xxxx}(x,t)-\left(\beta+\rho\int\limits_0^L u_x^2(x,t)\,dx\right)u_{xx}(x,t)$$

$$-\sigma\left(\int\limits_0^L u_x(x,t)u_{xt}(x,t)\,dx\right)u_{xx}(x,t)=0,\quad 0<x<L,\quad 0<t\le T, \tag{1}$$

$$\begin{gathered}u(x,0)=u^0(x),\quad u_t(x,0)=u^1(x),\\ u(0,t)=u(L,t)=0,\quad u_{xx}(0,t)=u_{xx}(L,t)=0,\end{gathered} \tag{2}$$

where α, γ, ρ, σ, β and δ are the given constants, among which the first four are positive numbers, while $u^0(x)$ and $u^1(x)$ are given sufficiently smooth functions.

Equation (1) obtained in [1] applying the Timoshenko theory describes the vibration of the beam. To approximate the solution of problem (1), (2) with respect to x and t variables a projection method and a difference scheme are used. As a result a system of nonlinear equations is obtained, which is solved by the Newton iterative method. The convergence conditions and error estimate of the iterative method are studied.

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Affine Planes with Polygons

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This concerns geometry, logic, and history.

With the publication of his *Geometry* in 1637, Descartes initiated our habit of doing geometry in terms of lengths, symbolized with minuscule Latin letters.

When Apollonius's change-of-basis theorem was reworked in Cartesian terms, Apollonius's own proof was ignored, presumably because it characterized conic sections by an equation of areas, namely a triangle and a trapezoid.

The proof works in an affine plane, which, as we know from Artin's *Geometrical Algebra*, can be axiomatized by Desargues's and Pappus's Theorems.

By considering an affine plane as comprising a sort of points and a sort of polygons, I axiomatize it with just enough axioms to allow Euclid's proof that triangles on the same base are equal, just in case the line joining their apices is parallel to that base.

The theorems of Desargues and of Pappus become theorems again, the latter by the proof that Pappus himself gave.

შალვა ფხაკაძის აღნიშვნათა თეორია და ქართული ენა

კონსტანტინე ფხაკაძე

ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი,
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ფხაკაძის აღნიშვნათა თეორია ჩამოყალიბდა მის მიერვე ამ მიზნით სპეციალურად შემუშავებული ჟ საკმარისად ზოგადი მათემატიკური ენის ფარგლებში. ამასთან, საზოგადოდ, აღნიშვნათა თეორია ფორმალური ენებისა და თეორიების ფორმალურად გაფართოების ანუ განვითარების ფორმალურ წესთა სისტემაა. ანუ, ის მათემატიკური ენა, რომლის ფარგლებშიც მას აღნიშვნათა თეორიის შემუშავება ჰქონდა ჩაფიქრებული ცხადია უნდა ყოფილიყო იმდენად ზოგადი, რომ მის ფარგლებში შემუშავებული სხვადასხვა განსაზღვრებებით შემოტანილი სხვადასხვა ტიპის შემამოკლებელი სიმბოლოები და მათი თვისებები გადატანადი ანუ ძალისმიერი უნდა ყოფილიყო თითქმის ნებისმიერ მათემატიკურად საინტერესო ენისათვის, როგორ ენათა შორისაც ცხადია არის ქართულიც.

ამგვარად, ზემოაღნიშნულთა გათვალისწინებით მოხსენებისას მიმოვიხლავთ შალვა ფხაკაძის ჟ საკმარისად ზოგად მათემატიკურ ენაზე დაყრდნობით განხორციელებული ქართული ენის პირდაპირი ფორმალურ-ლოგიკური აღწერის შედეგებს [1–3], რომელთაგან პირველი მთავრი შედეგი გახლდათ ქართული და მატემატიკური ენების ზოგადი ერთტიპობრიობის დასაბუთება [4], რაც მერე უკვე გაღრმავდა ენის მატარებელ ერთეულებში ამ ენების ბუნებრივი და განუწყვეტელი კავშირების დასაბუთებით [5].

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ქართული და აფხაზური ენებით ევროკავშირში ანუ ქართული და აფხაზური ენების სრული ტექნოლოგიური უზრუნველყოფის მიზნები და პრობლემები

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია, შალვა მალიძე, კონსტანტინე დემურჩევი

ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

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ზემომიმოხილული საკითხის მეტად მაღალი სახელმწიფოებრივი მნიშვნელობების გათვალისწინებით მოხსენებისას კიდევ ერთხელ შევეცდებით დავასაბუთოთ 17 მაისის მოხსენებისას სარეკომენდაციო სახით გამოკვეთილი ხედვის მართებულობა.

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About Weakly Consistent and Consistent Criteria for Hypotheses Testing

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Let (E, S) be a measurable space with a given family of probability measures $\{\mu_h, h \in H\}$, where H is the a set of hypotheses.

Definition 1. An object $\{E, S, \mu_h, h \in H\}$ is called a statistical structure.

Let S_n be an increasing sequence of σ -algebras such that $\bigcup\limits_{n=1}^{\infty} S_n = S$. Suppose that in the set of hypotheses H a metric $r(h, h')$ is introduced.

Definition 2. A sequence of S_n-measurable H-valued functions $g_n(x)$ is called a weakly consistent criterion for hypotheses testing if

$$\lim_{n\to\infty} \mu_h\{x:\ r(g_n(x), h) \geq \varepsilon\} = 0, \quad \forall\, \varepsilon > 0, \quad \forall\, h \in H.$$

Let H be a set of hypotheses and let $B(H)$ be a σ-algebra of subsets of H which contains all finite subsets of H.

Definition 3. We will say that the statistical structure $\{E, S, \mu_h, h \in H\}$ admits a consistent criterion for hypothesis testing if there exists at least one measurable mapping $\delta : (E, S) \to (H, B(H))$ such that $\mu_h(\{x : \delta(x) = h\}) = 1$, $\forall\, h \in H$.

Let E be a separable Hilbert space, and let H be the set of kernel operators in E with kernel norm

$$\|A\|_1 = \sum_{n=1}^{\infty} \lambda_n,$$

where λ_n are eigenvalues of operator $(A^*A)^{1/2}$. Then $L_2(E)$, equipped with the kernel norm, is a Banach space, and $\|A\| \le \|A\|_1$, where $\|\cdot\|$ is the operator norm.

Theorem. *Let $L_1(E)$, with the kernel norm, be a locally compact separable space bounded by this norm,*

$$\int r(g_n(x), h)\mu_h(dx) \ge \int r(g_{n+1}(x), h)\mu_h(dx), \quad \forall\, h,$$

and the functions $\int r(g_n(x), h_1)\mu_h(dx)$ are continuous with respect to h for all h_1. If g_n is a weakly consistent criterion for hypothesis testing then the statistical structure $\{E, S, \mu_h, h \in H\}$ admits a consistent criterion for hypothesis testing.

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On Algebraic K-Functors of Crossed Restricted Enveloping Algebras of Lie p-Algebras

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We construct a crossed universal Λ-enveloping (restricted) associative algebra of Lie (p)-algebra (Λ is a commutative algebra over a ground field k, on which Lie (p)-algebra

acts by derivations) which generalize the notions of universal enveloping (restricted) associative algebras of Lie (p)-algebras. Poincaré–Birkhoff–Witt theorems are proved for them, it is shown that such a crossed universal Λ-enveloping (restricted) associative algebras are particular cases of crossed Hopf algebras and its algebraic K-functors are Frobenius modules over Grothendieck functor of enveloping algebras of Lie (p)-algebras. Also it is proved that Sweedler's and Hopf's cohomologies with coefficients in algebras are isomorphic.

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Towards Formalization of ABAC$_\beta$ in PρLog

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Access control is a security technique that specifies which users can access particular resources in a computing environment. Formal description of access control is extremely important, since it should be defined, unambiguously, how rules regulate what action can be performed by an entity on the resource, how to guarantee that each request gets an authorization decision, how to ensure consistency, etc. It is also important that such a formal description is at the same time declaratively clear and executable, to avoid an additional layer between specification and implementation.

In this talk we discuss an Attribute-Based Access Control model, called ABAC$_\beta$, which is an extension of ABAC$_\alpha$ model with context, contextual attributes and meta-attributes. We present the current capabilities of PρLog system to formalize ABAC$_\beta$ operational model and discuss possible extensions of the system in this purpose.

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On an Existence of σ-Finite Invariant (Quasi-Invariant) Borel Measure in Polish Space

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It is well known that if E is an arbitrary infinite-dimensional Polish topological vector space, then there is no non-degenerate σ-finite Borel measure in E quasi-invariant under the group of all translations of the space. For an infinite-dimensional topological vector space E it is in general impossible to define a non-degenerate σ-finite Borel measure quasi-invariant under the whole additive group of the space. Under there circumstances it is natural to relax the requirement of quasi-invariance of the measure. In particular, one can ask about the existence in E of non-zero σ-finite Borel measure quasi-invariant with respect to some fairly "large" group G of translations of E. By "large" we can mean, for instance, that G is dense vector subspace in E (see, [1–4]).

In this respect the following question have been stated: Give a characterization (in purely algebraic and topological term) od all topological vector spaces E satisfying the condition: there exists in E at least one non-zero σ-finite Borel measure invariant (quasi-invariant) with respect to some dense vector subspace of E.

The following statement is valid.

Theorem. *Let E be a complete metric topological vector space. Then there is at last one non-zero σ-finite Borel measure in E invariant (quasi-invariant) under some dense vector subspace of E if and only if E is separable.*

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The curl and ∇ div Operators Eigenfields Properties

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The author studies global and local properties of the curl and ∇ div operator's eigenfields. The family of eigenfields of the curl operator forms a subspace $\mathbf{V}^0$ in the class of solenoid fields which are orthogonal to the kernel of the curl in $\mathbf{L}_2(G)$.

In turn, the space $\mathbf{V}^0$ contains subspaces $\mathbf{W}^m = \{\mathbf{f} \in \mathbf{V}^0, \dots, \operatorname{curl}^m \mathbf{f} \in \mathbf{V}^0\}$, $m \geq 1$, such that the $\mathbf{W}^{m+1} \subset \mathbf{W}^m \subset \mathbf{H}^m$, which are similar to Sobolev spaces $\mathbf{H}^m(G)$.

The curl operator's eigenfields generate vortex flows, which can be seen in some cases.

On the other hand, the ∇div operator's eigenfields forms a subspace $\mathcal{A}_\gamma$ in the class of potentional fields in $\mathbf{L}_2(G)$ wich contains subspaces $\mathbf{A}_\gamma^{2k} = \{\mathbf{f} \in \mathcal{A}_\gamma, \dots, (\nabla div)^k \mathbf{f} \in \mathcal{A}_\gamma\}$, $k \geq 1$, such that the $\mathbf{A}_\gamma^{2(k+1)} \subset \mathbf{A}_\gamma^{2k} \subset \mathbf{H}^{2k}$, which are similar to Sobolev spaces $\mathbf{H}^{2k}(G)$.

The first part concerns the operators curl and the gradient of divergence in a Hilbert space $\mathbf{L}_2(G)$, where $G \subset R^3$ is bounded region with smooth boundary. Operators S and $\mathcal{N}_d$ are their self-adjoint continuations in orthogonal subspaces $\mathbf{V}^0$ and $\mathcal{A}_\gamma$ of solenoidal and irrotational fields. In the main theorem we will demonstrate Fredholm properties of mappings $S + \lambda I$ and $\mathcal{N}_d + \mu I$. New spaces $C(2k, m) \equiv \mathbf{A}_\gamma^{2k} \oplus \mathbf{W}^m$ are introduced and properties of mappings $\operatorname{rot} + \lambda I$ and $\nabla \operatorname{div} + \mu I$ are proved.

In the second part the domain G is a ball B of radius R. Formulas for solenoidal basic fields are given by the author [1]. G. Islamov and V. Bobkov visualized the curl eigenfields corresponding to the minimum eigenvalue.

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Time-Frequency Representations of Multilinear Wigner Type Operators

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Multilinear localization operators in the context of modulation spaces were studied by many authors. In this work we also introduce a τ-dependent multilinear Wigner transform W_τ, $\tau \in [0,1]$. We prove that multilinear τ-Wigner transform is a multilinear short-time Fourier transform in disguise. Further, we show that multilinear τ-Wigner transform has covariance property and we calculated the short-time Fourier transform of the multilinear τ-Wigner transform. Through these results, we have examined the boundedness properties of multilinear τ-Wigner transform both on product Lebesgue spces and on modified version of modulation spaces.

Some key references are given below.

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Estimation of the Quadratic Risk of the Grenander Estimator at the Density Flat Regions

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We consider in this report the Grenander estimator of nonincreasing densities on the interval $[0,1]$ without any smoothness assumptions. For fixed number n of i.i.d random variables $X_1, X_2, \dots, X_n$ with values in $[0,1]$ and the nonincreasing density function $f(x)$, $0 \le x \le 1$, which is known to be flat on the interval $[a,b]$, $0 \le a < b \le 1$, we prove an inequality bounding the weighted quadratic risk of the Grenander estimator on the interval $[a,b]$ by the value $1/n$.

The Spaces of Spherical Polynomials and Generalized Theta-Series

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Let

$$Q(X) = b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{12}x_1x_2,$$

be a nondiagonal ternary quadratic form.

Consider all possible spherical polynomials P_{ki}, with even indices i and $k = \nu - 1, \nu$; their number is 5 for $\nu = 4$ $(k \ge i \ge 0)$:

$$P_{30} = \frac{b_{12}(b_{12}^2 - 2b_{11}b_{22})}{4b_{22}^3}\, x_1^4 + \frac{b_{12}^2 - b_{11}b_{22}}{b_{22}^2}\, x_1^3x_2 + \frac{3b_{12}}{2b_{22}}\, x_1^2x_2^2 + x_1x_2^3,$$

$$P_{32} = \frac{b_{12}(b_{12}^2 - 4b_{11}b_{22})}{24b_{22}^2b_{33}}\, x_1^4 + \frac{b_{12}^2 - 4b_{11}b_{22}}{12b_{22}b_{33}}\, x_1^3x_2 + \frac{b_{12}}{2b_{22}}\, x_1^2x_3^2 + x_1x_2x_3^2,$$

$$P_{40} = -\frac{b_{11}(b_{12}^2 - b_{11}b_{22})}{b_{22}^3}\, x_1^4 - \frac{4b_{11}b_{12}}{b_{22}^2}\, x_1^3x_2 - \frac{6b_{11}}{b_{22}}\, x_1^2x_2^2 + x_2^4,$$

$$P_{42} = \frac{(b_{12}^2 - 4b_{11}b_{22})(b_{12}^2 - 2b_{11}b_{22})}{24b_{22}^3 b_{33}}\, x_1^4 + \frac{b_{12}(b_{12}^2 - 4b_{11}b_{22})}{6b_{22}^2 b_{33}}\, x_1^3 x_2 + \frac{b_{12}^2 - 4b_{11}b_{22}}{4b_{22}b_{33}}\, x_1^2 x_2^2 - \frac{b_{11}}{b_{22}}\, x_1^2 x_3^2 + x_2^2 x_3^2,$$

$$P_{44} = \frac{(b_{12}^2 - 4b_{11}b_{22})^2}{16b_{22}^2 b_{33}^2}\, x_1^4 + \frac{3(b_{12}^2 - 4b_{11}b_{22})}{2b_{22}b_{33}}\, x_1^2 x_3^2 + x_3^4.$$

We form the basis of the space of generalized theta-series $T(4, Q)$ with spherical polynomial P_{ki} of order 4 and quadratic form $Q(X)$. We have proved the following

Theorem. $\dim T(4, Q) = 5$ and the generalized theta-series with spherical polynomials P_{ki} ($k = 3$ or 4; i is even):

$$\vartheta(\tau, P_{30}, Q);\ \ \vartheta(\tau, P_{32}, Q);\ \ \vartheta(\tau, P_{40}, Q);\ \ \vartheta(\tau, P_{42}, Q);\ \ \vartheta(\tau, P_{44}, Q)$$

form the basis of the space $T(4, Q)$.

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Oscillatory Properties of Solutions of Higher Order Nonlinear Functional Differential Equations

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Oscillatory properties of solutions of higher order nonlinear functional differential equati01tS are investigated. In particular, unir»provable in a certain sense c ditions are found under which all proper solutions of those equation are oscillatory.

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On the Representation of Numbers by Some Positive Ternary Quadratic Forms with Square-Free Discriminant

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Let $F = ax^2 + by^2 + cz^2 + rxy + sxz + tyz$ is a positive ternary quadratic form.

There is proved [2] that there are just 790 possibilities up for a positive ternary quadratic forms F, belonging to one-class genera. Just 82 are diagonals (i.e. $r = s = t = 0$) between them.

Formulas for the representation of numbers by these 82 forms were obtained and all the arithmetical progressions, which members and only them, are not represented by corresponding quadratic forms were found by G. A. Lomadze [1] and his disciples.

G. L. Watson [2] proved, that among the 790 there exist just 20 non-diagonal ternary quadratic forms with square-free discriminant belonging to one-class genera.

We obtained the formulas for the representation of numbers by these 20 forms and found all corresponding arithmetical progressions.

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The Equation of the Movement of a Uniform Viscoelastic Body when the Defining Relationship Contains Conformable Fractional Derivative

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It is studied the equation of the movement of a uniform, viscoelastic body of density ρ, infinite in both parties, the making longitudinal vibration under influence of external loading $f(t,x)$ when the constitutive relationship it is given in a look

$$\sigma(t) = E\eta^{\beta} D^{\beta} e(t)$$

and

$$f(t,x) = f_1(x)\cos\omega t,$$

where $\sigma(t)$ is stress, but $e(t)$ is strain. In a formula, the symbol D^{β}, $0 < \beta < 1$ represents conformable fractional derivatives. The solution of the equation of the movement is received.

Effective Solution of the one Basic Boundary Value Problem of Statics of the Theory of Elastic Mixture in an Infinite Domain with a Circular Hole

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In the paper we consider effective solution of the boundary value problem for homogeneous equation of static of the linear theory of elastic mixture in an infinite domain with a circular hole, when on the boundary are given differences of partial displacements and the sum of stress vector components.

To solve the problem we use the formulas due to Kolosov–Muskhelishvili and the method described in [1].

The solution of the problem is represented in quadratures (Poisson type formula), which is defined modulo an arbitrary summand constant vector.

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Uniqueness in the Inverse Problems of the Potential Theory

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Let $K(x,y)$ be fundamental solution to the Laplace operator in $\mathbb{R}^n$ $(n=2,3)$ and $V_\Omega(\mu)=\int\limits_\Omega K(x,y)\mu(y)\,dy$ be a Newtonian volume potential with a density μ.

Uniqueness problem. Let Ω_1 and Ω_2 be two bounded domains of $\mathbb{R}^n$ and μ be some density function defined in $\Omega_1\cup\Omega_2$. Moreover, let $V_{\Omega_1}(\mu)=V_{\Omega_2}(\mu)$ for all $x\in\mathbb{R}^n\setminus(\overline{\Omega_1\cup\Omega_2})$. By this condition we have to define the location of the domains Ω_1 and Ω_2 with respect to each other.

This problem, in general case, is not uniquely solvable. Under some additional restrictions we prove the following uniqueness theorems.

Uniqueness theorems. Let $\Omega_\infty^{(1,2)}$ be connected unbounded component of $\mathbb{R}^n\setminus(\overline{\Omega_1\cup\Omega_2})$ and $\Omega_0=\mathbb{R}^n\setminus\overline{\Omega_\infty^{(1,2)}}$.

Theorem 1. *Let Ω_1 and Ω_2 be two bounded piecewise smooth domains of $\mathbb{R}^2$ and let there exist a point $x_0\in\partial\Omega_\infty^{(1,2)}$ and a number $r>0$ such that $\sigma_0^r\cap\overline{\Omega_1}=\varnothing$, where $\sigma_0^r=\{x:\ |x-x_0|<r\}\cap\partial\Omega_\infty^{(1,2)}$. Assume that for all $r'\le r$ the arc $\sigma_0^{r'}$ contains a segment of some line L and that $\sigma_0^{r'}$ does not belong to L $(\sigma_0^{r'}\setminus L\neq\varnothing)$. if $\mu\in C^1(\overline{\Omega_1\cup\Omega_2})$, $\frac{\partial\mu}{\partial L}=0$ and $\mu(x_0)\neq 0$, then the potentials $V_{\Omega_1}(\mu)$ and $V_{\Omega_2}(\mu)$ do not coincide on $\Omega_\infty^{(1,2)}$.*

Theorem 2. *Let Ω_1 and Ω_2 be two bounded piecewise smooth domains of $\mathbb{R}^3$ and let there exist a point $x_0\in\partial\Omega_\infty^{(1,2)}$ and a number $r>0$ such that $\sigma_0^r\cap\overline{\Omega_1}=\varnothing$, where $\sigma_0^r=\{x:\ |x-x_0|<r\}\cap\partial\Omega_\infty^{(1,2)}$. Assume that for all $r'\le r$ the subsurface $\sigma_0^{r'}$ contains a part of some plane P and that $\sigma_0^{r'}$ does not belong to P $(\sigma_0^{r'}\setminus P\neq\varnothing)$. if $\mu\in C^1(\overline{\Omega_1\cup\Omega_2})$, $\frac{\partial\mu}{\partial L}=0$ for a line L parallel to P and $\mu(x_0)\neq 0$, then the potentials $V_{\Omega_1}(\mu)$ and $V_{\Omega_2}(\mu)$ do not coincide on $\Omega_\infty^{(1,2)}$.*

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Strong Convergence and Summability of Walsh–Fourier Series in Martingale Hardy Spaces

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Unlike classical theory of Fourier series which deals with decomposition of a function into continuous waves the Walsh functions are rectangular waves. Such waves have already been used frequently in the theory of signal transmission, multiplexing, filtering, image enhancement, codic theory, digital signal processing and pattern recognition. The development of the theory of Vilenkin–Fourier series has been strongly influenced by the classical theory of trigonometric series. Because of this it is inevitable to compare results of Vilenkin series to those on trigonometric series. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, which studies orthonormal systems from the point of view of the structure of a topological group.

This lecture is devoted to derive necessary and sufficient conditions for the modulus of continuity such that partial sums and some classical summability methods with respect to one- and two-dimensional Walsh–Fourier series converge in norm. Moreover, we also present some strong convergence theorems of partial sums and some classical summability methods of the one- and two-dimensional Walsh system.

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Existence of Angular Border Meanings of Analytic Function on Nonempty Set in the Unit Disk

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In the unit desk necessary and sufficient conditions about the existence of angular border meanings of analytic function on nonempty set are accepted.

School Differentiations on the Secondary Stage According Directions

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The structure, content and organization forms involved in every state educational system are different.

Therefore, different views of educational system all over the world functioned and functions differently.

Research was carried out in 16 public schools on secondary stage. The target of our investigation was to state students' opinion whether it is possible to pass exams at the university according getting knowledge at school; and what's their opinion about differential teaching in school.

Whether it would be any need of private teachers, if such system existed.

The answer to these questions gives my investigation about differential teachings at school.

Boundary Value Problem for the Bi-Laplace–Beltrami Equation on a Hypersurface

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We investigate the boundary value problems for the bi-Laplace–Beltrami equation on a smooth bounded surface $\mathcal{C}$ with a smooth boundary in the non-classical setting in the Bessel potential spaces $\mathbb{H}^s_p(\mathcal{C})$ for $s > \frac{1}{p}$, $1 < p < \infty$. To the initial BVP we apply a quasi-localization and obtain a model BVP for the bi-Laplacian. The model BVP on the half plane is investigated by the potential method and is reduced to an equivalent system in Sobolev–Slobodečkii space. Boundary integral equations are investigated in both Bessel potential and Sobolev–Slobodečkii spaces. The symbol of the obtained system is written explicitly and is responsible for the Fredholm properties and the index of the system. An explicit criterion for the unique solvability of the initial BVP in the non-classical setting is derived as well.

Innovative Teaching Methods in Information and Communication Technologies

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Today, many teachers apply modern technologies and innovative methods of teaching at school in order to achieve learning effectiveness. These methods include active and interactive forms used in training. I strive to teach schoolchildren to think, analyze, put forward ideas, to be creative in solving any problems, to appreciate the beautiful. Creative people adapt faster in society, at work, learn a better profession and do their job. The subject "Technology" is special. It not only forms the polytechnic outlook of children, introduces new technology, modern materials processing technologies, helps to orient in the world of professions, but also provides an opportunity to join creative work at school. The ability to do a lot and do it yourself is a pledge of self-confidence. Modern society, technical progress, difficult economic conditions in the country require the development of such human abilities as: mobility, professionalism, competence, ability to navigate in various fields of science, culture and production; constantly improving their competence, able and willing to learn.

In order to expand the students' outlook and increase interest in the subject, the technology is developing methodological material that allows using modern computer technologies in their lessons using PowerPoint, Publisher, Excel, Adobe Photoshop and others. Knowing about the big children to computer technology, I try to direct it in the right direction: I teach children to make presentations and routings, draw up drawings, crosswords, work with various electronic reference books and encyclopedias In my work I take into account the individual characteristics of students, I use communicative teaching methods, I spend various types of lessons, both standard and non-standard: lesson – competition, lesson – competition, lesson – presentation, integrated lessons. When teaching, I use different types of tasks of different levels of complexity, providing a differentiated approach to learning, which allows us to more fully discover the creative and intellectual abilities of students, to intensify their cognitive activity. Based on the student's opportunities, the conditions of training and material support, I choose the appropriate types of work, take into account the content of the program (some topics can be omitted or develop new ones for other types of work according to their image), and also determine the time needed to refine the content of the program topic.

What is today "innovative education"? – This is an education that is capable of self-development and which creates conditions for the full development of all its participants; hence the main thesis; innovative education is developing and developing education.

What is "innovative educational technology"? This is a complex of three interrelated components:

- Modern content, which is transmitted to students, implies not so much the development of subject knowledge, as the development of competencies that are adequate to modern business practice. This content should be well structured and presented in the form of multimedia educational materials that are transmitted through modern means of communication.
- The main goal of innovative education technologies is to prepare a person for life in an ever-changing world.
- Education should develop the mechanisms of innovation, find creative ways to solve vital problems, promote the transformation of creativity into the norm and form of human existence.

Generating Sets of the Complete Semigroups of Binary Relations Defined by Semilatices of the Class $\Sigma_8(X, 5)$

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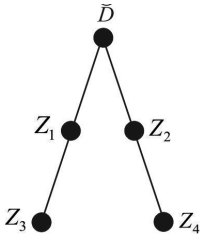
Let $\Sigma_8(X, 5)$ be a class of all X-semilattices of unions whose every element is isomorphic to an X-semilattice of unions $D = \{Z_4, Z_3, Z_2, Z_1, \breve{D}\}$, which satisfies the condition:

$$Z_4 \subset Z_2 \subset \breve{D}, \quad Z_3 \subset Z_1 \subset \breve{D}, \quad Z_4 \setminus Z_3 \neq \varnothing, \quad Z_3 \setminus Z_4 \neq \varnothing,$$
$$Z_2 \setminus Z_1 \neq \varnothing, \quad Z_1 \setminus Z_2 \neq \varnothing, \quad Z_2 \setminus Z_3 \neq \varnothing, \quad Z_3 \setminus Z_2 \neq \varnothing,$$
$$Z_4 \setminus Z_1 \neq \varnothing, \quad Z_1 \setminus Z_4 \neq \varnothing, \quad Z_4 \cup Z_3 = Z_4 \cup Z_1 = Z_3 \cup Z_2 = Z_1 \cup Z_2 = \breve{D}.$$



Let $C(D) = \{P_0, P_1, P_2, P_3, P_4\}$ is a family of sets, where P_0, P_1, P_2, P_3, P_4 are pairwise disjoint subsets of the set X and $\varphi = \begin{pmatrix} \widetilde{D} & Z_1 & Z_2 & Z_3 & Z_4 \\ P_0 & P_1 & P_2 & P_3 & P_4 \end{pmatrix}$ is a mapping of the semilattice D onto the family of sets $C(D)$. Then the formal equalities of the semilattice D have a form:

$$\begin{aligned} \widetilde{D} &= P_0 \cup P_1 \cup P_2 \cup P_3 \cup P_4, \\ Z_1 &= P_0 \cup P_2 \cup P_3 \cup P_4, \\ Z_2 &= P_0 \cup P_1 \cup P_3 \cup P_4, \\ Z_3 &= P_0 \cup P_2 \cup P_4, \\ Z_4 &= P_0 \cup P_1 \cup P_3. \end{aligned}$$

Here the elements P_4, P_3, P_2, P_1 are basis sources, the element P_0 is source of completeness of the semilattice D. Therefore $|X| \geq 4$, since $|P_4| \geq 1$, $|P_3| \geq 1$, $|P_2| \geq 1$ and $|P_1| \geq 1$.

In this paper we are studying irreducible generating sets of the semigroup $B_X(D)$ defined by semilattices of the class. $\Sigma_8(X, 5)$.

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Logical-Probability Model of Reliability of Complex Systems Based on Multifunctional Elements

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One of the ways to increase the reliability of economic, technical, human and other complex systems is the formation of flexible, structural rearranging solutions on the multifunctional elements (MFE). Such systems have a feature of maneuvering, and in the case of failure elements, can rearrange to ensure reliable operation [1].

It is important to form the highly efficient MFE systems based on quantitative assessment of reliability indicators (cheating, fraudulence, vulnerability). Big number of MFE systems and their functions, increase the number of conditions of these systems, which makes it hard to evaluate the reliability. In such cases it is crucial to use Logical-probability methods evaluation complex structural systems.

The algorithm of evaluation of reliability of complex systems, contains the following steps:

1. The description of MFE based systems structures and workability conditions by logical functions;
2. The conversion of logical functions into Orthogonal Dysfunctional Normal Form (ODNF), by using logical-probability methods;
3. Replacement of logical values by probability values and logical operation by arithmetical operations in ODNF;
4. Generating the polynomial of system reliability evaluation by simplifying algebraic expressions and numerical evaluation of system reliability by inputting probability data of the elements in this polynomial.

The work provides the logical-probability models of MFE-s and their parallelly functional systems. Specifically:

- Reliability model of system of $n = m \geq k$, $i \in [1, m]$ class, where n $(n > 1)$ is the number of MFE-s, m $(m > 1)$ – number of functions of the system, and k_i $(0 \leq k_i \leq m$, $i \in [1, m]$ – the functional capacity of the elements;
- The description of system workability condition by logical function and its transformation into ODNF by using Orthogonalization algorithm;
- Evaluation of reliability for $n = m \geq k_i$, $i \in [1, m]$, class systems using different k_i $(0 \leq k_i \leq m)$ and comparative analysis [2].

Based on the logical-probability models and algorithms, provided in this work, it is recommended to develop universal computer software, which will be used for the rich variety of complex systems.

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On Linguistics and Set Interpretations of Logical Operations with Corresponding 3-Input Circuits in Digital Electronics

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In the papers [4–9] there were discussed synthetic using of logical operations conjunction, nonexclusive and exclusive disjunctions and implication in case of three propositions. We made their classification and interpretations in Georgian language, so how these statements will be read by using of conjunctions "and", "or", "either... or ...", "if... then...". This kind of grammatical interpretations is not discussed in English, in German and in Russian languages up to now. Similar representations are also not included in the famous textbooks of discrete mathematics for 3-input gates circuits in digital electronics [1–3]. This paper discusses similar interpretations into English, German and Russian languages and their applications in the digital electronics.

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Complexity and Formal Analysis of ETRU Cryptosystem

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NTRU [1] is a public-key cryptosystem based on hardness of the closest vector problem over lattices. Its some variants are considered to be quantum resistant [2]. We study ETRU [3] which is a cryptosystem derived from NTRU. Similar to NTRU, ETRU is over of rings of polynomials, while its polynomials have coefficients in Eisenstein integers. In this study, we analysis ETRU and improve its running time. ETRU polynomials have Eisenstein integers coefficients and therefore we first try to reduce the cost of multiplication of two Eisenstein integers. This problem is inducted to the product of integers. We apply the Karatsuba–Ofman method into the multiplication operation. Implementation results show that this method reduces running time of ETRU. Secondly, we implement ETRU over some special primes q. Prime q is used in ETRU for the modulus operations. Hence, we choose very special q Eisenstein primes to accomplish a faster implementation of ETRU. For instance, we use Solinas Primes since they are widely used in cryptography to reduce number of field operations. Our implementation results show that the running time of ETRU over a Solinas prime is better than an ordinary prime. Finally, formal analysis of a cyrytosystem is important [4] and so we will present our preliminary results on formal analysis of ETRU cryptosystem.

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Modulus of Continuity and Boundedness of Subsequences of Vilenkin–Fejér Means in the Martingale Hardy Spaces

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The talk is devoted necessary and sufficient condition for the modulus of continuity for which subsequences of Fejér means with respect to Vilenkin systems are bounded from the Hardy space H_p to the Lebesgue space L_p, for all $0 < p < 1/2$.

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On an Ill-Posed Problem in the Hilbert Space of Finite Orbits

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Let H and M are Hilbert spaces, $A : M \to N$ is a compact operator and $\{\varphi_k\}$, $\{\psi_k\}$ are orthogonal systems in M and H, respectively. The notion of singular value decomposition (SVD) of A with respect to orthogonal systems is defined. The problem of approximate solution of an ill-posed equation $Au = f$, where the operator A admits a SVD is considered. We seek the generalized solution in the sense of Moore–Penrose. This means that if $f \in \operatorname{Im} A + \operatorname{Im}^{\perp}$, the generalized solution satisfied the equation $A^*Au = A^*f$ and belongs to the set $(\operatorname{Ker} A)^{\perp} = \operatorname{Ker} A^*$, where $A^* : M \to N$ is the conjugate to A operator in the sense of Hilbert spaces. We transfer the equation $A^*Au = A^*f$ to the space of n-orbits $D((A^*A)^{-n})$ and define the orbital operator $(A^*A)_n : D((A^*A)^{-n}) \to D((A^*A)^{-n})$ [3]. For the transferred equation $(A^*A)_n \operatorname{orb}_n((A^*A)_n^{-1}, u) = \operatorname{orb}_n((A^*A)_n^{-1}, A^*f)$ a linear spline central algorithm in the Hilbert space $D((A^*A)^{-n})$, is constructed. Similar problem for the transferred equation $(A^*A)_{\infty} \operatorname{orb}((A^*A)_n^{-1}, u) = \operatorname{orb}((A^*A)^{-1}, A^*f)$ in the Frechet space of all orbits $D((A^*A)^{\infty})$, which is the projective limit of the sequence os the spaces $\{D((A^*A)^{-n})\}$, was considered in [2]. In this space this equation become well-posed and linear spline generalized central algorithm was constructed. As an example, SVD of A. K. Louis [1] of Radon operator R, is considered.

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On Calculation of the Inverse of Harmonic Oscillator in the Space of Finite Orbits

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Classical harmonic oscillator operator $Au(t) = -u''(t) + t^2u(t)$, $t \in \mathbb{R}$ is a symmetric and positive definite operator in the Hilbert space $L^2(\mathbb{R})$. The equation $-u''(t)+t^2u(t) = f(t)$, $t \in \mathbb{R}$, defined on Schwartz nuclear space $S(\mathbb{R})$ of rapidly decreasing functions on one dimensional Euclidean space $\mathbb{R}$, is largely used in quantum mechanics for "white noise" analysis.

In the our previous paper the least squares method generalized by us for Frechet spaces was used for approximate calculation of the inverse of classical harmonic oscillator in $S(\mathbb{R})$. The convergence and some estimates of convergence of a sequence of the approximate solutions to the exact solution was also proved.

In this paper we transfer this equation in the space of n-orbits $D(A^n)$ with the norm

$$\|u\|_n = \left(\|u\|^2 + \|Au\|^2 + \cdots + \|A^n u\|^2\right)^{1/2},$$

where $\|u\|$ is the norm of the space $L^2(\mathbb{R})$. The transfered orbital equation has the following form

$$A_n(\mathrm{orb}_n(A, x)) = \mathrm{orb}_n(A, x),$$

where $\mathrm{orb}_n(A, x) := \{x, Ax, \ldots, A^n x\}$ and $A_n(\mathrm{orb}_n(A, x)) = \mathrm{orb}_n(A, Ax)$. For approximate calculation of orbital equation we prove that the least squares method is the linear central spline algorithm in the space $D(A^n)$.

Theoretical studies are confirmed by numerical experiments conducted by A. Tomash, R. Vacheishvili and Prof. M. Kublashvili.

On the Use of Fermat's Method in Study of Proof by Contradiction

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We give new examples of using the method of infinite descent in the study of mathematics at school. The purpose of our paper is to determine how the study of Fermat's method might affect school students' mathematical curiosity and attitudes toward mathematical proofs.

Solution of Some Contact Problems by Boundary Element Methods Based on the Singular Solutions of Flamant and Boussinesq's Problems

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Contact problems have a wide spectrum of applicability in building mechanics, mining mechanics, soil mechanics and engineering fields, such as wheel-rail contact in railway industry, investigation of friction and wear, rolling fatigue, ball bearing, etc. Many scientific works devoted to contact problems investigate the problem of indentation of an elastic body on an elastic foundation in the form of a halfspace with a constant modulus of elasticity, or they investigate the indentation on an elastic homogenous layer (approximate solutions). In the authors earlier works [1–3], the boundary value and boundary contact problems are solved by boundary element method (BEM) based on a singular solution of Kelvin problem.

The main goal of the present work is to find the solution of the contact problems for a half-space by BEMs based on two different singular solutions (one is the solution of Flamant problem, and another is the solution of Boussinesq's problem), and then to compare the obtained results. Thus we will consider the normal contact problems, which

are formulated as follows: indenter, whose weight may be neglected, presses the surface of the half space with a certain force, i.e. normal stress acts on the contact surface (here, we consider compressible stresses, while in case of the stretching tension, there will be an open contact or crack what we do not consider here), and the tangential stress is zero. In particular, we consider two types of the distributed load, which correspond to the following cases: 1) when a half-space is subject to the frictionless flat rigid indenter, and 2) when a half-space is subject to frictionless cylindrical rigid indenter. The article considers plane deformation. Problems are solved by BEM, which is based on the solutions of Flamant (BEMF) and Boussinesq's (BEMB) problems. Stress-strain state of the half plane, namely, constructed contours of stresses in half plane, is studied.

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Implementing Technological Developments in the Primary School Classrooms

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This work focuses on the importance of using the latest technological developments in the educational system. A special interest is paid to effective implementation of technology in primary school system. To achieve high quality teaching it is necessary to mix the traditional teaching methodology with new technological advances. Using computer technology in the primary classroom helps teachers provide better learning environment and helps students stay engaged. The study also examines the dangers and drawbacks of

technology use in young children. We should take into consideration children's age and the amount of time they spend working online.

Index